Reply

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Dr. O'Brien has shown for simple diffusion with cyclic boundary conditions that the DuFort-Frankel scheme (DFFS) is conservative in the large. This behavior is the result of the implicit term of the scheme being constructed so that the assumptions of the space differencing are not violated. O'Brien's analysis is also valid if the amount of scalar in the system is increased linearly with time by a source term, a zero increase being a special case. However, difficulties may arise when some of the values at the $(N+1)$st time level are determined only in part by the interior point DFFS.

Taylor (1970) discusses the instability of the DFFS when central difference approximations are made to boundary conditions involving first-order space derivatives. Taylor atributes the instability to the boundary conditions being inconsistent with the hyperbolic nature of the DFFS. Insolated boundaries defined by setting $2T_B^N-(T_{B-1}^{N+1}+T_{B-1}^{N-1})$ identically to zero produce an oscillation in the total amount of scalar which damps as steady state is approached.

In certain applications additional difficulties are associated with lattice separation. The solution of the DFFS proceeds along two distinct intermeshed lattice structures defined by the sum of $N+I$ being odd or even, with N the time level and I the space point. A necessary condition for conservation is a starting method suggested by O'Brien that distributes the total around of scalar between the two lattices. However, for accuracy, the solutions started on each lattice must be compatible or a checkerboard appearance occurs to the total solution because one lattice has more scalar than the other. For pure diffusion problems an alternative is to compute on only one lattice as originally suggested by DuFort and Frankel (1953).

Difficulties with the DFFS were encountered in problems which were started as suggested by O'Brien, but contained boundary conditions. It is likely that the property of the DFFS demonstrated by O'Brien, as well as accuracy, could be maintained if the boundary conditions and advective terms were completely consistent with the internal structure of the DFFS.

I wish to extend my appreciation to Dr. O'Brien and Dr. Djurić for clarifying conditions under which the DFFS may be a useful approximation.

REFERENCES

DuFort, E. C., and S. P. Frankel, 1953; Stability conditions in the numerical treatment of parabolic differential equations. *Math. Tables*, **7**, 135–152.

Taylor, P. J., 1970; The stability of the DuFort-Frankel method for the diffusion equation with boundary conditions involving space derivatives. *Computer J.*, **13**, 92–97.