The Wavenumber–Frequency Spectrum of Vortical and Internal-Wave Shear in the Western Arctic Ocean

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ABSTRACT

Upper-ocean velocity and shear data were obtained from Doppler sonars operated at the Surface Heat Budget of the Arctic Ocean (SHEBA) ice camp during the camp’s year-long drift across the western Arctic Ocean. These are used to estimate wavenumber–frequency spectra of shear $E(\kappa_z, \sigma)$ during three selected time intervals. The Arctic shear spectra are similar in form to typical oceanic spectra, except that they have roughly an order-of-magnitude less variance. The slope of the frequency dependence is also steeper ($\sigma^{-3}$ for $\sigma < -f$, where $f$ is the Coriolis frequency) in the internal-wave band, and the vertical wavenumber dependence is centered at higher wavenumber. Given the small vertical scales and low velocities of Arctic signals, a careful assessment of sonar precision is performed. Fluctuations at vertical scales $> 10$ m and time scales $> 1$ h are deemed significant. At subinertial frequencies, a vortical (quasigeostrophic) contribution to the shear spectrum is seen. The vertical wavenumber dependence of the shear spectrum in this frequency range is distinctly red, in contrast to the band-limited form of the superinertial spectrum; that is, $E(\kappa_z, \sigma) \sim \kappa_z^{-1}$ for $|\sigma| < f$. A fundamental characteristic of both the internal-wave and vortical spectral contributions is that the observed frequency bandwidth increases linearly with increasing vertical wavenumber magnitude. This is interpreted as the signature of the Doppler shifting of the observations by random “background” currents and by ice motion and is responsible for the distinctly “nonseparable” nature of the shear spectrum. As a consequence, the vertical wavenumber spectrum of the subinertial motion field is white: $E\text{vort}(\kappa_z) = \int E\text{vort}(\kappa_z, \sigma) d\sigma \sim \kappa_z^0$.

1. Introduction

It would be attractive to have a simple multidimensional spectral model of the oceanic internal-wave field that could be used to predict wave field statistics as a function of changing environmental conditions. The internal-wave analog of the Pierson–Moskowitz (1964), Phillips (1958), or Joint North Sea Wave Project (JONSWAP; Hasselmann et al. 1973) models would have a form that reflected the scales and rates of external forcing, the cross-scale transfers of wave energy, and the ultimate conversion of wave energy to the mean flow, to heat, and to the vertical buoyancy flux. The current standard in spectral models (Garrett and Munk 1972, 1975, 1979; Munk 1981) is “universal,” reflecting an open-ocean spectrum of unchanging level or form.

A sense of the evolution of the spectrum with changing wave field energy is slowly being formed (Müller et al. 1992). The effort is complicated by the presence of quasigeostrophic (vortical; Holloway 1983; Müller 1988) motions that occupy the same vertical scales as internal waves. When laterally or vertically advected, they can appear at the same temporal frequencies as well. Differentiating these signals from internal waves in observational records is challenging.

In the western Arctic Ocean, internal-wave energy levels are less energetic by a factor of 10–100 than in the open ocean (Levine et al. 1985; Morison 1986; Levine 1990; D’Asaro and Morison 1992; Merrifield and Pinkel 1996; Halle and Pinkel 2003; Pinkel 2005). Arctic observations thus yield a view of the spectral form at the low-energy extreme. Arctic mesoscale currents are also generally weak, and the Coriolis frequency is near its planetary maximum. The Arctic is thus an optimal region to attempt to distinguish internal-wave and vortical motions on the basis of observed frequency. The experimental challenge is to resolve these small-scale, low-variance features under the polar ice cap.
Measurements of upper-ocean velocity were obtained during the 1997/98 drift of the Surface Heat Budget of the Arctic Ocean (SHEBA) ice camp through the western Arctic Ocean (Uttal et al. 2002). Doppler sonars were mounted immediately under the ice, and they profiled to depths in excess of 250 m. Depth resolution was 3.4–3.69 m. Rotary wavenumber–frequency spectral analysis is used to decompose the motion field according to the sense of rotation in both depth and in time. The analysis objective is to quantify the spectrum of vertical shear in this low-energy environment and to identify the dynamical fields that contribute to it.

Instrumentation and analysis methods are discussed next. Given the low signal levels, the noise in the sonar measurements is explored in some detail. Then, representative wavenumber–frequency spectral estimates are presented for several selected measurement periods. From these 2D spectra, independent 1D vertical wavenumber spectra of vortical and internal-wave shear are estimated. In a companion work (Pinkel 2008), we explore the possibility that the apparently continuous frequency dependence of the spectrum results from the Doppler smearing of a few discrete spectral lines.

2. Methods

The SHEBA ice camp was deployed in September of 1997 from the Canadian icebreaker de Groseilliers. The ship was frozen into the ice at 75°N, 143°W in the central Beaufort Sea (Fig. 1). During November 1997, a 140-kHz Doppler sonar system, developed at Scripps Institution of Oceanography, was installed at the camp. The four downward-slanting beams were oriented 45° below horizontal. Repeat-sequence coded pulses (Pinkel and Smith 1992) were transmitted at 0.6-s intervals. Depth resolution was 3.27 m.

The initial SHEBA sonar was destroyed on 9 February 1998, during a period of ice convergence and underthrusting. It was replaced in March with a 160-kHz repeat-sequence coded system. The four beams of the replacement sonar were angled 60° downward, with a corresponding depth resolution of 3.69 m. This sonar ran for the duration of the experiment, with intermittent interruptions that were from camp power failures.
Throughout SHEBA, profiles of velocity and acoustic scattering strength were recorded at 1-min intervals. Edited profiles were formed into 1-h averages and were rotated into geographic coordinates. At depths above 15 m, the profiles were contaminated by nearby ice reverberation.

The measurement of finescale motion in the Arctic is challenging, given the contrast between the harsh operating environment above the ice and the relative tranquility below. Both the low level of biological scattering and the low ocean velocity variance render acoustic measurements difficult. In addition, the physical separation of the sonar beams can allow lateral variability to introduce unanticipated changes in signal phase. These compete with the vertical and temporal phase fluctuations that are the focus of this investigation. Lateral variability was credited with degrading the apparent vertical resolution of commercial ADCPs in a recent study (Polzin et al. 2002).

The issue of measurement precision is complicated by ambiguity in the definition of “signal” versus “noise.” In the Arctic, at the smallest scales, the sonars very precisely measure the swimming behavior of zooplankton and small fishes. The distinctions that can be made from acoustic data relate to the degree of correlation of the velocity estimates spatially and temporally. An analysis of these correlation properties follows. The conclusion is that, for vertical scales greater than 10 m and time scales greater than 1 h, the velocity measurements are correlated but are not uniform across the spatial extent of the array. The reader willing to accept this finding can skip the paragraphs below.

To focus on the issue of precision and variability, data are examined from an 8-day record in August of 1998, a time of active biological vertical migration. Here, $u_j(z)$, $v_j(z)$, and $w_j(z)$ represent the spatial means across the horizontal extent of the array, $\Delta u_j' = -\Delta u_j''$, $\Delta w_j' = -\Delta w_j''$, and so on, and $z = rb_3$.

For homogeneous data, $\langle \Delta u'' \rangle = \langle \Delta w'' \rangle = 0$ for all $i, j$. It is further assumed that $\langle \Delta u'_j \Delta w_j' \rangle = 0$ for all $i, j$, where $i \neq j$. Here the brackets indicate an average over time.

In terms of these variables, the “slant velocity” measurement at $X_i(r)$, for example, is

$$V_i'(r) = b_i \cdot U(X_i) + \varepsilon_i(X_i)^T$$

$$= b_i(u_0 + \Delta u_i) + b_3(w_0 + \Delta w_i) + \varepsilon_i.'$$ (1)

The noise $\varepsilon$ is assumed to be uncorrelated with the ocean velocity vector $U$, with the noise in other beams, and with itself over ranges greater than $\Delta z_{res}$. An estimate of horizontal velocity in the so-called JANUS approximation is proportional to the difference in slant velocity signals of back-to-back sonar beams at like values of range:

$$u_{JANUS} = (V_i'(r) - V_j'(r))/2$$

$$= b_1u_0 + b_3w + (\varepsilon_i' + \varepsilon_j')/2.$$ (2)

The mean horizontal contributions $u_0$ to the slant signals add while the vertical contributions $w_0$ cancel. The reverse applies to the spatial differences. The approximation would be exact, were it not for the finite separation between the sonar beams.

The variance of the JANUS velocity is

$$\langle u_{JANUS}^2 \rangle = b_1^2\langle u_0^2 \rangle + b_3^2\langle \Delta u^2 \rangle + \langle \varepsilon_i^2 \rangle/2.$$ (3)

The individual beam slant-velocity variance is, in contrast,

$$\langle V_i^2 \rangle = b_1^2\langle u_0^2 + \Delta u^2 \rangle + b_3w_0\langle w_0 + \Delta w^2 \rangle + \langle \varepsilon_i^2 \rangle.$$ (4)

Typically, $\langle w_0^2 \rangle$ and $\langle w_0w_0 \rangle$ are much smaller than $\langle u_0^2 \rangle$, and the slant variance closely approximates the JANUS variance, but with twice the noise. Thus, a simple way to identify noise (including spatially uncorrelated velocity) is to compare the spectra of slant and JANUS velocities.

In Fig. 2, vertical wavenumber spectra of JANUS and slant shear are presented. Shear is presented rather than velocity because shear spectra are “whiter” than those of velocity. Small details are more easily distinguished. When 10-min shear spectral averages are compared, the factor-of-2 relationship between slant and
JANUS estimates is apparent for all but the lowest wavenumbers. The implication is that a substantial fraction of the higher wavenumber variance is uncorrelated between the beams.

The identical data can be further averaged to form hourly profiles, whose slant and JANUS wavenumber spectra are given by the thick lines in Fig. 2. If the variance is uncorrelated in time as well as space, the hourly spectra should be smaller by a factor of 6 than their 10-min counterparts. In fact, the levels fall by less than a factor of 6, except at the highest wavenumbers. A significant fraction of the remaining variance is associated with temporal correlations greater than 1 h.

Spatial variability across the array can be quantified by comparing two metrics of precision. A four-beam sonar yields two independent estimates of JANUS vertical velocity from the back-to-back beam pairs:

\[
\begin{align*}
  w^I_{\text{JANUS}} &\sim \frac{\langle V^I + V^IV \rangle}{2} \\
  w^{\text{II} - \text{IV}}_{\text{JANUS}} &\sim \frac{\langle V^I + V^IV \rangle}{2}
\end{align*}
\]

The quantity \((w^{\text{II} - \text{IV}}_{\text{JANUS}})^2\) provides a measure of both the spatially uncorrelated noise in the measurement and any lateral variability in the hourly averaged signals. Its shear-equivalent wavenumber spectrum is given in Fig. 2 by the light-blue curve. Note that a white \((\kappa_2^z)\) velocity noise spectrum corresponds to a \(\kappa_2^z\) spectral slope in shear. In terms of velocity, the \(\Delta v\) metric corresponds to a difference of 0.0017 m s\(^{-1}\) across the array, for hourly averages.

A contrasting noise metric is the interlaced slant velocity \(V^EVEN\) and \(V^{ODD}\). This is obtained from a single sonar beam by averaging successive “single ping”
hour-averaged records.

(0.6 s) echo statistics into “even” and “odd” buffers alternately. One thus obtains two independent estimates of hourly averaged velocity at points collated in space and time. Their difference is a fundamental metric of system performance. The spectral difference in hourly averaged interlaced velocity \((V'_{\text{EVEN}} - V'_{\text{ODD}})^2/2\), normalized to have an expected noise variance of \((\sigma^2)/2\), identical to the expected noise of the JANUS velocity, is given in (terms of shear) by the green line in Fig. 2. \(^1\) The rms velocity error identified from “interlaced averaging” is 0.00 12 m s\(^{-1}\) for these hour-averaged records.

The difference between the \((\Delta w^2)\) metric and the interlaced noise metric is a factor of 1.5 and corresponds, in velocity, to a spatial variation of order 0.0005 m s\(^{-1}\) (rms) across the array for these hourly averages. The \(\Delta w\) signal can be plotted in depth and time. Coherent wavelike signals are seen in a very noisy background. Some fraction of this noise is presumably due to spatial variability in scatterer swimming.

Three sections of the SHEBA record (Fig. 1) have been selected for detailed analysis. In the first, 1–29 December 1997, the ice camp was drifting in the deep Beaufort Sea. Internal-wave kinetic energy and shear levels were as low as any seen in the Arctic. During the second period, from 6 to 17 June 1998, the camp was drifting over the Chukchi Cap. The water depth was 500–1000 m. Mesoscale eddies were occasionally present (Pinkel 2005, his Fig. 3). Third, an extended record from 23 August to 5 October is presented, representing deep-water (3 km) conditions close to the Chukchi slope.

For spectral analysis, hourly averaged velocity profiles from depths of 67–243 m in December and 52–275 m in June and August are used. The upper bounds are set to be deeper than the maximum of the buoyancy frequency in the halocline. The lower limits are set by maximum sonar range, which is a function of acoustic scattering conditions.

To estimate wavenumber–frequency spectra of shear \(E(\kappa_z, \sigma)\), depth–time records of complex current \(U(z) = u_{\text{JANUS}} + i\mu_{\text{JANUS}}\) are Vaisala (Wentzel–Kramers–Brillouin) stretched (Pinkel 1984) in both amplitude and vertical phase. The stretching is normalized such that the total kinetic energy and the vertical extent of the observation window are preserved. The stretched profiles are then first-differenced in depth and 2D Fourier transformed. Fourier coefficients are “recolored” in vertical wavenumber, dividing by the factor \(2 - 2 \cos(2\pi\kappa_z\Delta z_{\text{samp}})\), where \(\Delta z_{\text{samp}}\) is the depth difference between adjacent samples. The wavenumber dependence of the spectral estimates is then divided by the factor \(\text{sinc}(2\pi\kappa_z\Delta z_{\text{res}})^2\) to correct for the finite spatial extent of the transmitted pulse (of vertical extent \(\Delta z_{\text{res}}\)) and subsequent averaging by \(\Delta z_{\text{res}}\) in range. A modeled noise corresponding to 0.0005 m s\(^{-1}\) rms hourly averaged velocity uncertainty is then removed. The combined effect of these various corrections is minimal at scales greater than 10 m. To avoid presenting results that are overly sensitive to the corrections, spectral displays are here truncated at 10-m vertical scale (Fig. 2).

Shear spectra are obtained by weighting the velocity spectra by the factor \((2\pi\kappa_z)^2\). The observed time-mean shear is deemed significant and thus the “zero frequency,” \(\sigma = 0\), bands of the spectra are preserved for all \(|\kappa_z| > 0\).

3. Observations

A deformed surface representation of the shear wavenumber–frequency spectrum from June is presented in Fig. 3. The spectral estimate is formed at 16 degrees of freedom ( dof) in the lowest \(\sigma-\kappa\) bands, increasing to several hundred dof at high \(\sigma\) and \(\kappa\). \(^2\) Logarithmic averaging is employed. The anticyclonic spectral quadrants peak at the inertial frequency \(-f\) and diminish rapidly with increasing frequency. At low frequencies, the spectral peak is distributed across a broad range of wavenumbers. With increasing frequency, the spectral maximum moves to increasingly higher wavenumber. The cyclonic quadrants also appear to have a broadband wavenumber dependence at low frequency, again shifting to higher wavenumber with increasing frequency. There is no sign of an inertial peak in the cyclonic quadrants. The spectrum appears to be continuous across the subinertial–superinertial boundary. Variance associated with subinertial, nonpropagating motions is split evenly between upward and downward quadrants.

To compare the December, June, and August–September shear spectra in detail, it is useful to examine spectral cross sections as a function of wavenumber and of frequency, separately. Cross sections of the anticyclonic quadrants of the shear spectrum are given as a function of vertical wavenumber in Fig. 4. Despite the significant variability in energy and shear, the spectral density of the inertial peak is remarkably similar from record to record. Overall shear variance is proportional

\(^1\) So that the even and odd averages each have an independent “hour” of data, 2 h of input data are used here.

\(^2\) Wavenumbers and frequencies are represented in radial form in the equations but are given as cyclic quantities in the figures. Temporal frequencies are given in cycles per hour in the figures, rather than cycles per second.
FIG. 3. A representative wavenumber–frequency spectral estimate from June 1998 showing shear spectral density for (top) anticyclonic and (bottom) cyclonic rotation associated with (left) downward phase propagation and (right) upward phase propagation. For the internal-wave constituents in this record, upward phase propagation corresponds to downward energy propagation. Distinct inertial ridges are seen in the anticyclonic spectra, as indicated by the arrows pointing up at low wavenumber and the arrows pointing down at the peak of the ridge. An underlying continuum appears to stretch from super- to subinertial frequencies.
to the degree that the spectral density spreads from
the inertial peak to higher (and lower) frequencies. There
is a factor-of-3 contrast between the high-frequency–
low-wavenumber motions in December and June,
for example—a consequence of this differential spread-
ing.

In the frequency cross sections (Fig. 5), the low-
energy December frequency spectrum decays steeply,
exceeding $\sigma^{-4}$ for a broad range of vertical scales. The
decay rate falls as low as $\sigma^{-1}$ at high-wavenumber and
high frequency, in the energetic June data, correspond-
ing to a larger “frequency spread” in the spectrum.

These spectral forms are comparable to higher-
energy observations at midlatitude (Pinkel 1984; Sher-
man and Pinkel 1991). They can be compared with the
more common 1D vertical wavenumber spectra ob-

**Fig. 4.** Cross sections of the shear spectrum at the inertial frequency $-f$ (upper curve in each box), and at multiples $2\times -f$ for (a), (d) December 1997; (b), (e) June 1998; and (c), (f) August 1998. These sections correspond to motions with anticyclonic rotation in time. The shift of the spectral maximum to higher wavenumber with increasing frequency is apparent. The vertical asymmetry of the spectrum in December and August is most apparent in the inertial frequency band. The more symmetric June observations were obtained over the shallow Chukchi Cap.
tained from vertically profiling instruments by integrating over frequency (Fig. 6). Here, the shear variance density associated with upward and downward propagation is plotted separately, resulting in spectral levels that are a factor of 2 smaller than those that represent the sum of both propagation directions. These one-dimensional spectra reflect the combined contributions of the low- (observed) frequency components of the wave field, which have a high-wavenumber cutoff of order $k_z^{-1}$, and the high-frequency constituents, which in fact have a blue wavenumber dependence (Fig. 4). The contribution of high-frequency motions at high

![Fig. 5. Cross sections of the shear spectrum at wavenumbers corresponding to 100-, 33-, 20-, 14-, and 11-m vertical wavelength for the (a), (d) December; (b), (e) June; and (c), (f) August records. These sections correspond to motions with anticyclonic rotation in time. A $\sigma^{-1}$ reference line is drawn. The spectrum is sharply peaked about the inertial frequency at large vertical scale. The frequency bandwidth $\Delta \sigma$ increases with decreasing vertical scale.](image)
The wavenumber is so great that the “slope” of the apparent high-wavenumber cutoff is influenced more by the temporal filtering of these data than by the resolution of the sonar. Here, hourly averaged velocity profiles are being used: when 10- or 3-min profiles are used, the cutoff is less abrupt (Fig. 2, thin lines).

The reference lines in Fig. 6 indicate the spectral density $E(k_z) = N^2/k_z$. Wave packets of bandwidth $\Delta k_z/k_z \sim O(1)$ are self-unstable to convective instability if this limit is exceeded (Fritts 1984; Müller et al. 1992). These Arctic spectra demonstrate a high-wavenumber cutoff even though they fall significantly below the $N^2/k_z$ self-saturation limit. The steepness of the high-wavenumber cutoff is influenced by the time averaging of the acoustic data. High-wavenumber motions that are Doppler shifted to frequencies above 1 cph are attenuated by the time average.

To investigate this small-scale–high-frequency relationship, it is useful to return to the combined wavenumber–frequency spectrum, $E(k_z, \sigma)$, now viewing the spectrum in terms of linear frequency and wavenumber coordinates (Fig. 7). The four quadrants of each spectrum correspond to anticyclonic (left) and cyclonic (right) rotation of shear in time and upward ($\sigma/k_z$ positive) and downward ($\sigma/k_z$ negative) propagation of phase in time. For example, downward (energy) propagating inertial waves are associated with upward phase propagation and are represented in the lower-left quadrant of each spectral estimate. Variance associated with wave packets that are significantly superinertial is divided across diagonally opposite quadrants of the spectrum.

The spectral maps show distinct maxima at both the inertial and vortical (zero frequency) bands. The vortical shear peaks are centered at zero wavenumber. The inertial peaks, in shear, correspond to 20–50-m vertical-scale motion. The spectra are spread continuously in frequency, with the degree of spread increasing with increasing wavenumber magnitude. Indeed, it appears that the low-frequency cyclonic shears result from the spreading of the anticyclonic inertial peak across the cyclonic–anticyclonic boundary.

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**Fig. 6.** Vertical wavenumber spectra of (left) anticyclonic and (right) cyclonic rotating shears, for December (line a), June (line b), and August (line c). Anticyclonic rotating shears have 3–10 times the variance of the cyclonic shears at high wavenumber. Much of this shear is found at frequencies far from $f$ in Figs. 4 and 5. Reference lines for a limiting spectral form $E(k_z) = N^2/k_z$ are drawn for $N = 3, 4, \text{ and } 5$ cph. Wave packets of $\Delta k_z/k_z \sim O(1)$ are self-unstable to convective instability if this limit is exceeded. With increasing shear variance, the spectrum appears to “fill in” from high to low wavenumber (red arrow).
Equivalent spectra, normalized such that \( N(\kappa_z, \sigma) = E(\kappa_z, \sigma) / \int E(\kappa_z, \sigma) \, d\sigma \) (Fig. 8), emphasize the frequency dependence of the spectrum and provide a graphic view of the apparent spreading. Here the first frequency moment of the spectrum,

\[
\mu_1(\kappa_z) = \int \sigma E(\kappa_z, \sigma) \, d\sigma / \int E(\kappa_z, \sigma) \, d\sigma,
\]
is plotted in white, showing the tendency of the shear spectrum to center on the inertial frequency. The negative square root of the second moment,

\[
-\mu_2(\kappa_z) = -\int \sigma^2 E(\kappa_z, \sigma) \, d\sigma / \int E(\kappa_z, \sigma) \, d\sigma - \langle \sigma \rangle^2,
\]
plotted in black, demonstrates a linear growth in frequency bandwidth with increasing wavenumber magnitude. The spreading from the vortical peak is less apparent—at no point comparable to the inertial spread. In part, the dominant inertial signal obscures the view of the vortical peak.

The Garrett–Munk (GM) spectrum (Garrett and Munk 1972, 1975, 1979; Munk 1981; Gregg and Kunze 1991), is given in Figs. 7d and 8d. The GM spectrum is zero for \(-f < \sigma < f\).
Figures 7 and 8 exhibit some degree of frequency isolation between the internal-wave and vortical signals. Although a precise separation is thwarted by the Doppler shifting of the signals, it is of value to examine the two fields as independently as possible. In Fig. 9, vertical wavenumber spectra of internal-wave and vortical shear are presented. The modeled 2D inertial spectrum $E_I^M (\kappa_z, \sigma)$ is given the wavenumber dependence of the observations $E(\kappa_z, \sigma = -f)$ at $\sigma = -f$ and a frequency dependence that scales with vertical wavenumber magnitude, consistent with Fig. 8. When integrated over the modeled frequency dependence, the “total” inertial contribution to the spectrum is $E_I^M (\kappa_z) \sim \kappa_z E(\kappa_z, -f)$.

Using the model, the amount of inertial variance that has spread into the zero-frequency (vortical) band, $E_I^M (\kappa_z, 0)$, can be estimated. This amount is then subtracted from the observed spectrum, $E_V^M (\kappa_z, 0) = E(\kappa_z, 0) - E_I^M (\kappa_z, 0)$, to estimate the vortical contribution to the 2D shear spectrum at $\sigma = 0$. A 2D vortical spectrum, $E_V^M (\kappa_z, \sigma)$, is then formed. Again, the frequency bandwidth is proportional to wavenumber magnitude,
and the wavenumber dependence is set to match the observed $E^M_V(\kappa_z, 0)$. The total vortical contribution to the oceanic shear field is the integral over frequency:

$$E^M_V(\kappa_z) = \int E^M_V(\kappa_z, \sigma) d\sigma \sim \kappa_z E^M_V(\kappa_z, 0).$$

The 1D wavenumber spectra of both inertial and vortical shear are whiter than their 2D counterparts by the factor $\kappa_z$.

The spectral signatures of the wave and vortical fields are distinct. The vortical spectrum is essentially white. Differences between the upward- and downward-propagating vortical phases are negligible, as expected for a (vertically) nonpropagating phenomenon.

The wave spectrum is band limited, with a low wavenumber cutoff that becomes clear after the vortical signal is removed. From these and other observations, there is a sense that the shear spectrum accommodates increased variance by a “leftward” shift of this cutoff to lower-wavenumber, faster-propagating waves (Fig. 6).

**Fig. 9.** Vertical wavenumber spectra of observed total shear (blue) and modeled inertial (red), vortical (green), and total shear (black). Details of the modeling procedure are described in Pinkel (2008). Vortical shear variance levels change throughout the year, roughly in step with wave shear fluctuations. The wavenumber dependence of the vortical constituent is approximately white (dotted lines). At the smallest scales, the vortical contribution is masked by wave variance spreading out from the inertial band.
Rms shear levels for the wave and vortical constituents are given in Table 1. These result from integration of the spectrum over vertical wavenumbers $-0.1 < \kappa_z < 0.1$ cpms, with the “zero wavenumber” band excluded. The values are essentially identical for the deep-sea observations of December and August–September. Wave shears increase by 50% over the Chukchi Cap relative to the deeper basin data of December and August, and vortical rms shear doubles.

### 4. Summary and speculation

With the Coriolis frequency high and mesoscale advection low, the western Arctic provides a near-ideal opportunity to obtain an “undistorted” view of the finescale shear field. The drawback to working in the Arctic is that ambient shear levels are very low and the distinction between shear and measurement error can be small. From a close examination of data obtained from Doppler sonars at the SHEBA ice camp, motion at vertical scales greater than 10 m and time scales greater than 1 h is clearly above the measurement uncertainty. Contiguous records from the deep Canada Basin, the Chukchi Cap, and the Chukchi slope are used to estimate the vertical wavenumber–frequency spectrum of shear.

As initially hoped, the vortical signal can be distinguished, although imperfectly, from the internal-wave contribution with the help of a simple model (Pinkel 2008). The observed 2D spectrum of shear $E(\kappa_z, \sigma)$ has wavenumber dependence $\kappa_z^{-3}$ at frequencies $|\sigma| < f$. However, the model 1D wavenumber spectrum associated with purely vortical motion is apparently white; that is, $E^w(\kappa_z) = \kappa_z^0$. The analogous 1D wave spectrum $E^w(\kappa_z)$ is band limited in form. At low wavenumber, $\kappa_z < 0.02$ cpms, it rises roughly as $\kappa_z^{1/2}$ to a maximum “finescale” peak. The wave spectrum accommodates increases in wave-field variance through a broadening of the “inertial peak” toward lower wavenumber and an increase in peak magnitude (Fig. 6). The high-wavenumber cutoff of the wave shear spectrum occurs far below the single-wave self-saturation threshold (Fritts 1984; Müller et al. 1992). With reduced time averaging, the spectrum extends to higher wavenumber than the spectrum of the hourly-averaged shear reported here. However, the increased variance appears to be spatially incoherent between the sonar beams and is not clearly an oceanic signal.

In terms of both wavenumber and frequency, the form of the Arctic shear spectrum $E(\kappa_z, \sigma)$ is similar to that of mid- and low-latitude upper-ocean spectra. A broad band of vertical wavenumbers is energetic at the inertial frequency. At higher observed frequencies, there is progressively less variance density, with the lowest wavenumber bands decreasing most rapidly. In terms of frequency cross sections of the spectrum, the cross sections at the lowest wavenumbers are the “most red” in frequency. The spectral slope exceeds $\sigma^{-4}$ in the low-energy December data, at long vertical wavelength. At shorter vertical scales and higher-energy sites, the spectral slope is reduced.

A useful metric of spectral form is the frequency bandwidth as a function of vertical wavenumber. In contrast to spectral models with separable wavenumber and frequency dependence, the observed Arctic spectra demonstrate a frequency bandwidth that increases linearly with vertical wavenumber (Fig. 8). This property is also shared with lower-latitude wave fields. It is thought that this linear bandwidth increase is a consequence of the simple Doppler shifting of intrinsically near-inertial motions. Even in the comparative calm of the Arctic, advective effects dominate the time dependence of the small-scale motion field.

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