Residual Transport of Suspended Material by Tidal Straining near Sloping Topography

KIRSTIN SCHULZ AND LARS UMLAUF
Leibniz-Institute for Baltic Sea Research, Warnemünde, Germany

(Manuscript received 11 November 2015, in final form 14 March 2016)

ABSTRACT

Tidal straining is known to have an important impact on the generation of residual currents and the transport of suspended material in estuaries and the coastal ocean. Essential for this process is an externally imposed horizontal density gradient, typically resulting from either freshwater runoff or differential heating. Here, it is shown that near sloping topography, tidal straining may effectively transport suspended material across isobaths even if freshwater runoff and differential heating do not play a significant role. A combined theoretical and idealized modeling approach is used to illustrate the basic mechanisms and implications of this new process. The main finding of this study is that, for a wide range of conditions, suspended material is transported upslope by a pumping mechanism that is in many respects similar to classical tidal pumping. Downslope transport may also occur, however, only for the special cases of slowly sinking material in the vicinity of slopes with a slope angle larger than a critical threshold. The effective residual velocity at which suspended material is transported across isobaths is a significant fraction of the tidal velocity amplitude (up to 40\% in some cases), suggesting that suspended material may be transported over large distances during a single tidal cycle.

1. Introduction

Subtidal circulation and residual transport of suspended material in estuaries and the coastal ocean is known to be tightly connected to the presence of horizontal density gradients, generated by either river runoff, differential heating, or differential evaporation. The most obvious dynamical implication of such horizontal density differences is the generation of a gravitationally driven residual circulation with a landward transport of dense waters near the bottom and a seaward return current above (Pritchard 1952; MacCready and Geyer 2010).

Residual currents may also be triggered by periodic variations of stratification induced by the oscillatory vertical shear of a tidal flow acting on the horizontal density gradient. This mechanism leads to a periodic “tidal straining” of the horizontal density gradient (Simpson et al. 1990), resulting in weak or even convectively unstable stratification during flood and therefore in significantly larger turbulent diffusivities compared to the more stably stratified ebb period (Figs. 1a,c). Jay and Musiak (1994) proposed that this tidal asymmetry in turbulent mixing may have a profound impact on the tidally averaged momentum budget. They pointed out that the stronger vertical homogenization of momentum during the more turbulent flood phase leads to a residual current that augments the gravitationally driven residual circulation (see MacCready and Geyer 2010). Exploring the physically relevant parameter space with a one-dimensional numerical model, Burchard and Hetland (2010) concluded that the residual circulation due to tidal straining typically dominates over the gravitational circulation in tidally energetic environments.

Whereas the residual circulation described above largely determines the horizontal transport of dissolved substances, additional effects have to be considered when modeling the residual transport of suspended particulate matter (SPM) with a sinking motion relative to the moving fluid. During the more turbulent flood phase (Fig. 1a), SPM concentrations are typically larger due to enhanced erosion, and suspended material is mixed up higher into the water column compared to the less turbulent ebb phase (Fig. 1c). The correlation between these asymmetries in concentration and the oscillating current results in a “tidal pumping” mechanism.

Corresponding author address: Kirstin Schulz, Department of Physical Oceanography, Leibniz-Institute for Baltic Sea Research, Seestraße 15, 18119 Warnemünde, Germany.
E-mail: kirstin.schulz@io-warnemuende.de

DOI: 10.1175/JPO-D-15-0218.1

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that induces a net landward transport of suspended material (Uncles et al. 1985; Jay and Musiak 1994; Scully and Friedrichs 2007). Using a one-dimensional coupled sediment transport model, Burchard et al. (2013) showed that, for a broad range of parameters, tidal pumping is much more effective in transporting suspended material compared to advection by the residual current. This finding is in line with Scully and Friedrichs (2003), who pointed out that the transport due to tidal straining may even adverse the residual advective transport.

A related periodic straining process, not requiring any externally imposed horizontal density gradients, has recently been identified near the slopes of stratified lakes (Lorke et al. 2005). In this case, a cross-slope (i.e., an approximately horizontal) density gradient is generated by the projection of the purely vertical interior stratification onto the sloping topography of the lateral boundaries of the basin (Figs. 1b,d). In the presence of oscillating up- and downslope currents, associated with basin-scale internal wave motions in the case studied by Lorke et al. (2005), a shear-induced straining of the cross-slope density gradient is observed, analogous to classical tidal straining. During upslope flow, dense water is advected on top of lighter, near-bottom water, leading to a reduction of vertical stratification and ultimately to convection (Fig. 1b). Vice versa, during periods of downslope flow, stable stratification evolves because of the downslope advection of lighter water on top of denser near-bottom fluid, resulting in a suppression of turbulent mixing (Fig. 1d). This effect has been observed in lakes of different size and geometry (Lorke et al. 2005, 2008; Cossu and Wells 2013) and could be reproduced in three-dimensional numerical modeling studies (Becherer and Umlauf 2011; Lorrai et al. 2011).

A recent study by Endoh et al. (2016) provided first direct observational evidence for the occurrence of slope-induced tidal straining on the continental shelf, and suggested that this process may also be important for the cross-slope transport of suspended material.

Umlauf and Burchard (2011) used an idealized one-dimensional numerical model to study the relevance of these findings for the oceanographically relevant parameter space. Their simulations revealed that shear-induced periodic stratification (SIPS) in sloping bottom boundary layers (BBLs) is expected to occur for virtually any parameter constellation and triggers a residual circulation that is in many respects similar to classical tidal straining. The transport of suspended material was, however, not investigated in this study.

In view of these similarities, and the relevance of classical tidal straining for the transport of suspended material in estuaries and the coastal ocean, it would be interesting to understand under which conditions and by which mechanisms tidal straining near sloping topography may be able to transport suspended material across isobaths. Here, we investigate this question with the help of a combined theoretical and idealized modeling approach, focusing on the identification of the basic mechanisms that determine cross-isobath transport of suspended material. The model, described in detail in section 2, builds up on the model by Umlauf and Burchard (2011), extended here by adding a simple erosion and transport model for suspended material. In section 3, we investigate the basic SPM transport mechanisms due to tidal straining near sloping topography with the help of a few idealized examples. The relevant (nondimensional) parameters are identified in section 4, and the parameter space is then explored in section 5 before we draw some conclusions in section 6.
respectively. JULY 2016 SCHULZ AND UMLAUF 2085

vanish (see Fig. 2). Under these conditions, it is easy to show that the upslope buoyancy gradient is constant and given by

\[ \frac{\partial b}{\partial x} = N_z^2 \sin \alpha. \] (3)

The slope-normal buoyancy gradient is defined as

\[ N_z^2 = \frac{\partial b}{\partial z}, \] (4)

which does not depend on \( x \) and converges to

\[ N_z^2 = N_z^2 \cos \alpha \quad \text{for} \quad z \to \infty, \] (5)

far away from the bottom and thus outside the BBL.

b. Equations of motion

Starting from the Reynolds-averaged Boussinesq equations, Umlauf and Burchard (2011) showed that under the above assumptions, the equations of motion can be expressed as

\[
\frac{\partial u}{\partial t} = (b - b_e) \sin \alpha + \frac{\partial u}{\partial z} - \frac{\partial \tau}{\partial x},
\]

\[
\frac{\partial b}{\partial t} = -u N_z^2 \sin \alpha - \frac{\partial G}{\partial z},
\] (6)

where \( t \) denotes time, \( u \) is the cross-slope velocity, \( \tau \) is the slope-normal turbulent flux of momentum (normalized by some reference density \( \rho_0 \)), and \( G \) is the slope-normal turbulent buoyancy flux. The latter two are defined as

\[
\tau = \overline{uw'w'},
\]

\[
G = \overline{wb'w'}. \] (7)

where the primes indicate turbulent fluctuations, and the overbar is the Reynolds average (\( w \) is the slope-normal velocity). The turbulent diffusivities of momentum and buoyancy \( \nu \) and \( \nu_b \) are computed from a second-moment turbulence closure model with two transport equations for the turbulent kinetic energy \( k \) and the turbulence dissipation rate \( \varepsilon \) (see Umlauf and Burchard 2005). We assume high Reynolds numbers and therefore ignore the molecular fluxes of momentum and buoyancy in (6). The turbulence model and all model parameters are identical to the model used in Umlauf and Burchard (2011) and Umlauf et al. (2015), where a more detailed model description may be found.

The term \( \partial u/\partial t \) appearing in (6) is an integration constant that, in general, depends on time and plays the role of a prescribed external pressure gradient used to force the model. Internal pressure gradients are represented by the term \( (b - b_e) \sin \alpha \), which mirrors the tendency of isopycnals to relax back toward

Fig. 2. Schematic view of isopycnal structure (black lines) in the vicinity of a uniform plane slope with slope angle \( \alpha \). Suspended material is indicated by the gray-shaded region near the bottom. The double arrow indicates the oscillatory current up and down the slope; the gray lines show the equilibrium levels of the isopycnals. The upslope and slope-normal coordinates are denoted by \( x \) and \( z \), respectively.

2. Geometry and governing equations

a. Geometry

We investigate the motion of a Boussinesq fluid in a semi-infinite domain bounded by a uniform slope with slope angle \( \alpha \) (Fig. 2). For simplicity, we ignore the effects of rotation and assume that the geometry is two-dimensional with the horizontal and vertical coordinates referred to as \( \hat{x} \) and \( \hat{z} \), respectively. Vertical stratification is quantified with the help of the squared buoyancy frequency

\[ N_z^2 = \frac{\partial b}{\partial z}, \] (1)

where \( b \) denotes the buoyancy of the fluid (all quantities are Reynolds-averaged unless noted otherwise). Outside the BBL, we assume that \( b \) approaches the undisturbed equilibrium buoyancy \( b_e \), which is characterized here by strictly horizontal isopycnals (gray lines in Fig. 2) and constant stratification:

\[ N_e^2 = \frac{\partial b_e}{\partial z}. \] (2)

It should be noted that although \( N_e \) is constant by definition, \( b_e \) may vary in space and time due to up- and downslope advection as discussed in more detail below.

Umlauf and Burchard (2011) showed that this two-dimensional problem can be reduced to one dimension by introducing the rotated upslope and slope-normal coordinates \( x \) and \( z \) (\( z = 0 \) at the bottom) and assuming that all upslope gradients except the buoyancy gradient vanish (see Fig. 2). Under these conditions, it is easy to show that the upslope buoyancy gradient is constant and given by

\[
\frac{\partial b}{\partial x} = N_z^2 \sin \alpha. \] (3)

The slope-normal buoyancy gradient is defined as

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\]

\[
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\]

where \( t \) denotes time, \( u \) is the cross-slope velocity, \( \tau \) is the slope-normal turbulent flux of momentum (normalized by some reference density \( \rho_0 \)), and \( G \) is the slope-normal turbulent buoyancy flux. The latter two are defined as

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The term \( \partial u/\partial t \) appearing in (6) is an integration constant that, in general, depends on time and plays the role of a prescribed external pressure gradient used to force the model. Internal pressure gradients are represented by the term \( (b - b_e) \sin \alpha \), which mirrors the tendency of isopycnals to relax back toward
their equilibrium levels (see Fig. 2). The equilibrium buoyancy \( b_e \) is computed from an evolution equation of the form

\[
\frac{\partial b}{\partial t} + u_c N^2 \sin \alpha = 0, \tag{8}
\]

which represents the up- or downslope advection of the undisturbed buoyancy field (Umlauf and Burchard 2011).

At the lower boundary, we use a no-slip condition for the velocity and assume that the buoyancy flux vanishes:

\[
u = 0, \quad \frac{\partial b}{\partial z} = 0. \tag{9}
\]

Boundary conditions for the turbulence quantities are discussed in Umlauf and Burchard (2011), who assumed that the bottom is hydrodynamically rough (which introduces the bottom roughness \( z_0 \) as an additional external parameter) and that a logarithmic wall layer exists very close to the bottom. Far away from the lower boundary \((z \to \infty)\), boundary conditions are derived from the assumptions that \( u \) and \( N \) are uniform and that all turbulent fluxes vanish.

Assuming that there are no hydrodynamic feedbacks resulting from the suspended material, (6) and (8) fully describe the evolution of the unknowns \( u, b, \) and \( b_e \). This system of equations has been numerically solved for different parameters, using a time step and grid size small enough to exclude any significant numerical errors. Mathematical and numerical implementation details are described in Umlauf et al. (2005).

c. Boundary layer forcing and resonance

We restrict our analysis to the special case of an oscillatory outer flow

\[
u_c = U \sin \omega t, \tag{10}
\]

where \( U \) is the fixed velocity amplitude, \( \omega = 2\pi/T_f \) is the forcing frequency, and \( T_f \) is the forcing period. Here, we focus exclusively on motions at the \( M_2 \) tidal frequency \((T_f = 12.4\) h\) such that \( u_c \) provides a simple representation of the near-bottom currents induced by barotropic or baroclinic tides.

Umlauf and Burchard (2011) showed that BBL motions relative to the interior contain both kinetic and potential energy, which suggests the possibility of reversible BBL oscillations. It can be shown that these oscillations occur at the frequency

\[
\omega_c = N \sin \alpha, \tag{11}
\]

which happens to coincide with the frequency for the critical reflection of internal waves impinging on a slope with slope angle \( \alpha \) (see Thorpe 2005). Close to resonant forcing \((\omega \approx \omega_c)\), it is therefore likely that the geometric assumptions outlined above break down, and the model can no longer be applied (Umlauf and Burchard 2011). We therefore exclude this parameter range from the following analysis, and only consider the cases \( \omega \gg \omega_c \) (strongly supercritical forcing) and \( \omega \ll \omega_c \) (strongly subcritical forcing). Note that for given forcing frequency \( \omega \), (11) may be inverted to compute the critical slope angle \( \alpha_c \). Evidently, supercritical forcing corresponds to subcritical slopes and vice versa.

d. Erosion and transport of suspended material

Focusing on the basic physical mechanisms determining the transport of suspended material near sloping topography, we use a relatively simple erosion and transport model. Suspended material is assumed to sink vertically down with a constant settling velocity \( w_s \), but is otherwise considered to behave like a passive tracer (no hydrodynamical feedbacks). Processes like aggregation or disintegration of suspended particles are ignored. The evolution equation for the concentration \( c \) of suspended material is thus of the form

\[
\frac{\partial c}{\partial t} = -\frac{\partial}{\partial z} (F_z - c w_s \cos \alpha), \tag{12}
\]

where the slope-normal turbulent sediment flux is defined as

\[
F_z = -\nu_s \frac{\partial c}{\partial z}. \tag{13}
\]

Clearly, the cross-slope advection term \( \partial(uc)/\partial x \) does not appear in (12) because of the assumed cross-slope homogeneity in our geometry. This assumption is formally valid only if the length scale \( L_c \) of cross-slope variations in SPM concentration is much larger than the tidal excursion scale: \( L_c \gg U/T_f \). The concentration of suspended material, however, may vary on smaller scales in many situations, which should be kept in mind when interpreting the results from this study.

At the lower boundary, the turbulent SPM flux \( F_z \) in (13) equals the erosion flux, here determined from the classical expression

\[
F_z = \alpha_c \max \left( \left| \frac{\tau_b}{\tau_c} \right| - 1, 0 \right) \quad \text{for} \quad z = 0, \tag{14}
\]

where \( \alpha_c \) is the erosion coefficient, \( \tau_b \) is the bottom shear stress, and \( \tau_c \) is the critical shear stress for resuspension, both normalized by the constant reference density \( \rho_0 \) (see Krone 1962; Sanford and Chang 1997; Amoudry and Souza 2011). The availability of erodible material is assumed to be unlimited in this idealized study.
Note that, in general, the SPM parameters appearing in (12) and (14) are not independent. For example, for the relatively simple case of noncohesive material, van Rijn (1984b) suggests a relation between $w_s$, $\tau_c$, and the grain size. The situation becomes, however, considerably more complex if unsorted or noncohesive material is considered (e.g., Dade et al. 1992; Sassi et al. 2015) or if biological activity (bioturbation, biofilms) plays a significant role (Grant et al. 1986; Grant and Daborn 1994). Because a generally valid relation is not available at the moment, we consider $w_s$, $\tau_c$, and $\alpha_c$ as independent parameters in our study. The physically relevant range for these parameters is explored in detail in section 5.

e. Residual transports

To quantify the tidally averaged transport of SPM, we define the total cross-slope residual flux

$$F_x = \langle uc \rangle - \langle c \rangle w_s \sin \alpha,$$  \hspace{1cm} (15)

where the angular brackets denote the tidal average. The last term in (15) is recognized as the projection of the vertical sinking velocity onto the downslope direction. For the small slope angles considered here, however, this “sinking flux” is negligible and will therefore be ignored in the following.

Following Burchard et al. (2013), we further decompose the residual flux into contributions from the residual current and the tidal fluctuations

$$\langle uc \rangle = \langle u \rangle \langle c \rangle + \langle \tilde{u} \tilde{c} \rangle,$$  \hspace{1cm} (16)

where the tilde indicates deviations from the tidal average. The first term on the right-hand side in (16) represents the contribution of the residual current to the total residual flux, whereas the second term, referred to as the tidal pumping term in the following, mirrors the effect of tidal straining. Dividing (16) by the average concentration $\langle c \rangle$, we find an expression of the form

$$u_c = \langle u \rangle + \langle \tilde{u} \rangle \langle \tilde{c} \rangle / \langle c \rangle,$$  \hspace{1cm} (17)

which may be interpreted as the effective velocity at which suspended material is transported across the slope. Note from (17) that the transport velocity $u_c$ may be significantly different from the residual velocity $\langle u \rangle$ (both may even have opposite sign) if tidal forcing is important.

3. Boundary layer dynamics and sediment transport

As an example, to illustrate the basic processes, we consider the following a case with a relatively mild slope ($\alpha = 0.002$), a typical tidal velocity amplitude ($U = 0.5 \text{ m s}^{-1}$), and some other typical parameters summarized in Table 1 (case 1). According to (11), in this example the resonance period of the BBL is $T_c = 2\pi \omega_c^{-1} = 87 \text{ h}$ and the critical slope $\alpha_c \approx 0.014$, indicating that tidal forcing is strongly supercritical. Correspondingly, the slope is strongly subcritical. All results discussed here and in the following sections correspond to fully periodic conditions.

a. Boundary layer dynamics

As the sediment model has no feedback on the hydrodynamic part of the problem, the evolution of the near-bottom velocities, stratification, and mixing parameters is qualitatively similar to that discussed in Umlauf and Burchard (2011). Here, we only briefly summarize the main hydrodynamical characteristics of this type of flow to provide the context for the following discussion of resuspension and residual SPM transports.

Figures 3a and 3b show that, for the parameters compiled in Table 1, the oscillating tidal currents generate a BBL of approximately 40 m thickness, characterized by strongly reduced stratification below a sharp pycnocline that separates the BBL from the nonturbulent interior region. During periods of upslope flow, a large fraction of the BBL becomes gravitationally unstable (light gray lines in Fig. 3), whereas during downslope flow it remains stably stratified throughout. Umlauf and Burchard (2011) showed that the periodic destabilization of the BBL can be explained by a differential advection mechanism resulting from the interaction of the frictional near-bottom shear and the constant upslope density gradient, analogous to tidal straining. More specifically, they showed that during upslope flow, dense fluid may be transported on top of

<table>
<thead>
<tr>
<th>Case</th>
<th>Forcing</th>
<th>$U$ (m s$^{-1}$)</th>
<th>$\omega$ (s$^{-1}$)</th>
<th>$z_0$ (m)</th>
<th>$\alpha_c$ (kg s$^{-1}$ m$^{-3}$)</th>
<th>$\tau_c$ (m$^2$ s$^{-2}$)</th>
<th>$w_s$ (m s$^{-1}$)</th>
<th>$N_c^2$ (s$^2$)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Supercritical</td>
<td>0.5</td>
<td>$1.41 \times 10^{-4}$</td>
<td>$10^{-2}$</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$5 \times 10^{-3}$</td>
<td>$10^{-4}$</td>
<td>$2 \times 10^{-3}$</td>
</tr>
<tr>
<td>2</td>
<td>Subcritical</td>
<td>0.5</td>
<td>$1.41 \times 10^{-4}$</td>
<td>$10^{-2}$</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$2 \times 10^{-4}$</td>
<td>$10^{-3}$</td>
<td>$5 \times 10^{-2}$</td>
</tr>
<tr>
<td>3</td>
<td>Subcritical</td>
<td>0.5</td>
<td>$1.41 \times 10^{-4}$</td>
<td>$10^{-2}$</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$9 \times 10^{-4}$</td>
<td>$10^{-3}$</td>
<td>$5 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Note that supercritical forcing corresponds to subcritical slopes and vice versa.
the lighter, more slowly moving fluid in the immediate vicinity of the bottom, resulting in unstable stratification and thus turbulent convection. It is worth noting that Endoh et al. (2016) reported recent observations of this process for a tidal BBL of similar vertical scale on a sloping continental shelf.

The periodic changes in stratification visible in Fig. 3b are directly mirrored in the turbulent diffusivities shown in Fig. 3c. During periods of upslope flow, when weak or unstable stratification develops, diffusivities are strongly enhanced compared to the stratified period with downslope flow. This variability in $\nu_t^b$ caused by tidal straining induces a tidal asymmetry that is, besides gravitational forcing, the key trigger for residual cross-slope transports inside the BBL.

SPM properties shown in Figs. 3d and 3e are consistent with these tidal asymmetries in mixing. During periods of upslope flow, a larger amount of material is resuspended (Fig. 3e), SPM is mixed up higher into the water column, and SPM concentrations are somewhat larger compared to periods with downslope flow (Fig. 3d). These asymmetries can be interpreted as a consequence of the larger turbulent diffusivities during upslope flow and the larger bottom shear stress, which, according to (14), determines the resuspension of settled material.

b. Residual results

Figure 4 shows some tidally averaged key parameters like the residual velocity, the SPM concentration, and the cross-slope flux of SPM, based on the simulation shown in Fig. 3. The residual currents are of the order of 0.01 m s$^{-1}$, directed upslope near the bottom and downslope in the upper part of the BBL (Fig. 4a). In this context, it should be noted that for fully periodic conditions and negligible mixing outside the BBL, it can be mathematically shown that the total cross-slope transport

![Figure 3](https://example.com/figure3.png)

**Fig. 3.** Temporal variability of (a) velocity, (b) magnitude of the squared buoyancy frequency, (c) turbulent diffusivity, (d) SPM concentration, and (e) magnitude of the bottom shear stress (black), critical shear stress for resuspension (gray), and integrated sediment concentration (blue). Gray lines in (a)–(d) indicate gravitationally unstable regions. All results represent fully periodic conditions (time refers to the start of the simulation). Parameters correspond to case 1 in Table 1.
vanishes for the geometry investigated here (Umlauf and Burchard 2011):

$$\int_0^z \langle u \rangle \, dz = 0.$$  \hspace{1cm} (18)

This implies that the upslope residual current near the bottom is exactly compensated by the negative (downslope) residual flow in the upper part of the BBL.

The tidally averaged SPM concentration (gray line in Fig. 4a) shows a strong increase toward the bottom, as expected from the classical Rouse-type balance between sinking and upward turbulent transport of suspended material. Figure 4b shows that the residual upslope current in the lower part of the BBL leads to a weak upslope advection of suspended material. This transport, however, is negligible in the upper part of the BBL ($z > 20$ m), and several times smaller than the contribution from tidal pumping in the lower part. This is also mirrored in the effective SPM transport velocity defined in (17), which is several times larger than the residual current (see Fig. 4c). Similar to classical tidal straining, therefore, we find that tidal pumping is crucial for the total residual transport of suspended material.

The physical mechanisms inducing tidal pumping in this example are easily identified from Fig. 3. As discussed above, sediment concentrations during upslope flow are larger than during downslope flow due to enhanced erosion and stronger upward mixing of suspended material. This leads to a positive correlation between $\bar{u}$ and $\bar{c}$ and thus to a residual upslope transport of suspended material. The magnitude of the transport velocity ($u_c \sim 0.1$ m s$^{-1}$) suggests that suspended material is transported several kilometers upslope during a single tidal cycle in this example. This supports our main hypothesis that tidal straining in the BBL near sloping
topography constitutes an effective mechanism for the transport of suspended material across isobaths.

To investigate how this process is affected by the properties of the suspended material, we compare in Fig. 5 the effects of different sinking speeds and critical shear stresses, leaving all other parameters unchanged (see case 1 in Table 1). For the lowest sinking speed ($w_s = 10^{-2}$ ms$^{-1}$), shown in Fig. 5a, downward sinking of suspended material cannot compete with upward mixing, and SPM concentrations inside the BBL therefore fluctuate only marginally around the tidal average ($\sim \zeta c / h c i / C_2$), except in the upper few meters of the BBL (not shown). In this case, tidal pumping is not effective ($\sim u t / \zeta c i / C_2 h u i / h c i$) and transports are largely determined by the two-layer structure of the residual current shown in Fig. 4a.

This situation is contrasted by the case with the highest sinking speeds ($w_s = 10^{-2}$ ms$^{-1}$), displayed in Fig. 5c, which is physically similar to the classical tidal pumping mechanism described already in the context of Fig. 4b above: higher SPM concentrations during the less stratified and more turbulent upslope flow phase result in a residual upslope transport of suspended material. Because of the higher sinking velocity compared to Fig. 4b, however, the pumping process is now confined to the lowest few meters of the water column, as shown in Fig. 5c (note the different vertical scale in this panel).

Physically most interesting is the intermediate case shown in Fig. 5b, corresponding to a sinking velocity of $w_s = 10^{-3}$ ms$^{-1}$. Although the vertical structure of the profile suggests a similarity with the well-mixed case in Fig. 5a, the underlying processes are entirely different. Here, the upslope transport of SPM in the lower part of the BBL (see Fig. 5b) is driven by classical tidal pumping, analogous to the cases with higher sinking velocities shown in Figs. 4b and 5c. The downslope transport observed in the upper part of the BBL, however, is triggered by a modified (inverse) pumping mechanism resulting from a phase shift in the SPM concentrations.

FIG. 5. Vertical profiles of residual SPM transports for different critical shear stresses [as indicated in the legend in (c)] and settling velocities: $w_s = (a) 10^{-2}$, (b) $10^{-3}$, and (c) $10^{-2}$ m s$^{-1}$. All other parameters correspond to case 1 in Table 1.
This is most easily understood from Fig. 3b, showing that the well-mixed near-bottom layer reaches its maximum thickness not before the point of flow reversal. Because newly eroded material is confined to this turbulent near-bottom layer, maximum SPM concentrations in the upper part of the BBL are observed with a substantial delay compared to the near-bottom region (not shown). This phase shift results in a positive correlation between periods of downslope flow and high SPM concentrations and therefore in a residual downslope transport of suspended material in the upper part of the BBL.

The role of the critical shear stress is investigated in Fig. 5, where again all other parameters correspond to case 1 in Table 1. According to (14), a lower critical shear stress leads to a larger erosion flux and thus, for otherwise unchanged parameters, to a larger SPM concentration in the water column. Figure 5 reveals that the net effect is an increase of the residual SPM fluxes without, however, affecting the basic mechanisms described above. Similar effects are observed if the bottom roughness is changed (Fig. 6). Reducing or increasing the bottom roughness by a factor of 2 with respect to the reference case (case 1 in Table 1) results in significant changes in the bottom stress and, therefore, in a modified erosion flux. This results in strongly altered SPM concentrations and transport rates (Figs. 6a,b) but only in small modifications of the dynamics. The BBL thickness varies by less than 10% (not shown), and the transport velocity varies by less than 15% compared to the reference case 1 (Fig. 6c). Keeping these basic effects of variations in $t_c$ and $z_0$ in mind, we will therefore not investigate the impact of these parameters in further detail in the following.

c. Supercritical slopes

The parameters for our second example, used to illustrate the basic transport mechanisms for subcritical forcing (i.e., supercritical slopes), are compiled in Table 1 (cases 2 and 3). The BBL resonance period in this case is $T_c \approx 1.1$ h, indicating that tidal forcing is strongly
Correspondingly, the slope angle $\alpha = 0.05$ is more than a factor of 10 larger than the critical slope angle $\alpha = 0.0045$.

Despite these differences in forcing, however, the observed flow patterns exhibit many similarities with the case 1 (subcritical slope) discussed above. During upslope flow, a gravitationally unstable, vigorously turbulent, near-bottom layer evolves, whereas during downslope flow, turbulence is suppressed by the generation of stable stratification (Figs. 7a,b). Different from case 1, high-frequency oscillations at the BBL resonance period $T_c$ can now be distinguished in the cross-slope velocities (Fig. 7a), however, without significantly modifying the overall dynamics. More important for the following discussion is the observation that because of the stronger tendency for restratification, tidal asymmetries in the turbulent diffusivity and the BBL thickness are much more pronounced compared to the case with subcritical slope (Fig. 7b). For example, the maximum thickness of the turbulent BBL varies between $h_{up} = 10$ m during upslope flow and $h_{down} = 3$ m during downslope flow.

Figures 7c and 7d, showing SPM concentrations for two different sinking speeds, reveal that this asymmetry in BBL thickness strongly impacts the vertical SPM distribution. During upslope flow, SPM is mixed up higher into the water column, whereas during downslope flow, most of the suspended material is trapped in the much thinner turbulent BBL, resulting in extremely high near-bottom SPM concentrations. After the maximum BBL thickness $h_{up}$ during upslope flow has been reached, the turbulent BBL collapses quickly during the restratification process (Fig. 7b). Particles mixed up during the strongly turbulent upslope flow phase may therefore remain suspended in the nonturbulent region above the BBL if their sinking speed is small. Figure 7c shows that this effect leads to nonzero SPM concentrations above the BBL even after the reversal to downslope flow.
Quickly sinking particles, on the other hand, will always be confined to the turbulent BBL (Fig. 7d).

These extreme tidal asymmetries have a strong impact on the residual SPM fluxes as illustrated in Fig. 8. Both the near-bottom residual circulation (Fig. 8a) and the residual SPM flux (Fig. 8b) suggest a downslope transport of suspended material in the near-bottom region, which is exactly the opposite of the case with subcritical slope shown in Fig. 4. The reversal of the residual SPM flux close to the bottom \((z < h_{\text{down}})\) is easily understood from Figs. 7c and 7d, showing that near-bottom SPM concentrations are much higher during phases of downslope flow compared to upslope flow. The net effect is a downslope tidal pumping of suspended material. The reversal of the near-bottom residual circulation for supercritical slopes, also evident from Fig. 8a, was already noted by Umlauf and Burchard (2011), who showed mathematically that this phenomenon occurs if convective mixing (unstable stratification) during upslope flow dominates tidally averaged near-bottom mixing. Similar to the previous examples for subcritical slopes, however, the contribution of the residual current to the total residual SPM flux is small compared to the effect of tidal pumping.

The processes in the upper part of the BBL \((z > h_{\text{down}})\) are more similar to classical tidal pumping. Here, SPM concentrations are larger during periods of upslope flow, when strong turbulence results in enhanced re-suspension and upward mixing of suspended material (Figs. 7c,d). While many aspects of this process resemble the situation for subcritical slopes shown in Fig. 3, there is one important difference. For quickly sinking material, the concentration of suspended material is zero during the downslope flow phase for \(z < h_{\text{down}}\) (Fig. 7d), resulting in tidal pumping with maximum efficiency in this region.

The strength and even the direction of the vertically integrated residual transport are determined by the relative importance of downslope tidal pumping in the near-bottom region, and upslope tidal pumping in the region above. The crucial parameter that determines this interplay is the sinking velocity, as discussed in more detail below.

### 4. Nondimensional description

To investigate the physically relevant parameter space in a systematic way, it is useful to identify the key non-dimensional parameters that determine the solutions of
the equations described in section 2. To this end, we first note that there are eight dimensional parameters appearing in the equations and boundary conditions described above. They may be grouped into parameters related to the properties of the slope ($\alpha$ and $z_0$), the interior flow ($U_0$, $\omega$, and $N_c$), and the suspended material ($\alpha_c$, $\tau_c$, and $w_c$). These dimensional parameters can be combined into five nondimensional products, with one possible choice being

$$\alpha, \ Z = \frac{N_c}{\omega}, \ P = \frac{w_c}{U}, \ T = \frac{\tau_c}{U^2}, \ R = \frac{z_0 \omega}{U},$$  \hspace{1cm} (19)$$

where $Z$ is a frequency ratio, $P$ is a special form of the Rouse number, $T$ is the nondimensional critical shear stress, and $R$ is a nondimensional measure of the bottom roughness. Note that for dimensional reasons, the erosion parameter $\alpha$ does not appear in this or any other set of nondimensional products (it is the only parameter involving the dimension of a mass).

### a. Nondimensional equations

To understand how these nondimensional numbers appear in the governing equations, we start by defining nondimensional versions of the time and the slope-normal coordinate:

$$t^* = t \omega, \ z^* = \frac{z \omega}{U}.$$  \hspace{1cm} (20)$$

The dimensional variables appearing in the transport equations in (6) and (12) are then nondimensionalized according to

$$u^* = \frac{u}{U}, \ b^* = \frac{b}{\mu N_c}, \ \tau^* = \frac{\tau}{U^2},$$

$$G^* = \frac{G}{U^2 N_c}, \ c^* = \frac{c U}{\alpha_c}, \ F_z^* = \frac{F_z}{\alpha_c}.$$  \hspace{1cm} (21)$$

This yields the following nondimensional equations:

$$\frac{\partial u^*}{\partial t^*} = Z \alpha (b^* - b_z^*) + \cos \phi^*, \quad \frac{\partial b^*}{\partial t^*} = -Z \alpha u^* - \frac{\partial G^*}{\partial z^*},$$

$$\frac{\partial c^*}{\partial t^*} = -\frac{\partial}{\partial z^*} (F^*_z - P c^*),$$  \hspace{1cm} (22)$$

where we used the special form of the tidal forcing term in (10) and assumed mild slopes ($\alpha \ll 1$). It should be noted that the variables in (21), although nondimensional, are not scaled, that is, terms appearing in (22) are generally not of order one.

Only three of the five nondimensional parameters compiled in (19) are seen to appear in (22). Nondimensional boundary conditions, derived from (9) and (14), are of the form

$$u^* = 0, \ \frac{\partial b^*}{\partial z^*} = 0, \quad F_z^* = \max \left( \frac{\tau^*}{R}, 1, 0 \right) \quad \text{for} \quad z^* = 0,$$  \hspace{1cm} (23)$$

which reveals that the fourth nondimensional parameter, the nondimensional critical shear stress $T$, appears in the boundary condition of the erosion model. Umlauf and Burchard (2011) showed that parameter number five, the roughness number $R$, enters the problem via the lower boundary condition for the turbulence dissipation rate. These authors also pointed out that because the turbulence model does not involve any dimensional constants, no additional parameters enter the problem.

### b. Nondimensional fluxes

The total cross-slope transport of suspended material is easily quantified from the integral of the residual flux defined in (15):

$$F_{int} = \int_0^\infty F_z \, dz,$$  \hspace{1cm} (24)$$

which can be reexpressed in nondimensional form as

$$F_{int}^* = \frac{\omega}{\alpha_c U} F_{int} = \int_0^\infty F_z^* \, dz^*.$$  \hspace{1cm} (25)$$

To derive a bulk expression for the effective velocity at which SPM is transported, it may seem tempting to work with the BBL average of (17). In view of the constraint in (18), however, this would eliminate the contribution from the residual current, which is small but generally not negligible. Also, it should be noted that, according to (17), we find $|u_c| \rightarrow \infty$ in regions where $\langle c \rangle \rightarrow 0$. Regions with the smallest SPM concentrations would therefore provide the largest contributions to the integral of $u_c$, which therefore cannot be considered as a sensible bulk measure for the transport velocity.

Here, we use the following alternative approach to define the bulk transport velocity:

$$U_c = \frac{\int_0^\infty F_z \, dz}{\int_0^\infty \langle c \rangle \, dz},$$  \hspace{1cm} (26)$$

which does not exhibit the above problems. The nondimensional transport velocity is then defined as
The quantities $F^*_{\text{int}}$ and $U^*_c$ will be central to the following analysis of cross-slope SPM transport in nondimensional parameter space.

c. Parameter space

In the following, we investigate the variability of SPM transport across the physically relevant (nondimensional) parameter space. All computations were carried out in dimensional space, and results were then made nondimensional as described in the previous sections. Note that different simulations in dimensional space map onto identical nondimensional solutions if they correspond to the same nondimensional parameters. We used this fact to test the correctness of the mathematical and numerical implementation of our model.

Our simulations will be based on the numerical examples discussed in section 3, now, however, allowing for a broader range of variations of the key nondimensional parameters: the sinking speed $P$, the stratification parameter $Z$, and the slope angle $\alpha$. For the nondimensional sinking speed, we explored the parameter range $5 \times 10^{-4} \leq P < 5 \times 10^{-2}$ (sampled with 100 logarithmically spaced intervals), which represents a typical spectrum of sinking speeds in the ocean (e.g., Ferguson and Church 2004; Van Leussen 1988). The stratification parameter $Z$ is varied between $Z = 71$ and 224, corresponding to one order of magnitude change in the vertical density gradient. Finally, the slope angle $\alpha$ was varied over the range $10^{-3} \leq \alpha < 10^{-1}$, which covers virtually all oceanographically relevant slopes. This range was resolved with 200 logarithmically spaced intervals to account for the large variability of the SPM fluxes in the vicinity of the critical slope angle (see below).

5. Nondimensional simulations

In the following, we explore the nondimensional parameter space defined above, focusing on the effects of variations in nondimensional settling velocity, stratification, and slope angle. To this end, two groups of simulations with subcritical and supercritical forcing, respectively, were carried out based on the parameters summarized in Table 3 (cases I and II). The nondimensional parameters corresponding to cases 1–3, discussed in the previous sections, are shown in Table 2 for reference.

a. Supercritical forcing

Nondimensional transports and transport velocities for the simulations with supercritical forcing and therefore subcritical slopes (case I in Table 3) are shown in Fig. 9. This figure reveals that for the whole parameter range studied here, the nondimensional transport $F^*_x$ is positive, suggesting that suspended material is generally transported in the upslope direction for subcritical slopes. Transport rates exhibit a decreasing trend for increasing sinking speeds (Fig. 9a), which is explained, to the first order, by the fact that higher sinking speeds imply smaller SPM concentrations and thus smaller net transports. As this effect masks some of the dynamics, it is more instructive to consider the behavior of the bulk transport velocity $U^*_c$ defined in (26), which may be viewed as a normalized version of $F^*_x$ in which this direct concentration effect has been removed. Figure 9b shows that the dependency of this quantity on $P$ is similar for all values of $Z$, with low transport velocities observed for the smallest and largest sinking speeds, respectively, and a single maximum for moderately large values of the order of $P = 10^{-2}$.

The decay of $U^*_c$ for the largest values of $P$ is easily understood from the fact that in these cases most of the SPM is located very close to the bottom, that is, in a region with reduced velocities due to the effect of bottom friction. In the limiting case of very large $P$, we would therefore expect a total collapse of the upslope transport. Small values of $U^*_c$ are also found for the

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TABLE 2. Nondimensional parameters corresponding to cases 1–3 in Table 1. Note that supercritical forcing corresponds to subcritical slopes and vice versa.

TABLE 3. Nondimensional parameters for the simulations in section 5. Note that supercritical forcing corresponds to subcritical slopes and vice versa.
opposite case of very small sinking speeds, where SPM is distributed nearly homogeneously across the BBL. As discussed above in the context of Fig. 5a, the transport in this case is determined by the residual flow rather than tidal pumping, implying that the upslope transport near the bottom and the downslope transport above nearly cancel. Although $U^*_c$ tends to zero for small $P$, the same is not true for the transport $F^*_x$, which is largely determined by the strong accumulation of SPM in the BBL due to the reduced deposition of eroded material. These cases with very small sinking speeds, however, involve long transients before periodic conditions are reached, which may rarely be achieved in nature.

For intermediate sinking speeds, Fig. 9b reveals a continuous increase of $U^*_c$ for increasing $P$ until a maximum value of $U^*_c = 0.15–0.2$ is reached, depending on stratification. Assuming an M2 tide with a typical velocity amplitude of 1 m s$^{-1}$, this corresponds to a distance of $\Delta x = 6.7–8.9$ km over which suspended material is transported upslope during one tidal cycle. Physically, the increase in $U^*_c$ in this parameter range represents the transition between the situation shown in Fig. 5b, in which the transports in the upper and lower parts of the BBL largely compensate, and the more efficient single-layer transport due to classical tidal pumping in the near-bottom region as discussed above in the context of Figs. 4b and 4c and Fig. 5c (these cases are also marked in Fig. 9). It is remarkable that the maximum transport velocities for the different values of $Z$ exhibit only a small variability, despite the fact that the BBL thickness varies by a factor of 2.5. The overall conclusion from these numerical experiments is that the transport of SPM is positive (upslope) for supercritical forcing (subcritical slopes), with the highest transport velocities found for moderately high sinking speeds as a result of classical tidal pumping.

b. Subcritical forcing

Results for case II with subcritical forcing/supercritical slopes (Table 3) are displayed in Fig. 10. While the transports and transport velocities show the same trends, and do so for the same reasons, as for the cases with supercritical forcing for very small and large sinking speeds, respectively, there is one important difference. Below a certain threshold of the sinking speed $P$ that depends on the stratification parameter $Z$, the transport now changes direction and becomes negative (downslope). This transition was explained in the context of Fig. 8 as a result of the decreasing importance of upslope tidal pumping in the upper part of the domain for decreasing settling velocities. For slowly sinking material, part of the SPM that has been mixed up high into the water column during the energetic upslope flow phase remains suspended in the nonturbulent region above the much thinner BBL during downslope flow, thus partly compensating the previous upslope transport. Because of this compensating effect in the upper part of the domain, the total transport is therefore dominated by the near-bottom residual transport, which
is downslope due to the inverse pumping mechanism described above.

For quickly sinking material, however, no suspended material is found in the nonturbulent restratified upper part of the domain that develops during downslope flow. Tidal pumping above the level of the thin downwelling BBL is therefore extremely efficient, leading to bulk transport velocities that reach up to 40% of the tidal velocity amplitude (Fig. 10b). We conclude that for both super and subcritical forcing, the most efficient upslope transport occurs for material with moderately large sinking speeds due to classical tidal pumping. The transport velocities in the subcritical regime, however, are about twice as large compared to the cases with supercritical forcing.

c. The effect of the slope angle

The parameters in all previous examples have been carefully chosen to insure that the forcing frequency $\omega$ is not in the vicinity of the critical frequency $\omega_c$ for BBL resonance, where some of the model assumptions are likely to break down (see above). In the following analysis, however, we study the model behavior across the entire range of oceanographically relevant slopes, including the transition from sub- to supercritical slopes. The parameters for these simulations correspond to case III in Table 3.

To emphasize the model behavior near critical slopes, in Fig. 11 the slope angle has been normalized by the critical angle $\alpha_c = \omega/N_{\alpha_c}$, found from inverting (11). As shown in the following, the model exhibits a particular behavior near slopes that correspond to the critical ($\alpha/\alpha_c = 1$) or twice the critical slope ($\alpha/\alpha_c = 2$), which are therefore indicated in the figure. The non-dimensional sinking velocity in these examples corresponds to $P = 10^{-2}$, which is close to the efficiency maximum for tidal pumping, as discussed above.

Most obvious from these simulations is the strong increase of both the SPM transport and the transport velocities at the transition from sub- to supercritical slopes, consistent with the results discussed in the preceding sections. Remarkable are the high residual transport velocities for slightly supercritical slopes ($\alpha/\alpha_c = 1.4$), where $U_c$ may reach up to 60% of the tidal velocity amplitude. In the immediate vicinity of critical slopes, however, where the BBL resonantly oscillates at the forcing frequency, the transport breaks down. A related observation was made by Umlauf and Burchard (2011), who pointed out the irreversible upslope buoyancy flux collapses if the BBL is resonantly forced. Interestingly, our results also reveal a reduction of the transport for $\alpha/\alpha_c = 2$, which may be understood from the fact that this case represents the first harmonic, where the BBL oscillates at exactly twice the forcing frequency.

As suggested already by the results shown in Figs. 9 and 10, the interpretation of the influence of stratification is complicated by the strong dependency of the transport rates on the nondimensional settling velocity $P$. A robust result, however, seems to be that transport rates in the strongly supercritical range ($\alpha > 2\alpha_c$) show a systematic increase with increasing stratification.
6. Discussion and conclusions

As one of the most important results, our study suggests that periodic motions in the vicinity of sloping topography trigger a residual transport of suspended material that is, for most parameter combinations, directed upslope. The physical transport mechanisms resemble classical tidal pumping as described in the context of many previous studies on SPM transport in estuaries and regions of freshwater influence (ROFIs; Simpson 1997) in the coastal ocean. However, while classical tidal pumping critically depends on an externally imposed horizontal density gradient, tidal pumping near sloping topography only requires a slope, vertical stratification, and an oscillating, near-bottom current that may be associated with tidal motions or, likewise, with any other energetic periodic process including near-inertial waves, topographically trapped waves, and internal seiching motions. This combination of factors is nearly ubiquitous in the ocean and in lakes, and we expect that the same is true for the new transport mechanism described here.

Our analysis also showed that the residual transport is governed by five nondimensional parameters, among them the ratio \( \frac{P}{5} \frac{w_s}{U} \) of the sinking speed and the tidal velocity amplitude, which was found to determine the effectiveness of tidal pumping. The highest transport velocities are found for values of the order of \( \frac{P}{5} \times 10^{-2} \), that is, for material with moderately high sinking speeds, assuming a typical tidal velocity range. For even higher sinking speeds, suspended material remains in a thin, near-bottom layer, where the transport is inhibited by the effect of bottom friction. For lower sinking speeds, tidal pumping becomes less efficient.

Beyond the nondimensional sinking speed \( P \), also the roughness number \( R = \frac{z_0}{U} \), the topographic slope \( \alpha \), and the nondimensional stratification \( Z = N_w/\omega \) (the latter two defining the transition from super- to subcritical forcing) turned out to be important parameters. A direct comparison of these nondimensional parameters with those found in studies of classical tidal straining in estuaries and ROFIs is, however, complicated by the fact that the water depth \( H \) (one of the key parameters in classical tidal straining) does not appear in our geometry. Some analogies between both types of problems can nevertheless be identified, noting that the key parameters in classical tidal straining are the Simpson number, \( S_i = H^2 \frac{\partial b}{\partial x} / U^2 \), the unsteadiness number, \( U_n = \omega H / U \), and the length-scale ratio \( A = z_0 H \) (e.g., Burchard and Hetland 2010; Burchard et al. 2013). Recalling that the (quasi) horizontal density gradient in our case can be expressed as \( \frac{\partial b}{\partial x} = N_w^2 a \) for mild slopes \( \alpha \ll 1 \), it is easy to show that \( S_i A^2 = a \alpha z^2 R^2 \) and \( U_n A = R \), where the left- and right-hand sides represent classical and sloping tidal straining, respectively. Note that some authors identify the velocity scale \( U \) with a bulk friction velocity \( U_* \propto U \), which is qualitatively equivalent.

Our numerical experiments showed that due to the more pronounced tidal asymmetries for supercritical...
slopes, SPM transport by tidal pumping is substantially more efficient than for subcritical $\alpha$. The sensitivity of the cross-slope fluxes with respect to variations of the slope angle and ambient stratification implies that the transport of suspended material near real oceanic slopes, which are in general neither uniform nor uniformly stratified, exhibits regions of convergence or divergence of suspended material. This is analogous to the convergence/divergence of cross-slope buoyancy fluxes discussed by Garrett (1991). The results in section 5 have shown that the upslope SPM flux strongly increases during the transition from subcritical to supercritical slopes (see Fig. 11). It may therefore be speculated that the convergence of suspended material in the vicinity of critical slopes is balanced by isopycnal intrusions of SPM toward the interior, possibly promoted by the strong turbulence usually associated with the critical breaking of internal waves (Fig. 12a). Similarly, for supercritical slopes, a convergence of upslope transports is expected at the transition from a region of strong to weak stratification (see Fig. 11), for example, in the upper part of a pycnocline. As pycnoclines are often regions of enhanced internal wave activity, it may be argued that the accumulated material is resuspended and transported inside the pycnocline toward the interior (Fig. 12b). These speculations can, however, only be substantiated with the help of a two- or three-dimensional modeling approach, which is beyond the scope of our study.

In the present investigation, we have concentrated on the basic mechanisms of SPM transport due to tidal straining near sloping topography, ignoring, for simplicity, numerous effects that may become relevant in more realistic scenarios. These include, besides the effects of the cross-slope inhomogeneities mentioned above, the effects of Earth rotation, the role of secondary currents induced by topographic features like submarine channels or ridges, and the impact of turbulence on the properties of SPM that all have been shown to significantly modify the mechanisms of classical tidal straining and residual SPM transport (e.g., MacCready and Geyer 2010; Schulz et al. 2015; Scully and Friedrichs 2003). The impact of these processes on the dynamics of tidal straining near sloping topography will have to be clarified in future studies. We have also emphasized the role of suspended material here, but it should be clear that bed load transport (van Rijn 1984a) may also provide an essential contribution to the overall cross-slope transport of particulate matter. It is likely that the tidal asymmetry in the bottom stress that we observed in all our simulations (see, e.g., Fig. 3e) leads to a residual bed load transport, but this aspect also requires a further analysis.

Acknowledgments. This study was carried out in the context of the interdisciplinary project SECOS, funded by the German Federal Ministry of Education and Research under Grant 03F0666A, WP 2.1 (PI: Lars Umlauf). We are grateful to Henk Schuttelelaars (TU Delft), Hans Burchard, Ulf Gräwe, and Knut Klingbeil (all IOW) for modeling support and helpful comments on the manuscript. Computations were carried out with a modified version of General Ocean Turbulence Model (GOTM; www.gotm.net) with the SPM component implemented using the Framework for Aquatic Biogeochemical Models (FABM; www.sf.net/projects/fabm).

APPENDIX

Numerical Boundary Conditions for SPM Concentrations

In view of the strong, near-bottom gradients of suspended material, the implementation of the boundary conditions for the SPM transport equation [(12)] requires special attention. While (14) provides an exact expression for the upward turbulent flux $F_z$ at the bottom, the numerical implementation of the sinking flux

$$F_z(z = 0) = w_s c_0 \quad (A1)$$
reasonable to assume that the bed cell is well mixed (i.e., $c_0 = c_1$) if the bed stress $|\tau_b|$ is smaller than the critical stress for erosion $\tau_c$, it is likely that the bottom concentration $c_0$ is significantly larger than $c_1$ if active erosion takes place ($|\tau_b| > \tau_c$). In this case, the popular assumption $c_0 = c_1$ may lead to large numerical errors and to a grid dependence of the numerical solution. In the following, we show how a more consistent lower boundary condition can be derived.

We start from the observation that close to the bottom, the transport equation in (12) reduces to a balance between upward mixing and downward sinking of suspended material,

$$0 = \nu^b \frac{\partial c}{\partial z} + w^* c,$$

assuming small slopes ($\alpha \ll 1$) for simplicity. The turbulent viscosity in the near-bottom region is known to follow the law-of-the-wall relation $\nu^t = \kappa u_*(z + z_0)$, where $\kappa \approx 0.4$ is the von Kármán constant, and $u_*=|\tau_b|^{1/2}$ is the bottom friction velocity (e.g., Pope 2000). The turbulent diffusivity $\nu^b_t$ in this region is proportional to $\nu^t$ and therefore adopts the form $\nu^b_t = Pr_t^{-1} \kappa u_*(z + z_0)$, where the turbulent Prandtl number $Pr_t$ plays the role of a constant proportionality factor of
order one. The turbulence model used in our study has been shown to exactly reproduce this near-wall behavior for \( \nu \) and \( \nu^p \) (e.g., Umlauf and Burchard 2003, 2005).

Inserting the above law-of-the-wall relation for \( \nu^p \) into (A2), we find a solution of the form

\[
\frac{c}{c_0} = \left( \frac{z}{z_0} + 1 \right)^{-p}, \tag{A3}
\]

which is recognized as the classical Rouse profile (van Rijn 1984b), slightly modified here by the appearance of the turbulent Prandtl number in the definition of the Rouse number

\[
p = w_r Pr/(\kappa u_*).
\]

Recalling that in any conservative numerical scheme \( c_1 \) represents the average concentration inside the lowest grid cell, a relation between \( c_0 \) and \( c_1 \) may be found from integrating (A3) across the cell. If we denote \( h \) as the cell thickness, this yields

\[
c_0 = c_1/r \quad \text{for} \quad |\tau_p| > \tau_c, \tag{A4}
\]

where

\[
r = \frac{1}{(1-p)h/z_0} \left[ \left( \frac{h}{z_0} + 1 \right)^{1-p} - 1 \right]. \tag{A5}
\]

Figure A1 shows the ratio \( r = c_1/c_0 \) as a function of the normalized cell thickness \( h/z_0 \) for different Rouse numbers. The most important conclusion from this figure is that the naive approach of assuming \( c_0 = c_1 \) to compute the sinking flux in (A1) during erosional periods introduces large numerical errors except for small Rouse numbers and/or extremely fine grids with \( h \ll z_0 \). Below, we nevertheless discuss some reference solutions that satisfy this condition. We note, however, that the constraint \( h \ll z_0 \) implies \( \Delta t \ll z_0/w_r \) for the numerical time step according to the well-known CFL stability criterion for explicit advection schemes. It is easy to show that, even for idealized one-dimensional simulations, the numerical effort may become prohibitively large for small \( z_0 \).

Using the example from section 3 above, Fig. A2 illustrates that assuming \( c_0 = c_1 \) during both erosive and nonerosive periods leads to significant numerical errors and to a grid dependence of the results. Estimating \( c_0 \) based on (A4) during periods with active erosion, however, removes this dependency on the numerical grid and leads to stable results already for moderate vertical resolution. All results discussed in this manuscript are therefore based on this new expression. Convergence studies were carried out to insure that all our results are independent of the numerical grid size and time step.

REFERENCES


Sanford, L., and M.-L. Chang, 1997: The bottom boundary condition for suspended sediment deposition. J. Coastal


