Topographic Form Drag on Tides and Low-Frequency Flow: Observations of Nonlinear Lee Waves over a Tall Submarine Ridge near Palau

GUNNAR VOET, MATTHEW H. ALFORD, AND JENNIFER A. MACKINNON
Scripps Institution of Oceanography, University of California San Diego, La Jolla, California

JONATHAN D. NASH
College of Earth, Ocean and Atmospheric Sciences, Oregon State University, Corvallis, Oregon

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ABSTRACT

Towed shipboard and moored observations show internal gravity waves over a tall, supercritical submarine ridge that reaches to 1000 m below the ocean surface in the tropical western Pacific north of Palau. The lee-wave or topographic Froude number, \( N h_0/U_0 \) (where \( N \) is the buoyancy frequency, \( h_0 \) the ridge height, and \( U_0 \) the farfield velocity), ranged between 25 and 140. The waves were generated by a superposition of tidal and low-frequency flows and thus had two distinct energy sources with combined amplitudes of up to 0.2 m s\(^{-2}\). Local breaking of the waves led to enhanced rates of dissipation of turbulent kinetic energy reaching above \( 10^{26} \) W kg\(^{-1}\) in the lee of the ridge near topography. Turbulence observations showed a stark contrast between conditions at spring and neap tide. During spring tide, when the tidal flow dominated, turbulence was approximately equally distributed around both sides of the ridge. During neap tide, when the mean flow dominated over tidal oscillations, turbulence was mostly observed on the downstream side of the ridge relative to the mean flow. The drag exerted by the ridge on the flow, estimated to \( O(10^4) \) N m\(^{-1}\) for individual ridge crossings, and the associated power loss, thus provide an energy sink both for the low-frequency ocean circulation and the tidal flow.

1. Introduction

Oceanic internal lee waves and their associated breaking and energy dissipation are thought to play a considerable role in the energy and momentum budgets of the mean and mesoscale ocean circulation. Numerical model studies show that turbulent mixing generated by breaking lee waves, in addition to internal tides and near-inertial waves, are an important driver of the global overturning circulation and may account for up to one-third of the internal wave-driven water mass transformation in the deep ocean (Nikurashin and Ferrari 2013). Estimates for the global energy flux into lee waves vary between 0.2 TW (Nikurashin and Ferrari 2011) and up to almost 0.5 TW (Scott et al. 2011). Even though this is less than the 0.7–1.3 TW going globally into internal tides (Egbert and Ray 2000; Munk and Wunsch 1998; Nycander 2005; Garrett and Kunze 2007; Falahat et al. 2014), numerical model studies show that lee waves can have a significant impact on the ocean stratification (Melet et al. 2014). Lee waves seem to play a considerable role in the energy budget of mesoscale eddies, extracting as much as 0.12 TW from the eddy field in the Southern Ocean (Yang et al. 2018). Furthermore, where in the water column lee waves break and dissipate their energy—whether this occurs in the bottom layer or if energy propagates upward—matters for the overturning circulation (Melet et al. 2014). The fraction of lee-wave energy ultimately available for turbulent mixing depends on the presence of sheared background flows, which can, through conservation of wave action, lead to an appreciable amount of lee-wave energy feeding back into the mean flow (Kunze and Lien 2019).

Lee waves may also play an important role for the momentum balance of the global mean and mesoscale flow field. The interaction between flow and bathymetry, and subsequent lee-wave generation, is often expressed as a drag on the flow and therefore termed (topographic)
form drag. In this framework, the radiation of waves is a pathway for energy and momentum in the horizontal and vertical. Only where the waves break is energy dissipated and momentum deposited (e.g., Bell 1975b; Durran 2003). It is the asymmetry of lee waves generated by quasi-steady flow, as opposed to purely tidal lee waves, that allows for net momentum transports. This redistribution of horizontal momentum in the vertical has been shown to have effects on the large-scale ocean circulation (Naveira Garabato et al. 2013; Trossman et al. 2016). In the presence of background flows, tidal lee waves may also lead to net momentum fluxes (Shakespeare and Hogg 2019).

Observations of oceanic lee waves generated by quasi-steady or low-frequency flow are sparse. As they are arrested over bottom topography, these waves have zero frequency in a reference frame fixed to topography and thus in theory do not imprint a direct signature on moored time series. Cusack et al. (2017) observed lee waves over and behind a bathymetric feature in Drake Passage using Electromagnetic Autonomous Profiling Explorer (EM-APEX) floats and found vertical energy and momentum fluxes associated with these waves. Similarly, Meyer et al. (2016) observed lee waves using EM-APEX floats near the Kerguelen Plateau in the Southern Ocean. Using measurements from an underway CTD, Johnston et al. (2019) observed lee waves over a submarine ridge at the southern end of Palau. Eakin et al. (2011) observed vertically propagating lee waves in seismic reflection data over bathymetric features in the Caribbean Sea. A study of internal waves generated by mesoscale eddies by Clément et al. (2016) finds correlation between eddy decay and high-frequency energy content in moored velocity records. We are unaware of other direct low-frequency lee-wave observations in the ocean and one has to revert to observational studies of mountain waves for further observational studies. Tidal motions play only a subordinate role in the atmosphere, making the observation of lee waves easier than in oceanographic environments where the generation and propagation of tidal internal waves masks lee waves generated by the low-frequency flow.

A number of observational studies exist on internal tidal waves generated at steep, supercritical topography (Klymak et al. 2008; Legg and Klymak 2008; Nakamura et al. 2010; Pinkel et al. 2012; Alford et al. 2014). These studies show highly nonlinear, high-mode internal waves that lose part of their energy through breaking near the generation sites during certain phases of the tide. In addition to shear-driven mixing processes, these waves exhibit convective breaking mechanisms in transient features associated with critical flow conditions. Depending on the tidal excursion length scale in relation to the topographic length scale, a tidal wave may reach a quasi-steady state during certain phases of the tide (Musgrave et al. 2016), thus bridging the gap between tidal oscillatory waves and steady lee waves.

Here we present direct observations of quasi-steady internal lee waves that are generated both by tidal and lower-frequency flow over a tall submarine ridge. Observations taken at both spring and neap tide allow us to contrast conditions with dominating tidal flow and dominating mean flow. In the following, we describe the study region and observation techniques in section 2, discuss lee-wave parameters and present results in section 3, and discuss implications of the results in section 4.

2. Experimental details

a. Study region

The measurements were carried out over a tall submarine ridge north of the island group Palau in the tropical western North Pacific (Fig. 1). The northern end of Palau generally lies within the westward-flowing surface-intensified North Equatorial Current (NEC) with eastward-flowing undercurrents beneath it (Schönau and Rudnick 2015; Qiu et al. 2015; Zeiden et al. 2019). During the time of the survey at the onset of La Niña conditions following a relatively strong El Niño event, the large-scale currents deviated from this general picture and the eastward-flowing North Equatorial Counter Current (NECC) meandered around the northern tip of Palau, leading to general eastward surface flow.

The very shallow, island-like Velasco Reef at the northern end of Palau drops to about 1000 m depth within a horizontal distance of only a few hundred meters and extends as a long ridge around this depth for several kilometers northward (Fig. 1c). The ridge is mostly two-dimensional with variations in ridge height on the order of 100 m. The bottom depth drops to about 3000 m within just a few kilometers from the ridge crest on the eastern side and, less steep, to about 2000 m on the western side (Fig. 2).

Defining a general ridge width is complicated owing to the asymmetry of the ridge-normal bathymetry. Following Klymak et al. (2010), the Gaussian shape of the ridge may be described by \( h = h_0 \exp(-x^2/W^2) \) with the topography height over the ocean floor \( h_0 \) and the ridge width \( W \). A ridge width \( W = 3 \) km results in Gaussian shapes that describe the cross-ridge bathymetry relatively well when using \( h_0 = 1000 \) m on the western side and \( h_0 = 2500 \) m on the eastern side (Fig. 2). The topographic aspect ratio \( a_0 = h_0/W \) thus
varies between 0.3 on the western side and 0.8 on the eastern side.

The ridge is supercritical with respect to both diurnal and semidiurnal frequencies. The characteristic slope of an internal wave, or the ratio of horizontal wavenumber $k$ to vertical wavenumber $m$, is

$$\alpha = \frac{|k|}{m} = \sqrt{\frac{\omega^2 - f^2}{N^2 - \omega^2}}, \quad (1)$$

where $\omega$ is the tidal frequency, $N$ the buoyancy frequency, and $f$ the local Coriolis parameter. A topographic slope equal to $\alpha$ is usually described as critical (Garrett and Kunze 2007). With the buoyancy frequency at the depth of the ridge $N \approx 3 \times 10^{-3} \text{s}^{-1}$ and $f = 2.2 \times 10^{-5} \text{s}^{-1}$ at this latitude, tidal characteristic slopes are 0.025 and 0.05 for diurnal and semidiurnal frequencies, respectively (Fig. 2, dashed). They are thereby much smaller than the the steep slopes of the ridge; hence the ridge is supercritical.

b. Observations

All data were collected within the framework of the Office of Naval Research-sponsored research project Flow Encountering Abrupt Topography (FLEAT). The program was designed to study various aspects of flow-topography interactions, including their impact on energy and momentum balance of larger-scale flows. Various moored, shipboard, and autonomous measurement systems were deployed during the course of the experiment. Here, we focus on shipboard measurements during June 2016 and observations from one mooring that was deployed for 10 months in the vicinity of the ridge.

1) SHIPBOARD OBSERVATIONS

Shipboard measurements were collected during cruise RR1607 on R/V Roger Revelle between 2 and 24 June 2016. The survey over the ridge was designed to contrast conditions between spring and neap tide with the anticipation of low-frequency-flow-generated lee waves.
being less masked by internal tide generation over the ridge during neap tide. The ridge was therefore surveyed during two periods from 7 to 9 June at spring tide and again on 13 and 14 June during neap tide conditions as predicted by a global tidal model (TPXO; Egbert and Erofeeva 2002).

(i) Towyos

Towed profiling observations (towyos) were carried out using a Seabird 9/11-plus CTD on a standard rosette. The instrument package was towed at a speed of about 0.8 kt (1 kt ≈ 0.51 m s⁻¹) back and forth along a line crossing the ridge in zonal direction at 8.55°N. At this latitude, the ridge crest is located at a depth of 943 m and thereby slightly shallower than to the north and south. While towing, the instrument package was initially cycled at vertical speeds of 1 m s⁻¹ between the surface and 40 m above the seafloor, reaching to depths between 900 and 1500 m depending on position along the ridge. This resulted in a sawtooth-like sampling pattern with a horizontal resolution of about 500 m near the ridge crest and about 1000 m in deeper water. After a few ridge crossings, the upper turnaround depth was adjusted to about 500 m water depth to increase the horizontal resolution between individual profiles to about 300 m near the ridge crest and about 500 m in deeper water. The survey line was also slightly shortened to reduce the time needed for one ridge crossing from about 6 h during the initial operation to about 3 h or a quarter of the semidiurnal tidal phase space. This sampling pattern was kept for the remainder of both the spring and the neap tide survey. The resulting trajectory of the instrument package is shown in Fig. 2.

During the neap tide survey, one RBR Solo thermistor sampling at 1 Hz and one Seabird SBE37 Microcat, recording temperature, conductivity, and pressure at 6 s sampling interval, were clamped on the hydrowire to provide measurements of the upper 500 m of the ocean. The pressure record from the slower SBE37 was used to depth-grid temperature measured by the RBR Solo with a faster response time. Temperature and conductivity were then used to calculate density. Resulting density profiles have much larger error bars than using data measured by the 9/11-plus system and are only used to estimate the influence of upper ocean stratification on bottom-pressure perturbations (appendix B).

A constant offset was applied to the ship’s GPS position to account for the horizontal distance between ship and instrument package due to drag on the hydrowire. Matching multibeam bathymetry with altimeter readings from the instrument package, we determined an optimal offset of 250 m.

Turbulent dissipation rate $\epsilon$ was estimated from CTD data based on Thorpe scales (Thorpe 1977; Dillon 1982). Details of the Thorpe scale method are shown in appendix A.

(ii) LADCP

Two lowered acoustic Doppler current profilers (LADCP) were attached to the rosette. Both instruments were Teledyne RD Instruments (TRDI) Workhorse types operating at 300 kHz and 8 m bin size. The LADCPs provided measurements of shear that extended beyond the shipboard sonar system’s (HDSS, see below) reach and into the bottom layer masked by sidelobe contamination in the HDSS data. Unfortunately, bottom tracking on the LADCPs failed and we have relatively low confidence in absolute LADCP velocity profiles outside the reach of HDSS data due to missing reference absolute velocities near the bottom. Measurements of vertical velocity shear are not affected.

(iii) HDSS

Shipboard velocity measurements were obtained using the Revelle’s Hydrographic Doppler Sonar System (HDSS) consisting of a 50 kHz and a 140 kHz sonar system (Pinkel 2012). Data from the two systems were combined and reach to about 1000 m water depth at a vertical resolution of 15 m. While this puts velocities at ridge-crest level into reach of the measurement system, bottom reflection of the sidelobe contamination renders the near-bottom 10% unusable. Individual velocity profiles were averaged to a temporal resolution of 1 min.

2) MOORED OBSERVATIONS

An array of subsurface moorings was deployed near the northern end of Velasco Reef for ten months from June 2016 through April 2017. From this array, mooring F3 was deployed near the ridge at a water depth of 1666 m (Figs. 1, 2) and was equipped with a combination of 75 and 300 kHz TRDI ADCPs, providing velocity measurements over nearly the whole water column. Velocity was measured with a burst of 10 pings every 10 min that were subsequently averaged to one value every 10 min. In addition to velocity, temperature and conductivity was measured at various depths along the mooring using RBR Solo thermistors and Seabird SBE37 CTDs.

3. Results

a. Flow conditions

Moored and shipboard velocity observations near the ridge reveal a complex flow pattern with a combination of low-frequency flow divided by various shear layers,
tides, and a tidal fortnightly modulation. As the ridge is oriented in a north–south direction, zonal flow components correspond to ridge-normal flow in the following. The low-frequency ocean currents in the vicinity of the ridge were strongly layered with large vertical shear between the layers during the time of the experiment. The zonal velocity component from the moored observations (Figs. 3a and 4a) shows this banded structure.
with time-mean eastward flow in the upper 150 m, westward flow between 200 and 300 m, eastward flow between 350 and 700 m and westward flow in the range around the ridge crest from 800 to 1000 m. The time-mean meridional velocity component exhibits a similarly banded structure at smaller current amplitudes (Fig. 4b). HDSS observations, recorded during the two cross-ridge surveys, closely match the velocity pattern seen in the mooring data (Figs. 3b and 4). Mean eastward flow in the surface layer reflects the atypical situation of the NECC meandering around the northern end of Palau as discussed above. Mean flow near the ridge-crest level was mostly westward and northward at about 0.05 m s$^{-1}$ in each component. Average conditions deviate between mooring and HDSS at the depth of the ridge crest; as the HDSS observations were taken closer to the ridge (cf. Fig. 2) we consider these a more accurate estimate of the flow directly interacting with topography. The 36-h low-pass-filtered moored velocity time series in Fig. 3c shows that the banded structure lasted throughout the time of the surveys. The cause of the highly banded structure remains an open research question on which we do not further elaborate here. We only note that the strongly sheared layers show up in other observations near Velasco Reef (MacKinnon et al. 2019) but the number of layers is reduced in the farfield (Zeiden et al. 2019).

Tidal variability of the flow was dominated by a semidiurnal and, to lesser extent, diurnal signal that was modulated by a clear spring–neap cycle. Figure 3d shows the moored ridge-normal velocity time series bandpass-filtered around the semidiurnal and diurnal tidal frequencies. Spring–neap modulation is apparent with stronger tidal velocities around 7 June and weaker tidal velocities around 14 June. This is reflected in the barotropic or depth-averaged moored zonal velocity record showing tidal amplitudes of about 0.1 m s$^{-1}$ during spring tide and reduced tidal modulation during neap tide (Fig. 3e). The phase of the fortnightly cycle as predicted by a tidal model (TPXO; Egbert and Erofeeva 2002) matches relatively well with the spring–neap cycle observed here. However, amplitudes of the TPXO-predicted tide are reduced by about a factor of 2 when compared to the observations and the phase is offset by about 10°. We attribute this mismatch to the coarse resolution of the TPXO model of only 1/6° where local bathymetry is not resolved sufficiently. For example, bottom depth in the TPXO model at the mooring site is 2754 m compared to multibeam-measured depth of 1666 m.

Velocities around the ridge-crest level, calculated as a depth-mean over the range of 750–950 m, show higher tidal amplitudes than the depth-averaged velocities (Fig. 3f). Harmonic analysis of the moored velocity time series at ridge-crest level reveals peak tidal amplitudes of about 0.15 m s$^{-1}$ during spring tide and only 0.05 m s$^{-1}$ during neap tide. The semidiurnal tidal constituents $M_2$ and $S_2$ alone account for 55% and 20% of the tidal energy at the ridge-crest level. Tidal currents are mostly ridge normal with tidal ellipse aspect ratios of 4 and 8 for the $M_2$ and $S_2$ constituents, respectively.

The 4-h low-pass-filtered HDSS ridge-crest-level velocity time series in Fig. 3g sums up the flow conditions expected to be important for lee-wave generation. During spring tide, tidal motion back and forth across the ridge dominates. The westward mean flow is superimposed. The ratio of average semidiurnal (tidal) energy to overall kinetic energy in the flow above the ridge crest.
ridge crest is 0.58 in the spring tide case and only about 0.02 for the neap tide survey. In contrast, the westward mean flow dominates during neap tide with a ratio of lower-frequency flow energy to overall flow energy at 0.65. This ratio drops to 0.05 during spring tide. The moored time series does not show a clear mean westward flow in the range of 750–950 m during the neap tide survey (cf. Fig. 3f); however, as noted above, the HDSS data recorded closer to the ridge are expected to better reflect current conditions in the direct vicinity of the topography.

b. Lee-wave parameter space

Both tides and low-frequency flows generate internal lee waves as they interact with bottom topography. While the lee-wave response at any given time is dictated by the total velocity (sum of tides and mean flow), we first consider these in isolation. In the following we discuss a number of important parameters for each type of lee wave before we present observations of waves above the ridge.

A nondimensional lee-wave Froude number predicts a nonlinear flow–topography response. Various definitions for dimensionless numbers describing flow over topography are referred to as Froude number in the literature. Here we follow Mayer and Fringer (2017) to define the lee-wave Froude number

$$F_{r,\text{lee}} = Nh_0/U_0$$  \hspace{1cm} (2)

with buoyancy frequency $N$ at ridge-crest level defined through $N^2 = -(g/\rho_0)\partial\gamma/\partial z$, unperturbed flow amplitude $U_0$ and obstacle height $h_0$. Termed the topographic Froude number in Klymak et al. (2010) but defined in the same manner, this dimensionless number predicts the response of flow to bathymetry. At $F_{r,\text{lee}} \ll 1$ and above, the interaction results in a nonlinear wave response (Klymak et al. 2010). Here, with ridge height $h_0$ between 1000 and 2000 m, depending on the direction of the flow, stratification $N \approx 3 \times 10^{-3}$ s$^{-1}$ and flow speeds between 0.05 and 0.2 m s$^{-1}$, the lee-wave Froude number ranges between 25 and 140, thereby putting the flow into the regime where hydraulic effects and wave breaking are expected. A high lee-wave Froude number also implies partial blocking of the upstream flow (Klymak et al. 2010).

In shallow-water flow, the water depth $D$ above the obstacle is an important parameter in setting the wave response (e.g., Farmer and Smith 1980). Here, $D$ is not expected to play an important role in the response to topography as $U_0/(ND) \approx 0.02 \ll 1$. For systems with small $U_0/(ND)$, we do not expect waves to interact with the surface; we thus treat the ocean as infinitely deep (Mayer and Fringer 2017).

For the dominating semidiurnal tide, the observed tidal amplitudes translate into tidal excursion length scales

$$L_{\text{exc}} = \frac{U_{M_2}}{\omega_{M_2}}$$  \hspace{1cm} (3)

ranging from 400 m to 1 km for peak tidal velocities at neap and spring tides, respectively. The relevant topographic length scale with which to compare is the width of the ridge $W$ at the depth where topographic blocking is expected to occur. Water below this depth level lacks kinetic energy to be lifted across the ridge (Baines 1995, chapter 5). With maximum ridge crest tidal velocities $U_0$ about 0.15 m s$^{-1}$, $W$ is estimated to about 800 m at a depth level $U_0/N \approx 60$ m below the ridge crest. During neap tide when peak tidal velocities are only about 0.05 m s$^{-1}$, $W \approx 400$ m. The tidal excursion parameter

$$\xi = \frac{L_{\text{exc}}}{W}$$  \hspace{1cm} (4)

the ratio of tidal excursion length scale $L_{\text{exc}}$ and topographic length scale $W$, is thus $\mathcal{O}(1)$.

In his seminal work on the generation of tidal internal waves under linear conditions, Bell (1975a) shows that for small tidal excursion parameter $\xi$, the internal wave response is mainly at the fundamental forcing frequency. For increasing $\xi$ water particles are advected farther across the whole width of the ridge and waves at higher harmonics of the fundamental frequency are excited. Musgrave et al. (2016) show how under moderate conditions the time-dependent superposition of this multifrequency response generates a transient lee wave for $\xi = \mathcal{O}(1)$. For very large $\xi$ the solution converges to the steady lee-wave limit.

Steady lee waves have zero Eulerian frequency and are arrested over the topographic feature. Under linear conditions, their intrinsic frequency can be calculated as

$$\omega_{\text{lee}} = U_c k_{\text{topo}}$$  \hspace{1cm} (5)

with the ridge crest velocity $U_c$ and the topographic wavenumber $k_{\text{topo}} = 2\pi/W$. Here, with mean flow velocities of about 0.05 m s$^{-1}$ near the ridge crest, the intrinsic time scale based on $\omega_{\text{lee}}$ is about 6 h. In the linear case, this corresponds to the establishment time for the lee wave. As this is about half a semidiurnal period, at this time scale the steady lee wave will clearly feel the influence of the tide and, as the velocity observations show, will be advected across the ridge during spring tide. However, as shown above, Froude numbers suggest the problem to be nonlinear. Following Klymak et al. (2010), the time for the first wavelength response for
flow impinging on the ridge to form can be estimated as \\
\[ \Delta t = 2\pi (\alpha_{ef} U_C)^{-1} \] which is independent of the flow velocity. With the topographic steepness parameter \( \alpha_{ef} \) ranging between 0.3 and 0.8 on the eastern and western side, respectively, establishment time scales are on the order of 1–2 h and therefore much shorter than half a period of the semidiurnal tide. At these short time scales, the semidiurnal flow will behave like a quasi-steady current and it is the simple addition of tidal flow and mean flow that sets up the nonlinear, quasi-steady lee-wave response.

The vertical group speed of a nonlinear wave scales approximately as \\
\[ c_{gz} \alpha_{ef} U_C \] (Klymak et al. 2010). This translates to vertical propagation of (100) m within an hour with values for \( \alpha_{ef} \) and \( U_C \) cited above.

Quasi-steady lee waves and tidal lee waves react very differently when encountering background flows or shear layers. For tidal waves near the acoustic limit (Bell 1975b), the presence of a steady background flow means that the ray pattern radiating away from the ridge will not be symmetric but tilted owing to Doppler shifting. For quasi-steady lee waves, shear layers can reduce the wave frequency to the local inertial frequency. In these critical layers, phase propagation relative to the background current stalls and hence no upward energy propagation is possible (Bell 1975b; Booker and Bretherton 1967). Quasi-steady lee waves dissipate their energy in these critical layers. However, as the first shear layer is located about 300 m above the ridge crest it takes about 3 h for the lee wave to reach the critical layer at the vertical group speed \( c_{gz} \) estimated above. As this reaches tidal time scales where flow conditions above the ridge cannot be approximated as steady anymore, it is unclear whether much of the nonlinear lee-wave response reaches the critical layer and dissipates its energy there. We know of no observations of this.

From the above considerations, we expect a combination of tidal flow and mean flow at the ridge crest to form quasi-steady lee waves within relatively short time scales (faster than half a tidal cycle). During spring tide when oscillatory velocities are strong, lee waves can form on both sides of the ridge. The mean flow will bias the resulting lee waves by reducing the ridge crest velocity during eastward tidal flow and increasing it during westward flow. During neap tide when the mean flow dominates and inhibits oscillatory flow across the ridge, lee waves will be generated only on one side of the ridge.

c. Lee-wave observations

The measurements reveal the presence of breaking internal waves in the vicinity of the ridge. In the following we highlight one towyo section across the ridge in detail before we show all sections from both surveys. The example section shown in Fig. 5 was conducted at

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**Fig. 5.** Sample section across the ridge from the spring tide survey. Towyo data are depth averaged into 25 m size bins for better visualization. The ship moved in a westward direction as indicated by the arrow in (b). (a) Vertical isopycnal displacement \( \zeta \) (color along instrument package trajectory) and potential density anomaly \( \sigma_1 \) (black contours). The thick contour is the \( \sigma_1 = 27.25 \) isopycnal. Density contours are repeated in the following panels. (b) Zonal (cross-ridge) velocity \( u \) from HDSS and LADCP. LADCP velocity is shown in the bottom layer where sidelobe interference inhibits shipboard sonar measurements. Characteristic wave slopes are shown for the semidiurnal tidal frequency (dotted), intrinsic lee-wave frequency at 0.05 m s\(^{-1}\) flow speed (dash–dotted), and lee-wave frequency at 0.15 m s\(^{-1}\) flow speed (dashed). (c) Dissipation rate of turbulent kinetic energy \( e \). Only bins with \( \log_{10}(e) > -9.5 \) W kg\(^{-1}\) are shown. (d) Inverse Richardson number (\( R_i^{-1} \)). Only bins with \( R_i^{-1} > 0.25 \) are shown.
the beginning of the spring tide survey. The ship crossed the ridge in the westward direction during a phase of westward flow above the ridge crest. Isopycnals over the ridge are vertically displaced by up to 80 m relative to a survey-mean density profile. The dissipation rate of turbulent kinetic energy \( \varepsilon \) from Thorpe scales is highly elevated downstream of the ridge near topography. Elevated levels of \( \varepsilon \) are also found along the backward (relative to the flow direction) sloping band of wavy features, for example, clearly visible at \( x = 0.5 \) km around 750 m depth.

The vertical wavelength of a high-mode stationary wave implied by the velocity scale is \( \lambda_0 = 2\pi U_c/N \) (Klymak et al. 2010). For peak ridge crest velocities \( U_c \approx 0.15 \text{ m s}^{-1} \) the expected vertical wavelength \( \lambda_0 \) is approximately 350 m. Due to the background shear layers it is difficult to make out the velocity signature of the waves. In their model runs, Klymak et al. (2010) find vertical displacements of about 0.25\( \lambda_0 \). Vertical displacements of up to about 80 m observed here also scale as approximately 0.25\( \lambda_0 \). For comparison, Alford et al. (2014) found vertical wavelengths \( \lambda_0 \approx 300 \) m for high-mode oscillatory lee waves over Kaena Ridge near Hawaii.

The characteristic slope \( \alpha \) of an internal wave can be calculated following (1) based on the wave frequency \( \alpha \). Characteristics for the semidiurnal tide \( \alpha_{M_2} \) and steady lee-wave response \( \alpha_{\text{lee}} \), the latter calculated from (5), are shown in Fig. 6. The slope of the steepest characteristic, calculated from a ridge crest velocity of 0.15 m s\(^{-1} \), traces the upward and backward propagation of the wave pattern most closely.

Observations of turbulent dissipation rate and concurrent Richardson numbers suggest that both shear-driven mixing and convectively driven mixing are at play during the wave breaking. We show inverse Richardson number calculated as the ratio of the square of vertical current shear \( U^2_z = (\partial u/\partial z)^2 + (\partial v/\partial z)^2 \) to stratification \( N^2 \):

\[
\text{Ri}^{-1} = U^2_z/N^2;
\]

\( \text{Ri}^{-1} \) expresses the tendency of shear to overcome stratification. Turbulent mixing driven by shear instabilities is usually expected as the inverse Richardson number reaches beyond 4 (Miles 1961; Howard 1961). Due to relatively coarse vertical resolution of the shear measurements we expect shear-driven turbulence to occur for \( \text{Ri}^{-1} < 4 \). However, for a number of mixing events \( \text{Ri}^{-1} \) is well below unity. These regions of relatively low \( \text{Ri}^{-1} \) and high turbulent dissipation rate are indicative of convectively driven mixing as often found in hydraulic jumps (e.g., Alford et al. 2013).

All cross-ridge sections from both the spring and neap tide surveys are shown in Figs. 6 and 7, respectively. They show similar features discussed for the single section above. Isopycnals exhibit wave-like fluctuations over the ridge in many of the sections. Vertical displacements of isopycnals are \( \sim (50) \) m. The wave-like patterns appear to be arrested over the ridge with a backward-sloping characteristic relative to the dominant flow direction over the ridge crest. Waves of this type appear both during the spring and neap tide surveys, although vertical displacements are generally smaller during neap tide.

Turbulent dissipation rates \( \varepsilon \) reach up to \( 10^{-6} \text{ W kg}^{-1} \) and are thus elevated by several orders of magnitude over oceanic background levels. Enhanced levels of \( \varepsilon \) near topography are found to be shifting with the tidal flow back and forth across the ridge for the spring tide survey. For example, columns 1, 4, and 12 in Fig. 6 show enhanced \( \varepsilon \) on the western side of the ridge during westward flow. For the neap tide survey, overall levels of \( \varepsilon \) are weaker and high values are mostly found on the western side of the ridge. As for the example section, inverse Richardson numbers reach beyond 2 for some of the regions of elevated \( \varepsilon \), but not all, indicating the presence of both shear-driven and convectively driven turbulence processes.

d. Survey-average conditions

We highlight the differences between spring and neap tide survey by calculating time-mean of flow and turbulent dissipation rate. The left panels in Fig. 8 show the time mean over each survey with values bin-averaged on a 500 m width \( \times \) 50 m depth grid. The ridge-normal flow during both surveys is very similar with westward flow near the ridge crest and eastward flow aloft. The time-mean turbulence pattern for the spring tide survey is relatively symmetric across the ridge and thereby indicative of breaking internal tidal waves generating this pattern. This is as expected for tidal velocities dominating over the mean flow. The value of \( \varepsilon \) is slightly higher on the western side, possibly indicating that the superposition of the westward mean flow on top of the tidal flow leads to swifter flows and a stronger hydraulic response on the western side. In contrast, the survey-average turbulent dissipation rate pattern for the neap tide is highly skewed toward the western side of the ridge. We conclude that a steady lee wave, set up by the tidal flow leads to swifter flows and a stronger hydraulic response on the western side. In contrast, the survey-average turbulent dissipation rate pattern for the neap tide is highly skewed toward the western side of the ridge. We conclude that a steady lee wave, set up by the lower-frequency flow across the ridge, breaks and generates the asymmetric pattern in turbulent dissipation. Individual ridge crossings show that the tide, even though weaker than at spring tide, still modulates this time mean pattern (Fig. 7). However, as the tide is not strong enough to overcome the westward flow during
neap tide, we mostly see elevated turbulent dissipation rates associated with breaking internal lee waves on the western side. This is a central result of our study.

Absolute values of turbulent dissipation are higher during spring tide than during neap tide near the ridge and higher up in the water column, consistent with stronger internal tide generation, local breaking over the ridge and upward radiation where some of the waves also locally dissipate.

Viewed in terms of height above bottom (Fig. 9), time averages of turbulent dissipation rate $\varepsilon$ follow a familiar pattern of decreasing turbulence away from the bottom into the interior ocean (e.g., Polzin et al. 1997; Waterhouse et al. 2014). The lower 200 m above bottom exhibit an order of magnitude difference in turbulent dissipation on either side of the ridge during the neap tide survey. At 400 m above bottom and beyond, $\varepsilon$ is of comparable size between eastern and western sides with an order-of-magnitude difference between spring and neap tides. This may indicate that low-frequency flow-generated lee waves cannot pass through critical layers set up by the strong shear layers, only becoming apparent during neap tide when no tidal internal waves are breaking and causing higher $\varepsilon$ aloft.
Averaging the spring tide survey into different phases of the tide shows how tidal waves set up and break on either side of the ridge. The center panels in Fig. 8 show zonal velocity, turbulent dissipation rate, and vertical isopycnal displacement averaged into eastward and westward flows as indicated in the low-pass-filtered HDSS time series in Fig. 3g. The data are averaged on the same 500 m × 50 m grid as the overall survey means. Time-mean velocities show how the flow over the ridge crest changes direction. Turbulent dissipation rate is biased toward the west during westward flow and, although not as clear, stronger on the eastern side during eastward flow. Time-mean displacement shows generally elevated isopycnals on the upstream side of the ridge.

Further separation of the spring tide survey into peak tidal flows and slack tide is shown in the four right panels.
in Fig. 8. The bottom panel shows the separation into four tidal phases using the 8-h low-pass-filtered HDSS ridge-crest level ridge-normal velocity. Peak tidal flows are defined as ridge-normal flow magnitude above 0.04 m s$^{-1}$. During slack tides, regions of higher turbulent dissipation rate are concentrated around the ridge crest with a slight bias toward the downstream side relative to the preceding tidal flow. During peak tidal flow, the pattern is also biased toward the downstream side but less concentrated around the ridge crest. Displacement patterns during peak tidal flow exhibit the upstream lifting and downstream suppression seen in the center panels. Displacement during slack tide does not show a clear pattern.

e. Form drag

The ridge exerts a force on the flow that may be formulated as a form drag. Form drag manifests itself as a pressure drop across the obstacle and can be calculated from the horizontal integral over the product of bottom pressure $p_B$ and bottom slope $dh/dx$:

$$D_f = \int_{x_1}^{x_2} p_B \frac{dh}{dx} \, dx. \quad (7)$$

Form drag as defined here has units of newtons per meter since we are operating in only two dimensions. Integrating also in the north–south direction along the ridge would result in form drag expressed in newtons. We did not measure bottom pressure $p_B$ directly but can obtain the part of $p_B$ due to baroclinic motions (following Warner et al. 2013) based on the hydrostatic equation by integrating the density anomaly $\rho'$ vertically:

$$\rho' = \int_{z_0}^{z_1} \rho'(z) \, g \, dz. \quad (8)$$

Density anomaly profiles $\rho'$ are calculated from the CTD measurements by subtracting the survey-mean density profile for the spring and neap tide surveys, respectively. As CTD measurements were only obtained to within a safe distance from the bottom, density data are missing.
in the lower 20–60 m above bottom. In this bottom layer we assume a constant density profile as given by the deepest density estimate in each CTD profile. Pressure anomalies obtained by integrating \( \rho' \) from 480 m depth to the bottom are shown in Fig. 10. For most profiles, no density data exist at shallower depths and we have to restrict our analysis to the deeper part of the water column, assuming that variations in the density field higher up in the water column do not exert a leading order influence on the pressure field and hence form drag at depth. A comparison with bottom pressure calculated from full-depth density profiles shows that this introduces an uncertainty in individual bottom-pressure anomaly estimates of up to 100 N m\(^{-2}\). The uncertainty in form drag estimates introduced by only integrating density in the deeper part of the water column is about \( 1.5 \times 10^4 \) N m\(^{-1}\) (appendix B). In addition, we are missing the external component of the bottom-pressure anomaly caused by variations in sea surface height (see, e.g., Warner et al. 2013). Nonhydrostatic pressure perturbations, also missing in \( p_B \) as estimated here, have been found to play only a subordinate role for nonlinear internal waves (Warner et al. 2013; Moum and Smyth 2006).

To obtain meaningful values for the form drag, horizontal integration of bottom pressure in (7) has to start and end at the same bottom depth. We integrate over the part of the ridge shallower than 1250 m as for most of the ridge crossings the profiles do not reach deeper on both sides of the ridge. Form drag calculated this way for each ridge crossing is shown in Fig. 10 and plotted in Fig. 11; absolute values of form drag are in the range of \((0.01–4) \times 10^4 \) N m\(^{-1}\). For reference, Warner et al. (2013) find maximum form drag of \( 1 \times 10^4 \) N m\(^{-1}\) during
Various parameterizations for form drag exist both for steady and oscillatory flow. Here we focus on parameterizations for steady flow as the tidal currents reach a quasi-steady state as shown above. Results from two parameterizations are plotted in Fig. 11: a bluff body drag that is quadratic in velocity calculated as

$$ F_d = \frac{1}{2} \rho_0 C_D h_m U_C |U_C|^2 \cdotp (9) $$

with a drag coefficient $C_D = \zeta U_C$ (cf. Warner et al. 2013), and a form drag parameterization for high-mode nonlinear lee-waves quadratic in topographic height parameter $h_m$ (Klymak et al. 2010):

$$ F_D = \frac{3}{2} \rho U_C h_m^2 \frac{\pi}{2}. \cdotp (10) $$

Only the part of the topography that is subjected to flow plays a role for the form drag. As the flow is not barotropic and only part of the water column interacts with the topography, and we lack information on the flow conditions around the deeper part of the ridge, we apply the parameterizations with two different topographic height parameters (300 and 600 m). As also found by Warner et al. (2013), the nonlinear wave parameterization better captures the higher form drag values than the bluff body parameterization. Only with an unusually high drag coefficient of $C_D = 50$ does the bluff body parameterization capture the relationship between ridge crest velocity and form drag reasonably well (dash–dotted teal line in Fig. 11).

Kinetic energy of the flow lost to form drag is calculated as $U_C D_F$, where $U_C$ is the velocity of the flow interacting with the bathymetry. Here, we calculate $U_C$ as before as an average of HDSS ridge-normal velocity over the layer between 750 and 950 m depth. We note that $U_C$ is the superposition of tidal and mean flow. For a two-dimensional ridge and purely barotropic tidal flow, the power loss due to form drag would equal the tidal barotropic to baroclinic conversion term (Warner et al. 2013). For oscillatory flow, form drag can have a component termed inertial drag in Warner and MacCready (2009) and Warner et al. (2013) that is in quadrature with velocity and does no net work on the flow. In Warner et al. (2013) the inertial drag is associated with the sea surface tilt driving tidal barotropic flow. As we do not include any sea surface variability in the $p_B$ estimates, we do not expect inertial drag to play a large role. Survey-average power loss is 1450 and 200 W m$^{-2}$ for the spring and neap tide survey, respectively. Dividing by the horizontal integration distance $dx = 2.7$ km used in the form drag calculation, this translates to power losses per unit area of 538 and 76 mW m$^{-2}$ for
For reference, in their study in Puget Sound mentioned above, Warner et al. (2013) find a tidally averaged power loss to form drag of $200 \text{ mW m}^{-2}$. Peak power losses during strong tidal currents were $1.7 \text{ W m}^{-2}$ in Warner et al. (2013), compared to peak power losses of $1.2 \text{ W m}^{-2}$ during strong westward flow at spring tide found here.

The relationship between the expected energy loss $U_C D_I$ and the measured energy loss, calculated as the integral over the dissipation rate of turbulent kinetic energy $\epsilon$ below 500 m depth and over each ridge crossing $\int_{D} p' u' dz$, is shown in Fig. 12. The slope of a least squares fit through the data points is approximately 0.1; thus, on average we observe about 10% of the expected energy lost to form drag in the direct vicinity of the ridge. The data into spring and neap tide surveys results in slopes of 0.1 and 0.15, respectively, indicating that a larger portion of energy dissipates locally during neap tide. As turbulence is patchy, there is a wide spread around the fits and the difference between spring and neap tides is not statistically significant. However, a relative increase in local dissipation during neap tide is expected as steady lee waves may encounter critical layers aloft and their vertical radiation is inhibited.

Radiation of internal waves may explain the remaining part of energy lost to form drag that is not observed to dissipate locally as in Knight Inlet (Klymak and Gregg 2004). The horizontal wave energy flux can be calculated as

$$F = \left\langle \int_{D} p' u' dz \right\rangle,$$

with pressure anomaly $p'$, velocity anomaly $u'$, and water depth $D$. The angle brackets denote an appropriate time average, for example, a tidal cycle when calculating the tidal internal wave flux. Most of the towyo observations stop at a turnaround depth of 400 m and thus do not allow calculation of horizontal internal wave fluxes. However, full depth profiles were measured during the first two ridge crossings of the spring tide survey, spanning approximately 12 h. The data density from these two ridge crossings on the eastern side of the ridge is sufficient to calculate the horizontal internal wave energy flux $F_{M_2}$ for the semidiurnal $M_2$ tide. Harmonic analysis of HDSS velocity and pressure perturbation calculated from (8) provides velocity and pressure anomaly in the semidiurnal frequency range. The resulting tidal-averaged depth-integrated energy flux in the east–west direction is directed away from the ridge at a magnitude of about $750 \pm 250 \text{ W m}^{-1}$. Thus, the eastward $M_2$ internal tide flux alone accounts for about half of the 1.5 kW m$^{-1}$ energy loss expected due to form drag during spring tide. This result is similar to the energetics of Knight Inlet Sill where about two-thirds of the internal waves generated from the barotropic tide radiate away while one-third is lost to near-sill processes (Klymak and Gregg 2004). A fraction of the energy flux into the lee waves may also feed back into the mean flow as explored in a recent study by Kunze and Lien (2019). Their theory hinges on the conservation of wave action in the presence of sheared background flow and may well be at play in the strongly sheared environment considered here (e.g., Fig. 4).

4. Summary and concluding remarks

We observed internal waves over a tall ridge under a combination of tidal and low-frequency flow. In the parameter space set by a topographic Froude number, the waves formed within relatively short times compared to tidal scales. The resulting waves resembled quasi-steady topographic lee waves and were highly dissipative on the lee side of the ridge. Average conditions during spring and neap tides revealed a contrasting pattern in the resulting
lee waves. During spring tide, when the tidal flow dominated, breaking waves were observed on either side of the ridge. During neap tide, when the low-frequency westward flow at the ridge crest dominated over the tidal flow, breaking lee waves formed mostly on the western side of the ridge. We conclude that the observed lee waves are generated by a combination of tidal flow and mean flow. As the wave generation consumes energy and can carry momentum, it provides a sink term for both the low-frequency ocean circulation and the tides.

A few studies find similar patterns in the deep ocean. Musgrave et al. (2017) find that the superposition of a subinertial, topographically trapped diurnal tide and radiating semidiurnal tide leads to modulation of the tidal lee-wave response in a channel on the Mendocino Ridge. When the currents from the two tidal components interact constructively, a tidal lee wave forms and strong turbulent dissipation rates are observed in the lee of the topography. When interacting deconstructively, no tidal lee wave forms. The results presented here may be interpreted analogously: during neap tide, when tidal currents are weakened and do not dominate over the mean flow anymore, the constructive superposition with the mean flow leads to tidal lee-wave generation on the western side while no lee waves are observed on the eastern side. Dale and Inall (2015) observe an asymmetry in tidal lee waves and associated turbulence at a saddle amid steep and complex Mid-Atlantic Ridge topography. They speculate that the asymmetry could be due to background flow, a similar situation as presented here.

A recent theoretical study by Shakespeare and Hogg (2019) examines the influence of a mean flow on momentum fluxes associated with tidal-wave generation. Bell (1975b) already noted that for a mean background flow generating lee waves, the presence of tides can increase momentum fluxes by several tens of percent but did not elaborate further. Shakespeare and Hogg (2019) explore this phenomenon in more detail and show how the bottom background flow Doppler shifts the internal tide at generation and leads to a net momentum flux directed against the bottom flow. Unfortunately, form drag estimates presented here are too coarse to explore this phenomenon in further detail.

The observations show form drag on the order of $10^4$ N m$^{-1}$, acting both on tidal flow and mean flow. The form drag scales approximately linearly with ridge-crest velocity and is captured relatively well by the Klymak et al. (2010) parameterization when adjusting the topographic height parameter. A comparison of expected loss due to form drag with observed energy loss in the vicinity of the ridge suggests that between 85% (neap tide) and 90% (spring tide) of the energy lost to form drag radiates away and dissipates elsewhere. A rough estimate of about 1 kW m$^{-1}$ for the $M_2$ internal tide energy flux on the eastern side of the ridge supports this conclusion. A recent study by Zheng and Nikurashin (2019) shows, based on idealized numerical simulations and linear theory, that for the Drake Passage region about 30%–40% of lee waves generated by low-frequency flow dissipate locally near topography while the remaining part of the waves decay downstream of the topographic feature.

Mixing and drag induced by the ridge influence the low-frequency flow. A motivating question for this study is the general importance of submarine ridges for the energy and momentum budget of low-frequency flows. We do not have an estimate for the form drag averaged over a tidal cycle reliable enough to estimate the stress on the mean flow separated from that on the total flow. The kinetic energy of a 0.05 m s$^{-1}$ amplitude flow that extends over 500 m in the vertical is about $7 \times 10^3$ J m$^{-2}$. Power loss due to form drag was estimated to about 50 mW m$^{-2}$ for the neap tide survey. Assuming that during neap tide the low-frequency flow contributes the majority to the lee-wave formation on the western side of the ridge, the $(0.05$ m s$^{-1} \times 500$ m) flow would be consumed within about 3.5 h at this power loss. Since there are still tidal currents contributing at neap tide, this estimate is likely too fast, but it shows that the ridge exerts a strong influence on the flow around its ridge-crest level.

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APPENDIX A

Turbulence Estimates

Turbulent dissipation is estimated from CTD density profiles using the Thorpe scale method. Thorpe length scales $L_T$, calculated from the vertical displacements $d$ needed to sort a density profile into a stable state as $L_T = \sqrt{\langle d^2 \rangle}$, are linearly related to the Ozmidov scale (Thorpe 1977; Dillon 1982) and can be used to estimate the dissipation rate of turbulent kinetic energy $\epsilon$. The Ozmidov scale $L_O = \epsilon^{1/2} N^{-23}$ gives the size of the largest eddies that may overturn under stably stratified conditions.
Thus, it is a proxy for the vertical distance over which denser water may be carried by turbulent motion. Using the linear relationship between $L_O$ and $L_T$, turbulent dissipation may be calculated as

$$\varepsilon = cL_O^2N^3,$$

(A1)

with the proportionality constant $c$ ranging between 0.64 (Dillon 1982) and 0.9 (Ferron et al. 1998) in various studies. Turbulence estimates from Thorpe scales are usually within a factor of 2–5 of (more direct) microstructure observations of turbulence when sufficiently averaged over a number of turbulent events (Moum 1996; Ferron et al. 1998; Alford et al. 2006; Voet et al. 2015).

Spurious overturns resulting from noise in the density measurements are excluded using a method developed by Ferron et al. (1998). Before sorting the density profile into its stable state, an intermediate profile is constructed that takes the noise in the measurements into account by discretizing the measurements at the accuracy level of the density profile ($1 \times 10^{-3}$ kg m$^{-3}$).

**APPENDIX B**

**Bottom-Pressure Estimates and Form Drag Error**

Density anomalies $\rho'$ are integrated from 480 m depth to near the bottom to infer bottom-pressure perturbations $\rho_B'$ used in the form drag calculations. Constraints in profiling timing, as aliasing of the internal tide in the cross-ridge sections was to be avoided, led to only partial coverage of the water column with the shipboard CTD for most of the survey. However, full depth CTD profiles were recorded for the first two ridge crossings and allow for an error estimate of bottom pressure calculated from only the deeper part of the density profile (Fig. B1). Full-depth density profiles constructed from shipboard CTD measurements at depth and RBR Solo and SBE37 Microcat data from higher up in the water column provide additional error estimates during the neap tide survey. Absolute values of bottom-pressure anomalies differ on average by 20 N m$^{-2}$ between the two methods. We note that 65% of the values are within 40 N m$^{-2}$ of each other and only 3% differ by more than 100 N m$^{-2}$. The sign of the bottom-pressure anomalies is the same for both methods.

Form drag from the integration of (7) with $\rho_B$ calculated from full-depth density profiles differs from the partial-depth integrated profiles by $0.6 \times 10^4$ N m$^{-1}$ on average (see yellow markers in Fig. 11 for individual values). The maximum difference occurred during the first ridge crossing of the neap survey at $1.4 \times 10^4$ N m$^{-1}$. We assign a conservative uncertainty of $1.5 \times 10^4$ N m$^{-1}$ to the form drag estimates, thereby spanning all observed differences.

**REFERENCES**


