Revisiting Near-Inertial Wind Work: Slab Models, Relative Stress, and Mixed Layer Deepening

MATTHEW H. ALFORD

Scripps Institution of Oceanography, University of California, San Diego, La Jolla, California

(Manuscript received 15 May 2020, in final form 11 August 2020)

ABSTRACT: The wind generation of near-inertial waves is revisited through use of the Pollard–Rhines–Thompson theory, the Price–Weller–Pinkel (PWP) mixed layer model, and KPP simulations of resonant forcing by Crawford and Large. An Argo mixed layer climatology and 0.6° MERRA-2 reanalysis winds are used to compute global totals and explore hypotheses. First, slab models overestimate wind work by factors of 2–4 when the mixed layer is shallow relative to the scaling $H^* = u^*/(N f)^{1/2}$, but are accurate for deeper mixed layers, giving overestimation of global totals by a factor of $1.23 \pm 0.03$ compared to PWP. Using wind stress relative to the ocean currents further reduces the wind work by an additional $13 \pm 0.3\%$, for a global total wind work of 0.26 TW. Second, the potential energy increase $\Delta PE$ due to wind-driven mixed layer deepening is examined and compared to $\Delta PE$ computed from Argo and ERA-Interim heat flux climatology. Argo-derived $\Delta PE$ closely matches cooling, confirming that cooling sets the seasonal cycle of mixed layer depth and providing a new constraint on observational estimates of convective buoyancy flux at the mixed layer base. Locally and in fall, wind-driven deepening is comparable in importance to cooling. Globally, wind-driven $\Delta PE$ is about 11% of wind work, implying that >50% of wind work goes to turbulence and thus not into propagating inertial motions. The fraction into this “modified wind work” is imperfectly estimated in two ways, but we conclude that more research is needed into mixed layer and transition-layer physics. The power available for propagating near-inertial waves is therefore still uncertain, but appears lower than previously thought.

KEYWORDS: Internal waves; Oceanic mixed layer

1. Introduction

Near-inertial internal waves (NIW) are known to dominate internal wave kinetic energy and shear spectra at all depths in the ocean (Alford and Whitmont 2007; Silverthorne and Toole 2009). Because NIW energy (Alford and Whitmont 2007) and parameterized mixing (Whalen et al. 2018) both show strong seasonal cycles, a reasonable hypothesis is that wind-generated near-inertial waves contribute significantly to ocean mixing. In attempts to quantify the energy input of NIW, a number of authors (D’Asaro 1985; Alford 2001, 2003b; Watanabe and Hibiya 2002; Jiang et al. 2005; Rimac et al. 2013) have used the slab model of Pollard and Millard (1970, hereafter PM70) to estimate the work done by the wind on mixed layer inertial motions, in analogy to previous estimates of the work done by the wind on the mean circulation (Wunsch 1998). Global totals range from 0.3 to 1.3 TW, with the spread due to various methods and the temporal and lateral resolution of the wind products (Jiang et al. 2005). These totals are comparable to the ~1 TW of power converted from the barotropic tides to baroclinic tides, but the spread hampers quantitative certainty regarding the relative importance of near-inertial waves and internal tides in mixing the ocean. Because the climate change response of wind-forced NIW and internal tides should be quite different, resolution of this issue is important.

In addition to the spread in the slab model estimates, more fundamental issues exist. First, a number of seminal papers on mixed layer deepening in the late 1960s and 1970s (Kraus and Turner 1967; PM70; Pollard et al. 1973, hereafter PRT73; Niiler 1975) documented the role that inertial oscillations have in deepening the mixed layer. Specifically, they generate shear at the base of the mixed layer that entrains fluid from the interior. The power to the turbulence supporting this increase in potential energy is a “tax” that must be paid out of the wind work from the near-inertial motions in the mixed layer that will eventually propagate into the interior. How large is this loss?

Second, Plueddemann and Farrar (2006, hereafter PF06) showed that these mixing processes, which occur on time scales faster than the Rayleigh damping in the slab model, causes slab model estimates to be biased high by a factor up to 4 because inertial kinetic energy (IKE) grows much too quickly and the work is overestimated. At several buoys, they found that the Price–Weller–Pinkel (PWP) model (Price et al. 1986) reproduced observed estimates of the wind work much better. How large is this overestimation globally?

Finally, it has been shown that relative versus absolute stress makes a difference in wind-driven mean equatorial currents (Kelly et al. 2001). For wind work estimates, Liu et al. (2019), Zhai (2017), and Rath et al. (2013) found that estimates computed with stresses relative to the Earth rather than the inertial currents causes overestimates from 28% to 60%. We examine this here too, obtaining a more modest 13% ± 1% overestimate.

In this paper we revisit the mixed layer deepening literature in order to address these issues and obtain a global estimate of the near-inertial wind work that imperfectly addresses the shortcomings of the slab model and includes the losses due to turbulence. After presenting estimates of the wind work using the PWP model that are improved over adiabatic slab model estimates, we find that these must be further reduced by a...
poorly constrained further factor between 2 and 3 to account for the energy lost to turbulence. To contextualize these estimates of wind-driven potential energy increase, we compare them to the total observed potential energy (PE) increase computed from Argo and the surface cooling-induced buoyancy flux from the ERA-Interim reanalysis. In doing so, a powerful new constraint on the ratio of buoyancy flux between the surface and the mixed layer base (e.g., Ball 1960; Anis and Moum 1994) is obtained.

2. Methods

a. Approach

Various authors (Zilitinkevich 1972; PRT73; Niiler 1975; Davis et al. 1981; Large et al. 1994; Crawford and Large 1996, hereafter CL96; Skyllingstad et al. 2000; Grant and Belcher 2011) have presented energy budgets for the mixed layer. Owing to the complexity of the turbulent processes there and the long timeline over which the research has occurred, reconciliation of the different frameworks can be difficult. A central dichotomy between approaches is whether or not to treat the mixed layer as a “slab” in which momentum is assumed to be distributed infinitely quickly down the mixed layer depth \( H \). Observations show that shear exists in the mixed layer and that momentum penetrates beneath it into the so-called “transition layer” (e.g., Davis et al. 1981; Johnston and Rudnick 2009; Dohan and Davis 2011). The PWP model Price et al. (1986) is one of the simplest nonslab models, employing a bulk and a gradient Richardson number closure following PRT73. Comparisons with both more sophisticated 1D models, including large-eddy simulations (LES; Skyllingstad et al. 2000; Grant and Belcher 2011) and K-profile parameterizations (KPP; CL96), as well as observations (PF06) have demonstrated the skill of PWP in modeling deepening mixed layers.

Recognizing the PWP’s model’s neglect of a variety of mixed layer processes of known importance including lateral restratification by submesoscale motions (Hosegood et al. 2008; Fox-Kemper et al. 2008), Langmuir turbulence and surface breaking (e.g., McWilliams et al. 1997; Li et al. 2019), we leverage the numerous above-listed demonstrations of its accuracy for wind-driven mixed layer deepening in order to extend PF06’s examination of PWP at a few locations to the globe. Additionally, the potential energy increase of the mixed layer during the generation process of inertial motions is explicitly calculated.

b. Theory

The momentum equations for a slab-like mixed layer undergoing stress at its upper and lower boundaries (due to wind stress and turbulent processes at the mixed layer base, respectively) are

\[
\frac{dZ_I}{dt} + iZ_I = -\frac{1}{2} \frac{d}{dt} \left( \frac{g}{H} \right),
\]

(1)

where \( H \) is the mixed layer depth, \( Z_I = u_I + i v_I \) is the complex inertial current, \( u_I = \rho_o^{-1} \tau_w \), and \( v_I = \rho_o^{-1} \tau_R \). The terms \( \tau_w \) and \( \tau_R \) are the complex wind stress and turbulent stress, respectively, where \( \rho_o \) is the density of seawater. The bulk and gradient Richardson number closures employed by PRT73 will be introduced below. With a Rayleigh drag \( \lambda \), the turbulent stress becomes \( \tau_R = H \rho \nu \lambda \), where \( \rho \nu \lambda \) is the familiar wind work derived by D’Asaro (1985). Since PF06 concluded that an additional term \( \Pi_H = (\rho/2) Z_I^2 (dH/dt) \) associated with conversion from KE per unit mass to KE per unit area is small (1%–2% of the other terms), we follow them in neglecting it.

The shear production term \( P_M \) (equivalent to \( \Pi_R \) in PF06) represents energy lost to turbulent motions, or equivalently the production of turbulent kinetic energy. As with any turbulence in a stratified fluid, some of this energy goes into mixing (in this context, raising the potential energy by deepening the mixed layer) and some goes into kinetic energy dissipation. A central point of this paper is that the energy available for propagating near-inertial motions increases through \( (dIKE/dt) \), which is diminished relative to \( \Pi \) by the amount of energy that was lost to turbulence.

Absent lateral terms, \( P_M = e + J_k \), where \( e \) is the dissipation rate and \( J_k \) is the buoyancy flux (e.g., Tennekes and Lumley 1972). The buoyancy flux is traditionally related to the production via the flux Richardson number,

\[
R_f = J_y P_M.
\]

(3)

Equivalently, a mixing efficiency \( \Gamma = J_y/e \) can be defined. For stratified turbulence in the ocean interior, Osborn (1980) argued that \( R_f = \Gamma/(\Gamma + 1) \leq 1/e \), giving \( \Gamma \leq 0.2 \).

Since we follow CL96 in defining \( P_M \) and \( e \) as the integrals from the surface to a reference depth \( D \) well below the mixed layer, the buoyancy flux equals the rate of increase of potential energy, \( J_k = (dPE/dt) \), where

\[
PE = \int_0^D \rho(z)gz dz.
\]

(4)

Thus,

\[
P_M = R_f^{-1} \frac{d}{dt} PE.
\]

(5)

The increase in PE relative to the energy put into inertial motions was a central focus of the PRT73 theory and the KPP

\[1\] We will follow CL96’s notation of referring to \( \Pi \) and \( P_M \) in terms of their time integral over a specified period (here one month), such that IKE, \( \Pi \), and \( \Delta PE \) are directly comparable and all have units of joules per square meter (J m\(^{-2}\)). At times in the sections below, they will also be presented as rates in watts per square meter (W m\(^{-2}\)), comparable to past estimates of wind work, by dividing by the time interval.
simulations of CL96. For initial deepening of a mixed layer from zero, PRT73 found that potential energy was one-third of IKE. Simulating observationally motivated winds in the northeast Pacific, CL96 found that the partitioning was strongly dependent on how resonant the wind forcing was. Specifically, for resonant conditions, IKE was about half of $P$, with the rest lost to turbulent motions $PM$. Of this, $J_b$ associated with mixed layer deepening was about 12%, implying a flux Richardson number of about 25%. The fraction decreased for winds that were more off-resonance.

In the following, we will use the imperfect but useful tool of the PWP model. First, we will examine the energetics of the slab and PWP models during resonant and realistic forcing for a variety of mixed layer depths, and then will investigate the implications by extending the calculations to the globe.

c. Price–Weller–Pinkel (PWP) mixed layer model

We use the Price et al. (1986) mixed layer model as implemented by PF06, which contains the same physics as in PRT73. Specifically, the momentum equations are solved with specified initial conditions and prescribed surface fluxes, subject to a closure using both a bulk and a gradient Richardson number. When these thresholds are exceeded, momentum and buoyancy are redistributed until stability is restored. The model can be run with this closure on or off; with it off, the solution is identical to the damped slab model of PM70. That is, $T_R$ is the turbulent stress due to the turbulent closure in PWP; when mixing is turned off we reproduce the Rayleigh damping of the PM70 slab model intended to model the propagation of near-inertial waves over a time scale of days.

Our approach is to run the model with mixing on and off at each location. The wind work estimates computed with mixing on and off are then referred to as $\Pi_{PWP}$ and $\Pi_{slab}$, respectively. The model is run at 0.5-m vertical resolution and forced with wind stress from the Modern-Era Retrospective Analysis for Research and Applications Version 2 (MERRA-2) product (Gelaro et al. 2017), which are gridded hourly at about 0.6° resolution from the year 2019. Hourly output avoids complications with underresolved inertial response for coarse sampling (D’Asaro 1985; Alford 2003b). Detailed information about the MERRA project can be found at http://gmao.gsfc.nasa.gov/research/merra/. Because our focus here is wind forcing and it dominates buoyancy forcing on time scales of days (CL96), we set surface heat and freshwater fluxes to zero for these calculations, but will return to this issue in section 4e by comparing our wind-forced $\Delta PE$ estimates to total observed $\Delta PE$ from Argo and cooling-induced surface buoyancy fluxes from the ERA-Interim reanalysis. Additional comparisons are done in section 4c by modifying the model to allow forcing with stress computed from current-relative rather than absolute winds.

One-month simulations were conducted at each location and for each month of the year 2019. Initial stratification profiles were computed at each time and place, beginning with the World Ocean Atlas 2001 (Conkright et al. 2002). Mixed layer depth $H$ (Fig. 1) is from the 1° × 1° monthly climatology computed from Argo profiles published by Holte et al. (2017). At each location, the climatological temperature and salinity
values were made uniform down to a depth $H$, and the calculation was run with winds for that month. Following PF06, a Rayleigh damping was implemented, though our value of $r = 1/3.7$ days is slightly greater than the value used by PF06. We did not find significant sensitivity of our results to the value of $r$ used.

3. Slab model and PWP estimates as functions of mixed layer depth

Sample calculations forced with resonant inertially rotating winds ($\tau_x + i\tau_y = e^{-i\eta t}$, where $f$ is the inertial frequency) for one-half period are presented (Fig. 2) for shallow and deep mixed layers. The results reproduce familiar behavior. At left, an initially shallow mixed layer rapidly deepens (Fig. 2b) in response to the input of momentum by the wind stress on 2 July. Because the slab calculation has no mixing and cannot reproduce this deepening, kinetic energy rapidly increases (Fig. 2c, blue) much more than for the PWP case, so that the wind work $\Pi$ (Fig. 2d) is much greater for the slab model as shown by PF06. The energy input by the wind (Fig. 2c), computed as the time integral of each flux term) is overestimated in the slab model, with a significant portion of the wind work in the PWP model put into potential energy (black).

The deepening extends to about 50 m in this case, following the PRT73 prediction:

$$H^* = \frac{\nu^*}{\sqrt{\eta N}},$$

where $\nu^* = \sqrt{\tau_0 \rho_{\text{air}}}$ is the friction velocity of the imposed stress and $N$ is the stratification immediately below the mixed layer. For a constant wind, $H^*$ is the maximum depth to which the deepening occurs after half an inertial period, when the ocean currents oppose the wind and therefore arrest the wind work. For steady winds, large-eddy simulations (Ushijima and Yoshikawa 2020) show that the deepening continues after half a period, and for purely inertial winds, deepening would continue indefinitely. However, the parameter is a useful indicator of a long-known observation (e.g., Turner and Kraus 1967) that deepening greatly slows in the fall (i.e., deep mixed layers are hard to deepen further). We can indeed see that for a mixed layer deeper than $H^*$ (Fig. 2, right), deepening is much smaller. Since a much smaller fraction of energy goes to deepening the mixed layer, the slab model once again becomes a good approximation.

The $H^*$ scaling is demonstrated by rerunning the resonantly forced simulations keeping forcing the same for each run (giving $H^* = 40$ m) but for a range of mixed layer depths from 5 to 80 m (Fig. 3). At left, PWP and slab wind work are plotted versus $H$, with $H^*$ indicated (vertical dashed line). At right, the ratio is plotted versus $H/H^*$. The slab model estimate of $\Pi$...
exceeds that of the PWP model by factors up to 4 for \( H < H^* \), in agreement with the results of PF06 that were all at locations with \( H < H^* \). For \( H > H^* \), the two estimates agree well, since the deepening of the mixed layer by turbulence is less important.

We next examine the energetics of these resonant PWP calculations in more detail, with the goal of gaining insight but also to demonstrate that the PWP model broadly reproduces the KPP results of CL96. For deep mixed layers, little energy goes into increasing potential energy (Fig. 4, left, black), such that \( \Pi \) (red) and IKE (blue) are nearly equal. As \( H \) becomes smaller than \( H^* \), relatively more energy goes into PE and IKE is only about 50% of \( \Pi \) (fractions plotted at center), consistent with the simulations of CL96 for resonant winds. A flux Richardson number (Fig. 4, right) can be computed as \( \Delta PE/(\Pi - \Delta IKE) \), again consistent with the CL96 calculations for resonant winds: 50% of wind work goes to increase IKE, 50% is lost to turbulence, and 3%–20% of the turbulent kinetic energy is expended in increasing the potential energy, giving \( R_f = 0.05–0.35 \). These calculations indicate that for resonant conditions, even the PWP wind work overestimates the energy going into IKE by a variable factor, up to a factor of 2 or so for shallow mixed layers.

The calculation is next repeated for real winds and stratification taken from 30°N, 175°W, but with mixed layer depth varying from 5 to 80 m as before (Fig. 5). Though the winds are more complicated, the simulations show the same behavior as the resonant case with significant episodic deepening seen with the shallow initial mixed layer (Figs. 5a–e), and much less for the deep case (Figs. 5f–j). As in the resonant case, the slab model overpredicts IKE and \( \Pi \) for shallow \( H \) but they agree well for deep \( H \).

Energetics are more complicated in the real case as now both \( \Pi \) and \( (dIKE/dt) \) can have either sign depending on its phasing relative to existing inertial currents. Determination of the energy deposited into IKE is correspondingly more complicated, and is approximated as the work done by the linear damping on the inertial kinetic energy, \( \Pi_{Rayleigh} = 2\pi IKE \). Since positive \( \Pi \)

![Fig. 3. Slab and PWP wind work for \( H/H^* \), for resonant forcing with varying \( H \) but \( H^* = 40 \) m held constant. (left) Slab vs PWP wind work plotted vs \( H \) with \( H^* \) marked as a dashed line. (right) The ratio plotted vs \( H/H^* \).](image)

![Fig. 4. Energetics in the PWP model vs \( H/H^* \) for resonant forcing. (left) Wind work \( \Pi \) (red), IKE (blue), and \( \Delta PE \) (black) from the PWP model are plotted vs \( H/H^* \). (center) Fractions IKE/\( \Pi \), \( P_{eff}/\Pi \), and \( \Delta PE/\Pi \). (right) Flux Richardson number from (3) vs \( H/H^* \).](image)
can increase $IKE$ but negative $II$ cannot remove energy that has already propagated away, $IKE_{Rayleigh}/II$ is an overestimate for real forcing. Indeed, for the example shown the behavior is similar to the resonant case with $\Delta PE/II$ decreasing and $IKE/II$ increasing for deeper $H$ (Fig. 6). However, the implied mixing efficiency is too high to be physically reasonable because $IKE$ is overestimated. For this reason, we use other methods below to estimate $IKE$ for the realistic winds.

### 4. Global calculations

#### a. Slab model results

Global slab model calculations are very similar in spatial, temporal patterns and overall total of 0.37 TW (Fig. 7a, Table 1) as previous work, showing the familiar wintertime enhancement in midlatitudes and in the western portion of each basin where the storm tracks are. Since these patterns have been presented elsewhere, we simply focus on the

---

**Fig. 5.** As in Fig. 2, but for MERRA-2 winds from September 2019. Stratification and forcing are the same in each case, but initial $H$ is 20 m at left and 60 m at right.

**Fig. 6.** As in Fig. 4, but for MERRA-2 winds from September 2019. $IKE$ is computed, unsuccessfully, as the energy removed by the Rayleigh drag, giving unrealistically high values of $R_f$ (see text).
annual-mean map, which shows broad maxima in both hemispheres.

b. Slab model versus PWP model

Because the PWP calculation is computationally expensive compared to the slab model owing to its recursive adjustments, the PWP estimates are computed every fourth location in latitude and longitude (1/16 of the total slab model locations), for a resolution of about 2.4° (Fig. 7b). Overall features are nearly identical, with a reduction of spatial resolution owing to the subsampling of PWP locations. To ensure that the coarser sampling did not bias the results, means were computed from the full half-degree-resolution slab calculations and from the same quantities subsampled at the locations of the PWP calculations; the two differed by less than 5%. Globally, the total wind work computed from the slab model overestimates the PWP estimate by a factor of 1.23.

The spatial patterns and dependence on mixed layer depth support the theoretical ideas presented in the previous section. A sample scatterplot of the ratio of slab to PWP wind work versus \( H/H^* \) (Fig. 8a, gray) and the binned average (black) shows the same behavior as for the resonant case; that is, when mixed layer exceeds \( H^* \) (see Fig. 1 for the geography of \( H \) and \( H^* \)) the slab model reproduces the PWP estimate well, but overestimates it by up to a factor of 4 for shallower H. Because the distribution of \( H \) includes many values near and less than \( H^* \), these factors combine to give the above average overestimation factor.

The \( H^* \) scaling also produces understandable zonal-mean patterns (Fig. 9). In March (top panels), zonal-mean \( H \) (Fig. 9a) is significantly greater than \( H^* \) in the Northern Hemisphere, giving very similar slab and PWP estimates (Figs. 9b,c). In the Southern Hemisphere, \( H \) is comparable to \( H^* \), giving rise to slab estimates 1.5–2 times higher than PWP. In September, the patterns are essentially reversed.

In summary, these findings support the conclusions of PF06 that the slab model overestimates the PWP model, and contextualize and quantify their results. By scaling \( H \) versus \( H^* \), it is seen that the overestimation occurs only when \( H \sim 3 H^* \); the distribution of \( H/H^* \) is such that the global total is 1.23 ± 0.03 times higher for the slab model, where the error estimate is one

Table 1. Monthly and annual total values for all quantities. First two columns: wind work \( \Pi \) computed from absolute wind stress and from the slab and PWP model. Next four columns are PWP wind work, near-inertial kinetic energy IKE estimated from the degree of resonance of the forcing (CL96) and from the increase in potential \( \Delta \)PE assuming a constant flux Richardson number \( R_f = 1/6 \), and \( \Delta \)PE computed from current-relative wind stress. The next two columns are the ratios of the slab to the PWP model wind work and of the absolute to the current-relative wind work. The final three columns are the fractions of the total wind work for each IKE estimate and \( \Delta \)PE.

<table>
<thead>
<tr>
<th>Month</th>
<th>( \Pi_{\text{slab}} ) (GW)</th>
<th>( \Pi_{\text{PWP}} ) (GW)</th>
<th>( \Pi_{\text{IKE}} ) (GW)</th>
<th>( \Pi_{\text{IKEPE}} ) (GW)</th>
<th>( \Delta )PE</th>
<th>Slab/( \Pi )</th>
<th>Absolute/relative</th>
<th>( \Pi_{\text{IKECL96}} ) (GW)</th>
<th>( \Pi_{\text{IKEPE}} ) (GW)</th>
<th>( \Delta )PE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>465</td>
<td>330</td>
<td>293</td>
<td>56</td>
<td>122</td>
<td>29</td>
<td>1.37</td>
<td>1.13</td>
<td>0.19</td>
<td>0.44</td>
</tr>
<tr>
<td>Feb</td>
<td>422</td>
<td>317</td>
<td>280</td>
<td>50</td>
<td>115</td>
<td>28</td>
<td>1.34</td>
<td>1.13</td>
<td>0.18</td>
<td>0.44</td>
</tr>
<tr>
<td>Mar</td>
<td>351</td>
<td>303</td>
<td>263</td>
<td>45</td>
<td>90</td>
<td>27</td>
<td>1.27</td>
<td>1.15</td>
<td>0.17</td>
<td>0.39</td>
</tr>
<tr>
<td>Apr</td>
<td>332</td>
<td>282</td>
<td>250</td>
<td>45</td>
<td>87</td>
<td>27</td>
<td>1.15</td>
<td>1.13</td>
<td>0.18</td>
<td>0.37</td>
</tr>
<tr>
<td>May</td>
<td>289</td>
<td>262</td>
<td>233</td>
<td>42</td>
<td>83</td>
<td>25</td>
<td>1.14</td>
<td>1.13</td>
<td>0.18</td>
<td>0.38</td>
</tr>
<tr>
<td>Jun</td>
<td>309</td>
<td>272</td>
<td>244</td>
<td>42</td>
<td>111</td>
<td>21</td>
<td>1.12</td>
<td>1.12</td>
<td>0.17</td>
<td>0.49</td>
</tr>
<tr>
<td>Jul</td>
<td>311</td>
<td>271</td>
<td>242</td>
<td>40</td>
<td>116</td>
<td>21</td>
<td>1.13</td>
<td>1.12</td>
<td>0.16</td>
<td>0.51</td>
</tr>
<tr>
<td>Aug</td>
<td>323</td>
<td>252</td>
<td>226</td>
<td>37</td>
<td>105</td>
<td>19</td>
<td>1.19</td>
<td>1.12</td>
<td>0.17</td>
<td>0.5</td>
</tr>
<tr>
<td>Sep</td>
<td>371</td>
<td>276</td>
<td>244</td>
<td>40</td>
<td>95</td>
<td>25</td>
<td>1.26</td>
<td>1.13</td>
<td>0.16</td>
<td>0.42</td>
</tr>
<tr>
<td>Oct</td>
<td>372</td>
<td>294</td>
<td>259</td>
<td>44</td>
<td>72</td>
<td>31</td>
<td>1.22</td>
<td>1.14</td>
<td>0.17</td>
<td>0.31</td>
</tr>
<tr>
<td>Nov</td>
<td>405</td>
<td>308</td>
<td>270</td>
<td>44</td>
<td>76</td>
<td>33</td>
<td>1.27</td>
<td>1.14</td>
<td>0.16</td>
<td>0.3</td>
</tr>
<tr>
<td>Dec</td>
<td>445</td>
<td>322</td>
<td>283</td>
<td>47</td>
<td>91</td>
<td>32</td>
<td>1.34</td>
<td>1.14</td>
<td>0.17</td>
<td>0.34</td>
</tr>
<tr>
<td>Year</td>
<td>366</td>
<td>291</td>
<td>257</td>
<td>44</td>
<td>97</td>
<td>26</td>
<td>1.23</td>
<td>1.13</td>
<td>0.17</td>
<td>0.41</td>
</tr>
</tbody>
</table>
c. Relative versus absolute wind

As the ocean responds to the wind, the stress is reduced relative to what it would be in a fixed reference frame, which should correspondingly reduce the wind work. Liu et al. (2019) found that globally integrated wind work computed from surface drifters was 60% higher using stress in a fixed reference frame relative to that computed using relative currents. Zhai (2017) found a smaller overestimation of 25% in numerical simulations of the North and Atlantic Oceans, while Rath et al. (2013) estimated a 28% overestimation in the Southern Ocean, again from numerical simulations. To assess the effect in our calculations, wind speed and direction were computed from the absolute MERRA-2 wind stresses, and the ocean current from the PWP simulations was subtracted at each time step in order to compute the ocean-relative wind stress for the next time step. We find that the absolute-stress wind work values are overestimated by about 13% ± 0.3% relative to those computed from current-relative stresses, with no significant spatial patterns (Fig. 10, Table 1). The error estimate is one standard error of the mean of the monthly totals.

d. Mixed layer deepening and energy available for NIW

A central point of this paper is that if one is concerned with the energy available for propagating near-inertial waves, then the correct quantity is the “modified wind work” \( W\) rather than \( P\). We make two attempts at estimating \( W\) here; neither is entirely satisfactory. First, we use our estimates of potential energy increase and some assumptions about mixing efficiency to estimate \( P_M \) and therefore \( W\) from (2). However, the necessary assumptions are primitive.

![Figure 8](image1)

**Fig. 8.** (top) Individual monthly values of \( \Pi_{\text{slab}}/\Pi_{\text{PWP}} \) (gray) and binned averages (black) plotted vs \( H/H^* \) for the whole year of 2019. (bottom) The probability density function of \( H/H^* \).

![Figure 9](image2)

**Fig. 9.** Zonal-mean plots vs latitude of (left) \( H \) and \( H^* \), (center) wind work from slab and PWP, and (right) their ratio, for (a)–(c) March and (d)–(f) September.
and unsatisfying, so we obtain a second estimate using the KPP results from CL96.

1) IKE FROM \( \Delta PE \)

Our first estimate of IKE estimates \( P_M \) from \( \Delta PE \) computed from the PWP simulations. \( \Delta PE \) is straightforward to calculate from the density profiles at the beginning and end of each monthly simulation, using a reference depth of \( D = 350 \text{ m} \) to ensure integration below the deepest mixed layers. The time-mean map of \( \Delta PE/\Pi \) (Fig. 11a) shows that the regions of strongest forcing tend to put the smallest fraction of energy into deepening the mixed layer, consistent with CL96. Integrated globally, the fraction is consistent throughout the year, with an annual mean of 11% (Table 1).

The turbulence production can then be crudely estimated as \( P_M = R_f^{-1} \Delta PE \), where \( R_f \) is taken as 1/6 following Osborn (1980). Though \( R_f \) is a parameter of the flow rather than a constant, substantial observational evidence supports a mean value near 1/6 in the ocean interior, including agreement with dye and microstructure estimates of turbulent diffusion (Ledwell et al. 1993) and comparison of thermal and kinetic energy dissipation rates (Moum 1996; Monismith et al. 2018). For deepening mixed layers, CL96 and Skyllingstad et al. (2000) found values spanning this number, but with considerable spread depending on \( H \) (as shown in Fig. 2), stratification at the mixed layer base, and the degree of resonance of the forcing (motivating the next section). Intuitively, deep mixed layers might have less efficient mixing owing to the large fraction of “unmixed” water with buoyancy flux only occurring in the transition layer, as suggested by Fig. 2.

Glossing over these variations in \( R_f \), whose parameter dependencies need to be explored more fully, we here simply estimate turbulence production for each location and month from (5), taking a constant value of \( R_f = 1/6 \). Then \( IKE_{PE} = \Pi - R_f^{-1} \Delta PE \).

The interpretation of \( P_M \) determined in this way from the monthly estimates is somewhat complicated because \( \Delta PE \) can only increase in time as the PWP model cannot “unmix” the fluid. The integrated wind work, however, can increase and decrease depending on the phasing of the wind (e.g., Fig. 5). The ratio of \( \Delta PE/\Pi \) is usually 5%–20% (CL96 and Fig. 12, black), but values \( R_f^{-1} \Delta PE > \Pi \) occur about 18% of the time.

\[ \text{Fig. 10. Comparison of fluxes from wind stress computed from the PWP model using Earth-relative winds ("absolute," thin black) and those relative to the inertial currents ("relative," thick gray), for April. (a) Absolute and relative wind work. (b) Their ratio.} \]

\[ \text{Fig. 11. Annual-mean maps of (a) the ratio of potential energy increase to wind work, } \Delta PE/\Pi, \text{ and (b) the associated ratio of near-inertial kinetic energy increase to the wind work, } IKE_{PE}/\Pi. \]
mostly where II is small or negative. Since these values are not energetically consistent with our understanding of turbulence, we discard them, giving the distribution plotted in gray in Fig. 12.

With these assumptions, IKE_Pe/II and IKE_Pe are plotted in Figs. 11b and 13b, respectively. The mean value of IKE_Pe/II is 0.34 (Table 1).

2) IKE FROM CL96

Our second method of estimating IKE employs results from a different model than PWP to give IKE/II for nonresonant conditions and a range of H. CL96 showed that for resonant forcing, about 53% of II goes into IKE, with the fraction falling off as winds become less resonant:

\[
\frac{\text{IKE}_{\text{CL96}}}{\text{II}} = 0.532(1 - e^{-2.8C}),
\]

where we use C rather than CL96’s Π, to avoid confusion with the mixing efficiency. Parameter C is the ratio of energy at the inertial frequency relative to that of a time series of identical amplitude but rotating purely inertially. Nearly resonant winds put maximal energy into IKE, while less resonant winds must have greater wind stress to achieve the same wind work and therefore give rise to greater production of turbulence in the mixed layer and at its base.

A variable percentage of this energy, decreasing with increasing H, goes to deepening the mixed layer. For the near-inertial waves however, the fraction of PM that goes to increasing PE does not matter; it has already been lost and cannot force propagating near-inertial motions by pumping the base of the mixed layer (Gill 1984). At best, therefore, approximately half of the wind work goes into IKE, and less where winds are less resonant.

To quantify CL96’s estimates of IKE/II in the context of the global totals, C(x, y, t) is computed by calculating the rotary spectrum of wind stress at each location and for each month. Because traveling midlatitude storms dominate the wind work and are more resonant than other atmospheric systems (D’Asaro 1985), we calculate the spectrum only for data spanning ±3 days of the strongest wind event in each one-month record (Fig. 14a). Because of the finite length of this record relative to the time scale of the forcing, this is likely an underestimate. The fraction IKE/II computed from (7) is 40%–50% for the most resonant events but otherwise hovers much lower (Fig. 14b), with a global mean of about 17% (Table 1).

The power input to NIW, IKE_{CL96} = (IKE_{CL96}/II)P_{WP} (Fig. 13a), with a global total of only 44 GW (Table 1), is reduced significantly compared to the wind work shown in Fig. 7b. As noted, this estimate is likely biased low owing to the method used to compute the wind rotation. Additionally, the CL96 results have only been verified observationally at the Station Papa region.

3) COMPARISON

Maps of the fraction of the wind work available for NIW, IKE_Pe/II, and IKE_{CL96}/II are shown in Figs. 14b and 11b, while the overall power into IKE from the two quantities is shown in Fig. 13. Though the two quantities have similar patterns, mostly set by those in II, IKE_Pe is clearly greater than the CL96 estimate (which we acknowledged earlier was an underestimate). The spatial patterns are quantified and contextualized by plotting zonal means versus latitude for March and September (Fig. 15). For March, Northern Hemisphere mixed layers are deep (Fig. 15a), so that a large percentage of energy goes into IKE_Pe (Figs. 15b,c). IKE_{CL96} peaks at the same latitude as II, as expected given the greater degree of resonance of the traveling midlatitude storms (D’Asaro 1985). IKE_{CL96}, which is a lower bound as indicated above, is substantially lower than IKE_Pe everywhere. IKE_Pe in March is smaller in the Southern Hemisphere owing to shallower mixed layers (Fig. 15a) and correspondingly greater mixing. In September, the situation is largely reversed.
the best agreement, we take obtained a range of 0.07–0.18 with a mean of 0.13. Because it gives fluxes and mixed layer turbulence profiles, Anis and Moum (1994) cited in CL96. From direct measurements of surface buoyancy based on theoretical arguments by Ball (1960) and observations for September. (b) The annual-mean average fraction of wind work going to IKE computed from (7).

e. Potential energy and cooling

COMPARISON WITH CLIMATOLOGICAL COOLING

Wind, surface wave breaking, and surface buoyancy losses are the dominant energy sources for mixed layer turbulence and therefore the primary means of deepening the mixed layer via entraining the denser fluid below. While resonant wind forcing is known to be important for midlatitude mixed layer deepening in the fall (e.g., Turner and Kraus 1967, CL96), it is generally understood that buoyancy forcing, particularly the heating/cooling cycle, sets the cycle of mixed layer properties on seasonal time scales (e.g., Moisan and Niiler 1998; Giglio and Roemmich 2014). However, the author is not aware of a quantitative comparison of the cooling-induced buoyancy flux and the buoyancy flux measured from observed differences in potential energy associated with mixed layer depth deepening, and so this is examined here to contextualize the previous discussions of wind-induced potential energy increase.

Assuming a one-dimensional balance with no storage, the entrainment flux at the mixed layer base is thought to equal a constant $a$ times the surface buoyancy flux, where CL96 take $a = 0.2$ based on theoretical arguments by Ball (1960) and observations cited in CL96. From direct measurements of surface buoyancy fluxes and mixed layer turbulence profiles, Anis and Moum (1994) obtained a range of 0.07–0.18 with a mean of 0.13. Because it gives the best agreement, we take $a = R_f = 1/6$ for our comparisons.

Since the buoyancy flux at the mixed layer base equals the rate of potential energy increase, the equivalent expression can be obtained for $\Delta PE$:

$$a^{-1}\Delta PE = J_p^i H,$$

where $J_p^i = -(g/\rho_o)(a/c_p)Q_{\text{net}}$, where $g$ is acceleration due to gravity, $\rho_o$ is a seawater density at the surface, $a = 2 \times 10^{-4} \text{K}^{-1}$ is the thermal expansion constant, and $c_p$ is the thermal capacity of seawater at constant pressure.

The downward heat flux $Q_{\text{net}}$ at the surface is taken from the ERA-Interim climatology (Dee et al. 2011), which performs better than most other products (Brunke et al. 2011). As with most heat flux climatologies, errors are $O(5–10) \text{ W m}^{-2}$ (equivalently $J_p^i \approx 2.5–5 \times 10^{-9} \text{ W kg}^{-1}$).

The total observed change in potential energy may be obtained from the monthly Argo climatology. For each month and location, $PE$ is estimated from (4), and $\Delta PE$ from the time derivative. Since we only consider cooling, negative values (decreasing $H$) are discarded.

Maps of $a^{-1}\Delta PE$ from the PWP simulations, $a^{-1}\Delta PE$ from the Argo climatology, and $J_p^i H$ from ERA-Interim and Argo are shown in Fig. 16 (top, middle, and bottom, respectively) for June, October, and December, with zonal means shown in Fig. 17. Time series for the Northern Hemisphere, Southern Hemisphere, and Station Papa are plotted in Fig. 18.

Potential energy increases from Argo (Fig. 16, middle) represent the observed mixed layer deepening due to all processes. $a^{-1}\Delta PE$ is greatest in each hemisphere in its respective winter, exceeding $6 \times 10^{-3} \text{ W m}^{-2}$. The observations closely match $J_p^i H$ from ERA-Interim in both pattern (Fig. 16) as well as magnitude (Figs. 17a and 18), consistent with the overall dominance of cooling in deepening the mixed layer on seasonal time scales (Moisan and Niiler 1998), and also providing a strong new constraint on inferred values of $a$, supporting the value of $a = 0.13$ from Anis and Moum (1994). For reference, a heat flux error of $20 \text{ W m}^{-2}$ would give errors in $J_p^i H$ of $0.5 \times 10^{-3} \text{ W m}^{-2}$ for a 100-m-deep mixed layer, small compared to these values. Though beyond the scope of this work, similar analyses could be performed to examine dependence of $a$ on mixed layer parameters such as $H$, Monin–Obukhov length, etc.

The PWP estimates of $a^{-1}\Delta PE$ (Fig. 16, top) are the equivalent modeled buoyancy fluxes associated with wind-forced mixed layer deepening. They show the same patterns as $H$ (Fig. 7); e.g., maxima in midlatitudes in the western portion of each basin, and are generally somewhat lower than the cooling terms. The zonal means for June and December (Figs. 17a,c) confirm the dominance of cooling in hemispheric winter as stated above; however, in fall (Fig. 17b), resonant wind forcing is comparable to the cooling term. Though we did not compute Monin–Obukhov length $L$, these statements are equivalent to the Monin and Obukhov (1954) theory, namely, when $H > L$, convective effects will dominate wind mixing in further deepening. Near 30°N, contributions to observed deepening (red) by wind and cooling are approximately equal. In summertime for each hemisphere, predicted wind deepening (blue) exceeds observed and climatological cooling-induced deepening, possibly implicating the importance of solar heating.

5. Discussion

The primary implication of this work is the reduction of the energy available for propagating near-inertial waves compared
to previous estimates, some of which were by the author, by a combination of the slab overestimate (1.23 ± 0.03), the absolute/relative fraction (1.13 ± 0.003), and—most significantly and uncertainly—the fraction lost to mixed layer turbulence (~2–6), for a total estimate of IKE, 0.1 TW. If we take the estimate IKEPE, only 26% of the wind work estimated from a slab model using absolute winds is available for propagating NIW, or 37% of that estimated from the PWP model with current-relative winds. For IKECL96, the corresponding percentages are 12% and 17%, respectively.

While the statement that it is IKE rather than II that is available to propagating NIW is robust, the low confidence the author has in the mean value and parameter sensitivity of IKE/II cannot be overstated. Many aspects of mixed layer physics that are known to impact turbulence and deepening, such as breaking waves and Langmuir circulations (e.g., Smith 1998; Kukulka et al. 2010; Belcher et al. 2012; Li et al. 2019), are absent. Additionally, it is well known that mesoscale motions impact NIW propagation by modulating the effective inertial frequency (Kunze 1985; Lee and Niiler 1998; Zhai et al. 2008). They also modulate H and therefore H/H*, potentially indirectly impacting energy lost to turbulence.

If true, our conclusion that 60%–85% of the wind work is lost to mixed layer turbulence is in line with modeling studies by Furuichi et al. (2008) and Zhai et al. (2009). With high-passed wind fields and thus no mesoscale field, the former study found 85% of wind work was dissipated in the upper 150 m. By including a mesoscale field, Zhai et al. (2009) found twice as much penetration of NIW to depth (nearly all of it in anticyclonic eddies), but still concluded that 70% of the energy was dissipated in the upper 200 m. The present study provides a physical interpretation for these results; namely, that the energy is lost to mixed layer turbulence leading to entrainment at the mixed layer base, that plays an important role in the annual cycle of mixed layer depth.

A rough constraint on the accuracy of these estimates can be had from the calculations by Whalen et al. (2018) of strongly wintertime-enhanced dissipation estimated from Argo floats using the Gregg (1989) parameterization. If all of the wintertime enhancement between latitudes 30°–45°N shown in Fig. 2b of Whalen et al. (2018) is attributed to breaking downward-propagating near-inertial waves, the dissipation integrated from 250 to 1250 m and from 1250 to 2000 m are 0.5 and 0.13 ± 0.02 W m⁻², respectively. If IKE ~ 1–1.5 × 10⁻² W m⁻² (Fig. 13, local dissipation between 250 and 2000 m accounts for approximately one-third to one-half of IKE, leaving the rest (12%–50%) to propagate in the form of low modes (Alford 2003a; Simmons and Alford 2012; Alford 2020) and radiate to the deep sea (12%–33%; Alford et al. 2012).

Seasonal cycles of NIW (Alford and Whitmont 2007; Silverthorne and Toole 2009) and downward-propagating NIW (Alford 2010; Alford et al. 2017) are known to occur in the deep sea. Clearly, some wind-forced energy makes it to depth, but direct calculation of NIW energy flux from observations is very challenging. In terms of energetics of deep-sea mixing, this study appears to support the suggestions of the above modeling studies that such NIW may be somewhat reduced in importance relative to the tides, though their greater shear for the same energy relative to internal tides (Alford 2010) may still allow them to play a role.

FIG. 15. Comparison of IKE computed using the potential energy and resonance methods to the wind work. (a),(d) H and H*. (b),(e) Zonal-mean II and IKE from each method. (c),(f) The ratio of IKE/II for each method. Top panels are for March; bottom panels are for September.
Regardless, the high degree of uncertainty in the mixed layer turbulent losses requires further study. Regional/global models with KPP mixed layer physics and realistic mesoscale fields may be useful tools for exploration of hypotheses and parameter space. Targeted observational process studies of transition-layer turbulence, such as highly resolved Lagrangian thermistor chains (A. Kaminsky and E. D’Asaro 2020, personal communication) during resonant and off-resonant forcing will provide crucial constraints. More observations of the energy flux, particularly the vertical component, and mixing of propagating near-inertial waves are needed. Finally, this work has only addressed wind-generated NIW; the role of mechanisms in which NIW gain energy from the mean flow (Polzin 2010; Vanneste 2013; Alford et al. 2013; Nagai et al. 2015; Barkan et al. 2017; Whalen et al. 2018) is still unknown. There is much work to do.

6. Conclusions

By driving the one-dimensional PWP mixed layer model with idealized and realistic winds, and initializing with Argo-based climatologies for mixed layer depth and interior stratification, we have revisited the topic of wind work and explored a variety of concepts regarding the energetics of the mixed layer, finding the following:

- Using the PWP model, we have verified and contextualized the assertions by PF06 that slab models overestimate wind
work, finding that slab models are accurate for deep mixed layers \( H > H^* \) where \( H^* = u^*/(Nf)^{1/2} \), but significantly overestimate the wind work in shallow ones, resulting in slab model total gains being biased high by a factor of 1.23 \( \pm 0.03 \).

- Calculating wind work using Earth-relative rather than current-relative stresses results in global totals higher by a factor of 1.13 \( \pm 0.003 \).

- Wind work II is not all available for propagating near-inertial waves; rather, the “modified wind work” \( \text{IKE} = H - P_{M} \) equal to the work minus the turbulent production is the relevant quantity [as pointed out by CL96, Jochum et al. (2013), and other studies, but underappreciated in the near-inertial literature].

- The fraction \( \text{IKE}/\text{II} \) was estimated globally from potential energy increase calculated from PWP and using KPP results from CL96, giving fractions of 37% and 19%, respectively.

- These wind-driven potential energy increases were compared to potential energy estimates from Argo and cooling-induced buoyancy fluxes from the ERA-Interim climatology. Cooling dominates the seasonal cycle of mixed layer depth, but wind-driven deepening is important in midlatitudes during fall.

- The comparison between Argo-measured potential energy and the ERA-Interim surface buoyancy flux provides a valuable global constraint on the ratio of buoyancy flux at the surface and the mixed layer base. We find \( a = 0.17 \), close to the value of \( a = 0.13 \) estimated by Anis and Moum (1994).

Acknowledgments. This paper was written during the early stages of the COVID-19 outbreak and is dedicated to the heroic hospital workers and the many families that have lost loved ones. The author is grateful to Eric D’Asaro for many inspiring conversations from 1998 to 2020 as well as for very useful comments on an earlier draft of this work. The author thanks Jim Moum for a series of helpful conversations, the scientists and students of the Multiscale Ocean Dynamics group for support and feedback, Tom Farrar for providing PWP MATLAB code and useful discussions, Madeleine Hamann and Sam Kelly for assistance in downloading the MERRA-2 winds, and Matt Mazloff for providing the ERA-2 analyses. This work was supported by ONR Award N000141812404.

REFERENCES


FIG. 18. Time series of buoyancy flux from \( R_{n} \Delta P_{E} \) and from climatological ERA-Interim heat fluxes during cooling conditions. (a) Northern Hemisphere. (b) Southern Hemisphere. (c) Station Papa region.


