Global Patterns of Low-Mode Internal-Wave Propagation. Part I: Energy and Energy Flux

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ABSTRACT

Extending an earlier attempt to understand long-range propagation of the global internal-wave field, the energy $E$ and horizontal energy flux $F$ are computed for the two gravest baroclinic modes at 80 historical moorings around the globe. With bandpass filtering, the calculation is performed for the semidiurnal band (emphasizing $M_2$ internal tides, generated by flow over sloping topography) and for the near-inertial band (emphasizing wind-generated waves near the Coriolis frequency). The time dependence of semidiurnal $E$ and $F$ is first examined at six locations north of the Hawaiian Ridge; $E$ and $F$ typically rise and fall together and can vary by over an order of magnitude at each site. This variability typically has a strong spring–neap component, in addition to longer time scales. The observed spring tides at sites northwest of the Hawaiian Ridge are coherent with barotropic forcing at the ridge, but lagged by times consistent with travel at the theoretical mode-1 group speed from the ridge. Phase computed from 14-day windows varies by approximately $\pm 45^\circ$ on monthly time scales, implying refraction by mesoscale currents and stratification. This refraction also causes the bulk of internal-tide energy flux to be undetectable by altimetry and other long-term harmonic-analysis techniques. As found previously, the mean flux in both frequency bands is $O(1 \text{kW m}^{-1})$, sufficient to radiate a substantial fraction of energy far from each source. Tidal flux is generally away from regions of strong topography. Near-inertial flux is overwhelmingly equatorward, as required for waves generated at the inertial frequency on a $\beta$ plane, and is winter-enhanced, consistent with storm generation. In a companion paper, the group velocity, $\hat{c}_g = F E^{-1}$, is examined for both frequency bands.

1. Introduction

It is becoming clear that not only the magnitude but also the geography of deep mixing, which is primarily driven by internal-wave breaking, is required to run accurate models of the ocean’s circulation (Samelson 1998; Simmons et al. 2004; Hasumi and Suginohara 1999). Furthermore, simulation of a future ocean requires an understanding of the mechanisms of mixing, rather than just parameterizations. Therefore, a comprehensive picture of internal-wave sources, propagation, and dissipation would represent substantial progress.

The wind (which forces near-inertial waves near the earth’s Coriolis frequency $f$) and the barotropic tides (which flow over sloping topography and generate propagating internal-tidal motions) dominate the power input into the internal-wave field. Their totals have been estimated and appear to sum to a sizable fraction of the 2 TW argued necessary to keep the deep oceans stratified (Munk and Wunsch 1998). Maps of the depth-integrated energy input into the near-inertial motions (Alford 2001, 2003b) and the semidiurnal internal tide (Egbert and Ray 2000, 2001) have been constructed. Assuming no propagation or wave–wave interactions, the distribution of internal-wave dissipation would be given by the sum of these two source distributions, $S$.

However, recent observations indicate that both internal tides (Cummins et al. 2001; Ray and Mitchum 1997; Dushaw et al. 1995; Kantha and Tierney 1997) and near-inertial waves (D’Asaro et al. 1995; Alford and Gregg 2001; Alford 2003a) can propagate far from their sources. Construction of a depth-integrated internal-wave dissipation map $D$ would therefore require an understanding of the divergence of the horizontal internal-wave energy flux $F$:

$$D = S - \nabla \cdot F. \tag{1}$$

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(In this view, $D$ includes wave–wave interactions and all other processes that transfer energy out of each frequency band; these are assumed to lead to motions that dissipate locally.) The propagating flux $\mathbf{F}$ and the processes that affect it are the subject of this paper.

The fate of the low-mode internal tides has long been a topic of study, since they carry most of the energy converted from the barotropic tide and interact with the rest of the internal-wave spectrum only very slowly (St. Laurent and Garrett 2002). One exception to this may be near a latitude of 28.8°, where $2f = M_2$, and the parametric subharmonic instability (PSI) transfers energy rapidly to small (breaking) scales. Though long known to be a potential sink of energy for the internal tide, it was thought to be too slow to be significant [transfer time scales $O$(100 days); Olbers and Pompfrey 1981] until more recent numerical studies (Hibiya and Nagasawa 2004; MacKinnon and Winters 2005; H. Simmons 2005, personal communication) indicated significantly faster transfers for low-mode internal tides near 28.8°. Alternately, topographic scattering may direct energy into higher (and more unstable) modes (Gilbert and Garrett 1989; Muller and Liu 2000).

[Simple calculations by St. Laurent and Garrett (2002) indicate about 10% energy loss per wavelength, but more realistic calculations taking real bathymetry into account are needed.] Continental slopes are a third possible “graveyard,” where the slope is nearly critical to the $M_2$ tide over broad regions. Recent observations (Nash et al. 2004; Moum et al. 2002; Nash et al. 2006) of elevated, tidally modulated mixing rates above critical regions of continental slopes possibly support this scenario, but more work is required. Given the sensitivity of models to the distribution of mixing, an ocean where all the energy dissipated at latitude 28.8° would differ sharply from one where all the mixing occurred at the continental margins.

In the case of near-inertial waves, downward-propagating energy is commonly observed following storms. Their high wavenumber and strong shear make them dramatic upper-ocean features. However, the bulk of the energy input appears to rapidly escape the forcing region as low-mode near-inertial waves (D’Asaro et al. 1995; Alford 2003a). These waves also interact only slowly with the rest of the spectrum, and thus can propagate far, as suggested by D’Asaro (1991) and observed in the precursor to this paper (Alford 2003a, henceforth A03). Since near-inertial waves are generated near the local inertial frequency, they are constrained to propagate toward the equator toward lower local $f$ (Gill 1984; D’Asaro 1989; Garrett 2001), as observed by A03 and here. In addition, they may be subject to rapid PSI when they have reached a latitude where their frequency equals 2 times the local Coriolis frequency (Nagasawa et al. 2000; H. Simmons 2005, personal communication).

Both internal tides and near-inertial waves are subject to refraction by mesoscale currents (as described by Rainville and Pinkel 2006) and variable stratification (Alford et al. 2006). Owing to their higher phase speed, low modes are proportionally less subject to refraction (D’Asaro 1991; Mitchum and Chiswell 2000; Rainville and Pinkel 2006). Still, both the path and the speed of propagation of even the first mode are strongly affected by moderate mesoscale fields (Chiswell 1994, 2002; Rainville and Pinkel 2006). For example, Rainville and Pinkel (2006, henceforth RP06) measured phase changes of approximately ±20° just 500 km from the Hawaiian Ridge, in approximate agreement with ray-tracing calculations. Thus, measurements at a fixed location are expected to detect changes in both magnitude (as different beams are “swept” past the mooring) and arrival time (phase). These are superimposed upon modulations in the amplitude of the generated wave due to low-frequency heaving of the thermocline (Mitchum and Chiswell 2000).

These phase modulations cause the “coherent” internal tide (which is phase locked with the astronomical forcing) to become “incoherent” with increasing distance from the source, and thus invisible to detection by altimetry (which requires long-term harmonic analysis to detect the 12.4-h tide with its 9.9-day repeats). The importance of this distinction is illustrated in Fig. 1, which shows energy flux estimated from altimetry (black; Ray and Cartwright 2001) for the region between the Hawaiian Ridge and the Aleutians (digital flux vectors were kindly provided by R. Ray). Fluxes from a Princeton Ocean Model (POM) simulation of $M_2$ internal-tide generation at the Hawaiian Ridge domain (yellow; Merrifield et al. 2001) are plotted merely to indicate the primary generation regions. (Their scale is greater by a factor of 10 owing to their proximity to the ridge.) Beams are evident in the altimetric fluxes radiating from the Hawaiian and Aleutian Ridges. Their extension hundreds of kilometers away from their generation regions has motivated much of the recent work in the region; however, a clear reduction in flux is seen both in the northward and southward fluxes. A vital question is whether the observed decays are due to (i) actual loss of energy or (ii) loss of coherence with altimetric signals, rendering the beams undetectable (RP06).

Moorings, at the cost of greatly reduced spatial coverage, can detect the incoherent portion of the flux since they do not rely on harmonic analysis (Chiswell 2002, 2006; A03). In addition to documenting the meth-
ods and findings of A03 in more detail than was possible there, this paper focuses on six moorings in this region (Fig. 1). They complement the measurements of Chiswell (2002) in their much greater distance from the ridge. Specifically, we present four new pieces of evidence (none possible with altimetric methods):

1) Mean energy-flux estimates from moorings at 28° and 42°N (red arrows) show northwestward fluxes of $O(1 \text{ kW m}^{-1})$, well past where the altimetric fluxes have faded out.

2) Flux and energy vary strongly, as is clear from the differences between the three annual-mean esti-
mates at 28°N. As will be shown, the magnitude and phase of velocity and isopycnal displacement signals at these locations are strongly variable (by an order of magnitude and ±45°, respectively).

3) Strong spring–neap cycles are clearly detectable at each site, with peaks lagging those in barotropic forcing at the generation site by times consistent with travel at the theoretical mode-1 group speed.

4) The group velocity \( \hat{c}_g = FE^{-1} \) (presented in Alford and Zhao 2007, henceforth Part II) is consistent with that expected from linear theory, indicating a linear, unidirectional tide.

These observations are suggestive of a linear, anisotropic internal tide that is sufficiently strongly refracted by mesoscale fields to be rendered invisible to altimetric methods after \( O(1000 \text{ km}) \), but that still is able to propagate >2400 km away without losing all of its energy to PSI, wave–wave interactions, and topographic scattering.

Data and methods are discussed (in more detail than was possible in A03) in section 2. Time series of energy and energy flux for the semidiurnal tide are next presented and discussed in detail for the region north of Hawaii. We employ several methods of diagnosing wave propagation characteristics, and examine the phasing of the spring–neap cycle relative to barotropic forcing. The global distribution of \( E \) and \( F \) for both the semidiurnal and near-inertial bands is examined and documented, extending the work of A03 by adding 1) more moorings, 2) energy in addition to flux, and 3) probability density functions of each. In Part II, we continue by examining the group velocity \( \hat{c}_g = FE^{-1} \), with an ultimate goal (not attained here) of describing the processes affecting the propagation and dissipation of these low-mode internal waves.

2. Data and methods

a. Mooring data

Data from more than 2200 historical moorings (Fig. 2) were examined. Among them, 1064 are archived by the Oregon State University Buoy Group, 445 from the Woods Hole Oceanographic Institution (WHOI) database (kindly provided by C. Wunsch), 245 from the National Oceanic Data Center (NODC), 446 from the World Ocean Current Experiment (WOCE), and 416 from Autonomous Temperature Line Acquisition System (ATLAS) moorings.

Each mooring provides time series of temperature and velocity at discrete water depths, from which we wish to compute low-mode energy and flux. Only 80 moorings proved suitable for our purposes (Table 1; Fig. 2), according to five criteria:

1) Only deep-water moorings (\( H > 3200 \text{ m} \)) were considered.

2) Moorings were required to have at least four usable instruments (\( N \geq 4 \)), of which the first one is shallower than 800 m and the deepest one is within 1000 m from the bottom. These ensured stable resolution of the first two baroclinic modes (except for five moorings, discussed later, which only resolved the first mode).

3) The sampling time interval \( \Delta t \) is \( \leq 3 \text{ h} \), for adequate resolution of near-inertial and tidal signals.
4) The record length $T$ is $> 180$ days. This allows a stable time mean to be computed and ensures sufficient spectral resolution for separation by band-pass filtering.

5) Fifteen records contaminated by excessive mooring motion [diagnosed following Fu (1981), by the correlation between shallow pressure and temperature in each frequency band], which could contaminate flux and energy estimates, were discarded.

For each mooring, energy and energy flux are computed, as described in the ensuing subsections. Briefly, isotherm displacements are computed at each depth from the bandpassed mooring temperature records and climatological stratification profiles. Normal-mode displacement and current profiles are determined from climatological ocean stratification data. Full-depth profiles of isopycnal displacement and current in the two lowest modes are calculated from the vertically discrete measurements by a least squares inverse. This allows the baroclinic pressure anomaly $p'$ to be computed by vertically integrating $N^2 \eta$. Calculation of energy and energy flux is then straightforward, the latter given by the covariance of mode-fit velocity and baroclinic pressure anomaly.

b. Bandpass filtering

Near-inertial and semidiurnal components of the historical data time series are extracted via bandpass filtering. A fourth-order Butterworth filter is designed with zero-phase response and quarter-power points at $[c^{-1} \omega_0, c\omega_0]$ around the center frequency $\omega_0 = \{f, M_2\}$. The bandwidth parameter $c = 1.25$ was chosen narrow enough to maximally isolate the two bands, while wide enough to avoid filter ringing.

c. Computing vertical displacement

Ocean stratification profiles are calculated using the $1^\circ \times 1^\circ$ annual-mean climatological temperature and salinity data from the World Ocean Atlas 1994 (Levitus and Boyer 1994; sample shown in Fig. 3). At each instrument, the vertical temperature gradient $T_z(z_i)$ is computed. Vertical displacement $\eta(z_i, t)$ is then computed via the relation

$$\eta(z_i, t) = T(z_i, t)/T_z(z_i),$$

(2)

where $T(z_i, t)$ is the bandpassed temperature measured at depth $z_i$ and time $t$. This assumes that temperature signals are due to vertical advection of a constant gradient. To avoid introducing spurious displacements, instruments where $T_z < 3 \times 10^{-5} \, ^\circ C \, m^{-1}$, or where $T$ increases toward the seafloor, are removed from the calculation.

d. Modal decomposition

Over a flat bottom, the waves’ vertical structure can be represented by a superposition of discrete vertical modes that depend only on the buoyancy frequency $N(z)$. For low-frequency internal waves ($\omega \ll N$), they are the solutions to

$$\frac{\partial^2}{\partial z^2} \eta(z) + \frac{N^2(z)}{c_n^2} \eta(z) = 0$$

(3)

and are subject to the boundary conditions $\eta(0) = \eta(H) = 0$, where $H$ is the water depth, $n$ is the mode number, and $c_n$ is the eigenspeed. The buoyancy frequency $N(z)$ is computed from the climatological hydrographic profiles (sensitivity calculations indicate that typical seasonal cycles have negligible impact on either the modal shapes or the isopycnal displacements). Full-depth profiles of modal velocity, $u_n(z, t)$, and displacement, $\eta_n(z, t)$, are obtained, for the first two modes, from the filtered time series at discrete depths $z_i$ by performing a least squares inverse at each time $t$.

In general, the ability of a mooring to resolve the modes depends on the vertical wavenumber spectrum and the mooring’s geometry. Nash et al. (2005) provide a means for estimating errors for temporally and/or spatially deficient moorings. The moorings span a wide range of geometries and number of instruments. For this reason, only two modes were solved for, providing the most stable calculation for sparse geometries. [The geometry at five moorings (Table 1) was only sufficient to resolve the first mode.] For the worst case of a four-instrument mooring with a large gap at the top, errors approach 100% for a single-wave-period flux estimate. However, bias remains near zero, suggesting that time averaging aids in reducing errors. For more optimal moorings, errors are $O(10\%–20\%)$.

e. Energy density

The depth-integrated energy density, $E$, is computed for each mode and each band as the sum of the horizontal kinetic energy density

$$KE = \frac{1}{2} \bar{\rho} \int_{-H}^{0} |\bar{u}(z)|^2 dz,$$

(4)

and available potential energy density

$$PE = \frac{1}{2} \bar{\rho} \int_{-H}^{0} N^2(z) \bar{\eta}(z)^2 dz,$$

(5)

where $\bar{\rho}$ is the vertically averaged water density, $N(z)$ is the buoyancy frequency, and $\bar{u}(z)$ and $\bar{\eta}(z)$ are the full-depth profiles of velocity and displacement calculated...
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<td></td>
</tr>
<tr>
<td>45</td>
<td>ACM6</td>
<td>1993</td>
<td>42.72</td>
<td>312.62</td>
<td>5</td>
<td>515</td>
<td>3894</td>
<td></td>
</tr>
<tr>
<td>46</td>
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<td>1993</td>
<td>42.57</td>
<td>313.31</td>
<td>6</td>
<td>483</td>
<td>4392</td>
<td></td>
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<tr>
<td>47</td>
<td>ACM6</td>
<td>1993</td>
<td>42.35</td>
<td>314.00</td>
<td>7</td>
<td>514</td>
<td>4701</td>
<td></td>
</tr>
</tbody>
</table>
above. The resultant energy density has units of joules per meter squared.

f. Flux calculation

For each mode, the depth-integrated internal-wave energy flux (W m\(^{-1}\)) is computed as the covariance of the normal-mode velocity and baroclinic pressure anomaly,

\[ F = \int_{-H}^{0} \langle \mathbf{u}(z')p'(z') \rangle \, dz', \]  

where \( \langle \rangle \) indicates an average over one wave period.

In Eq. (6), the baroclinic pressure anomaly \( p'(z) \) is calculated from the normal-mode vertical displacement \( \eta(z) \) and buoyancy frequency \( N(z) \) (Kunze et al. 2002). For each mode, \( p'_n(z) \) is obtained as

\[ p'_n(z) = \bar{p} \int_{-H}^{0} N^2(z') \eta_n(z') \, dz' - \bar{p}_n, \]  

where \( \bar{p}_n \) is the mean defined as

\[ \bar{p}_n = \bar{p} \int_{-H}^{0} N^2(z') \eta_n(z') \, dz'. \]  

g. Sample calculation

Figure 4 shows a typical calculation of energy flux at mooring 2 (28°N, 208°E), over a 5-day period. This location is northeast of the Hawaiian Island chain and is dominated by fluxes toward the northeast (Fig. 1). Figures 4a,b give the filtered meridional velocity and displacement, represented by colored vertical bars at each instrument depth. Magnitudes are \( O(0.05 \text{ m s}^{-1}) \) for velocity and 50 m for displacement. At this location, the barotropic tide dominates the deep velocities. While some vertical discontinuities are visible (owing to the coarseness of the data relative to the wavenumber content of the signals), a field composed of baroclinic plus barotropic motions can be pictured.

The modal solutions (Figs. 4c,d) are generally representative of the discrete filtered data. In the case of velocity, from which the barotropic signal has now been removed, velocity now decreases with depth owing to the weaker stratification. For displacement, the large shallow values cannot be represented by a sum of mode 1 and 2, and so are absent in the modal solutions.

Baroclinic pressure anomaly (Fig. 4e) is of order 200–400 Pa, resulting in surface elevations of 2–4 cm, as seen in altimetry (Ray and Mitchum 1997). It displays similar depth dependence as velocity, such that the flux \( F \), which is the covariance of the two, is extremely surface enhanced (Fig. 4f) and has frequency of 2 times that of \( M_2 \). Figure 4g shows the depth-integrated meridional energy flux (black, running-averaged over an \( M_2 \) cycle) and the time cumulative energy passage (northward, red).

The phasing of the signals that produces positive and negative depth-integrated fluxes (Fig. 4g) is evident. Early in the record, displacements are moderate (20 m)
but result in pressure in phase with velocity (Figs. 4c,e), yielding northward fluxes. Displacements later in the record are stronger, but pressure is nearly out-of-phase with velocity, resulting in weak southward fluxes. This phase modulation will be discussed in detail below.

3. Moored observations of semidiurnal energy and flux north of the Hawaiian Ridge

a. Sample time series

In this section, we present time series at two moorings as examples to examine temporal characteristics of the internal tide energy density and energy flux, as well as the amplitude and phase of the mode-1 velocity and displacements (Figs. 5 and 6). The mooring shown in Fig. 5 is located at 28°N, 208°E, in a region of moderate northeastward fluxes northeast of the Hawaiian Ridge. The one in Fig. 6 is located at 41°N, 197°E, in a confluence of northward fluxes from Hawaii and southward from the Aleutian Islands (Fig. 1). Together with the time series, several diagnostics are also plotted that allow determination of both internal self-consistency of the modal solutions, as well as characteristics of wave propagation.

In each figure, the spring–neap envelope of the barotropic tidal currents from TPXO.6 is plotted in Figs. 5a and 6a as an indication of forcing strength. The strength of the barotropic currents varies across the North Pacific Ocean, but the timing of the spring–neap cycle does not; consequently, the central location (30°N, 200°E) is used for these figures and the lag calculations in the next section.

Flux and energy are plotted in Figs. 5b–d and 6b–d. For flux, each mode (thin) and the total (thick) is plotted, together with the time integral (gray; label at right). At these and most moorings, the flux is dominated by the first mode, as expected given the strong weighting of baroclinic pressure anomaly toward low modes (Nash et al. 2005). At 28°N (Fig. 5) time-mean flux is \( \approx 1 \text{ kW m}^{-1} \) toward the northeast, consistent with generation at the Hawaiian Ridge (most likely the Kauai Channel), as noted before. At 41°N, time-mean flux is weakly eastward, with a modulated north–south component.

Potential energy density (PE; thin), kinetic-energy density (KE; thick), and the sum (E = PE + KE; gray) are plotted next. Total energy varies by an order of magnitude (as discussed below) but is of order 1 kJ
Fig. 4. Calculation of the semidiurnal internal-tide flux over 5 days at mooring location (28°N, 208°E). (a), (b) Filtered northward velocity and isopycnal displacement plotted at each measurement depth; (c) the sum of mode-1 and -2 meridional velocity; (d) isotherm displacement; (e) pressure anomaly profiles; (f) meridional energy flux; and (g) the depth-integrated energy flux (black) and the time cumulative energy passage (red).
Fig. 5. Time series of semidiurnal energy, flux, modal amplitude, and phase at mooring location 2 (28°N, 208°E). Mode-1 ellipses of $u$, $u\eta$, and $v\eta$ are plotted at top for selected 4-day periods indicated on the time series below with vertical gray shading (see text). Time series of (a) the spring–neap cycle envelope of barotropic tidal currents in the North Pacific according to TPXO.6; (b) depth-integrated eastward energy flux in mode 1 (thin), 2 (dashed), their sum (solid), and the cumulative energy passage (gray); (c) as in (b) but for the northward flux; (d) mode-1 kinetic energy (black), potential energy (thin gray), and the total (thick gray); (e) the phase of $u$ (solid) and $v\eta$ (dashed) with 95% confidence limits shaded; where hidden, white lines indicate the edge of the shading; (f) as in (e) but for $v\eta$; (g) the phase difference between $u\eta$ (solid) and between $v\eta$ (dashed); and (h) the relative phase between $u\eta$. Phases are computed using harmonic analysis over overlapping 14-day windows.
Energy is somewhat higher at 28°N than at 40°N, perhaps owing to the greater distance from the Hawaiian Ridge.

KE generally exceeds PE at 28°N as expected for a freely propagating, linear internal tide. For \( \omega \ll N \), the ratio of KE and PE for a free internal wave is

\[
 r = \frac{\text{KE}}{\text{PE}} = \frac{\omega^2 + f^2}{\omega^2 - f^2}, \tag{9}
\]

where \( \omega \) is the wave frequency (\( M_2 \) for these examples) and \( f \) is the local inertial frequency. However, two waves of equal amplitude propagating in opposite directions display offset nodes and antinodes in KE and PE, invalidating (9) and leading to a periodic pattern in \( r \) (Nash et al. 2004; Alford et al. 2006). At 41°N, regions where KE and PE each dominate are seen, consistent with standing-wave contributions expected at this location. (In theory, \( r \) could be used to compute intrinsic...
frequency, providing another consistency check; in practice, it is far too noisy to be useful for this purpose.)

Both flux and energy are highly variable at both sites. In rough terms, energy and flux magnitude rise and fall together, a characteristic that will be taken advantage of in Part II to compute group velocity. Variability consists of a spring–neap component, as well as lower-frequency variability. For example, little flux at 28°N occurs for the first 50 days of the record at 28°N, becoming relatively constant after this. In the following year (mooring 3; time series not shown), annual-mean fluxes are substantially weaker than the other two deployments at this location (Fig. 1). Zonal flux is relatively constant at 41°N for the first part of the record, but is nearly zero after yearday 450. The sign of the meridional flux changes with both a spring/neap and a lower-frequency modulation.

The qualitative differences between the internal tide, viewed through these two moorings, and the deterministic barotropic tide are obvious. To better understand the variable fluxes, we examine the phase of the mode-1 velocity and displacement signals via hodographs (Figs. 5 and 6, top panels), and the phase of \(u, v, \) and \(\eta\), computed with standard harmonic analysis (Foreman 1996) in 14-day windows. Together, these two techniques allow a variety of features of the propagation to be diagnosed.

Hodographs of mode-1 \(uv\) are displayed in the top panels of Figs. 5 and 6, with the limits indicated on the leftmost one. Each panel represents data from a four-day window, indicated with gray shading in Figs. 5b,c,g,h and 6b,c,g,h. A linear free wave demonstrates ellipses aligned in the direction of propagation, with the ratio of semimajor to semiminor axes given by \(O(0.01 \text{ m s}^{-1})\). Observed ellipses at 28°N are generally aligned with the general flux direction [e.g., east–west at yeardays 323 and 473 when fluxes (b), (c) are eastward and north-eastward, respectively]. At 41°N, the magnitude, ellipticity, and orientation of the ellipses are more variable than at 28°N, again possibly owing to multidirectional fluxes.

The next two hodograph rows are current-displacement ellipses of \(u\eta\) (middle) and \(v\eta\) (lower). A new diagnostic technique, these are computed by normalizing mode-1 current and displacement by their respective maximal value during each 4-day period. In this view, in-phase variables (such as \(u\) and \(\eta\) for an eastward-propagating wave) appear as a line oriented along the diagonal \(y = x\); out-phase variables are oriented along \(y = -x\). Variables in quadrature (as for \(u\) and \(\eta\) for a standing wave) appear as circles. These diagnostics provide additional information to the fluxes; for example, strong eastward fluxes near yearday 323 at 28°N (Fig. 5b) display \(u\) and \(\eta\) aligned along \(y = x\). Southward fluxes near yearday 254 at 41°N (6) show \(v\) and \(\eta\) out of phase (aligned along \(y = -x\)). However, weak fluxes resulting from weak motions (north–south direction at 28°N near yearday 273) and from motion in quadrature (standing wave, as in the north–south direction 41°N near yearday 284) can be distinguished in this manner.

It is clear from the hodographs that the phase of each variable can vary substantially from one window to the next. Since the passband of our filter includes both \(M_2\) and \(S_2\) constituents, it is of interest to evaluate the phase variations of \(M_2\) alone. To do this, harmonic analysis is required over 14-day windows to separate \(M_2\) and \(S_2\). The phases of the mode-1 velocity and displacements (Figs. 5e,f and 6e,f), and their differences (Figs. 5g,h and 6g,h), are another diagnostic of wave propagation that isolates \(M_2\). 95% confidence limits, computed according to Koopmans (1995), are indicated with shading. The \(M_2\) phase of \(u, v, \) and \(\eta\) all vary \(\pm 90°\) at both the 28°N and the 41°N sites—contrasting sharply with the constant phase for a free mode-1 wave propagating at a time-invariant speed.

The phase differences are another measure of propagation characteristics; an eastbound/westbound free wave exhibits \(\Phi_u - \Phi_\eta = [0°, 180°]\), respectively, and \([90°, 270°]\) for standing waves. The calculated phases are generally in agreement with the hodographs and fluxes (e.g., \(\Phi_u - \Phi_\eta\) at 28°N hovers near \(0°\) during the strong eastward fluxes at yearday 386, and early in the record at 41°N), but differences exist as well, owing to both the exclusion of \(S_2\) and the longer temporal window used in the phase calculation.

As stated above, we hypothesize that the observed phase variation owes to refraction by mesoscale features between the supposed generation and detection sites. The expected phase modulation can be computed for refraction by currents (RP06) and time-varying stratification (Alford et al. 2006). RP06 found that phase differences of \(\pm 45°-90°\) were expected 1000 km north of the Hawaiian Ridge as a result of refraction by remotely sensed mesoscale currents (though the mesoscale during the time of their fieldwork was weaker on the north side of Hawaii than on the south).

For stratification changes, Alford et al. (2006) showed that observed arrival times of large \(M_2\) internal tides at Hawaii’s Mamala Bay were consistent with those expected because of seasonal changes in stratification detected at the Hawaii Ocean Time series (HOT) site. A similar comparison is not attempted here. However, seasonal effects can affect phase changes of the observed magnitude. The presence or
Table 2. Travel time of the mode-1 internal tide computed from lags in spring–neap cycle of energy flux observed at North Pacific moorings, relative to barotropic forcing at the Hawaiian Ridge. Travel times, and group and phase speeds, are also shown for a mode-1 linear internal tide propagating in climatological stratification. Distance is computed along the great circle from French Frigate Shoals (24°N, 193°E) for moorings 0, 4, and 14 and the Kauai Channel (22°N, 201°E) for 2, 3, and 15.

<table>
<thead>
<tr>
<th>Mooring No.</th>
<th>Distance from generation site (km)</th>
<th>Travel time (h)</th>
<th>Mean speed (m s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Observed</td>
<td>Group</td>
</tr>
<tr>
<td>0 (41°N, 197°E)</td>
<td>1921</td>
<td>180</td>
<td>229</td>
</tr>
<tr>
<td>2 (28°N, 208°E)</td>
<td>970</td>
<td>104</td>
<td>102</td>
</tr>
<tr>
<td>3 (28°N, 208°E)</td>
<td>970</td>
<td>104</td>
<td>102</td>
</tr>
<tr>
<td>4 (42°N, 208°E)</td>
<td>2430</td>
<td>248</td>
<td>296</td>
</tr>
<tr>
<td>14 (42°N, 208°E)</td>
<td>2430</td>
<td>289</td>
<td>296</td>
</tr>
<tr>
<td>15 (28°N, 208°E)</td>
<td>2430</td>
<td>—</td>
<td>296</td>
</tr>
</tbody>
</table>

The absence of a 100-m-deep mixed layer, for example, changes the mode-1 speed by 2%—sufficient to alter the arrival time 900 km away by 1.6 h (changing the phase by 45°). At 41°N (over 2 times as far from the generation site), phase changes would be 2 times as large.

b. Spring–neap cycle

Spring–neap cycles are observed in most of the semidiurnal-band records. To illustrate, we focus on moorings 0, 2, 3, 4, 14, and 15, NW of the Hawaiian Ridge (Table 2; Fig. 1). We plot moorings 2 (Fig. 7) and 4 (Fig. 8). Internal-tide energy-flux magnitude $F (|F_x i + F_y j|)$; Figs 7a and 8a, gray) and energy $E$ (Figs 7b and 8b, gray—note linear scale) are plotted. A pronounced fortnightly modulation is obvious in energy and flux at both sites, which is confirmed in the frequency spectrum of each quantity to the right. A prominent peak appears near 0.07 cpd (~1 cycle per 14 days), which, together with the peaks at lower frequency (monthly time scales), dominates the variance. The modulation is very strong, with the spring magnitude typically exceeding that at neap by a factor of 5–10.

Both of these moorings exhibit northwestward fluxes (Fig. 1), consistent with propagation from the Hawaiian Ridge as expected from models (Merrifield et al. 2001) and observations (Rudnick et al. 2003). Because the signals are dominated by mode 1, this energy would be expected to propagate at the group speed for the mode-1 semidiurnal tide. Can the spring tides be associated with barotropic forcing maxima at the ridge, lagged by an appropriate travel time? To investigate, the spring–neap component of flux magnitude and energy are extracted via bandpass filtering (Figs. 7a’ , 7b’ , 8a’, 8b’, gray patches), and plotted as black curves in Figs. 7a and 8a. It is clear that the maxima at the spring tides are nonsinusoidal and their magnitudes are thus underestimated, but otherwise the filtered quantity is an adequate representation of their timing.

The fortnightly filtered flux magnitude (thick) and energy time series (thin) are next normalized by their maximum values and plotted in the next panel on the same axes as the normalized barotropic current amplitude at the Hawaiian Ridge from the TPXO.6 model (gray). To account for the travel time of the tide from Hawaii to the mooring, the forcing curve is shifted to the right by 104 h at mooring 2 and 248 h at mooring 4, determined as the lag producing the maximal correlation (Figs. 7c’, 8c’).

Temporal agreement between the fortnightly phasing of lagged barotropic forcing and both energy and flux is excellent when energy and flux are strong. The strength of individual spring tides, however, appears not to be simply related to the forcing [perhaps consistent with low-frequency modulation of the generation process (Mitchum and Chiswell 2000)]. During weaker signals, energy and flux at the mooring are still in phase with each other but are not in phase with the forcing.

The inferred travel times to these sites (as well as to moorings 0, 3, and 14; not shown) are consistent with those expected for linear mode-1 waves. Assuming the signals at 28°N (mooring 2, 3) and 41°–42°N (mooring 0, 4) are generated in the Kauai Channel and French Frigate Shoals, respectively (see Fig. 1), the travel time of a linear mode-1 tide is simply computed as $t_r = f c_g^{-1} (s)$, where $s$ is distance along the propagation path (assumed to be a great-circle route, which differs only little from computed ray paths; Fig. 1). For each site, the group speed $c_g$ was computed for mode 1 using Levitus climatology, as in RP06, and described in more detail in Part II. Note that a proper integration is required since substantial slowing occurs toward the north; for example, $c_g = 2.7$ m s⁻¹ near Hawaii but only 1.7 m s⁻¹ at 42°N.

The results are summarized in Table 2. For each mooring, a 10% uncertainty in the distance (owing to uncertain generation site and propagation path) and 10% in the estimated time (owing to a broad peak in the correlation versus lag) are assumed, yielding 20% error bounds on the speed. Agreement with the linear...
mode-1 group speed is excellent for moorings 2, 3, and 14. (No significant correlation was obtained at mooring 15, precluding determination of a travel time there. The observed strong mesoscale field and weak spring–neap cycle at that mooring may be responsible.) At moorings 0 and 4, observed speeds are between the group and phase speed (right column). In all cases except mooring 0 (which may suffer contamination by southward fluxes from the Aleutians), the inferred speeds are within our computed error bounds of the linear group speed, and inconsistent with propagation at the phase speed (as expected; Lighthill 1978, 313–337).

4. Global distribution of energy and flux

a. Maps

Flux maps from an initial set of 60 moorings were presented in A03. Here (Fig. 9), an improved view is offered by 1) plotting all 80 moorings and 2) plotting energy as well as flux. Depth-integrated annual-mean internal tide energy fluxes at 80 mooring locations are shown for the tidal (Fig. 9b) and near-inertial bands (Fig. 9a). Each mooring is represented by an arrow and a dot, with the length of the arrow indicating the energy flux and size of the dot indicating the energy density (scales indicated at upper left). As in Alford (2003a), the fluxes are plotted atop a color map indicating the conversion rate from surface tide to internal tide from the TPXO.6 model (Egbert and Ray 2001, Fig. 9b) and the work done by the wind on mixed-layer near-inertial motions (Alford 2003b, Fig. 9a).

The flux maps qualitatively resemble those presented in Alford (2003a), because the extra moorings are mostly from the Atlantic so do not greatly expand coverage. We therefore only briefly restate A03’s conclusions here:

- Tidal and near-inertial fluxes are of the same order of magnitude, $\mathcal{O}(1 \text{ kW m}^{-2})$.
- A substantial fraction of energy is fluxed by low-mode waves out of control volumes placed around

Fig. 7. (a), (a’) Flux magnitude (thin), its spring–neap component (thick), and its frequency spectrum at mooring 2. The fortnightly frequency band used in the filtering in (a)–(c) is shaded in gray in the spectrum at right. (b), (b’) As in (a), (a’) but for mode-1 energy density. (c) Spring–neap curves of the energy density (thin black), the flux (thick), and barotropic currents from TPXO.6, computed at 30°N, 200°E (gray). Barotropic tide forcing is shifted 104 h behind in time (to the right). (c’) Correlation coefficient between barotropic forcing and the spring–neap components of flux magnitude. The maximum correlation occurs at a time shift of 104 h.
tidal-conversion regions and storm-track regions, respectively.

- Semidiurnal fluxes generally point away from strong conversion regions, for example, Hawaii and the west Pacific trench.
- Near-inertial fluxes are overwhelmingly equatorward, consistent with generation at the turning latitude on a $\beta$ plane.

Fluxes were originally computed in A03 for modes 1–2 for all moorings. Subsequent analysis has revealed that the geometry of five moorings (Table 1) was marginal for resolving the second mode. Hence, we have here conservatively elected to solve for only mode 1 at these sites. Given the dominance of mode-1 fluxes at most sites, this is not of great concern for these few moorings. The mode-1-only flux is in the same quadrant as before, but reduced slightly in magnitude. At the equatorial moorings (16–19), these questionable mode-2 fluxes introduced substantial directional spread in A03’s fluxes (their Fig. 3). Here, the spread is greatly reduced.

Time-mean energy in both bands (dots) shows substantial variability. Semidiurnal energy is often higher near conversion regions (dark regions in the grayscale map). Numerical simulations of the North Pacific (Niwa and Hibiya 2001) and of the globe [assimilating Ocean Topography Experiment (TOPEX) altimetry data; Kantha and Tierney 1997] show heightened energy near these regions, and our values $\mathcal{O}(1 \text{ kJ m}^{-2})$ are generally consistent with theirs, though a point-to-point comparison has not been conducted.

In the near-inertial band, a slight tendency toward higher flux and energy values in the western North Atlantic mirrors the stronger forcing there (grayscale; Alford 2001, 2003b). Alford and Whitmont (2007) found a similar pattern in situ energy determined from 695 individual current meters (representing all modes, but allowing much greater spatial resolution), complementing the low-mode quantity examined here.

**b. Probability density functions of energy and flux**

The global and seasonal patterns of near-inertial and tidal energy and flux are efficiently summarized through conditional probability density functions (PDFs) evaluated for all 80 moorings (Fig. 10). For each quantity, separate PDFs are constructed using

![Fig. 8. As in Fig. 7 but for mooring 4 (42°N, 208°E). The time series in (c) has been lagged by the maximal-correlation lag of 248 h (c’).](image-url)
summertime-only and winter-only data. (Summer/ winter is defined as yeardays 90–270 in the Northern/ Southern Hemisphere, respectively.) For flux (Figs. 10a,c), equatorward and poleward data are considered separately; for energy (Figs. 10b,d), separate PDFs are made for kinetic and potential energy. Energy and flux for both bands, regardless of season and direction, are strongly peaked toward small values. The near-inertial distributions extend to higher energy and flux values than do the semidiurnal distributions, indicating a more intermittent field, as generally observed (cf. Fu 1981), and consistent with the intermittent forcing (a small number of storms account for a large portion of the forcing; D’Asaro 1985; Alford 2001). However, substantial spread is seen in the semidiurnal field as well, as indicated by the time series (previous section).

For the semidiurnal band, neither energy nor flux displays a dependence on season, consistent with the quasi-constant forcing. A slight preference is, however, seen for poleward fluxes. Since the high turning latitude for $M_2 (75^\circ)$ allows energy propagation from sources, this is contrary to expectations. Apparently, the moorings are preferentially located poleward of sources, as qualitatively apparent from Fig. 9b.

The near-inertial fluxes, by contrast (Fig. 9a), are...
much stronger during winter. Moreover, the winter fluxes display an equatorward preference in both hemispheres, as found by Alford (2003a). During summer months, near-inertial fluxes are weak and without poleward or equatorward preference.

In energy, the semidiurnal band displays no seasonal cycle, while near-inertial energy (like flux) is winter-enhanced. In a complementary study, Alford and Whitmont (2007) found from a much larger database that in situ near-inertial energy (as opposed to the low-mode component seen here) was winter-enhanced by a factor of 3–4.

Both semidiurnal and near-inertial fluxes exhibit KE > PE, as expected for free internal waves. Fluxes from multiple directions at a location can yield standing waves that demonstrate PE > KE at half-wavelength intervals (Nash et al. 2004), as considered in more detail in Part II. The observation that KE > PE at most locations suggests that open-ocean standing waves are relatively rare, consistent with group velocity calculations for latitudes <35° (Part II).

5. Summary

The energy density and energy flux of the mode-1 and -2 semidiurnal internal tides are computed and analyzed for 80 globally distributed historical moored data. We observe the following:

- Time series of energy, flux, and phase, examined in several different ways at representative locations, provide an internally consistent view of wave propagation. These demonstrate that low-mode fluxes can be reliably resolved using discretely sampled moorings, and that several aspects of their propagation are observable, namely, 1) strong fluxes result when velocity and displacement are in phase; 2) standing-wave behavior can be distinguished at 41°N in both $v-\eta$ hodographs and $v-\eta$ phase differences; and 3) energy, phase, and flux are variable on fortnightly and seasonal time scales.
- Northeastward semidiurnal fluxes ~1 kW m$^{-1}$ are seen northeast of the Hawaiian Ridge at 28° and 42°N. These show a prominent spring–neap cycle,
whose timing relative to the barotropic spring–neap cycle is consistent with propagation of the linear tide at the group speed northeastward from the ridge. Signals take about 10 days to reach 42°N.

- Global maps and probability distributions indicate that near-inertial and tidal energy and energy flux are the same order of magnitude (1 kJ m⁻² and 1 kW m⁻¹, respectively) and, in both cases, are directed away from forcing regions. Near-inertial flux is winter-enhanced and directed toward the equator. For both bands, a significant portion of the power input is carried far from the forcing region.

6. Discussion

This work has focused on the lowest-wavenumber, lowest-frequency aspects of the internal-wave field. It has long been known (Briscoe 1975) that these elements are not isotropic (D’Asaro 1991; Polzin 2004), in contrast to the rest of the internal-wave spectrum (Garrett and Munk 1975). This work, which indicates that the internal tide propagates at least 2400 km northeast from Hawaii (taking ~10 days to do so), and that near-inertial energy propagates overwhelmingly toward the equator, provides strong support for these ideas. These historical moorings provide a rich view of this propagation not possible with altimetry and models, though at a great loss of spatial coverage.

The factor-of-5 spring–neap modulation in semidiurnal tidal energy and energy flux and consistency of the observed lags with propagation at the linear group speed are consistent with the notion of internal-wave groups (Thorpe 1999). Like ripples from stones thrown in a pond, internal-tide energy appears to travel in packets at the group velocity, as expected from linear theory (Lighthill 1978, 317–337). For horizontally propagating internal waves, individual wave crests would appear at the rear of the group, overtake it, and disappear at the leading edge. For the region north of Hawaii, the 10-day travel time to 42°N and the spring–neap cycle of forcing would be expected to produce alternating spatial maxima and minima in energy; for example, at spring tides, a maximum would lie near Hawaii and a minimum (from the previous neap) at a distance of \( \int_{0}^{t_{\text{days}}} c_{d} |x(t)| \, dt \) to the north. In Part II, we compute the group velocity at each mooring observationally (from \( \hat{\mathbf{c}}_{g} = \mathbf{F}E^{-1} \)) and find that it obeys—remarkably—the expected latitudinal dependence for linear waves, providing further support for this interpretation.

The variability of phase, energy, and flux indicates a wave field that is strongly refracted by mesoscale currents and stratification. Operationally, this limits the usefulness of altimetric estimates of the internal tide, which cannot detect the portion of the signal that has lost coherence with astronomic forcing. (Near-inertial waves, of course, have no “coherent” component and are thus always invisible to altimetry.) However, it gives rise to the possibility, at least in theory, of “internal-tide tomography,” that is, using arrival times to invert for path-integrated mesoscale fields.

One envisions an ocean with discrete generation regions, which give rise to internal-tide rays that propagate away. Both the ray paths and along-ray propagation time are time variable, giving rise (at any remote observation site) to a complicated interference pattern between all rays incident on that location. The first steps toward modeling this refraction have been taken (RP06), but much work remains.

Dissipation of the internal tide north of Hawaii is still not well addressed in this study, which used sparse and suboptimally placed moorings. We have argued, however (as have RP06), that the perceived attenuation of altimetric amplitudes northward from Hawaii is at least partly due to wave refraction and resulting loss of coherence with astronomic forcing. While we have not addressed the role of PSI, the persistence of strong fluxes at 42°N suggests that at least a fraction of the energy survives the strong wave–wave interactions at the “critical latitude.” Whether it undergoes a slow loss of energy due to topographic scattering and other processes in the open ocean or suddenly “breaks” at the continental slope remains an open question.

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