NOTES AND CORRESPONDENCE

A Dynamic Roughness Equation and Its Application to Wind Stress Determination at the Air-Sea Interface

S. A. Hsu

Coastal Studies Institute, Louisiana State University, Baton Rouge 70803

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ABSTRACT

A dynamic roughness equation including major wind and wave interaction parameters (wind shear velocity, wave height, and phase velocity) is derived. Because this equation implicitly incorporates the effects of wave steepness, relative water depth, and wind duration and fetch, it may be applied to a wide variety of natural conditions. This equation was used to construct a nomogram which can be utilized to determine the wind stress at the sea surface, given phase velocities, wave heights, and the wind speed at any height in the atmospheric boundary layer. The proposed relationships are verified by the available field and laboratory data under near-neutral atmospheric stability conditions from which the appropriate parameters could be determined.

1. Introduction

The quantity Z_0 (commonly referred to as the aerodynamic roughness length or dynamic roughness) may be interpreted physically in a manner such that it specifies the scale of the turbulent eddies that are generated by the surface roughness elements (Kraus, 1972).

The relationship between dynamic roughness (Z_0) and shear velocity (U_*) over the water surface has been investigated for more than 30 years (e.g., see von Kármán, 1934; Charnock, 1955; Vinogradova, 1960; Deacon, 1962; Roll, 1965; Kitaigorodskii, 1968; Wu, 1969a, b; Leichtmann and Snopkov, 1970; Manton, 1971; Sheppard *et al.*, 1972). Charnock's hypothesis (developed by dimensional analysis) that

$$Z_0 = a\frac{U_*^2}{g}, \tag{1}$$

where g is the gravitational acceleration and a is supposedly a constant, is the most commonly applied. The fact that a is not a constant, but exhibits variations dependent on the characteristics of the water surface, has led investigators such as Kitaigorodskii and Volkov (1965) to postulate an added dependence of Z_0 on wave height. However, as yet no explicit relationship has been developed which is acceptable under a variety of conditions.

The primary purpose of this note is to propose an explicit relationship between the dynamic roughness and the wind shear velocity, wave height, and phase velocity.

2. Formulation of the dynamic roughness equation

Phillips (1966) has given support to (1) on theoretical considerations, but Manton (1971) has further suggested that wave steepness is also important. Manton theorized that $Z_0 \propto H/L$, where H and L are the wave height and wave length, respectively. Thus, it would not be unreasonable to postulate that a is a function of these wave parameters and not necessarily a constant. The values of a determined from various studies over water surfaces have, in fact, exhibited scatter over a very wide range (e.g., see Roll, 1965). Therefore, it is suggested from dimensional considerations that

$$Z_0 \propto \frac{H}{L}\cdot\frac{U_*^2}{g}, \tag{2}$$

in which H/L is the wave steepness of the dominant waves, is appropriate. Because $C \propto (gL/2\pi)^{\frac{1}{2}}$, where C is the phase velocity, (2) may be rewritten as

$$Z_0 = \frac{a^*}{2\pi}\left[\frac{H}{(C/U_*)^2}\right], \tag{3}$$

where a^* is an unknown value. The phase velocity in

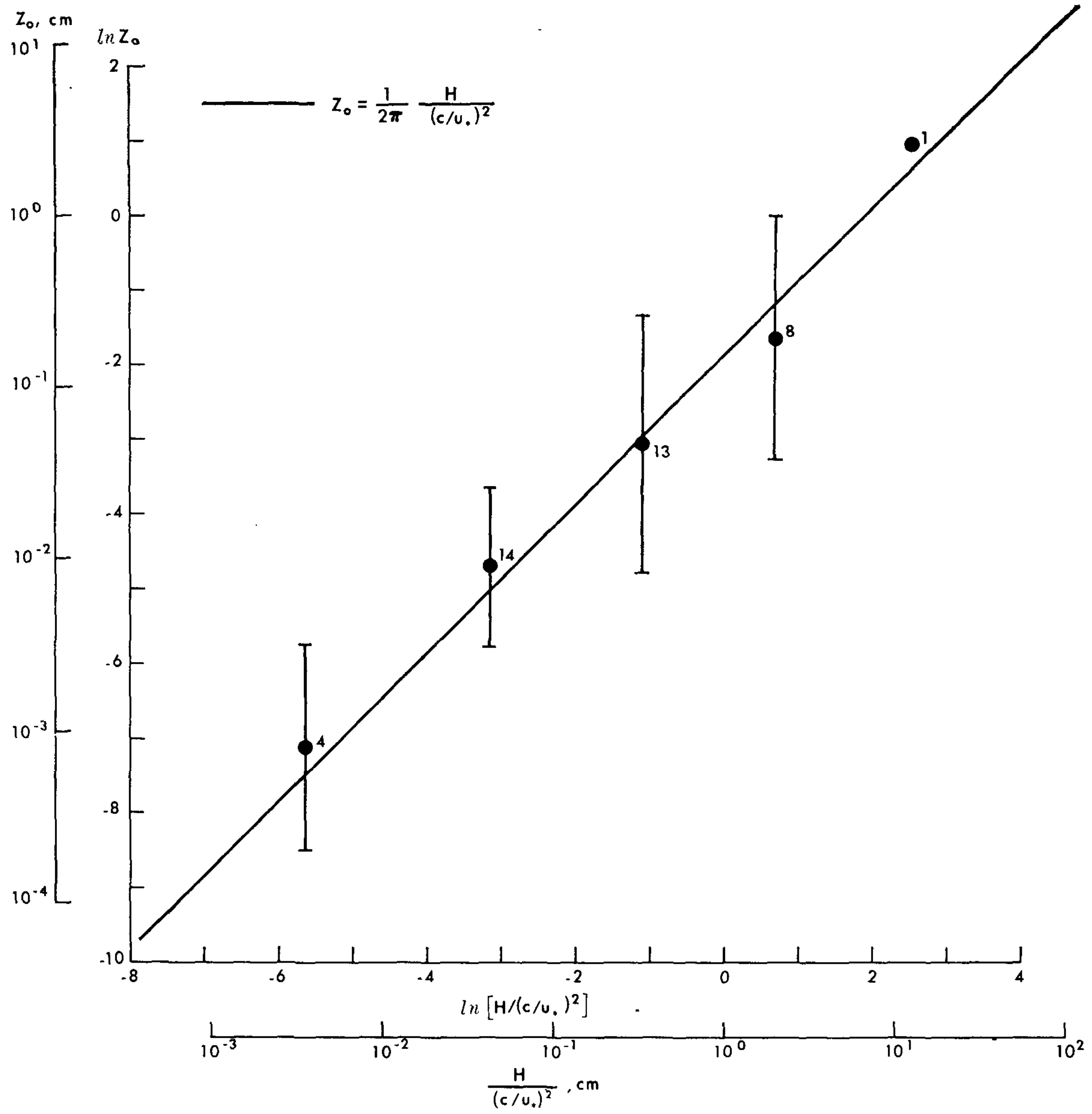

Fig. 1. Test of Eq. (4) based on appropriate field and laboratory experiments available as summarized in Table 1. Numerical values are number of investigators and vertical bars are the standard deviations for the class intervals as indicated in Table 1.

deep water may also be written as $C=gT/(2\pi)$. The values of Z_0 and $H/(C/U_*)^2$ were determined from 19 sets of available field and laboratory data, most of which were taken in and over deep water. It should be emphasized that these sets represent an attempt to collect data from every study from which the pertinent parameters could be calculated. Therefore, the data should not be interpreted as biased. These data sets contained observed wave parameters such as H, L and T (wave period), in addition to Z_0 and U_* calculations based on the logarithmic wind profile method and/or direct measurement of U_*. The calculation of Z_0 and/or U_* using the logarithmic wind profile method is valid only where neutral atmospheric stability conditions exist (e.g., see Kraus, 1972).

Fig. 1 illustrates the range of the observed Z_0 values versus the values predicted from $[H/(C/U_*)^2]$ in (3). These values are listed in Table 1. It is evident from Fig. 1 that the predictions are in excellent agreement with these observed values of Z_0 and that the slope of the fitted line very closely approximates $1/(2\pi)$, implying that $a^*\approx 1$. Therefore, (3) may be written as

$$Z_0=\frac{1}{2\pi}\frac{H}{(C/U_*)^2}. \tag{4}$$

TABLE 1. Summary of the observed wind and wave interaction parameters used to test Eq. (4) (cf. Fig. 1).

	$H/(C/U_*)^2$ 0.001–0.0099		0.01–0.099		0.1–0.99		1.0–9.99		10.0–99.99	
Investigators[a]	Z_0	$\frac{H}{(C/U_*)^2}$	Z_0	$\frac{H}{(C/U_*)^2}$	Z_0	$\frac{H}{(C/U_*)^2}$	Z_0	$\frac{H}{(C/U_*)^2}$	Z_0	$\frac{H}{(C/U_*)^2}$
Davidson (1970)			0.0064	0.063						
Deacon *et al.* (1956)[b]			0.031	0.088	0.088	0.23				
DeLeonibus (1971)			0.0073	0.055						
Dobson (1971)[c]			0.0039	0.033						
Hasse (1970)[b]	0.00019	0.0066	0.0014	0.0055	0.010	0.17				
Hidy and Plate (1966)	0.00036	0.0051	0.0044	0.043	0.022	0.43	0.081	2.89		
Kitaigorodskii and Volkov (1965)[d]			0.017	0.078	0.20	0.23	4.09	1.92		
Kuznetsov (1965)[e]					0.00084	0.24				
Kunishi (1963)	0.0039	0.0041	0.012	0.045	0.034	0.42	0.076	2.38		
Lai and Shemdin (1971)			0.013	0.033	0.088	0.68				
Rossby and Montgomery (1936)[b]			0.0070	0.071						
Shemdin and Hsu (1967)			0.10	0.042	0.18	0.35	0.19	2.05		
Sibul (1955)			0.0049	0.063	0.0075	0.41	0.019	1.95		
Snopkov (1965)[e]					0.25	0.82	0.11	1.68		
Takahashi (1958)			0.0041	0.028	0.13	0.30	0.63	1.05		
Takeda (1965)					0.042	0.11				
Walters (1973)[f]			0.022	0.044						
Wilson (1972)	0.0016	0.0013								
Wu (1968)					0.39	0.55	0.43	2.41	2.70	12.46

[a] Only neutral and near-neutral atmospheric stability conditions are listed in the table.

[b] The wave velocities C and/or wave heights H were estimated by the significant wave forecasting method (see Bretschneider, 1963).

[c] The Z_0 value was obtained from Smith (1967) and Weiler and Burling (1967) at the same site (S. D. Smith, personal communication).

[d] Data obtained during joint expeditions of the Institutes of Atmospheric Physics and of Oceanography of the U.S.S.R. Academy of Sciences in 1963–64 and reduced from the authors' Figs. 1 and 2.

[e] Data reduced from Figs. 1 and 2 of Kitaigorodskii and Volkov (1965).

[f] Wave information was obtained from J. N. Suhayda (personal communication), who conducted experiments on waves in the same area.

3. The determination of surface wind stress

Because (4) is valid under neutral atmospheric stability conditions, it may be used in the familiar logarithmic wind law

$$U_Z=\frac{U_*}{\kappa}\ln\frac{Z}{Z_0},$$

or

$$Z_0=\frac{Z}{\exp(\kappa U_Z/U_*)},\qquad(5)$$

where κ is the von Kármán constant ($\sim$0.4). Using this expression in (4), one obtains

$$\frac{Z}{\exp(\kappa U_Z/U_*)}=\frac{U_*^2}{2\pi}\frac{H}{C^2}$$

or

$$\frac{2\pi Z}{(H/C^2)}=U_*^2\exp(\kappa U_Z/U_*).\qquad(6)$$

A nomogram was constructed as a direct result of (6) and is shown in Fig. 2. It is used as follows:

(i) After values for H, T (wave period) and U (wind speed) at some height Z, above the surface, are determined from field experiments, calculate C from T using common methods, determine the value of $(2\pi Z)/(H/C^2)$, and locate it on the left-hand axis.

(ii) Determine the length of the right-hand axis (labeled U_Z) which is equivalent to the wind speed (m sec^{-1}) measured at height Z.

(iii) Draw a line perpendicular to the left-hand axis from the point located in (i) of length equivalent to that determined in (ii).

(iv) The right-hand limit of the line drawn in (iii) will intersect the appropriate value of U_* (m sec^{-1}) which is labeled in Fig. 2.

From this value of U_* the surface wind stress τ may be obtained from

$$\tau=\rho U_*^2,$$

where ρ is the air density.

An example for determining U_* given a value of $(2\pi Z)/(H/C^2)=3524$ m^2 sec^{-2} and $U_Z=10$ m sec^{-1} is shown in the lower right-hand corner of the figure.

Since it is most common for the lapse rate to be near neutral (Kraus, 1972), the nomogram constructed in this study may be used for general oceanographic applications such as wind-generated current studies.

4. Concluding remarks

It should be noted that Eq. (4) is an extension of the Charnock form [Eq. (1)]. Since the constant a in

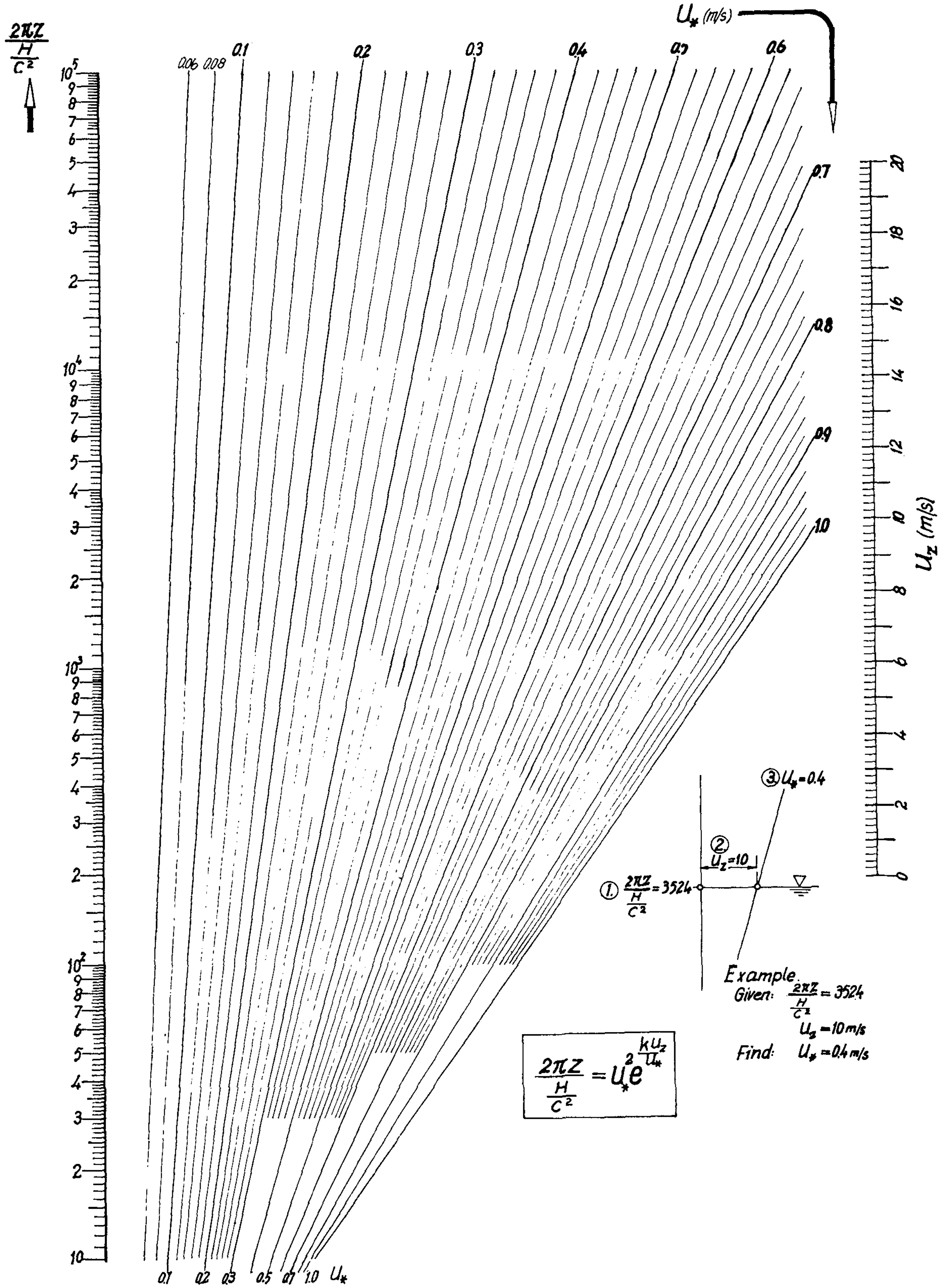

FIG. 2. Nomogram for determining the wind stress at the sea surface from some commonly measured parameters. See text for explanation.

(1) varies, as an example, from approximately 0.01 (Wu, 1968) to 0.08 (see Roll, 1965, p. 138), the modified form (4) is thus proposed. Furthermore, a variety of papers dealing with wave growth of a single wave component under turbulent boundary-layer conditions suggest that the rate of growth is a function of C/U_* (see, e.g., DeLeonibus and Simpson, 1972). One would thus expect

$$Z_0 = Hf(C/U_*),$$

and (4) is in support of this relation.

As the sea state evolves, C develops toward "saturation," and the Charnock form might be expected to appear as an asymptotic limit. A special form for which this is true is the one presented in this paper.

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