Modelling the Impact of Squall on Wind Waves with the Generalized Kinetic Equation

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ABSTRACT

This is a first study of short-lived transient sea states, arising from fast variations of wind fields. This study considers the response of a wind-wave field to a sharp increase of wind over a short time interval (a squall). Conventional wind-wave models based on the Hasselmann equation assume quasi stationarity of a random wave field and are a priori inapplicable for such transient states. To describe fast spectral changes, the authors use the generalized kinetic equation (GKE) derived without the quasi-stationarity assumption. A novel efficient highly parallelized algorithm for the numerical simulation of the GKE is presented. Simulations with the GKE and the Hasselmann equation are examined and compared. While under steady wind, the spectral evolution in both cases is shown to be practically identical, but after the squall the qualitative difference emerges: the GKE predicts formation of a transient sea state with a considerably narrower peak.

1. Introduction

At present the intrinsic variability of winds over the sea at time scales of tens of minutes is not taken into account in wind-wave modeling. In particular, very little is known about transient sea states with characteristic life spans of hundreds of dominant wave periods, resulting from sharp changes of wind. Wind squalls over the sea fall into this range of time scales and are an important natural hazard, with poorly understood implications for wind waves. The unpredictability of squalls makes them particularly dangerous in lakes where vessels are not designed to withstand suddenly increased waves. One such incident is described in the Gospels (Mark 4:37); in recent years the "white squall" phenomenon, especially common on the Great Lakes, has been blamed for a few tragedies. Experimental study of such short-lived states in the sea is extremely difficult since the events are rare and hence not reproducible. For example, in the observations reported by Bastianini et al. (2012), wind increased from approximately 15 to 42 m s^{-1} in less than 5 min and lasted about 20 min, while the significant wave height increased sixfold. To model such events there have been no adequate tools. At present all wave modeling is based on the numerical integration of the kinetic (Hasselmann) equation for the energy spectra of wave fields. The equation takes into account wind input, dissipation, and interaction between waves of different scales and directions and describes the slow evolution of wind-wave spectra in time and space (Komen et al. 1994; Janssen 2004). In terms of wave action density $n(\mathbf{k}, \mathbf{x}, t)$, where $\mathbf{k}$ is the wavevector, while $\mathbf{x}$ and t are "slow" spatial and temporal variables, it can be written as

$$\frac{dn(\mathbf{k},\mathbf{x},t)}{dt} = S_{\text{input}} + S_{\text{diss}} + S_{\text{nl}},$$

where S_{input}, S_{diss}, and S_{nl} describe contributions to dn/dt due to a variety of physical processes grouped as input, dissipation, and nonlinear interactions, respectively. The interaction term S_{nl} is dominant for energy-carrying waves (Zakharov and Badulin 2011). The expression for S_{nl} is derived under the assumption that the random wave field in hand has a slow $O(\varepsilon^{-4})$ time scale, which allows wind forcing to vary only on the same or larger time scale. Therefore, strictly speaking, the Hasselmann equation is not applicable to situations with rapid changes of wind. Because of the lack of alternatives, this

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difficulty is usually ignored, and the Hasselmann theory (Hasselmann 1962) is currently being used beyond the domain of its applicability. In the detailed study by Young and van Agthoven (1998), the Hasselmann equation was used to model the response to a sharp increase or decrease of wind. It is not clear to what extent these results can be trusted since such an increase clearly violates the equation conditions of validity. It is worth noting that both sea (van Vledder and Holthuijsen 1993) and wave tank observations (Autard 1995; Waseda et al. 2001) and direct numerical simulations (Annenkov and Shrira 2009) show a faster response of a wave field to an instant perturbation, which suggests a discrepancy with the strict $O(\varepsilon^{-4})$ evolution time scale embedded into the Hasselmann equation. One of the motivations for this work is to quantify applicability of the Hasselmann equation to changing winds. We specifically address the spectral response of a wind-wave field to a squall, defined as a sharp increase of wind speed, which lasts for a short time period before the wind speed returns to near its previous value. Our aim is to perform the modeling of wave field under squall with the generalized kinetic equation (GKE), which was derived without the quasi-stationarity assumption (Annenkov and Shrira 2006). The GKE differs from the Hasselmann equation by the form of S_{nl}, which for the GKE is nonlocal in time. In contrast to the Hasselmann equation, the GKE includes not only resonant wave–wave interactions but all nonresonant interactions as well, although only those not too far from resonance contribute significantly to spectral evolution. The first attempt at the numerical solution of the GKE was made by Gramstad and Stiassnie (2013), who have obtained the evolution of model spectra without wind forcing or dissipation and demonstrated the conservation of the invariants. Gramstad and Babanin (2014) performed preliminary numerical simulations that compared the GKE with the Hasselmann equation in the absence of dissipation and wind. Here, we present a new efficient and highly parallelized algorithm for the numerical simulation of the GKE and perform a direct comparison of simulations of a squall with the parallel simulations of the Hasselmann equation. We show that, as expected, under steady wind the wave field evolution predicted by the GKE and Hasselmann equation practically coincides. Then, a squall is modeled by an instant increase of wind, by a factor of 2–4, without changing its direction, followed by the return to the initial wind speed after $O(10^2)$ characteristic wave periods. We find that during the squall, the spectral evolution obtained with the GKE is close to that obtained with the Hasselmann equation. However, after the end of the squall, the GKE shows that a transient spectrum is formed, which has a considerably narrower peak than the Hasselmann equation predicts. This transient spectrum predicted by the GKE exists for a few hundred characteristic wave periods after the squall.

2. Theoretical background

We consider gravity waves on the surface of deep ideal fluid, described in Fourier space by the physical variables $\zeta(\mathbf{k}, t)$ and $\varphi(\mathbf{k}, t)$ (position of the free surface and the velocity potential at the surface, respectively), where $\mathbf{k}$ is the wavevector and t is time. Statistical description of the wave field is sought in terms of correlators of complex amplitude $b(\mathbf{k}, t)$, linked to $\zeta(\mathbf{k}, t)$ and $\varphi(\mathbf{k}, t)$ through an integral power series (Krasitskii 1994), assuming that wave slopes are $O(\varepsilon)$ small. The classical derivation (e.g., Zakharov et al. 1992) leads to the kinetic (Hasselmann) equation for the second statistical moment (spectrum) $n(\mathbf{k}, t)$:

$$\frac{\partial n_0}{\partial t} = 4\pi \int T_{0123}^2 f_{0123} \delta_{0+1-2-3} \delta(\Delta\omega)\, d\mathbf{k}_{123} + S_f\,, \tag{1}$$

where $n_0 = \langle b_0^* b_1 \rangle = n_0 \delta_{0-1}$, angular brackets mean ensemble averaging, $f_{0123} = n_2 n_3(n_0 + n_1) - n_0 n_1(n_2 + n_3)$, $\omega(\mathbf{k}) = (gk)^{1/2}$ is the linear dispersion relation, g is gravity, $k = |\mathbf{k}|$, $\Delta\omega = \omega_0 + \omega_1 - \omega_2 - \omega_3$, δ is the Dirac delta function, S_f is the forcing/dissipation term, and integration is performed over the entire $\mathbf{k}$ plane. The compact notation used designates the arguments by indices, for example, $T_{0123} = T(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$, $\delta_{0+1-2-3} = \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3)$, and so on. The derivation of Eq. (1) assumes a random wave field proximity to Gaussianity and stationarity since its derivation includes taking a large time limit. Without the latter approximation, the GKE derived by Annenkov and Shrira (2006) reads

$$\frac{\partial n_0}{\partial t} = 4\mathrm{Re} \int \left\{ T_{0123}^2 \left[\int_0^t \mathrm{e}^{-\mathrm{i}\Delta\omega(\tau - t)} f_{0123}\, \mathrm{d}\tau \right] - \frac{\mathrm{i}}{2} T_{0123} J_{0123}^{(1)}(0) \mathrm{e}^{\mathrm{i}\Delta\omega t} \right\} \delta_{0+1-2-3}\, d\mathbf{k}_{123} + S_f\,. \tag{2}$$

Here, $J_{0123}^{(1)}(0)$ is the initial value of the fourth-order cumulant, usually assumed to be zero at the start of the evolution. During the evolution, small but nonzero correlators emerge because of nonlinear interactions, so that

$$J^{(1)}_{0123}(t) = 2\mathrm{i}T_{0123}\int_0^t e^{-\mathrm{i}\Delta\omega(\tau-t)}f_{0123}\,d\tau + J^{(1)}_{0123}(0)e^{\mathrm{i}\Delta\omega t}.$$

Unlike the Hasselmann Eq. (1), which takes into account exact resonances only, the GKE Eq. (2) includes all interactions, although only those not too far from resonance contribute to spectral evolution. The GKE reduces to the Hasselmann equation in the large time limit $t\to\infty$.

3. Numerical algorithm

Although the GKE [Eq. (2)] is nonlocal in time, it can be solved iteratively by specifying the current value of $J^{(1)}_{0123}$ as the new initial condition, so the time integration is performed over the current time step only. The right-hand side is computed over all interacting quartets $\mathbf{k}_0 + \mathbf{k}_1 = \mathbf{k}_2 + \mathbf{k}_3$, where $\mathbf{k}_j$, $j = 0, 1, 2$ are chosen at the grid points, and the value of the amplitude for k_3, which is required to lie inside the grid but not necessarily at one of the grid points, is found by bilinear interpolation. For the simulations reported below the computational grid has 101 logarithmically spaced points in the range $0.5 \le \omega \le 3$ and 31 uniformly spaced angles $-2\pi/3 \le \theta \le 2\pi/3$. The high resolution of the grid was chosen to study the effects of the squall in detail; the analysis of required resolutions and of other properties of the algorithm will be presented elsewhere. All interactions satisfying $\Delta\omega/\omega_{\min} \le 0.25$, where $\omega_{\min}$ is the minimum frequency of waves within the interacting quartet, were taken into account. Thus, the algorithm accounts for all resonant and approximately resonant interactions, allowing a large mismatch. The total number of interactions exceeds 3×10^9. Time stepping is performed by the Runge–Kutta–Fehlberg algorithm with absolute tolerance 10^{-10} and the time step limited from above by the approximately ⅓ characteristic wave period. A typical computation takes 1–3 days on 64 computational cores. Initial conditions were specified as the Donelan et al. (1985) spectra for $2 \le U_{10}/c \le 5$, where c is the phase speed of the spectral peak, and U_{10} is wind at 10 m; wind forcing is according to Hsiao and Shemdin (1983). The specific choice of the wind generation model is of little importance here, since our focus is on the nonlinear interactions. The spectral evolution was traced with the same initial value of U_{10}/c for a few hundred characteristic wave periods, then an instant increase of wind forcing was applied, simulating the squall, for approximately 100 periods, after which the wind forcing was set back at the initial value.

Simulations of the Hasselmann Eq. (1) were performed using the Gurbo Quad 5 set of subroutines based on the Webb–Resio–Tracy (WRT) algorithm (van Vledder 2006), kindly provided by Gerbrant van Vledder. The same computational grid was used with the same maximum time step and absolute tolerance (10^{-6}) and the same initial conditions. To perform a more detailed and accurate comparison of the simulations with the GKE and Hasselmann equation, in a number of runs the spectra obtained with the GKE were used as initial conditions for the Hasselmann equation.

4. Results

First, the new algorithm for the numerical integration of the GKE was thoroughly validated on a number of model situations without forcing or dissipation, with the analysis of the invariants conservation properties and the large time asymptotics of the spectral evolution. The results of these preliminary simulations are not reported in this paper. Here, for the algorithm validation, we consider the development of the wave spectrum under the action of constant wind and compare the results with the simulations of the Hasselmann equation for the same forcing and dissipation and the same initial conditions. In Fig. 1a, results of the simulations for the GKE and Hasselmann equation are shown for a rather high wind (initially $U_{10}/c = 5$). Evolution is traced for approximately 500 characteristic wave periods. The results of the numerical simulation of the two equations are in close agreement, with a slightly faster downshift of the spectral peak in the case of the Hasselmann equation. However, in both cases the evolution tends to the theoretical asymptotic of the spectral peak $k_p = t^{-6/11}$ (Fig. 1b).

Our primary interest is in the effects of the squall, here understood as an instant increase of wind (start of squall) followed by its sharp decrease back to the initial value after $O(10^2)$ characteristic wave periods (end of squall). Since the GKE is nonlocal in time, its numerical modeling is performed as one continuous computation under changing wind conditions. Numerical simulation of the Hasselmann equation can be performed in the same way, or, to facilitate comparison between the two equations, the spectra obtained with the GKE at the start and the end of the squall can be used as initial conditions for the Hasselmann equations. In this study, we use both approaches.

In this way, we consider the development of the wind-wave field under moderate constant wind forcing ($U_{10}/c = 3$ initially) for approximately 800 wave periods, followed by an instant increase of wind to $U_{10}/c = 7.5$ for approximately 100 periods. Simulations with the Hasselmann equation during the squall are performed with the initial condition corresponding to the numerical solution of the GKE at the start of the squall. Since the

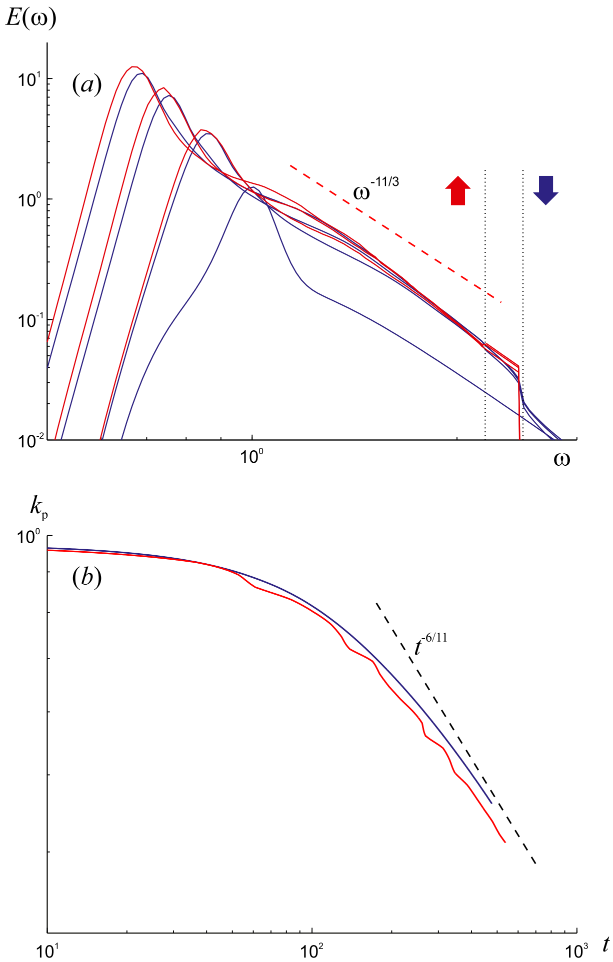

FIG. 1. (a) Comparison of the energy spectra $E(\omega)$ obtained by the numerical solution of the GKE (blue curves) and the Hasselmann equation (red curves) under constant wind forcing. Initial condition is taken as an empirical spectrum by Donelan et al. (1985) for $U_{10}/c = 5$, wind forcing (red arrow) corresponds to $U_{10}/c = 5$ at the initial moment, dissipation (blue arrow) is applied to $\omega > 2.5$, initial peak is at $\omega = 1$, $k = 1$, and gravity $g = 1$. Spectra are plotted every 160 characteristic periods. (b) Evolution of the wavenumber of the spectral peak, obtained with the GKE (blue) and with the Hasselmann equation (red). Time is measured in periods of the spectral peak of the initial condition.

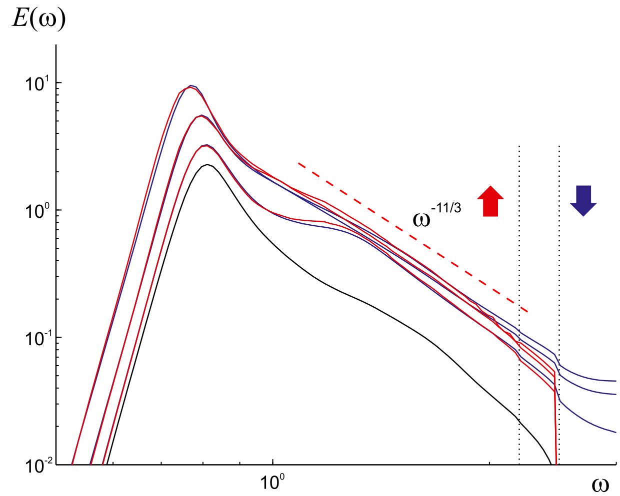

FIG. 2. Comparison of the GKE (blue curves) and Hasselmann equation (red curves) solutions during the squall. Wind forcing is instantly increased from $U_{10}/c = 3$ to $U_{10}/c = 7.5$. Initial condition for the Hasselmann equation is taken as the GKE spectrum at the beginning of the squall. Spectra are plotted every 32 characteristic periods.

squall instantly drives the wave system out of equilibrium with forcing, it is reasonable to expect that the simulations with the GKE and the Hasselmann equation give considerably different results since this nonequilibrium situation is clearly beyond the domain of applicability of the Hasselmann equation. However, the results shown in Fig. 2 demonstrate that the agreement between the two equations is nearly perfect.

After approximately 100 characteristic periods, the wind drops to its value before the squall. The subsequent evolution obtained with both equations is plotted in Fig. 3. Both equations demonstrate that the spectrum changes its form, developing a wide and flat "gap" on the slope at slightly higher frequencies than the frequency of the spectral peak. However, the GKE spectrum develops a considerably narrower peak. Here, the width of the peak is understood as the width of the spectrum at half the peak amplitude. As shown in Fig. 4a, the squall causes the peak of the spectrum to become narrower, this effect being captured by both equations. However, immediately after the end of the squall the peak of the spectrum obtained with the GKE continues to narrow down, while within the Hasselmann equation the peak widens considerably. This discrepancy between the two equations persists for about a hundred characteristic wave periods.

To compare the form of the spectrum obtained with different equations, it is instructive to perform a fit of the obtained spectrum to the JONSWAP spectral parameters α and γ. The dependence of γ on time during the evolution of the spectra obtained with both equations is shown in Fig. 4b. The squall causes the increase of γ in both cases, and after the end of the squall, γ continues to increase for a few dozen wave periods. However, this increase is much sharper for the spectrum obtained with the GKE. The angular spectra, prescribed at the initial moment according to Donelan et al. (1985) by the $\mathrm{sech}^2\beta\theta$ distribution, remain similar when simulated by both equations.

5. Concluding remarks

In this paper, we report the key first step toward the modeling of short-lived transient sea states. It has been demonstrated that by employing the GKE, which allows

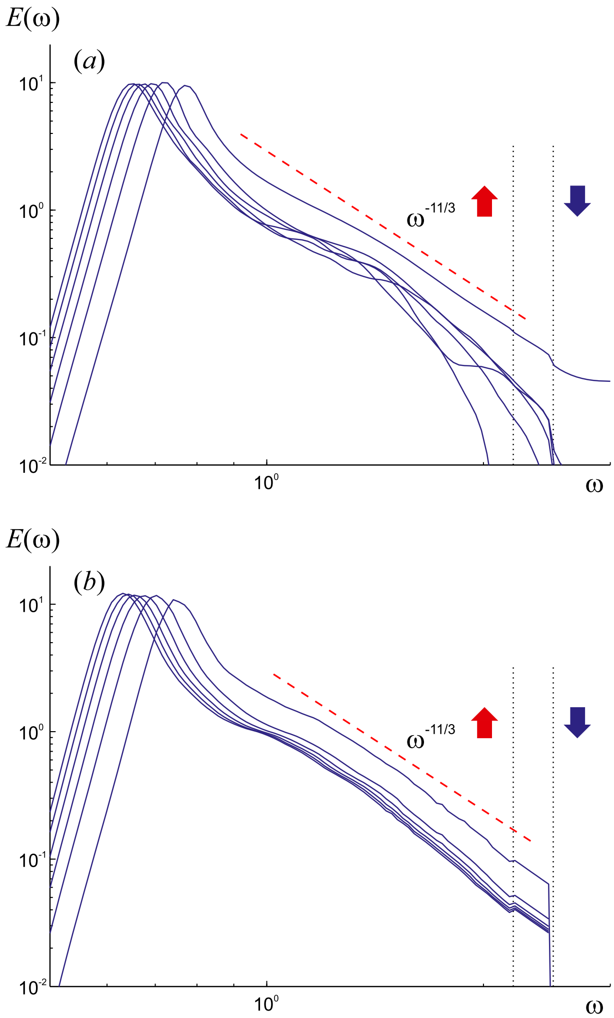

FIG. 3. Evolution of energy spectrum $E(\omega)$ after the squall, obtained by the numerical solution of (a) the GKE and (b) the Hasselmann equation. Spectra are plotted approximately every 60 characteristic periods.

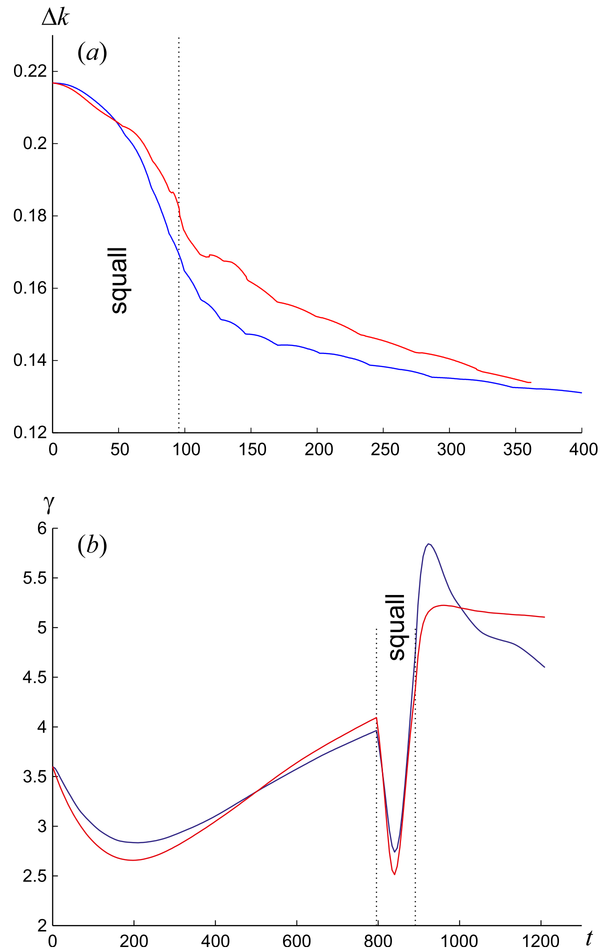

FIG. 4. (a) Evolution of spectral width during and after the squall, with the GKE (blue) and the Hasselmann equation (red). Initial condition in both cases corresponds to the GKE solution at the start of the squall. (b) Evolution of peakedness parameter γ with the GKE (blue) and the Hasselmann equation (red).

the fast dynamic of wave fields and takes into account nonresonant interactions, it is possible to simulate their evolution quite effectively. The simulations aimed at modeling wave field dynamics during an idealized squall event, apart from being the first simulations of squall, brought in unexpected results. First, both the field evolution under a moderate wind and the rapid growth of the wave spectrum during the squall as modeled with the GKE proved to be nearly identical to the predictions made with the Hasselmann equation. While such an agreement was expected for steady wind situations, the negligible discrepancy found during the rapid growth contradicts earlier direct numerical simulations (Annenkov and Shrira 2009). A direct comparison for the same initial conditions and spectral resolution is not possible at present, and hence there is a fundamental open question that requires a dedicated study. At the rapid growth stage we also observed a peculiar shape of the spectra with a dip on the spectral slope not reported in the literature. Second, counterintuitively, the predictions of the GKE and the Hasselmann equation diverge qualitatively just *after* the squall, that is, when the wind drops to its presquall level; the GKE predicts noticeably more narrow spectra characterized by a substantially higher peakedness parameter γ. At present there are no observations allowing a detailed quantitative test of the simulations. In the future the most perspective way to test the simulations against reality is to examine specially designed tank observations in the spirit of Autard (1995) and Waseda et al. (2001); however, a quantitative comparison is not straightforward and a sustained effort from two sides is required to bridge the gap between the simulations and observations.

The results suggest, and this is one of the main unexpected implications of the work, that the Hasselmann equation has a much wider range of applicability than follows from the assumptions adopted for its derivation. This adds solidity to the already existing simulations with the Hasselmann equation carried out outside the range of its formal validity, for example, in Young and van Agthoven (1998). Probably the most far-reaching implications stem from the demonstrated efficiency of the proposed highly parallel algorithm for solving the GKE. We have got the tool not only for studying a variety of short-lived transient processes; in the long term, it has a potential to replace the Hasselmann equation as the basis of wave modeling. Since the GKE algorithm is highly parallel, and has a lot of room for efficiency improvement, with a sufficient number of parallel processors it could be even made much faster than the existing codes for the Hasselmann equation.

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