Coastal-Trapped Waves in the East China Sea Observed by a Mooring Array in Winter 2006

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ABSTRACT

Using five mooring array observations in the coastal water of the East China Sea (ECS) in winter 2006, the authors identify three kinds of low-frequency waves using the ensemble empirical mode decomposition (EEMD) method. The analysis indicates that the periods of the waves varied from 2 to 10 days, which are consistent with coastal-trapped wave (CTW) modes: the Kelvin wave (KW) mode, the first shelf wave (SW1) mode, and the second shelf wave (SW2) mode. An analytical model is established and the dispersion relation of the waves from the analytical method agrees well with the observations. The wind-forced, coastal-trapped wave theory is then applied. The calculation shows that over a wide shelf, the forcing term of wind stress curl plays an important role in shaping the CTW. Numerical solutions reproduce the sea level variation and the alongshore current. The results show that the sea level variation mainly resulted from the KW mode, but the alongshore current resulted from both the KW and SW1 modes.

1. Introduction

The coastal-trapped waves (CTWs) are subinertial waves with energy confined to the continental shelves. The waves typically have amplitudes on the order of 10 cm and wavelengths of a few thousand kilometers (Mysak 1980). Their periods are between 2 and 20 days, and their phase speeds are from on the order of 1 m s\(^{-1}\) to on the order of 10 m s\(^{-1}\) for different wave modes (Clarke 1977). The CTWs are one of the major components of the mesoscale, subinertial sea level and alongshore current variability on the continental shelf and slope regions. Thus, they make important contributions to coastal circulation, exchange, and mixing (Huthnance 1995). Brooks (1978) found that the CTWs forced by the atmosphere contribute to the Gulf Stream meander off North Carolina. The CTWs can also lead to deep upwelling events off Kangaroo Island and the Bonney coast that occur over 3–10 days and 2–4 times each season based on Middleton and Bye (2007).

Hamon (1962) observed the low-frequency wave motions along the east Australian coast, and later Robinson (1964) defined the wave motions in terms of continental shelf waves. By using nondimensional equations in an orthogonal curvilinear coordinate, Feng (1981) studied the trapped waves over a broad continental shelf. The following investigations showed that the CTWs may be generated by different mechanisms, including atmospheric pressure variation (Noble and Butman 1979), ocean eddies (Huthnance et al. 1986), tidal forcing (Flather 1988), wind stress (Gill and Clarke 1974), and so on. Observations indicate that the CTWs have nearly the same periods as large-scale weather systems, thus the research on the wave generation has focused on wind-forced motions; meanwhile, wind-forced CTW theories are well established (Clarke and Van Gorder 1986;
These theories have been applied to continental shelves around the world by Clarke (1977), Mysak (1980), Hsueh and Pang (1989), Brink (1991), and Jordi et al. (2005).

The East China Sea (ECS) is a marginal sea in the northwest Pacific. Its bottom topography is characterized by a broad continental shelf and a steep slope along the Okinawa Trough. In winter, the atmospheric circulation over the ECS is controlled by the Mongolia high pressure system. In the northern ECS, the pressure shows a meridional distribution and the winds are predominantly north–south oriented. In the southern ECS, the pressure shows a zonal distribution and the winds are predominantly northeast–southwest oriented. The seafloor features are beneficial to the propagation of the CTWs; meanwhile, the atmospheric features provide favorable conditions for the generation of CTWs in the ECS. As the CTWs are an important dynamic component of the sea, several studies have previously investigated them. Liu and Qin (1990) studied the influences of bottom friction, the Coriolis parameter, coastline orientation, and shelf width on the CTWs using an idealized exponential profile in the vertical direction. Chen and Su (1987) noted that the signals have a phase speed around 15 m s\(^{-1}\) and have frequencies around 0.21 and 0.32 cycles per day (cpd) in the Yellow Sea (YS) and the ECS, which can be explained by the lowest-mode nondispersive free CTW. Wang et al. (1988) claimed that there were two free CTWs, whose phase speeds were between 14 and 19 m s\(^{-1}\), in the YS and ECS using a two-dimensional numerical model. Using a two-layer free CTW model, Kong et al. (1992) studied the interaction between CTWs in the ECS and the Kuroshio. Using the altimeter data of the Ocean Topography Experiment (TOPEX)/Poseidon, along with a numerical mode, Jacobs et al. (1998) detected the propagation of the coastal shelf waves with the phase speed of about 12 m s\(^{-1}\).

Although there are several studies on the CTWs in the ECS, none of them focused on the generation mechanism and propagation property or the contribution of the wind. The ECS is located in the East Asian monsoon region and its shelf is very wide, so the local winds constitute a main forcing to drive ocean dynamical processes. It is important to know how the ocean and the low-frequency motions in the ECS respond to the atmospheric forcing because they have great influences on the sea level and alongshore currents. In addition, previous investigations used the tide gauge data or the satellite altimeter data, which may miss some information on the wave properties.

In this paper, we use the mooring array data to study the CTWs in the ECS. We aim to identify the presence of the CTWs in the ECS and their generation mechanisms. In section 2, the study area and the data used in this paper are described. Based on the observed sea level data, the CTWs are derived using the ensemble empirical mode decomposition (EEMD) method (Wu and Huang 2009). Then, we apply the wind-forced CTW theory (Brink 1991) to the ECS in section 3. The discussion and summary are given in section 4.

2. Data, data processing, and analysis

Figure 1 shows the study area, the ECS, which connects with the YS in the north and with the South China
Sea in the south through the Taiwan Strait. The bathymetric data are derived from 5-Minute Gridded Global Relief Data Collection (http://www.ngdc.noaa.gov/mgg/global/etopo5.HTML). The isobaths in the study region are approximately parallel to the Chinese coastline (Fig. 1). According to the bottom topography, the ECS shelf can be divided into two portions: the wide continental shelf where the maximum depth is no deeper than 160 m and the continental slope where the principal body is the Okinawa Trough. In winter, atmospheric frontal systems, which are associated with strong wind bursts, pass through the area from the north to the south, which induces strong mixing and makes the vertical stratification not notable (Hsueh and Romea 1983). The tides and tidal currents are the major dynamic components of the ECS circulation. The major tidal constituents in the ECS are the M2, S2, K1, and O1 tides. The M2 tide is the dominant tidal constituent, whose amplitude is more than 1 m. The amplitudes of S2, K1, and O1 tides are more than 0.5, 0.3, and 0.2 m, respectively (Xiong 2012). The circulation system is also characterized by seasonally reversed coastal currents. In winter, the Zhejiang–Fujian coastal current flows southwestward from the Yangtze River estuary along the coast with a velocity of about 10–30 cm s\(^{-1}\) and affects a region of about 75–90 km offshore. In summer, the coastal current flows northeastward along the coast due to the reversed monsoon forcing (Xiong 2012).

### a. Field data

The sea level and alongshore current data measured by the mooring array constitute a baseline for this study. We use the data to extract CTW signals and verify the theoretical analysis results. The locations of the mooring stations CE3, CE4, JZ0604, JZ0903, and JZ0906 are shown in Fig. 1. Instruments, sampling frequencies, water depths, and observation periods are listed in Table 1. All five stations have sea level records, while four of them have current records. The sampling frequencies at these stations are different, so we resample the data records using the same frequency of 1 h\(^{-1}\).

<table>
<thead>
<tr>
<th>Station</th>
<th>Location (lat, lon)</th>
<th>Data type</th>
<th>Start, end depths from bottom and sampling interval (m)</th>
<th>Water depth (m)</th>
<th>Period (start–end date)</th>
<th>Sampling frequency</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>CE3</td>
<td>31.050°N, 123.000°E</td>
<td>Sea level</td>
<td>3.3–31.3 and 2</td>
<td>51</td>
<td>9 Jan–24 Mar</td>
<td>1 h</td>
<td>RDCP</td>
</tr>
<tr>
<td>CE4</td>
<td>31.0733°N, 123.300°E</td>
<td>and current</td>
<td>3.3–49.3 and 2</td>
<td>62</td>
<td>9 Jan–25 Mar</td>
<td>1 h</td>
<td>RDCP</td>
</tr>
<tr>
<td>JZ0903</td>
<td>27.862°N, 121.646°E</td>
<td>Sea level</td>
<td>3.3–31.3 and 2</td>
<td>31.3</td>
<td>1 Jan–6 Feb</td>
<td>10 min</td>
<td>Sontek–ADP</td>
</tr>
<tr>
<td>JZ0906</td>
<td>27.828°N, 122.156°E</td>
<td>Sea level</td>
<td>5.3–69.3 and 4</td>
<td>77.8</td>
<td>1 Jan–5 Feb</td>
<td>10 min</td>
<td>Sontek–ADP</td>
</tr>
<tr>
<td>JZ0604</td>
<td>28.826°N, 122.123°E</td>
<td>Sea level</td>
<td>23</td>
<td>7</td>
<td>1 Jan–6 Feb</td>
<td>10 min</td>
<td>Sontek–ADP</td>
</tr>
</tbody>
</table>

To ensure the accuracy of extracted CTWs, we process the sea level data in two steps. First, the atmosphere pressure adjustment is done. The atmosphere pressure has a direct contribution to the sea level, thus we use the National Centers for Environmental Prediction (NCEP) reanalysis atmosphere pressure data and the commonly used atmospheric pressure adjustment method to eliminate the effect of atmosphere pressure on the sea level data. Second, the time series are detided and detrended. The tidal constituents are removed from the sea level data by means of the EEMD decomposition. The trend of EEMD decomposition is also removed, as the long-term trend is not a research target for this study. From the tide-filtered, detrended sea level dataset, components with a higher frequency than 2 cpd are also removed. As an example, Fig. 2b shows the autospectra...
derived from the processed sea level data measured at station JZ0604. Figure 2a is the adjusted sea level before being detided and detrended. It is clear that the long-term trend and the low-frequency variability of the sea level fluctuation have been filtered out, while the signals with frequencies between 0.05 and 3 cpd are preserved.

Ocean current data were collected at stations CE3, CE4, JZ0903, and JZ0906. The vertical resolution of the current data is 2 m at CE3, CE4, and JZ0903, and 4 m at JZ0906. The horizontal currents are projected onto the cross-shelf and alongshore directions. Figure 3 shows the alongshore currents measured at station CE3. The other stations have similar patterns. The cross-shelf and alongshore components of the currents are averaged over the whole depth, respectively. The high-frequency signals are first removed by 25-h moving averaging. Then the currents are decomposed using the EEMD method, the tidal signals are removed thoroughly by dropping the first two components, and the mean and trend are removed by dropping the last component.

b. Meteorological products

The wind at 10 m is obtained from the NCEP–National Center for Atmospheric Research (NCAR) reanalysis data (http://www.esrl.noaa.gov/psd/data/gridded/data.ncep.reanalysis.surfaceflux.html). The temporal resolution of the data is 6 h. This dataset uses the T62 Gaussian grid with 192 by 94 points in a spatial coverage of 88.542°S–88.542°N, 0°–358.125°E, with a zonal spatial resolution of about 140–190 km and a meridional resolution of 118 km in the studying area. The wind stress is calculated using a bulk formulation of \( \tau = \rho_a C_D U_{10}^2 \), where \( \rho_a \) is the atmospheric density (kg m\(^{-3}\)), \( U_{10} \) is the wind speed (m s\(^{-1}\)) at 10-m height, and \( C_D \) is the drag coefficient based on Large and Pond (1981). The gridded wind stress field for the integral calculation in section 3d is generated by a bilinear interpolation. To check the reliability of the wind data, we collect the observed wind data from two meteorological stations in the ECS (Fig. 1). By interpolating the NCEP wind to the observation stations, we compare the NCEP winds with the observed winds. The comparison shows that the NCEP winds have a high correlation with the observed wind, with the lead correlation coefficient of about 0.9, and the amplitudes of the two datasets are nearly the same (Fig. 4a).

Besides, the NCEP wind leads the observed wind by a 6-h leading time at station Shengshan and by a 9-h leading time at station Dachen.

The NCEP–NCAR reanalysis atmospheric pressure at sea level, which was downloaded from the same website as the wind data, is used for correcting the observational sea level data. The data are spatially interpolated to the mooring stations as shown in Fig. 1 and Table 1 and then temporally interpolated to generate a time series dataset with 1-h resolution.

c. EEMD analysis of the sea level data

The EEMD method is used to analyze the time series of sea level data. The method is to overcome the mode mixing problem in the empirical mode decomposition (EMD), which is defined by a single intrinsic mode function (IMF) consisting of either signals of widely disparate scales or a signal of a similar scale residing in different IMF components (Wu and Huang 2009). It is an adaptive method to decompose a time series into a set of IMFs, which represent different scales of the original time series and form the adaptive and physical basis of the data. The EMD scheme is able to extract physically meaningful modes from a time series that is both non-stationary and nonlinear, and thus has been used in various fields from medicine (Cummings et al. 2004) to finance, including oceanographic applications in studying coastal seiches (Huang et al. 2000), internal waves (Ezer et al. 2011), and sea level rise (Ezer et al. 2013). A recent review of this theory can be found in Huang and Wu (2008).

Based on the EMD theory, the major steps of the EEMD method is as follows: First, add white noise to the time series. Second, decompose this new time series into IMFs. Third, repeat the first two steps for enough times with different white noise each time. And finally, calculate the ensemble mean of corresponding IMFs obtained from the third step.

The tide-filtered, detrended sea level data observed at stations CE3, JZ0604, and JZ0903 from 10 January to 6 February 2007 are decomposed into IMFs with added white noise. The significance test (Wu and Huang 2009) of all IMFs shows that all IMFs are significant compared to white noise and at the 95% confidence level. This procedure is repeated 500 times, and the ensemble means of corresponding IMFs of the decomposition is the final
result. Figure 5 shows the first three IMFs and the corresponding 95% confidence intervals. The sea level fluctuation has three main characteristic periods centered at 2, 4.5, and 11 days. The maximum amplitude for each IMF is different at the three stations. The IMF1 values at CE3 and JZ0604 have a maximum amplitude of about 12 cm, while that at JZ0903 comes up to 20 cm. The IMF1 accounts for about 50% of the whole fluctuation. The IMF2 values at the three stations have a maximum amplitude of about 7 cm (about 14 cm at JZ0903) and account for about 29% of the whole fluctuation. The maximum amplitude of all IMF3 is 5 cm, accounting for about 20% of the whole fluctuation. These results indicate that the lower the wave modes, the more energy they have.

Cross correlations are calculated for the same IMF between two stations. The lag correlation between any two stations shows a high correlation. Table 2 lists the distance between two stations, the maximum cross correlation, and its corresponding time lag between the two stations. Using the distance and the time lag, we calculate the phase speed of wave propagation. These time lags were associated with southward-propagating waves with phase speeds of about 16–18 m s$^{-1}$ for IMF1,
2.8–4.5 m s\(^{-1}\) for IMF\(_1\), and 1.7–3.7 m s\(^{-1}\) for IMF\(_2\). The waveforms detected in the ECS in previous studies (Wang et al. 1988; Chen and Su 1987; Jacobs et al. 1998) are similar to the IMF\(_1\) in our study. Comparing phase speeds of the free wave modes derived in section 3b, one can see that observed and modeled phase speeds are close to each other, but the modeled values underestimate the observed by up to 11% for IMF\(_1\), 13% for IMF\(_2\), and 53% for IMF\(_3\).

The above results hint that the IMFs of the sea level fluctuation in the ECS may be corresponding to different modes of CTWs. To confirm this, we calculate the dispersion relation of the CTWs and compare it with the observation. For the finite width sloping shelf profile such as that in ECS, Mysak (1980) gave a cubic implicit dispersion relation:

\[
\sigma^3 - \left[1 + \left(\frac{2\nu + 1}{\delta}\right)\right] \sigma + \frac{\ell}{\delta} = 0,
\]

where \(\delta = f^2 L_1^2 / gH\), \(\ell = IL_1\) can be regarded as the nondimensional wavenumber, \(L_1\) is the width of the continental shelf in segment A2, \(\ell\) is the wavenumber in the \(y\) direction, \(g\) is the acceleration due to gravity, \(\sigma = \omega / f\) is the nondimensional frequency, \(\omega\) is the frequency, \(f\) is the Coriolis parameter, and \(H\) is the water depth. The \(\nu\) should satisfy the Laguerre function \(L_n(2\ell) = 0\), for a countable infinite number of discrete \(\nu = \nu_0, \nu_1, \nu_2, \ldots\) for a given \(\ell\). Using the linear fitted bottom topography in the ECS, we calculate the first three modes \(\nu_0, \nu_1, \nu_2\). Then, corresponding to each \(\nu_n\), the cubic equation can be solved for the frequency function \(\sigma = \sigma(j, \nu_n, \ell)\). For each mode of \(\nu_n\), the roots of \(j = 1\) and 2 correspond to the dispersion relations, whereas the root of \(j = 3\) corresponds to the quasigeostrophic wave frequency function (Mysak 1980). In this paper, we only consider the low-frequency waves, so only the frequency function \(\sigma = \sigma(3, \nu_n, \ell)\) is solved. The first three modes for \(j = 3\) are plotted in Fig. 6. On the other hand, according to the definition of phase speed \(c_i = \omega_i / l_i\), in which \(i\) represents the \(i\)th mode of the waves, and using the frequency data derived from the IMFs and the phase speed listed in Table 2, we can obtain the wavelength of each mode and subsequently derive the observed dispersion relation of each IMF. Plotting these points onto the analytical dispersion relation curves, one can see that the three points are close to the curves of the corresponding modes.

The above analysis indicates that the IMFs from the EEMD decomposition are in some way, though not thoroughly, coincident with the wave modes derived from the CTW theory. We also decompose the observed wind at the two meteorological stations (Fig. 4b) using the EEMD and calculate the cross correlations of the same IMF between stations. Results of the cross correlations indicate that there is a 22-h time lag between two stations for the IMF6, corresponding to the propagation speed of 3.4 m s\(^{-1}\), which is close to the phase speed of 2.8 m s\(^{-1}\) for the IMF2 of the sea level variation in Table 2. In addition, the periods of the wind and the sea level variation of IMF2 are also close to 4.5 days.

Previous investigators have pointed out that the CTWs are mainly excited by the alongshore wind (Gill and Schumann 1974; Brink 1989). In the ECS, we use the time-dependent intrinsic cross-correlation (TDICC) method (Chen et al. 2010) to clarify the relationship between the sea level fluctuation and the local alongshore wind. The TDICC represents a major advance in
statistical analysis of data involving nonlinear and nonstationary processes. Figure 7 shows the TDICC analysis results of the first three IMFs at station JZ0604 (other stations show similar patterns). Note the significant time-dependent correlation of the signals; that is, while there was strong alongshore wind on 25 January 2007, a positive or negative sea level anomaly occurred and the correlation was high. At the other times, the correlation between the sea level and the wind was weak if the wind was not strong enough. This implies there is a threshold of wind speed, which is about 3 m s\(^{-1}\) for all three IMFs, for local CTW generation. The numerical results in section 3d will confirm these results.

A remarkable feature of the sea level variation is that the maximum amplitudes of IMF2 at the three stations get larger from north to south, which is from 5 cm at station CE3 to about 20 cm at station JZ0903. The consistency of propagation speed and frequency and the amplification of IMF2 imply that there may be resonance between wind and the low-frequency sea level variations.

### 3. Theoretical analysis

The data analysis in section 2c encourages us to apply the wind-forced CTW theory to the continental shelf of the ECS. In this section, we solve the barotropic CTW equations derived from the shallow water equations on a piecewise linear bottom topography.

#### a. Governing equations and boundary conditions

According to Huthnance (1975), if the square of the Burger number \( Bu = \frac{HN}{fL} = \alpha Nf \), where \( \alpha \) represents the bathymetric slope and \( N \) is the Brunt–Väisälä frequency) is much smaller than 1, the stratification of the ocean is less important than the rotational effect. In the ECS case, \( O(N^2) \sim 10^{-3} \text{ s}^{-2}, f^2 \sim 10^{-8} \text{ s}^{-2}, \) and \( \alpha^2 \sim 10^{-8} \), so \( Bu \sim O(10^{-3}) \), which is much smaller than 1. Choosing a right-handed system of axes \((x, y, z)\) as shown in Fig. 8 and ignoring the effects of nonlinearity and turbulence, the free surface shallow water equations with surface and bottom frictions are

\[
\begin{align*}
\tau_x + f u &= -\frac{1}{\rho} \frac{\partial}{\partial x} (p' + p_a) + \frac{\tau^x}{\rho H}, \\
\tau_y + f v &= -\frac{1}{\rho} \frac{\partial}{\partial y} (p' + p_a) + \frac{\tau^y - \tau_{\text{bot}}}{\rho H}, \quad \text{and}
\end{align*}
\]

\[
(Hu)_x + (Hv)_y + \frac{p'_f}{\rho g} = 0,
\]

where \( t \) is the time; \( x \) and \( y \) are the cross-shelf and alongshore coordinates, respectively; \( u \) and \( v \) are the depth-averaged velocities in the \( x \) and \( y \) directions; and \( p_a \) and \( p' \) are the atmospheric pressure at the sea surface

### Table 2. Distance between two stations, max correlation, time lag, and the observed and free wave phase speed for the first three modes.

<table>
<thead>
<tr>
<th>Stations</th>
<th>Distance (km)</th>
<th>Max correlation</th>
<th>Time lag (h)</th>
<th>Observed Phase speed (m s(^{-1}))</th>
<th>Free wave modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>JZ0604 and CE3</td>
<td>261</td>
<td>IMF1 0.81</td>
<td>4</td>
<td>18 (18, 18)</td>
<td>Mode 1: 16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IMF2 0.51</td>
<td>16.2</td>
<td>4.51 (4.49, 4.53)</td>
<td>Mode 2: 3.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IMF3 0.92</td>
<td>19.6</td>
<td>3.69 (3.67, 3.71)</td>
<td>Mode 3: 1.2</td>
</tr>
<tr>
<td>JZ0903 and JZ0604</td>
<td>117</td>
<td>IMF1 0.93</td>
<td>2</td>
<td>16 (16, 16)</td>
<td>Mode 1: 14.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IMF2 0.72</td>
<td>11.6</td>
<td>2.8 (2.77, 2.83)</td>
<td>Mode 2: 2.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IMF3 0.7</td>
<td>19.5</td>
<td>1.66 (1.63, 1.7)</td>
<td>Mode 3: 1.0</td>
</tr>
</tbody>
</table>

Fig. 6. (a) Dispersion relation for the linearly fitted bottom topography of the ECS and (b) comparison of theoretical dispersion relation with the observation. Circle is the first, cross is the second, and plus sign is the third mode. The x-axis label is the nondimensional wavenumber, namely, the wavenumber multiplied by the \( L_1 \) in segment A2; and the y-axis label is the nondimensional frequency, namely, frequency divided by the Coriolis parameter \( (f = 0.68 \times 10^{-4} \text{ rad s}^{-1}) \).
and perturbation pressure. The terms \( \rho, g, f, \) and \( H \) represent the water density, gravitational acceleration, Coriolis parameter, and water depth, respectively. The values \( t^x \) and \( t^y \) are cross-shelf and alongshore wind stress, respectively; \( t_{\text{bot}} \) is the bottom stress in the \( y \)-direction that can be linearized as \( t_{\text{bot}} = r_r(\tau^x) \), in which \( r_r(\tau^x) \) is the linear bottom friction coefficient and taken to be constant for simplicity.

Letting \( p = p' + p_0 \) be the perturbation pressure adjusted by the atmosphere, under the low-frequency, long-wave approximation, Eq. (1) can be combined and simplified into the following equation:

\[
(Hp_x)_x + (r p_x)_x + f H_x p_y - \frac{f^2}{g p} t_{\text{bot}} + f (\tau^y - \tau^x),
\]

which is subject to the following boundary conditions:

\[
p_{xt} + \frac{r(x)p_x}{H} + f p_y = f \tau^y/H, \quad \text{at} \quad x = -B,
\]

and

\[
p^{0-} = p^{0+}, \quad (H u)^{0-} = (H u)^{0+}, \quad \text{at} \quad x = 0, \quad \text{and} \quad (4)
\]

\[
p \to 0, \quad \text{at} \quad x \to \infty.
\]

In Eq. (4), \( 0^- \) and \( 0^+ \) indicate the variable values to the left and right of \( x = 0 \), respectively. Equation (4) shows the pressure and cross-shelf flow continuity at the junction points of different topographic segments, while Eq. (3) states that the normal flow at the coast vanishes at \( x = -B \), where the water depth is 3 times the Ekman layer e-folding scale (Mitchum and Clarke 1986). Boundary condition [Eq. (5)] indicates that the energy of CTW is mainly confined on the continental shelf (i.e., coastally trapped). Equations (2)–(5) can be solved by expanding the pressure as the product of the free wave eigenfunction \( F \) and the corresponding amplitude \( Y \):

\[
p(x, y, t) = \sum_{n=1}^{\infty} F_n(x) Y_n(y, t),
\]

where \( F \) is the solution of the free wave, which only changes with \( x \) and represents the cross-shelf structure, and \( Y \) changes with \( y \) and \( t \), representing the amplitude of \( F \).

b. Free wave modes

The water depth of the ECS can be approximated by the following piecewise linear functions:

\[
H(x) = \begin{cases} 
\alpha_1(x + L_1), & -L_1 < x < 0 \\
\alpha_1 + \alpha_2 x, & 0 < x < L_2 \\
\alpha_1 + \alpha_2, & L_2 < x < \infty
\end{cases}
\]

where \( L_1, L_2, d_1, \) and \( d_2 \) are defined in Fig. 8 and \( \alpha_i = d_i/L_i \) (\( i = 1, 2, \) and 3). These parameters are determined by a piecewise linear approximation using the

FIG. 7. Cross section of the three-dimensional time-dependent cross-correlation plot along the axis of the sliding window of 200-h length with the max lead–lag window of 50 h for (top) IMF1, (middle) IMF2, and (bottom) IMF3 of sea level and the alongshore wind at station JZ0604. The blue line represents the alongshore wind speed.

FIG. 8. Coordinates incorporated in the study. Dots trace the bottom depth across the ECS passing through station CE3: \( L_1 = 553 \) km, \( L_2 = 74 \) km, \( L_3 = 92 \) km, \( d_1 = 134 \) m, and \( d_2 = 713 \) m.
ETOP05 data, which is described in section 2. The width of the continental shelf in the ECS decreases from north to south, while the maximum depth increases from north to south. This topography variation may have some influence on the properties of the free waves, such as their phase speeds, and may influence the parameters that are used for the forced wave calculation.

Without considering the forcing term, Eq. (2) can be simplified to

\[
(H_{px})_{y} + fH_{p}{y} - \frac{f^2}{g} p_{t} = 0. \tag{8}
\]

Assume there exists an alongshore wave solution in the form of

\[
p = F(x) \exp[i(ly + \omega t)], \tag{9}
\]

where \( F(x) \) represents the cross-shelf variability of the amplitude, \( i \) indicates the imaginary part, \( l \) is the wave-number in the \( y \) direction, and \( \omega \) is the frequency. Substituting Eq. (9) into Eq. (8) yields an eigenvalue problem of \( F \). Its solution for the topography function [Eq. (7)] is the Bessel function for the continental shelf/break and an exponential function for a flat bottom. Using the boundary conditions [Eqs. (3)–(5)], where we set \( r(x) \) to zero, the solution to the eigenfunction yields a dispersion relation, which is an implicit function of the phase speed \( c \) (\( c = \omega / l \)). There are countable infinite solutions for \( c \), each corresponding to a wave mode. Once the phase speed is determined, the eigenfunction \( F(x) \) can be determined.

The first three modes of \( F(x) \) for segment A2 are shown in Fig. 9, which includes the cross-shelf eigenmode structure of the free waves. The lowest mode shown in Fig. 9 is the Kelvin wave (KW) as identified by Hsueh and Pang (1989). The extra node appears at the shelf break because of the sharp deepening of water depth (Hsueh and Pang 1989). The second and third modes are known as the shelf waves (SW1 and SW2), of which the phase speeds and the amplitudes are much smaller than the Kelvin wave. Figure 9 also shows that the energy of the waves is mainly confined to the continental shelf and negligible beyond the shelf. In other words, the configuration of the CTW is mainly controlled by the bottom topography of the continental shelf. The phase speeds of KW for segments A1, A2, and A3 are 17.3, 15, and 13.6 m s\(^{-1}\), respectively (Table 3), which are equal to the gravity wave speeds with water depths of 30.5, 23, and 18.8 m, respectively. One can see that the phase speeds decrease from north to south, corresponding to the variation of the shelf width. Our test run results (not shown) and previous work (Clarke and Van Gorder 1986; Jordi et al. 2005) indicate that this fast-propagating wave can only exist over a wide continental shelf. Figure 9 shows that the \( n \)th mode of the eigenfunctions has \( n \) zero nodes, which is different from the ordinary structure of the CTW, of which the \( n \)th mode has \((n - 1)\) zero nodes (Hsueh and Pang 1989). The extra node appears at the shelf break because of the sharp deepening of water depth at the shelf break in the bottom topography, which is much different from that in Hsueh and Pang (1989).

### c. Forced wave model

Now, let us expand the perturbation pressure as the sum of the product of free wave eigenfunctions \( F \) and corresponding amplitudes \( Y \):

\[
Y = \sum_{n=1}^{\infty} a_{n} \cos \left( \frac{n \pi x}{L} \right) + b_{n} \sin \left( \frac{n \pi x}{L} \right) \exp \left[ -\frac{d}{y} \right].
\]

![Fig. 9. Cross-shelf eigenmode structure of the first three free wave modes. The amplitudes are normalized with values set to 1 at the coast.](image)

<table>
<thead>
<tr>
<th>Segment</th>
<th>Mode</th>
<th>( c_{n} ) (m s(^{-1}))</th>
<th>( b_{n} ) (10(^{-2}) m(^{-1}))</th>
<th>( a_{1n} ) (10(^{-7}) m(^{-1}))</th>
<th>( a_{2n} ) (10(^{-7}) m(^{-1}))</th>
<th>( a_{3n} ) (10(^{-7}) m(^{-1}))</th>
<th>Decay distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1</td>
<td>17.3</td>
<td>2.15</td>
<td>-2.18</td>
<td>-4.65</td>
<td>-3.72</td>
<td>4576</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.8</td>
<td>1.18</td>
<td>-2.42</td>
<td>-16.16</td>
<td>-17.95</td>
<td>619</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.4</td>
<td>0.57</td>
<td>-1.16</td>
<td>-8.33</td>
<td>-42.85</td>
<td>233</td>
</tr>
<tr>
<td>A2</td>
<td>1</td>
<td>15.0</td>
<td>1.90</td>
<td>-2.54</td>
<td>-4.77</td>
<td>-3.64</td>
<td>3942</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.9</td>
<td>1.26</td>
<td>-3.01</td>
<td>-20.9</td>
<td>-23.02</td>
<td>477</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.0</td>
<td>0.61</td>
<td>-1.31</td>
<td>-10.96</td>
<td>-57.09</td>
<td>175</td>
</tr>
<tr>
<td>A3</td>
<td>1</td>
<td>13.6</td>
<td>1.59</td>
<td>-2.45</td>
<td>-4.35</td>
<td>-3.46</td>
<td>4088</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.7</td>
<td>1.28</td>
<td>-3.29</td>
<td>-20.62</td>
<td>-22.92</td>
<td>485</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.0</td>
<td>0.68</td>
<td>-1.51</td>
<td>-12.23</td>
<td>-55.45</td>
<td>180</td>
</tr>
</tbody>
</table>
Substituting Eq. (10) into the perturbation pressure governing Eq. (2) and using the same procedure as in Hsueh and Pang (1989), the alongshore and time-dependent amplitude \( Y_n(y,t) \) satisfies the following coupled first-order wave equation:

\[
p(x,y,t) = \sum_{n=1}^{\infty} F_n(x) Y_n(y,t).
\]  

(10)

Thus, only the first three modes are computed by Eq. (11). (1987) the higher modes are more sensitive to the inputs, the first three modes accounts for about 90.3% of that of the first six modes. In addition, according to Chapman (1987) the higher modes are more sensitive to the inputs, thus only the first three modes are computed by Eq. (11).

Usually the contribution of the wind stress curl and the change of atmospheric pressure are inessential, thus they are dropped from Eq. (11). However, as the shelf in the ECS is very wide, the integral of the last two terms in Eq. (11) may be considerable. To check the relative importance of the three terms on the right-hand side of Eq. (11), a test run with realistic wind stress and atmospheric pressure is carried out. A total of three experiments are implemented, each with only one forcing term from Eq. (11), that is, the alongshore wind stress, the wind stress curl, or the \( p_{at} \) term. The wind field and pressure data are introduced in section 2a. The time span is from 1 January to 28 February 2007. Taking the boundary condition of Eq. (11) to be zero to make sure that all the waves are locally forced, the results show that under the condition of a wide shelf, the magnitude of the wave generated by the wind stress curl is about 30%–50% of what is generated by \( \tau^x \), which is quite different from a narrow shelf where the effect of the wind stress curl term is smaller than that of wind stress term by about two orders of magnitude. Even though the shelf is wide, the contribution of \( p_{at} \) is small, which accounts for about 1%–2% of the contribution of the wind stress term. Thus, \( p_{at} \) will be dropped from Eq. (11) in the following calculation.

### d. Application in the ECS

We apply the wind-forced CTW theory to the ECS to calculate the low-frequency sea level fluctuation and alongshore currents using a numerical model forced by the NCEP wind stress field described in section 2. According to definitions of \( a_{mn} \) and \( b_n \), the frictional decay and wind coupling are both correlated with the shelf width and the slope gradient. The test run for the impact of the topography by the free wave parameters indicates that these parameters are also sensitive to the topography, especially the shelf width. In the ECS case, the shelf width changes from 500 km in the north ECS to 400 km in the south ECS; thus, we divide the coast zone of the study area into three segments, A1, A2, and A3 (Fig. 1). Within each segment, the topography and Coriolis parameter are assumed to be uniform. The depth profile for each segment is approximated using the lines perpendicular to the coastline, which go through stations CE3, JZ0604, and JZ0903, respectively. The water depth at the coast of each segment is set as 20 m, and \( r(x) \) is taken as \( 3 \times 10^{-4} \text{m s}^{-1} \). To allow the slowest-propagating wave to have enough time to pass through the alongshore integral interval (Clarke and Van Gorder 1986), we set the time step \( \Delta t = 6 \text{ h} \) and the alongshore interval \( \Delta y = 1000 \text{ m} \). The free wave parameters for the first three CTW modes for segments A1, A2, and A3, as well as the decay distance of the \( n \)th wave mode in the alongshore direction, are shown in Table 3.

The decay scales for the same mode of different segments are similar, except that segment A1 has a longer...
decay distance than the other two segments for all three wave modes, due to the greater shelf width of A1. For the same segment, the decay scales for different modes range from $O(4000)$ km for the lowest mode to $O(200)$ km for the third mode. The distance from the point QD shown in Fig. 1 to CE3 is about 600 km, which is far less than the lowest-mode wave decay distance. This means the KW activity in the YS can propagate to the ECS without much energy loss, while the second and the third modes cannot approach the stations shown in Fig. 1; that is, only the KW can propagate from the YS into the ECS. To examine the low-frequency wave activity in the ECS, we first choose the point QD as the origin of integral of Eq. (11) to solve for the time series of sea level and alongshore current at the stations in segment A1, where the initial amplitude is assumed to be zero. Second, we use the observations in this segment as the origin of the integral of Eq. (11) to simulate the time series at the stations in segments A2 and A3.

1) RESULTS FOR SEGMENT A1

The observed and simulated sea level and alongshore currents at CE3 and CE4 are shown in Fig. 10. Both the observation and simulation indicate that the northerly wind pulses induce positive sea level anomaly and downwind alongshore current. The agreement is better when the major northerly wind pulses appear on 1 February, 13 February, and 5 March. This agreement is consistent with the TDICC analysis in section 2; that is, there exists a minimum threshold of wind speed for the forced CTW generation. After the strong northerly wind, there is sea level relaxation, which contributes to the free CTWs, mainly the KW mode.

Figure 11 shows the correlations between the observations and the simulations of sea level and alongshore currents at five stations. The time interval for each station is shown in Table 1. The correlations without wind are calculated in the time interval from 9 January to the end time of the record at each corresponding station. The maximum correlation coefficients for the sea level are 0.59 at CE3 and 0.51 at CE4, for which the observations lag the simulations for 6 h. The coefficients of alongshore currents are 0.68 at CE3 and 0.62 at CE4, which are higher than those for the sea level, with zero time lag for the two stations.

Both Figs. 10 and 11 indicate that the simulation results of alongshore currents are better than those of sea level. The sea level fluctuations are mainly controlled by the KW mode, the alongshore currents are mainly controlled by the KW and SW1 modes, and the low-frequency waves in the ECS are mainly the free KW. In fact, Hsueh and Pang (1989) pointed out that the KW generated at the west coast off the Korean Peninsula can propagate to the Chinese coast. This wave will make contribution to the free wave activity in the YS. In addition, there may be other free wave sources. The integral
length for Eq. (11) is far less than the KW mode decay distance but larger than the SW1 mode; thus, if there is free wave activity happening at the origin of integral, the KW is the main part that can propagate to the ECS. In other words, the no free wave condition at the origin will have more impact on the sea level fluctuation than on the alongshore current.

2) RESULTS FOR SEGMENTS A2 AND A3

Clarke and Van Gorder (1986) pointed out that if adequate measurements were available at \( y = 0 \), \( Y_n(0, t) \) can be obtained by a least squares fit of the alongshore current measurements, or \( Y_1(0, t) \) can be approximated if only coastal sea level is available. However, the former procedure needs a large dataset of the alongshore current and the latter would not contain the contribution to the currents from the boundary area. To find more accurate \( Y_n(0, t) \), we use both the alongshore current and the sea level at CE3 and CE4 with a procedure that is similar to the one used in Church et al. (1986).

Because only the first three modes will be considered in Eq. (11), we truncate Eqs. (10) and (13) at three. Substituting the observation at CE3 and CE4 into truncated Eqs. (10) and (13), we can get a Moore–Penrose equation about \( Y_n(0, t) \) (\( n = 1, 2, \) and 3):

\[
MY = U, \tag{14}
\]

where

\[
M = \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{11x} & F_{12x} & F_{13x} \\ F_{21x} & F_{22x} & F_{23x} \end{pmatrix}, \quad Y = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}, \quad \text{and} \quad U = \begin{pmatrix} \rho g \eta_{\text{CE3}} \\ \rho g \eta_{\text{CE4}} \\ \rho f v_{\text{CE3}} \\ \rho f v_{\text{CE4}} \end{pmatrix}.
\]

The \( F_{ii} \) and \( F_{2i} \) (\( i = 1, 2, \) and 3) are the first three eigenfunctions at CE3 and CE4, respectively. Subscript \( x \) represents partial differentiation. The values \( \eta \) and \( v \) are the sea level and alongshore current at CE3 and CE4, respectively, and \( \rho \) and \( f \) are the water density and Coriolis parameter, respectively.

Eq. (14) can be solved by a least squares fit. The term \( Y \) is the Moore–Penrose solution:

\[
Y = (M^T M)^{-1} M^T U, \tag{15}
\]

where \( Y_1 \), \( Y_2 \), and \( Y_3 \) are now determined and will be used for the integral of Eq. (11).

Figure 12 shows a comparison between the observations and simulations. The sea level at JZ0906 is divided into two periods: one from 9 to 23 January and the other from 24 January to 5 February. The correlations between the observations and simulations during the two periods are both 0.9, so only one coefficient is plotted in Fig. 11. One can see that the sea level fluctuations in segments A2 and A3 are significantly affected by the free wave propagation. In fact, the correlations are higher than that for segment A1. In addition, one can see from Fig. 12 that the fitting is much better after 9 January; from then on, the realistic boundary condition is added. Furthermore, the simulated sea level amplitude is about 105% of the observation at the three stations, while the amplitude of the simulated alongshore currents is about 116% and 67% of the observation for JZ0903 and JZ0906, respectively.

To analyze the effect of the local wind quantitatively, we calculate the time series without the wind stress and
wind stress curl forcing. The correlations with the observations are plotted in Fig. 11. The correlations, especially the correlations for the alongshore current, are smaller if the wave motions are treated as free waves. It can be concluded that the alongshore currents are affected by the local forcing. This implies that the wind may not only modulate the periods of the waves, but also their phases. This explains why the phase speeds of the second and third IMFs of the sea level variation we derived from the observations are larger than, or sometimes equal to, the analytical results (Table 2). The wind amplifies the fluctuations, especially for the alongshore currents. If the local wind is removed, the sea level amplitude at JZ0604 is about 78% of that with the wind, 51% at JZ0903, and 68% at JZ0906, while the alongshore current amplitude is about 29% and 40% of that with the wind at JZ0903 and JZ0906, respectively.

4. Summary

Using theoretical and numerical models as well as data, we investigate the existence and characteristics of CTWs in the ECS and analyze their generation mechanisms. The main results are summarized as follows.

From the mooring array observations of the sea level at five stations, a group of low-frequency waves in the coastal area of the ECS are detected using the EEMD method. The waves have amplitudes of $O(10 \text{ cm})$ and periods centered at 2, 4.5, and 11 days. The comparison between the analytical and observational dispersion relations indicates that the waves are corresponding to different CTW modes: the free KW mode and the forced first and second shelf wave modes.

In winter, the fluctuation of the northerly and north-easterly winds in the ECS is strong, which provides a suitable condition for the generation of the CTWs. Applying the wind-forced CTW theory to the ECS, results show that for the wide shelf of the ECS not only the alongshore wind stress but also the wind stress curl plays an important role in the generation of the CTWs. Numerical model can successfully reproduce the low-frequency sea level variation and alongshore current. The simulation results indicate that the sea level variation mainly results from the Kelvin waves, while the alongshore current is affected by both the Kelvin waves and the first shelf wave.

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APPENDIX

The EMD Theory

The EMD theory includes empirical mode decomposition and Hilbert spectral analysis (Huang et al. 1998;
Huang and Wu 2008). In this paper we only use the EMD decomposition so only the EMD decomposition is introduced.

At any given time, the data may have many different coexisting simple oscillatory modes of significantly different frequencies, one superimposed on the other. Each component is defined as an IMF, with the following definitions:

1) in the whole dataset, the number of extrema and the number of zero crossings must either equal or differ at most by one; and
2) at any data point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

With the definition of the IMFs, one can decompose any function \( x(t) \) through the following “sifting” process: first, identify all the local extrema (minima) and then connect all the local maxima (minima) by a cubic spline line to form the upper (lower) envelope. The upper and lower envelopes should encompass all the data between them. Calculate the mean of the envelopes and designate it as \( h_1 \). The difference between the input data and \( m_1 \) is the first protomode \( h_1 \):

\[
x(t) - m_1 = h_1,
\]

where \( h_1 \) is expected to be an IMF through the above processing. However, that is not so because new extrema may be introduced when a local zero is changed from a rectangular to a curvilinear coordinate system. To eliminate background waves on which the pursuing IMF is riding and to make the wave profiles more symmetric, a repeat of the above procedure, namely the sifting, is necessary.

Treat \( h_1 \) as the data in the next iteration:

\[
h_1 - m_{1(1)} = h_{1(1)}.
\]

After \( k \) times of the above iterations, \( h_1 \) becomes an IMF, that is,

\[
h_{1(k-1)} - m_{1(k)} = h_{1(k)}.
\]

Designate \( h_{1(k)} \) as \( c_1 \):

\[
c_1 = h_{1(k)}.
\]

So far, the first IMF is derived. The term \( c_1 \) contains the highest-frequency component of the original signal. Separate \( c_1 \) from the rest of the data:

\[
r_1 = x(t) - c_1.
\]

Because \( r_1 \) still contains lower-frequency components, we can treat it as the new data and carry out the same sifting process described above. This procedure should be repeated to obtain the subsequent \( r_n \) until it becomes a constant, a monotonic function, or a function with only one maximum and one minimum from which no more IMF can be extracted:

\[
r_1 - c_2 = r_2,
\]

\[
\vdots
\]

\[
r_{n-1} - c_n = r_n.
\]

By summing Eqs. (A5) and (A6), we can get

\[
x(t) = \sum_{j=1}^{n} c_j + r_n.
\]

Finally we obtain decomposition with \( n \) IMFs and a residue \( r_n \).

The IMFs obtained through the sifting processes preserve the full nonlinearity and nonstationary in physical space. Moreover, they constitute an adaptive basis. This basis usually satisfies empirically all the major mathematical requirements for a time series decomposition method, including convergence, completeness, orthogonality, and uniqueness (Huang et al. 1998).

REFERENCES


