Internal-Tide Spectroscopy and Prediction in the Timor Sea

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ABSTRACT

Spectral analyses of two 3.5-yr mooring records from the Timor Sea quantified the coherence of mode-0 (surface) and mode-1 (internal) tides with the astronomical tidal potential. The noncoherent tides had well-defined variance and were most accurately quantified for tidal species (as opposed to constituents) in long records (>6 months). On the continental slope (465 m), the semidiurnal mode-0 and mode-1 velocity and mode-1 pressure variance were 95%, 68%, and 56% coherent, respectively. On the continental shelf (145 m), the semidiurnal mode-0 and mode-1 velocity and mode-1 pressure variance were 98%, 34%, and 42% coherent, respectively. The response method produced time series of the semidiurnal coherent and noncoherent tides. The spectra and decorrelation time scales of the semidiurnal tidal amplitudes were similar to those of the barotropic mean flow and mode-1 eigenspeed (~4 days), suggesting local mesoscale variability shapes noncoherent tidal variability. Over long time scales (>30 days), mode-1 sea surface displacement amplitudes were positively correlated with mode-1 eigenspeed on the shelf. At both moorings, internal tides were likely modulated during both generation and propagation. Self-prediction using the response method enabled about 75% of semidiurnal mode-1 sea surface displacement to be predicted 2.5 days in advance. Improved prediction models will require realistic tide–topography coupling and background variability with both short and long time scales.

1. Introduction

The Earth–moon–sun system produces a gravitational tidal potential, which forces the ocean at combinations of discrete astronomical frequencies (Doodson 1921). The tidal potential, which is uniform with depth, drives surface (i.e., mode-0/barotropic) tides, which propagate as linear shallow-water waves (Hendershott 1981). In the presence of density stratification, surface tides scatter at topographic features and transfer energy to internal (i.e., baroclinic) tides (e.g., Garrett and Kunze 2007). Internal tides comprise a superposition of vertical modes that individually satisfy Laplace’s tidal equations, with allowances for an equivalent depth (Miles 1974) and terms that account for further intermodal topographic scattering (Griffiths and Grimshaw 2007). In the deep ocean, most internal-tide energy and energy flux occur in modes 1 and 2 (e.g., Alford and Zhao 2007). Like the surface tide, mode-1 and mode-2 tides are well-described by linear dynamics in deep water because they have small Froude numbers [i.e., their induced currents, \(O(0.1)\) m s\(^{-1}\), are usually much smaller than their eigenspeeds, \(O(1)\) m s\(^{-1}\)], where eigenspeed is defined as \(c_n = \sqrt{gH_n}\), \(g\) is gravity, and \(H_n\) is the equivalent depth].

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a. Tidal prediction

Surface- and internal-tide predictions are useful for a variety of applications, which include (i) ocean engineering (Osborne et al. 1978), (ii) quantifying sediment, nutrient, larvae, and pollutant transport (e.g., Noble et al. 2009), (iii) forecasting the generation of nonlinear internal waves (Nash et al. 2012a), (iv) decontaminating observations of nontidal processes (Chavanne and Klein 2010), and (v) modeling sound propagation (Dushaw et al. 1995).

Because of their linearity, low-mode tides have sharp spectral peaks at the frequencies in the tidal potential. Even if there is nonlinear resonance, there can still be sharp spectral peaks at the forcing frequencies (e.g., Boegman and Ivey 2012). The presence of sharp spectral peaks provides motivation for using the harmonic method of tidal prediction, which consists of fitting harmonic functions at the tidal frequencies to data (e.g., Doodson 1921). The harmonic method contains a built-in assumption that all observed variability at the tidal frequencies is associated with the tides (i.e., the observations are noise free); however, the ocean contains an unpredictable spectrum of random “background” motion at a continuum of frequencies that includes the tidal bands (Munk and Bullard 1963; Munk et al. 1965). Random background motions at the tidal frequencies contaminate tidal predictions obtained via the harmonic method. To separate tides from background motions, Munk and Cartwright (1966) proposed the response method of tidal prediction, which isolates those motions at the tidal frequencies that maintain a constant amplitude and phase relationship to the tidal potential (i.e., motions that are “coherent” with the tidal potential).

Munk and Cartwright (1966) applied the response method to the Honolulu tide gauge and found both spectral peaks, associated with coherent tidal motion, and a smooth continuum, associated with noncoherent background motion. The analysis also revealed anomalous noncoherent spectral peaks at the tidal frequencies, that is, “noncoherent tides,” which were unpredictable using classical tidal analyses.

Since the initial speculation by T. Sakou and G. Groves (cited as personal communication in Munk and Cartwright 1966) and observational analysis by Radok et al. (1967), noncoherent tidal variance has been almost exclusively attributed to the presence of internal tides, which tend to “come and go” (Wunsch 1975). For instance, Mitchum and Chiswell (2000) showed that noncoherent records from tide gauges along the Hawaiian Ridge were consistent with in situ acoustic observations of noncoherent mode-1 tides. Additional evidence of noncoherent mode-1 tides has come from satellite altimetry (Ray and Zaron 2011), global numerical simulations (Shriver et al. 2014), and numerous in situ observations (e.g., Hendry 1977; Holloway 1988; Dushaw et al. 1995; Chiswell 2002; Eich et al. 2004; Chavanne et al. 2010; Zilberman et al. 2011; Nash et al. 2012a,b). Notably, a local study of internal tides by Eich et al. (2004) and reanalysis of the Honolulu tide gauge by Colosi and Munk (2006) confirmed that the noncoherent peaks identified by Munk and Cartwright (1966) were primarily because of mode-1 tides.

Despite numerous observations of noncoherent mode-1 tides, it is difficult to quantitatively compare results from different studies. Noncoherent variability depends on the type of data (i.e., sea surface displacement, velocity, temperature, or acoustic travel time) and the precise method of analysis (e.g., the definitions of the coherent tide and tidal bandwidth). For instance, Nash et al. (2012a), Chiswell (2002), and Colosi and Munk (2006) discuss noncoherent tidal dynamics in terms of 3–5 day, intra-annual (60–120 day), and interannual time scales, respectively. It is also difficult to quantitatively compare noncoherent mode-0 and mode-1 tides because mode-0 tides are either not analyzed, not separable from mode-1 tides, or assumed to be perfectly coherent.

Here, we apply the methods of Munk and Cartwright (1966) to precisely quantify the mode-0 and mode-1 coherent and noncoherent responses to the tidal potential. The methods unambiguously identify the bandwidth of the noncoherent tide and highlight both the difficulty in defining noncoherent variances for individual tidal groups and the uncertainties in estimating coherent tides from short data records.

b. Dynamics

Munk and Cartwright (1966) presented the dynamical hypothesis that tidal sea surface displacements are linearly related to the tidal potential at each frequency. This hypothesis is not controversial with respect to mode-0 tides because surface tides are both highly linear and forced directly by the tidal potential. The hypothesis is more nuanced with respect to mode-1 tides because internal tides propagate more slowly (allowing more time for nonlinear processes to develop) and are forced by mode-0 tide interactions with stratification and topography, not merely by the tidal potential. Nevertheless, observations indicate that a significant fraction of mode-1 variance is coherent with the tidal potential (e.g., 50% of temperature variance; Hendry 1977). Furthermore, idealized numerical models that simulate local coherent internal-tide generation and propagation have accurately predicted the observed coherent mode-1 internal tides at the Hawaiian Ridge (Chavanne et al. 2010) and New Jersey continental slope (Nash et al. 2012a). Accurately simulating coherent internal tides is
more difficult in regions of complicated topography because solutions are sensitive to the precise distances between generation regions (e.g., Buijsman et al. 2012) and the numerical domain size (Ponte and Cornuelle 2013).

Noncoherent internal tides are not simply linearly related to the tidal potential and, therefore, require a more complete (i.e., nonlinear) dynamical model for optimal predictions. Historically, there have been two categorical dynamical explanations for why internal tides are partially noncoherent with the tidal potential.

One explanation for noncoherent internal tides is that internal-tide generation depends on stratification (e.g., Baines 1982), and this is altered by interannual ocean warming, seasonal atmospheric forcing, mesoscale eddies, and storms. In other words, the internal tide–generating force is itself noncoherent with the tidal potential. For example, Mitchum and Chiswell (2000) showed that observed internal-tide amplitudes along the Hawaiian Ridge are positively correlated with interannual changes in stratification (as inferred from changes in mean sea level). On intra-annual time scales, Gerkena et al. (2004) showed that seasonal stratification alters the simulated internal-tide generation in the Bay of Biscay. At shorter time scales, Zilberman et al. (2012), Powell et al. (2012), and Zaron and Egbert (2014) used observations, an adjoint model, and an ensemble of simulations, respectively, to demonstrate that mesoscale stratification also alters internal-tide generation at the Hawaiian Ridge.

A second explanation for noncoherent internal tides is that temporally varying background currents and stratification alter internal-tide propagation (e.g., Rainville and Pinkel 2006; Park and Watts 2006). In the language of signal processing, the internal-tide is a carrier signal and low-frequency variability is a modulating signal (Munk et al. 1965). Observations by Chiswell (2002), and simulations by Zaron and Egbert (2014), support this explanation by showing that internal-tide amplitudes and phases offshore of Hawaii are consistent with modulation by mesoscale variability. Similarly, Colosi and Munk (2006) conclude that internal tides are noncoherent at the Honolulu tide gauge because Rossby waves and ocean warming alter their propagation from distant sources. On shallow continental shelves, noncoherent internal tides can also arise from highly nonlinear dynamics where, for example, strong barotropic tidal currents sweep slowly propagating internal tides across the shelf (O. Fringer et al. 2014, unpublished manuscript).

It is difficult to separate these two explanations in regions of complicated topography because (i) they are not mutually exclusive (i.e., changes in stratification and background currents typically modify both internal-tide generation and propagation) and (ii) internal-tide generation and propagation are coupled because propagating internal tides affect internal-tide generation where they encounter sloping topography (e.g., Kelly and Nash 2010; Zilberman et al. 2011). Moreover, both explanations for noncoherent tides can produce similar features in observational records. For example, a change in stratification can affect the location and intensity of an internal-tide generation region, in turn altering both the amplitude and phase (through travel time) of the internal tide observed at a fixed remote location. A change in stratification can also affect the direction of the internal tide (by changing the group speed), which can shift internal-tide interference patterns (Rainville et al. 2010) and alter both the amplitude and phase of the internal tide observed at a fixed remote location.

c. Setting and motivation

Here, we examine two 3.5-yr full-water-column mooring records of velocity, temperature, and salinity in the Timor Sea (Fig. 1). The tides in this region are among the most energetic in the world (Ray et al. 2005). At each measurement time, the depth-variable data were projected onto orthogonal vertical modes to produce modal-amplitude time series. The advantage of this transformation is that mode-0 and mode-1 tides can be independently analyzed for coherence using the original methods of Munk and Cartwright (1966).

Background conditions in the Timor Sea are likely to give rise to noncoherent internal tides. The region has energetic low-frequency and mesoscale variability (e.g., the Indonesian Throughflow and mesoscale eddies; Condie and Andrewartha 2008; van Sebille et al. 2014) and seasonally variable near-surface stratification, which can produce seasonal variability in internal-tide generation and propagation (e.g., Rayson et al. 2012). The region also has complicated topography, producing multiple generation regions, internal-tide interference patterns, and feedbacks between propagating internal tides and internal-tide generation. Here, we address the following questions:

- How is noncoherent variance distributed across tidal frequencies and between modes 0 and 1?
- How are noncoherent tides statistically and dynamically related to the local barotropic mean flow and stratification?

We also briefly discuss the global consequences of variable mode-0 to mode-1 energy conversion.

2. Methods

a. Observations

As part of Australia’s Integrated Marine Observing System (IMOS), several permanent moorings have been
deployed in the Timor Sea. Here, we analyze two of these moorings, which have now returned 42 months of velocity, temperature, and salinity measurements (Fig. 1).

The first mooring is located at 127.55°E, 9.82°S in 65 m of water on the south continental slope of the Timor Trough. The trough is roughly 150 km wide and has a maximum depth of 3000 m. It is bordered to the north by a narrow continental margin and the island of Timor. It is bordered to the south by the Australian North West shelf, which is approximately 400 km wide and 100 m deep. The slope mooring is instrumented with

(i) an upward-looking 75-kHz acoustic Doppler current profiler (ADCP) 6 m above the bottom, which measures velocity every 20 min at 10-m vertical resolution between approximately 70- and 440-m depth; (ii) thermistors at 20-, 30-, * 40-, 50-, * 60-, 70-, * 80-, 100-, 115-, * 130-, 155-, * 180-, 200-, * 250-, * 300-, 350-, * 400-, and 460-m depth, which sample every 2 min or less (here and below, an asterisk indicates the instrument was only deployed for the last 30 months); and (iii) conductivity sensors at 20-, 40-, 60-, 80-, and 100-m depth, which sample every 5 min.

The second mooring is located at 128.00°E, 11.00°S in 145 m of water on the Australian North West shelf. This geological region of the shelf is known as the Bonaparte basin. The mooring is situated in the center of a small canyon that runs northeast–southwest and is about 20 km wide and 50 m deep. The mooring is instrumented with

![Fig. 1](image-url)
(i) an upward-looking 150-kHz ADCP 6 m above the bottom, which measures velocity every 10 min at 4-m vertical resolution between approximately 15- and 139-m depth;

(ii) thermistors at 20-, 30-,* 40-,* 50-,* 60-, 70-,* 80-, 100-, 120-, and 140-m depth, which sample every 2 min or less; and

(iii) conductivity sensors at 40-* and 100*-m depth, which sample every 5 min.

Both moorings were serviced every 6 months, which resulted in 2–5-day data gaps because of the mooring recovery and redeployment. Full-water-column ship-based CTD profiles were obtained during the mooring services, which aid in determining the local functional relationship between temperature and density.

Collocated conductivity and temperature sensors (moored and ship based) indicate an unambiguous relationship between temperature and potential density (referenced to 100 m; Fig. 2). This temperature–density relation is extrapolated to slightly higher and lower temperatures using a second-order Taylor expansion. Using the temperature–density relation, all temperature measurements are assigned densities that are accurate to within ±0.27 kg m⁻³.

Buoyancy was computed as gravity multiplied by the difference between annual-mean and instantaneous densities.¹ Baroclinic pressure was obtained by depth-integrating buoyancy and subtracting the depth mean (e.g., Kunze et al. 2002). Because sea surface displacement was not measured, barotropic pressure cannot be determined.

Mooring velocity and pressure were fit to mode-0 and mode-1 vertical structure functions for time series analysis. The zeroth mode is constant with depth. Higher modes are determined by numerically solving the eigenvalue problem:

\[
\frac{\partial^2 \Phi_n}{\partial z^2} + \frac{N^2}{c_n^2} \Phi_n = 0,
\]

where \(N(z)\) is the buoyancy frequency, \(\Phi_n\) are the modes for vertical velocity, and \(c_n\) are the eigenspeeds. The modes for horizontal velocity and pressure \(f_n\) are the vertical derivatives of \(\Phi_n\), which are orthogonal and scaled so that

\[
\frac{1}{H} \int_{-H}^{0} \phi_m^* \phi_n \, dz = \delta_{mn},
\]

where \(H\) is water depth, and \(\delta_{mn}\) is the Kronecker delta. In practice, we solved (1) using a second-order finite difference matrix, rigid-lid and flat-bottom boundary conditions, and the buoyancy frequency determined from annual-mean stratification. The mode shapes were not sensitive to monthly variability in stratification, but eigenspeeds varied by about ±25 cm s⁻¹. Mode shapes computed from the observed \(N^2\) are similar to those computed using an \(N^2\) from climatological datasets [e.g., Commonwealth Scientific and Industrial Research Organisation (CSIRO) Atlas of Regional Seas (CARS); Ridgway et al. 2002]. Velocities and pressures were fit to modes by linearly interpolating the measurements onto a 1-m vertical grid and performing a least squares regression.

The analyses here focus on vertical modes 0 and 1 because they are the most energetic and accurately observed modes at the moorings. Because higher modes are neglected in this analysis, (i) energy and momentum transfer from modes 0 and 1 to higher modes cannot be quantified and (ii) vertical propagation of the internal tide cannot be diagnosed.

b. Spectral analysis

Time series of mode-0 and mode-1 amplitudes were analyzed for coherence with the equilibrium tide (Munk and Cartwright 1966). The equilibrium tide, which corresponds to the tidal potential divided by a constant, was calculated for second-degree spherical harmonics via the formulas provided by Munk and Cartwright (1966). In preparation for coherence analysis, all time series were low-pass filtered at

¹If buoyancy is defined as deviations from shorter averages, the low-frequency buoyancy spectrum is attenuated, but there is little effect on the mode-1 diurnal or semidiurnal spectrum.
20 cycles per day (cpd) and resampled using linear interpolation into exactly 30 measurements per $M_2$ tidal period. The data were then broken into 42 segments of 58 $M_2$ tidal periods (i.e., 30.02 days). The segment length was chosen so that the Fourier frequencies had 1-cycle-per-month (cpm) resolution and were centered over the tidal groups. At 1-cpm resolution, tidal groups are resolved but not constituents [which have a defined resolution of 1 cycle per year (cpy)]. Therefore, we refer to the tidal group that contains the $M_2$ constituent as the “$M_2$ group.”

The Fourier transforms $\mathcal{H}(f)$ were separated into coherent and noncoherent components

$$\mathcal{H} = Z^* G + \mathcal{H}_{\text{noncoh}},$$

where $\mathcal{H}_{\text{coh}} = Z^* G$ is the component of the observations that is coherent with the equilibrium tide, and $G(f)$ is the Fourier transform of the equilibrium tide. The frequency-dependent constant $Z(f)$, which linearly relates the forcing spectra to the observed spectra, is known as the admittance. The admittance is computed as a least squares fit of the time series of observed Fourier coefficients to the time series of the tidal potential Fourier coefficients:

$$Z = \langle GH^* \rangle/\langle GG^* \rangle,$$

where brackets indicate an ensemble average over the 42 time segments.

Horizontal velocity and pressure Fourier coefficients (i.e., $u_n$ and $p_n$) were separated into total, coherent, and noncoherent components using (3). Next, these coefficients were used to compute total, coherent, and noncoherent depth-integrated kinetic energy ($KE$), potential energy ($PE$), energy flux ($F$), and mode-0 to mode-1 energy conversion ($C_{01}$):

$$KE_n = \frac{H \rho_0}{2} u_n u_n^* \quad (J \text{ m}^{-2}),$$

$$PE_n = \frac{H}{2 \rho_0} P_n P_n^* \quad (J \text{ m}^{-2}),$$

$$F_n = H u_n^* p_n \quad (W \text{ m}^{-1}),$$

and

$$C_{01} = VH \cdot u_0^* p_1 \phi_1 |_{z=-H} \quad (W \text{ m}^{-2}),$$

respectively (e.g., Kelly et al. 2012), where $\omega$ is the tidal frequency, $f$ is the inertial frequency, $\rho_0$ is the reference density, and only the real parts of (5)–(8) have physical meaning. In calculating $C_{01}$, bathymetric gradients were computed using a central difference and averaged over 10 km.

c. The response method

Time series (as opposed to spectra) of coherent and noncoherent tides are necessary for practical tidal predictions. Coherent and noncoherent tides are separated in the time domain using the response method. Using this method, each observation is modeled as the response to a series of impulses, that is, a weighted sum of an input function at past and future times. Following Munk and Cartwright (1966), the input functions are the (real and imaginary) coefficients for the second-degree spherical harmonic of the tidal potential, and the weights are the responses to the tidal potential at lags $\Delta t = 0, \pm 2, \pm 4$ days, which are computed from a least squares fit. To form an analogy with the harmonic method, the lagged tidal potentials replace the trigonometric functions as the inputs and the weights at each lag replace the amplitudes and phases of each constituent as the station constants.

Here, the coherent and noncoherent semidiurnal tides are separated by (i) bandpassing the raw time series at $f_{M2} \pm 4$ cpm, (ii) predicting the coherent tide via the response method, and (iii) defining the noncoherent tide as the residual.

In an attempt to “self-predict” the noncoherent tide, we also use the time-lagged noncoherent tide itself as the input function. In this case, station constants are defined as the input weights for lags $\Delta t = \Delta T, \Delta T + 2$, and $\Delta T + 4$ days, where $\Delta T$ is the forecast interval (i.e., 0–20 days).

d. Numerical simulations

An atlas of data-assimilating surface-tide simulations [Ocean Topography Experiment (TOPEX)/Poseidon version 8 (TPXO8); Egbert 1997] provides an estimate of local mode-0 tides. A simulation using the MIT general circulation model (MITgcm; Marshall et al. 1997) with a configuration similar to Kelly et al. (2012) provides information about regionally generated $M_2$-frequency internal tides. The MITgcm simulation was initiated with horizontally constant stratification, obtained by combining annual-mean data from the mooring above 450 m and CARS (Ridgway et al. 2002) below 450 m. The MITgcm domain and bathymetry [obtained from the General Bathymetric Chart of the Oceans (GEBCO)] are shown in Fig. 1. The grid spacing is 2 km in the horizontal and 5 to 100 m in the vertical. Simulations were forced at the boundaries with $M_2$ mode-0 velocities from the TPXO8 atlas. Constant horizontal and vertical eddy viscosities of $10^{-1}$ and $10^{-2} \text{ m}^2 \text{ s}^{-1}$, respectively, provided a rudimentary turbulence closure, which stabilized the model without greatly affecting the low-mode tide. Sponge conditions were applied at the edge of the domain to prevent the reflection of outward-radiating internal tides.
3. Results

a. Velocity time series

Semidiurnal tides dominate demeaned mode-0 and mode-1 velocity amplitudes (Fig. 1). Mode-0 velocities lag the equilibrium tide by about 12 h and mode-1 velocities, in turn, lag mode-0 velocities by a couple of hours. Mode-0 velocities are correlated with the equilibrium tide because the astronomical forcing generates mode-0 tides. Mode-1 velocities are correlated with the equilibrium tide because mode-0 tides convert astronomical work to mode-1 tides through topographic internal-tide generation.

In general, mode-0 velocities are larger on the shelf (145 m) than the slope (465 m) because the barotropic tide in both locations has nearly the same volume transport. In contrast, mode-1 velocities are much smaller on the shelf than the slope because the most energetic internal tides (in a depth-integrated sense) are confined to deeper water. Mode-1 velocities at the slope mooring do not have obvious seasonal variability, but mode-1 velocities at the shelf mooring (145 m) are larger during February than July (i.e., internal tides are more energetic during strong Austral summer stratification).

b. Mode-1 sea surface displacement spectra

Dividing mode-1 surface pressures by \( \rho g \) produces mode-1 sea surface displacements, which are analogous to mode-1 sea surface displacements that are obtained from tide gauges and satellite altimetry. Mode-1 displacement variances at the slope mooring (465 m) are plotted in Fig. 3 using the same format as Fig. 1 of Munk and Cartwright (1966). The heights of the columns in Figs. 3a and 3b indicate the total variances of the equilibrium-tide and mode-1 tides, respectively. Equilibrium-tide variances are confined to nine tidal groups within the diurnal and semidiurnal species (long-period tides, i.e., <0.5 cpd, are neglected in this analysis). Equilibrium-tide
variances are at maximum in the semidiurnal group that contains the M2 constituent. Observed surface displacement variances are peaked at the diurnal and semidiurnal frequencies, but also exhibit a smooth background continuum between the tidal frequencies.

Figures 3b and 3c display coherent and noncoherent variances as filled and unfilled columns, respectively. The panels contain the same information, but the filled and unfilled columns are stacked in reverse order, which gives them different heights because of the logarithmic scale. Coherent variances have abrupt peaks in the tidal groups associated with the major diurnal and semidiurnal constituents. The amplitudes and phases of the admittances (Fig. 3d,e) vary between groups, but are generally smooth. Spikes in the admittance curves are due to admittances that have large uncertainties, which occur in weakly forced tidal groups.

Mode-1 sea surface displacements at the shelf mooring (145 m) are much smaller than those at the slope mooring, resulting in less total and coherent variance (Fig. 4). Because the equilibrium tide is nearly the same at both locations, smaller displacements on the shelf dictate smaller admittances.

At both moorings, noncoherent variances at the diurnal and semidiurnal frequencies are an order of magnitude larger than the high-frequency (>0.5 cpd) background continuum. These peaks, distinctly associated with tidal bands, are the noncoherent tides. One manifestation of the noncoherent tide is that the Fourier coefficients computed over 30-day segments vary in amplitude and phase. For example, M2 group coefficients at the slope mooring vary by ±1 cm and ±45° from the coherent amplitude and phase, respectively (Fig. 5a). The M2 group coefficients at the shelf mooring (Fig. 5b) have a narrower range of amplitudes (±0.3 cm) and broader range of phases (about ±90°).

The ratio of noncoherent-to-coherent surface displacement variance within the M2 group is about 1:4 and 3:2 at the slope and shelf moorings, respectively (Table 1). By comparison, these ratios are an order of magnitude larger than the ratios for total sea surface displacement from the Honolulu tide gauge (e.g., Colosi and Munk 2006). Large noncoherent mode-1 variance is consistent with the theory that noncoherent tidal cusps in total sea surface displacement are predominantly due to internal tides (Colosi and Munk 2006).
c. Velocity spectra

Observed velocity spectra contain a background continuum and both coherent and noncoherent tidal peaks (Figs. 6, 7). Mode-0 admittances are slightly smoother than those of mode 1. Noncoherent-to-coherent velocity variance ratios within the M2 group at the slope mooring are 1:28 and 1:3 for modes 0 and 1, respectively (Table 1). Variance ratios at the shelf mooring are 1:216 and 1:1 for modes 0 and 1, respectively. Noncoherent M2 group velocities at the moorings manifest as slight changes in the amplitude and inclination of ellipses during each 30-day segment (Fig. 8).

Noncoherent mode-0 variance at the slope mooring is small (0.92 cm$^2$ s$^{-2}$) but significant. Uncertainty in estimates of noncoherent variance predominantly arises because mode-0 (i.e., depth averaged) velocities are estimated over 70% of the water column because of the limited ADCP coverage. Over the sampled depths, higher-mode velocities will also have nonzero depth averages. The magnitude of contamination depends on the mode shapes, amplitudes, and phases of the noncoherent high-mode tides and the number of 30-day segments that are used to obtain the admittances. Monte Carlo simulations composed of 100 000 sets of 42 randomly phased ensembles of modes 1–4 (all with amplitudes equal to the observed noncoherent mode-1 tide) indicate that noncoherent mode-0 variance at the slope mooring has an uncertainty of $\pm 0.24$ cm$^2$ s$^{-2}$, about $\frac{1}{4}$ of the observed noncoherent tide. At the shelf mooring, mode-0 uncertainties are $\pm 0.05$ cm$^2$ s$^{-2}$, about $\frac{1}{10}$ of the observed noncoherent tide at that location.

When applying the formulas derived by Munk and Cartwright (1966), the uncertainty in the calculation of mode-0 admittance is negligible [e.g., uncertainties are $O(0.01)$ cm$^2$ s$^{-2}$ within the M2 group]. Random instrument noise is also negligible, as evidenced by low energy levels at the frequencies between the noncoherent tidal peaks. We also found that instrument sway does not produce significant errors in mode-0 velocities because the instrument is mounted just above the bottom. Last, velocity errors due to speed of sound fluctuations are corrected via manufacturer (Teledyne RD Instruments) protocols and do not significantly contribute to noncoherent mode-0 velocities.

d. Regional simulations

At both moorings, mode-0 velocities are predominantly coherent, nearly constant (because the tidal potential varies

![Fig. 5. Histograms of total mode-1 sea surface displacement (left) amplitude and (right) phase in the M2 group during each 30-day segment at the (a) slope mooring and the (b) shelf mooring. The stars mark the observed coherent amplitude and phase, where phase is referenced from the coherent phase. The dots mark the MITgcm amplitudes.](image-url)
little over the 3.5-yr record), and similar to those in the regional simulation (Fig. 8). The TXPO8 surface tides, therefore, provide suitable forcing to examine the average $M_2$ internal tide.

The MITgcm simulation suggests that mode-1 energy and mode-0 to mode-1 energy conversion are spatially heterogeneous (Fig. 9). Two-dimensional autocorrelations indicate that mode-1 energy near the slope mooring has along- and across-trough decorrelation length scales (full width at half amplitude) of about 35 and 15 km, respectively. Therefore, the moorings cannot provide information about tidal dynamics beyond these distances.

Because the simulation does not contain variable stratification or mean flows, it does not produce noncoherent internal tides and likely overestimates tidal coherence. The simulation also has a limited numerical domain, which may not encompass every generation site that contributes internal tides to the observational records (Ponte and Cornuelle 2013). With these caveats in mind, the simulations underestimate (overestimate) observed total mode-1 velocities and energy at the slope (shelf) mooring (Figs. 8, 9). In the vicinity of the moorings, the simulated mode-1 energy flux and mode-0 to mode-1 energy conversion are highly spatially variable and have magnitudes that are similar to those observed (Fig. 9), although the simulated values and observations do not agree well at the precise locations of the moorings.

The MITgcm simulations, along with previous surface-tide simulations, provide a basic picture of regional tidal dynamics. Surface-tide simulations by Ray et al. (2005) indicate that an energetic mode-0 tide propagates eastward across the Indian Ocean and bifurcates at the island of Timor. Most regional internal-tide energy and generation are concentrated in the straits north of Timor (Fig. 9). However, approximately $600 \text{ kW m}^{-2}$ of eastward mode-0 energy flux passes through the Timor Trough. Eastward mode-0 propagation is consistent with the observed slope mooring ellipses, which are oriented in the along-trough direction (Fig. 8). The MITgcm simulation indicates that the eastward-propagating mode-0 tide scatters to higher modes as it passes over three-dimensional sloping topography in the trough. Mode-1 generation on the slope is consistent with observed mode-1 velocity ellipses that are oriented across the slope (Fig. 8).

e. Tidal variability

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<th>Table 1. Observed tidal variances.</th>
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<td><strong>Total</strong></td>
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<td><strong>Slope mooring (465 m)</strong></td>
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<td>$u_0$ ($\text{cm}^2 \text{s}^{-2}$)</td>
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<td><strong>Shelf mooring (145 m)</strong></td>
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At both moorings, coherent mode-0 and mode-1 energy show strong spring–neap variability, while total mode-1 energy varies more dramatically and less predictably (Figs. 10, 11). Mode-1 eigenspeed and barotropic mean flow (calculated from average conditions over the 2-day segments) vary over multiple time scales. Mode-1 eigenspeeds are typically much larger than background velocities, except during low stratification periods at the shelf mooring (Figs. 11d,e).

Harmonic analysis identifies significant annual variability in the noncoherent Fourier coefficients and background fields (Table 2). The largest annual trends appear in noncoherent mode-1 pressure and mode-1 eigenspeed at the shelf mooring, where mode-1 energy is visibly enhanced during strong summer stratification. After removing the annual harmonics, noncoherent mode-1 surface displacement variance is equally explained by amplitude modulation (53% and 50% at the shelf and slope moorings, respectively) and amplitude-weighted phase modulation (47% and 50% at the shelf and slope moorings, respectively) based on the decomposition defined in Shriver et al. (2014). Total semidiurnal mode-1 energies have spectral shapes that are similar to those of the barotropic mean flow and mode-1 eigenspeed (Fig. 12). Conversely, the spectra of coherent mode-1 energies are dominated by discrete peaks associated with the beat frequencies of different tidal constituents. Tidal spectra at both moorings have peaks at 2 cpm, the frequency of the M_2 and S_2 spring–neap cycle.

At both moorings, autocorrelations indicate that mode-1 sea surface displacement, barotropic mean flow, and mode-1 eigenspeed have decorrelation time scales of 4–7 days (Fig. 12). The decorrelation time scales of mode-0 and mode-1 semidiurnal velocities (not shown) are similar to those of mode-1 sea surface displacement. Self-prediction of noncoherent mode-1 semidiurnal sea surface displacement using the response method also indicates a 3–5-day decorrelation time scale (Fig. 13). Secondary peaks, at around 15 days, in the mode-1 sea surface displacement autocorrelation (Fig. 12) and noncoherent self-prediction curves (Fig. 13) are associated with the spring–neap cycle, which is more pronounced at the shelf mooring than at the slope mooring.

Mitchum and Chiswell (2000) identified a positive correlation between M_2 mode-1 sea surface displacement...
amplitude and interannual-mean sea level at the Hawaiian Ridge. Subsequently, Chiswell (2002) identified a negative correlation between $M_2$ mode-1 amplitude and the 60–120-day mean sea level at the Hawaiian Ridge. Here, after low-pass filtering the data at 1 cpm, we find that total semidiurnal mode-1 sea surface displacement amplitude and mode-1 eigenspeed (which is positively correlated with mean sea level; Mitchum and Chiswell 2000) are significantly positively correlated ($r = 0.70, \text{ where } r_{95} = 0.31$ indicates 95% significance) at the shelf mooring, but are not significantly correlated at the slope mooring (Fig. 14). Additional analysis does not identify significant correlations between tidal amplitudes and the barotropic mean flow.

At the slope mooring, there is also a significant positive correlation ($r = 0.4$) between the noncoherent along-slope mode-0 velocity amplitude and the across-slope mode-1 velocity amplitude (Fig. 15). Other correlations between different combinations of along- and across-slope velocities at the slope and shelf moorings are insignificant.

4. Discussion

a. Quantifying noncoherent tides

Spectral analyses of the mooring data reveal practical information about quantifying the noncoherent tide. First, mode-0 and mode-1 noncoherent tides in the Timor Sea are one to two orders of magnitude more energetic than the background continuum. Therefore, there is some flexibility when designing a bandpass filter to isolate tidal energy. For instance, one will measure similar amounts of noncoherent variance in a time series whether it is bandpassed from 1.75 to 2.25 cpd or 1.25 to 2.75 cpd. Alternatively, the background continuum can be patched across the tidal bands and removed altogether (Munk and Bullard 1963).

Second, noncoherent tides have wider, smoother peaks than coherent tides, suggesting noncoherent tides forced at each constituent are smeared into surrounding groups. For example, there is almost as much noncoherent energy where forcing is weak (e.g., between the $M_2$ and $S_2$ groups) as there is where forcing is strong (e.g., in the $M_2$ and $S_2$ groups). As a result, estimating
noncoherent energy from a narrowband integral across the M2 group (i.e., $f_{M2} \pm 0.5$ cpm) will underestimate the total noncoherent energy associated with M2 forcing. Moreover, noncoherent peaks that are forced at neighboring constituents overlap, so one must either carefully separate the cusps for each group/constituent (Colosi and Munk 2006) or simply quantify noncoherent energy for an entire tidal species. Here, we choose the latter. Expanding the tidal bandwidth from the M2 group to the semidiurnal species increases the ratio of noncoherent-to-total energy by anywhere from 2% to 23% (Table 1).

Third, observed admittances vary slightly from group to group, indicating that, for example, the ocean responds differently to M2 and S2 forcing. Therefore, accurate estimates of the coherent spring–neap cycle require admittances for each group. If an observational record is too short to accurately determine admittances in each group, the spring–neap cycle cannot be properly separated into coherent and noncoherent components. In this sense, time series $\geq$ 6 months are preferable because admittances at the major semidiurnal groups, M2 and S2, can be computed from an ensemble of 12 (or more) 15-day segments, where 15 days is the minimum record length needed to separate the M2 and S2 groups.

### b. Noncoherent dynamics on the shelf

Low-frequency changes in stratification on the shelf have a first-order impact on the observed mode-1 internal tide (Fig. 11). In particular, mode-1 pressure, which can only exist when there is stratification, has a strong seasonal cycle (Table 2) and is highly correlated to low-frequency changes in mode-1 eigenspeed (Fig. 14).

The short (4–7 day) decorrelation time scale of the mode-1 tide on the shelf is similar to that of the barotropic mean flow and mode-1 eigenspeed (Fig. 12). It is not obvious why mode-1 tides are correlated with background conditions (i.e., mode-1 eigenspeed) over long, but not short, time scales. One possibility is that background conditions over long time scales have large spatial extents, which may extend across the region relevant for producing the observed internal tides. Conversely, background conditions over short time scales may be associated with eddies, fronts, and meanders that only affect a portion of the relevant region. As a result, internal tides generated a few kilometers from the mooring may be modulated by background anomalies that are statistically similar, but not precisely the same as those observed at the mooring.
There is evidence that suggests noncoherent mode-1 energy at the shelf mooring is primarily generated and modulated a few tens of kilometers from the mooring. First, the MITgcm simulation predicts generation on the walls of the small local canyon (Fig. 9f). Generation at these locations is consistent with mode-0 ellipses that are perpendicular to the canyon axis (Fig. 8b) and observations of negative internal-tide generation in the base of the canyon (Fig. 11), which arises because of remotely generated mode-1 pressure perturbations (e.g., Kurapov et al. 2003; Kelly and Nash 2010). Second, the noncoherent tide has a strong spring–neap cycle indicating it is made up of internal tides that are the same “age” and are likely generated in a single region (Holloway and Merrifield 2003). Third, the simulation indicates that the shelf mooring is surrounded by regions of low mode-1 energy, suggesting that internal tides generated in the Timor Trough do not efficiently transit the broad shallow continental shelf (Fig. 9).

c. Noncoherent dynamics on the slope

The annual cycle in mode-1 energy at the slope mooring is weak because internal-tide generation and potential energy in the deep ocean depend on the permanent thermocline (e.g., Gerkema et al. 2004). The lack of a clear spring–neap signal in the noncoherent tide and the presence of intermittent negative internal-tide generation (see section 4b) are consistent with the combined presence of local and remotely generated mode-1 tides. Again, we speculate that the short decorrelation time scale of the internal tide is due to mesoscale variability with small horizontal scales.

Noncoherent mode-1 tides cannot be attributed solely to their modulation during propagation because internal-tide propagation and generation are coupled in the Timor Trough. An internal tide generated on one flank can propagate across the trough and alter generation on the opposing flank (e.g., Kelly and Nash 2010).
For instance, analytical solutions indicate that, depending on its width, a top-hat trench can be resonant with respect to internal-tide generation (St. Laurent et al. 2003). The precise resonance conditions (if they exist) for the Timor Trough are unknown and nontrivial because they depend on the three-dimensional geometry and variable slopes of the trough flanks [e.g., see the semi-idealized numerical experiments of Drijfhout and Maas (2007)]. However, we do not observe spectral peaks in mode-1 energy that might indicate internal-tide resonance. The lack of sharp peaks in tidal admittance is consistent with Munk and Cartwright’s (1966) empirical “credo of smoothness,” which rejects the possibility of tidal resonance. However, diurnal mode-1 admittances are anomalously larger than semidiurnal mode-1 admittances (Figs. 3, 6), which may be at least partially due to trough geometry. Kinetic to potential energy ratios $R$ vary between the semidiurnal groups, indicating the presence of frequency-dependent partially standing waves (e.g., Nash et al. 2004). For example, $R$ in the $M_2$ group is 1.3, similar to that of a propagating plane wave, $(\omega^2 + f^2)/(\omega^2 - f^2) = 1.1$, but $R$ in the $S_2$ group is 4.2, consistent with a partially standing wave (note that PE is small because mode-1 sea surface displacement does not peak in the $S_2$ group; Fig. 3). Last, the correlation between the low-passed noncoherent mode-0 and mode-1 velocities indicates that local tide–topography coupling is partially noncoherent (Fig. 15). Although, it is impossible to say whether noncoherent mode-0 tides generate noncoherent mode-1 tides or vice versa (e.g., Kelly et al. 2013).

d. Global impacts of noncoherent generation

Because mode-0 tides can propagate more than 30,000 km during their lifespan (Munk 1997), their energy budgets depend on global integrals of internal-tide

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**Fig. 10.** Time series of total (black line) and coherent (gray shading) semidiurnal (a) mode-0 kinetic energy, (b) mode-1 energy, and (c) mode-0 to mode-1 conversion at the slope mooring. Mode-1 eigenspeed and (d) across-isobath (black) and (e) along-isobath (gray) barotropic mean flow. Isobath orientations are shown in Fig. 8; positive across-slope velocity is in the direction of shallower water.
generation. A global increase in mode-0 to mode-1 conversion of 1 mW m\(^{-2}\) could, ignoring feedbacks, deplete all mode-0 energy (4 \times 10^5 TJ or 1100 J m\(^{-2}\)) in about 2 weeks. However, point measurements of mode-1 to mode-0 conversion indicate that variability is much larger than \(\pm 1\) mW m\(^{-2}\), while mode-0 energy is approximately constant. For example, semidiurnal mode-0 to mode-1 conversion at the slope mooring varies by \(\pm 30\) mW m\(^{-2}\) (Fig. 10), and M\(_2\) conversion at the Hawaiian Ridge varies by \(\pm 5\)–100 mW m\(^{-2}\) (Zilberman et al. 2011; Powell et al. 2012). We therefore hypothesize that significant local temporal variability in mode-0 to mode-1 conversion must be completely uncorrelated at spatial scales large enough to influence the mode-0 tides. This conclusion is consistent with observations and numerical simulations that indicate variability in mode-0 to mode-1 conversion is spatially heterogeneous, both positive and negative (e.g., Kelly and Nash 2010; Nash et al. 2012a), and typically arises from random mesoscale variability (e.g., Zilberman et al. 2011; Powell et al. 2012).

5. Summary

Coherence analysis of tides in the Timor Sea indicates the following:

(i) Noncoherent tidal peaks in each species have a well-defined bandwidth (i.e., \(\Delta f \approx 7\) cpm or a 4-day time scale) and are much larger than the background continuum.

(ii) The total noncoherent variance is more easily quantified for tidal species than constituents because noncoherent cusps at neighboring constituents overlap because of their finite bandwidth (i.e., \(\Delta f \approx 2\)–4-cpm or a 7–14-day time scale).

<table>
<thead>
<tr>
<th>TABLE 2. Fraction of variance explained by an annual harmonic fit to the complex amplitude of the noncoherent tide, barotropic mean flow, and mode-1 eigenspeed (all calculated over four M(_2) periods).</th>
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<tbody>
<tr>
<td>Slope mooring</td>
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<td>Shelf mooring</td>
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(iii) Where $M_2$ and $S_2$ admittances differ, the coherent spring–neap cycle is most accurately extracted from long (e.g., ≥6 month) time series.

In situ measurements from the Timor Sea indicate that both mode-0 and mode-1 tides are predominantly coherent with the tidal potential. Noncoherent variance in mode 1 is much greater than that in mode 0, supporting the approximation that surface tides are entirely coherent (e.g., Mitchum and Chiswell 2000; Colosi and Munk 2006). As observed by Ray and Zaron (2011), noncoherent mode-1 sea surface displacement variances are measurable, but less than 1 cm$^2$ (Table 1). Mode-1 semidiurnal velocities are 32% and 66% noncoherent at the slope and shelf moorings, respectively. These values are consistent with those of Nash et al. (2012b), who used 90-day harmonic fits to estimate that total internal-tide velocities (not just mode 1) were 41% and 60% noncoherent at the slope and shelf moorings, respectively. Nash et al. (2012b) also found that the slope mooring in the Timor Sea had the most predictable internal tides of the 16 coastal locations that they examined. Therefore, the observations from the slope mooring may be more representative of the deep ocean than continental shelves.

Like Katsumata et al. (2010), we have observed that internal tides in the Timor Sea are temporally variable. Our central findings are as follows:

(i) At both moorings, noncoherent tides have 4–7-day decorrelation time scales, which are consistent with

![Graph](image_url)
those of the barotropic mean flow and mode-1 eigenspeed.

(ii) On the shelf, noncoherent tides are positively correlated with seasonal and low-frequency changes in stratification, have a strong spring–neap cycle, and are likely generated locally.

(iii) On the slope, noncoherent tides do not have a strong spring–neap cycle and are likely generated by complicated tide–topography coupling in the Timor Trough.

(iv) At the slope mooring, noncoherent mode-0 and mode-1 velocities are correlated.

(v) At both moorings, local mode-0 to mode-1 energy conversion is temporally variable but does not noticeably modulate mode-0 energy.

Presently, the response method (using the tidal potential and self-prediction) can be used to forecast about 75% of semidiurnal mode-1 variance 2.5 days in advance (Table 1; Fig. 13). Because noncoherent internal tides in the Timor Sea have short decorrelation time scales and complicated dependencies on topography, improved internal-tide predictions will require models that include realistic tide–topography coupling and background variability with both short and long time scales.

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REFERENCES


