Near-Surface Reflection and Nonlinear Effects of Low-Mode Internal Tides on a Continental Slope

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ABSTRACT: Two sets of mooring data were collected at two sites (MA and MB) along a cross-slope section on the northeastern continental slope in the South China Sea (SCS). These data are used to investigate evolution and energy decay of low-mode semidiurnal (M2) internal tides on a subcritical slope with respect to M2. At the deep portion of the slope (~1250 m; MA), the M2 internal tides show upward energy propagation, while vertically standing M2 internal tides are often observed at shallow MB (~845 m). A two-dimensional linear internal tide model with realistic topography and stratification reproduces the observations, suggesting that low-mode M2 internal tides incident on subcritical slopes evolve into vertically propagating internal waves due to topographic scattering, propagate upward to the boundary, and reflect from the sea surface. The reflection point largely depends on the phase between the modal components of the incoming flux. In the near-surface reflection region, two kinds of nonlinear effects are observed to decay energy of the incoming internal tides. One is the resonant parametric subharmonic instability which transfers M2 internal tides to diurnal subharmonics M1 (=M2/2), but the instability is found to mainly depend on the incident waves. The other one is the nonresonant wave–wave interaction, producing two higher-harmonic M4 (=5M2) rays with opposite vertical propagation. A strong westward mean flow is observed in the interacting region, with amplitudes comparable to that of the incident waves. This mean flow also appears to be generated by the nonlinear reflection of the M2 internal tides.

KEYWORDS: Internal waves; Nonlinear dynamics; Ocean dynamics

1. Introduction

The conversion from surface tidal energy to internal tides at rough topography is now widely recognized as a major mechanism for turbulent mixing in the deep ocean that drives the overturning circulation (Huang 1994; Munk and Wunsch 1998). The conversion energy partly goes into high-mode internal tides that locally dissipate to turbulence (e.g., St. Laurent and Garrett 2002; Waterhouse et al. 2014; Alford et al. 2015), but the larger fraction radiates to low-mode internal tides which can transport energy for thousands of kilometers away from the generation sites (Klymak et al. 2006; Carter et al. 2008; Zhao and Alford 2009). The breaking and associated mixing of these radiated internal tides affect the ocean properties (e.g., stratification, heat and nutrients) and the strength of the overturning circulation (de Lavergne et al. 2016; Ferrari et al. 2016; McDougall and Ferrari 2017). Therefore, it is important to understand where and how low-mode internal tides lose energy to dissipation scales available for mixing.

One potential mechanism for sink of low-mode internal tides is that these waves impact continental margins and their energy can scatter into higher modes via wave–topography interaction (Müller and Liu 2000; Klymak et al. 2011; Kelly et al. 2013a; Xie et al. 2018). In the interaction, topography is often classified as three types, which depends on the slope β of the bottom boundary and the slope α of the wave group velocity to the horizontal plane. The β > α, β = α, and β < α correspond to supercritical, critical and subcritical slopes, respectively. The slope α depends on the inertial frequency f, wave frequency ω, and buoyancy frequency N:

$$\alpha = \sqrt{(\omega^2 - f^2)/ (N^2 - \omega^2)}.$$

Combining the linear theory and numerical experiments, Müller and Liu (2000) found that more incoming energy is transferred to higher modes on near-critical slopes than on other slopes. Therefore, large internal wave overturning and strong dissipation are often observed at bottom boundary above critical–supercritical slopes (Eriksen 1982; Moun et al. 2002; Nash et al. 2004, 2007; Klymak et al. 2011; Martini et al. 2013; van Haren et al. 2015). Although the linear models estimated that little wave energy is dissipated on subcritical slopes (Müller and Liu 2000), the theory developed by Thorpe and Haines (1987) and Thorpe (1992) suggested that nonlinear interaction between incident waves and reflected waves at bottom boundaries can cause considerable wave steepening, form thermal fronts and enhance wave dissipation and mixing. In the ocean, upward traveling internal tide rays may be formed when low-mode internal tides scatter into higher modes on subcritical slopes (Kelly et al. 2013b). When these wave rays propagate upward to upper boundaries, they are also subject to nonlinear processes to decay energy. New and Pingree (1992) observed mode-1 internal solitary waves in the central Bay of Biscay and they attributed generation of these waves to interaction between upward traveling internal tide beams and pycnocline in the upper ocean. Thorpe (1998) suggested that nonlinear

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reflection of internal waves at the base of the mixed layer can cause generation of higher harmonics. Using a two-dimensional numerical model, Lamb (2004) found that internal tide beams reflect from the upper boundary and cause nonlinear interaction between incident and reflected beams, leading to generation of new internal wave beams with supertidal frequency. Subsequent theoretical studies (Tabaei et al. 2005) showed that when two internal wave rays with the same frequency \( \omega \) meet in the vicinity of surface reflection region, their nonlinear interaction can generate a mean flow (\( -\omega \) – \( \omega \)) and higher-harmonic disturbances with frequency \( 2\omega \) (\( \omega + \omega \)). This type of wave–wave interaction is not a result of classic resonant triad interactions but is nonresonant because the resonant interactions have a relatively slow energy transfer rate while new waves generated by nonresonant interactions can fast propagate away from the generation source (Lamb 2004). Mercier et al. (2012) conducted a series of laboratory experiments for the near-surface reflection of internal waves and observed the mean flow and higher harmonics generated by the nonresonant interactions.

Using numerical simulations, Gayen and Sarkar (2013) not only reproduced the nonresonant interactions generating higher harmonics during the near-surface reflection of internal tide beams, but showed the occurrence of parametric subharmonic instability (PSI), a nonlinear triad resonant interaction that transfers energy from a propagating wave with \( \omega \) to two smaller-scale subharmonics with \( \omega/2 \). Zhou and Diamessis (2013) also found these two nonlinear interactions in their wave reflection models but it is unclear which nonlinear process may become dominant. These studies have highlighted the importance of the near-surface reflection in energy decay of internal tides but most of them focus on internal tide beams (high mode internal tides) and are rarely linked to low-mode internal tides.

In this paper, we focus on low-mode internal tides impinging on the continental slope on the east side of Dongsha Island in the northeastern South China Sea (SCS). This slope is primarily subcritical for semidiurnal internal tides (Klymak et al. 2011). Xie et al. (2013) analyzed a set of mooring data collected in this region and simultaneously observed the nonresonant and resonant (PSI) interactions of internal tides during a semidiurnal spring tide. They hypothesized that these enhanced nonlinear effects result from the near-surface reflection of semidiurnal internal tides despite the internal tide beams were not observed. One motivation of this study is to test this hypothesis and clarify dynamics of the wave reflection on subcritical slopes. Since dissipation of low-mode internal tides is still poorly understood on subcritical slopes, our second object is to confirm the occurrence of nonlinear effects during the near-surface reflection of internal tides and suggest that nonlinear reflections of internal waves can be important for energy decay of low-mode internal tides impacting continental slopes.

2. Methods

a. Data

Mooring current data were obtained during the summer months (31 July–26 September) of 2014 at two fixed sites (MA and MB) on the continental slope on the east side of Dongsha Island (Fig. 1a). The site MA is located on the deeper portion of the continental slope, with the water depth of \( \sim 1250 \) m, and the water depth at MB is \( \sim 845 \) m. The mooring at MA was equipped with three acoustic Doppler current profilers (ADCPs): one upward-looking WHS75K ADCP moored near 440-m depth, one downward-looking WHS75K ADCP moored near 450-m depth, and another downward-looking WHS150K ADCP moored near 950-m depth. At MB, two sets of upward and downward looking WHS75K ADCPs were fixed at depths of 510 and 521 m, respectively. All WHS75K ADCP recorded data every 120 s from 16-m vertical bins, while the WHS150K ADCP at MA collected a sample every 90 s with a vertical resolution of 8 m. After 18 September, the top ADCP at MA had large swing that caused its depth variation more than 30 m due to the effects of Typhoon Fung-Wong. Data at depths above 160 m were unreliable during the period and not used in this study.

The bathymetry used in the paper is obtained from GEBCO’s gridded bathymetric data with a 30-arc-s interval (http://www.geobco.net/). The slope crossing the mooring sites is dominantly critical-supercritical to the \( K_1 \) internal tide but is subcritical to the \( M_2 \) internal tide (Fig. 1b). The ray paths for \( K_1 \) and \( M_2 \) are computed based on a profile of \( N_u(z) \) from the climatological temperature and salinity from the 2009 World Ocean Atlas (https://www.nodc.noaa.gov/OC5/WOA09) (Fig. 1c). At MB, temperature was also measured at depths of 110, 120, 140, 160, 200, 240, 280, 360, 460, 660, 760, and 825 m (Xie et al. 2018). The time-average temperature, as well as a yo-yo CTD profile (providing salinity data) near this site, is used to compute a stratification profile \( N_u(z) \). The \( N_u(z) \) shows similar results with \( N_u(z) \) at available depths (Fig. 1c).

The paper also uses the daily mean current data from a global ocean monitoring and forecasting system: the Copernicus Marine Environment Monitoring Service (CMEMS). The CMEMS used the NEMO (Nucleus for European Modeling of the Ocean) model that assimilated remotely sensed and observed in situ temperature, salinity and sea level, with a horizontal resolution of \( 1/12^\circ \times 1/12^\circ \) and vertical resolutions increasing from 1 m near the surface to 450 m in the deep ocean (>3000 m). These modeled currents mainly represent wind-driven currents and geostrophic motions.

b. Decomposition of signals

We focus on internal tides and low-frequency flows. A bandpass filtered technique based on a second-order Butterworth filter is used to extract the diurnal and semidiurnal components in raw current time series. Cutoff frequencies of the diurnal and semidiurnal bands are set to \([0.85, 1.15]\) cpd (cycles per day) and \([1.7, 2.2]\) cpd, respectively. Using the low-pass filter with a cutoff frequency of 2.5 days, the low-frequency component is extracted.

The bandpassed diurnal and semidiurnal horizontal velocities \( (u, v) \) are further separated as coherent and incoherent components by applying a one-dimensional \( \omega \) Fourier filter (van Haren 2004). The coherent components containing diurnal \( O_1 \) and \( K_1 \) and semidiurnal \( M_2 \) and \( S_2 \) can be obtained by

\[
\begin{align*}
u_c = & \sum_n U_n \cos(\omega_n t + \phi_n), \\
\omega_n = & O_1, K_1, M_2, S_2, \\
(2)
\end{align*}
\]
where $U_n$ is the amplitude of the $n$th constituent and $\phi_n$ is its phase, similarly for $v$. The incoherent component is then defined as the filtered velocity minus $u_c$. In addition, a two-dimensional ($u$ and $k_z$) Fourier filter is also used to decompose the velocity into $u^+$ with upward energy propagation ($k_z z + \omega t$) and $u^-$ with downward energy propagation ($k_z z - \omega t$), where $k_z$ is the vertical wavenumber (Pinkel 1984).

To investigate modal contents of internal waves at two mooring sites, the bandpass filtered velocity signals are projected into vertical modes. The baroclinic modes of the vertical displacement $\Phi(z)$ can be determined by the following eigenvalue equation (Gill 1982):

$$d^2\Phi(z)/dz^2 + N_c^2(z)\Phi(z)/C_n^2 = 0,$$

subject to the boundary conditions $\Phi(0) = \Phi(H) = 0$, where $C_n$ is the eigenspeed of the $n$th mode and $H$ is the water depth. The corresponding modal structure of horizontal velocity $\psi_n(z)$ is

$$\Psi(z) = d\Phi(z)/dz.$$

In Eqs. (2) and (3), $N_c(z)$ is used to calculate the modal function and the eigenspeed of each mode. The velocity of each mode can be extracted from the observed velocity profiles by least squares modal fitting (Nash et al. 2005).

c. Numerical model

A two-dimension Coupling Equations for Linear Tides model (CELT) developed by Kelly et al. (2013b) is used to investigate reflection of the $M_2$ internal tides on the subcritical slope. The model is configured with realistic stratification, topography, and planetary rotation. To reduce the effect of small-scale topography, a low-pass-filtered $H$ with a 30-km cutoff wavelength are applied to the east–west profile of the slope on the eastside of Dongsha Island. A section crossing the mooring sites is input into the model (Fig. 1b). The stratification is set to be horizontally constant across the entire section, which is computed from climatological data near the mooring sites (Fig. 1c). The model incorporates normally incident internal tides, twenty vertical modes, 1-km horizontal resolution, and radiation boundary conditions. The viscosity is set at $10^{-3} \text{m}^2 \text{s}^{-1}$. The deep sea and continental shelf are truncated at isobaths of 3200 and 250 m, respectively.

3. Near-surface reflection of semidiurnal internal tides

a. Observations

Kinetic energy spectra computed from horizontal velocities at two sites show the peaks at inertial ($f$), diurnal ($O_1$ and $K_1$)
and semidiurnal ($M_2$ and $S_2$) frequencies (Figs. 2a,b), which is a common feature in the South China Sea. In the diurnal band, two peaks at tidal frequencies $O_1$ and $K_1$ can be distinguished at most depths at MA, while the $O_1$ peak becomes ambiguous at MB, especially in the upper 120 m where the nontidal component $M_1$ ($= M_2/2$) is also energetic and comparable to $O_1$ (Fig. 2b). Semidiurnal energy at both sites shows a near-surface enhancement. At MB, most $M_2$ energy is concentrated in the upper layer. From 100 to 200 m, spectral energy of $M_2$ is reduced tenfold (Fig. 2b). The reduction is much larger than that at MA. The bandpass filtered semidiurnal velocities confirm the near-surface enhancement of $M_2$ internal tides. At MB, large $M_2$ velocities are often confined to the upper 120 m, below which the velocities are largely reduced (Fig. 3b), as shown in kinetic energy spectra (Fig. 2b).

The vertical propagation of internal tides can be identified in the velocity field. At MA, dominant downward (upward) phase (energy) propagation for $M_2$ internal tides is observed (Figs. 3a,c), while these waves often become vertically standing at MB (Figs. 3b,d). Hereafter, the vertical propagation refers to energy propagation. At MB, the enhanced semidiurnal velocities in the upper layer show an approximately 14-day spring–neap cycle due...
to the interference of coherent $M_2$ and $S_2$ internal tides, except in the period before 11 August during which the alternation of spring and neap is not obvious (Fig. 4a). During the first, second and fourth semidiurnal springs, the elevated $u_1$ and $u_2$ show comparable amplitude (Figs. 4b,c). The superposition of two opposite vertically propagating waves with comparable amplitude forms the vertically standing pattern near the sea surface (Fig. 3b). In other periods, the semidiurnal internal tides are often dominated by the upward traveling signals, but the enhanced downward propagating signals are also visible during the third spring.

The modal decomposition computed from the current records shows that mode-1 and mode-2 can explain more than 80% of total energy for semidiurnal internal tides, while their phase difference ($P_d = \phi_{mode-1} - \phi_{mode-2}$) may be associated with different propagation features of semidiurnal internal tides at the mooring sites. To investigate this process, we computed the phases of mode-1 and mode-2 in each 12.5 h based on Eq. (2) with $\omega = M_2$. At MA, mode-1 always lags mode-2 ($P_d < 0$) except on 17 September with very small internal tides (Figs. 5a,c). Although mode-1 often leads mode-2

![Figure 4](image1.png)  
**Fig. 4.** (a) Time series of semidiurnal current variance ($\sqrt{u^2 + v^2}$) averaged over depths above 120 m at MB. The green, blue, and red curves are total, upward, and downward components, respectively. The red vertical lines indicate neap tides identified from the $M_2 + S_2$ current variance. Time–depth maps of semidiurnal (b) $u_1$ and (c) $u_2$. The black vertical lines mark four semidiurnal springs (1–4) based on the maximum current variance.

![Figure 5](image2.png)  
**Fig. 5.** Time series of mode-1 (blue) and mode-2 (red) semidiurnal velocities at (a) MA and (b) MB. (c) Phase difference of mode-1 and mode-2 semidiurnal velocities at MA (green) and MB (black). The velocity is the zonal component.
(P_d > 0) during four semidiurnal springs at MB (Figs. 5b,c). P_d becomes negative when the upward propagating signals dominate over downward propagating signals, for example in the periods around 6 August, 5 September, and 15 September (cf. Figs. 5c and 4).

b. Numerical models

The mooring observations above have shown the near-surface intensification and bidirectional propagation of semidiurnal internal tides at MB, suggesting the potential near-surface reflection of internal tides on the subcritical continental slope. To confirm the surface reflection of the M_2 internal tides, we run the CELT model. The ratio (R) of observed mode-1 and mode-2 semidiurnal velocity magnitude is generally 1 to 3 except for those observed on 7–13 September at MB (Figs. 5a,b). Therefore, the M_2 internal tides with the energy flux F = 15 kW m^{-1} (F_{mode-1} = 2F_{mode-2}) are input into the model, propagating leftward from the deep basin to the slope (Fig. 6a). In this experiment (hereafter referred as E1), R is approximately equal to 1 at input boundary but it is between 1 and 3 at the mooring sites (Fig. 7), which is consistent with the observations. Since the local barotropic semidiurnal tide is very weak (Klymak et al. 2011), it is not introduced to the model. The model estimates that only ~1% of incoming internal tides are reflected back to the deep basin since the slope is dominantly subcritical with respect to M_2. Therefore, we mainly focus on the incoming waves that are transmitted onto the shelf. The linear evolution of these waves on the slope is given in Fig. 6a. A single mode wave is a purely horizontally propagating disturbance in a flat-bottomed ocean. However, the superposition of mode-1 and mode-2 shows the feature of

Fig. 6. (a) Horizontal velocities of incoming internal tides from the CELT numerical experiment E1 at t = 0 along the east–west profile of the slope shown in Fig. 1. The red and blue curves indicate incident (I_1-I_3) and reflected (R_1-R_3) rays computed from Eq. (1), respectively. The vertical lines indicate the locations of two mooring sites. Horizontal velocities from E1 at (b) MA and (c) MB in two M_2 periods.

Fig. 7. Time series of the modeled mode-1 (blue) and mode-2 (red) velocities from the CELT numerical experiment E1 at (a) MA and (b) MB.
vertical propagation on the gentle bottom ($x < -150\, \text{km}$), forming weak surface reflection at several positions (e.g., at $x = -215\, \text{km}$). The vertical propagation and surface reflection are enhanced on the steep portion ($-150 < x < 0\, \text{km}$) of the slope since low-mode internal tides are scattered to higher modes. Three incident internal tide rays ($I_1$, $I_2$, and $I_3$) that travel upslope and three corresponding downward propagating reflection rays ($R_1$, $R_2$, and $R_3$) are identified on the continental slope (Fig. 6a). The rays $I_3/R_1$ and $R_2/R_3$ are out of phase, corresponding to wave crest and trough, respectively. The strongest reflection appears at $x = -48\, \text{km}$ ($I_3/R_2$), with the reflection point nearly centering at MB. The surface reflection significantly enhances the velocities in the upper layer (above 120 m) at MB, which is consistent with the observations (Fig. 3b). Furthermore, $I_3$ and $R_1$ with opposite vertical propagation direction collide near MB. Therefore, internal tides appear to be vertically standing waves at this site (Fig. 6c), as shown in observations (Fig. 3d). On the contrary, upward traveling $I_2$ and $I_3$ pass through MA, except at depths very close to the sea surface where $I_1$ reflects from the upper boundary. Therefore, the $M_2$ internal tides at MA show dominantly upward propagation at most depths (Fig. 6b). The vertical structure of the modeled velocities at both sites is largely consistent with the observations (cf. Figs. 6b,c and Figs. 3c,d).

The numerical simulations also reproduce the observed phase difference between mode-1 and mode-2 semidiurnal internal tides at two mooring sites. In E1, mode-1 lags mode-2 at MA ($P_d = -45^\circ$; Fig. 7a), oppositely at MB ($P_d = 30^\circ$; Fig. 7b). The modeled $P_d$ are very close to the observed values ($P_d = -61^\circ$ at MA and $P_d = 34^\circ$ at MB) averaged over the first and second semidiurnal springs (Fig. 5c). The mooring observations have suggested that $P_d$ may affect vertical propagation of internal tides. Here, we conduct another two numerical experiments (E2 and E3) with different $P_d$ to explore the potential mechanism. In E2 and E3, the phase of the incoming mode-1 internal tides keeps the same to that in E1 while the phase of the incoming mode-2 internal tides at input boundary is increased by $\pi/3$ (E2) and $2\pi/3$ (E3). In these two experiments, $P_d$ at MB becomes $-30^\circ$ (E2) and $-90^\circ$ (E3), respectively. Because of $P_d < 0$, upward traveling signals become dominant at MB in these two experiments (Figs. 8c,d), as shown in the observations.

Two control experiments show that the reflection location of internal tides largely depends on the phase difference between mode-1 and mode-2 internal tides at input boundary (Fig. 8), which is essential to variable vertical propagation of internal tides at the mooring sites. As the mode-2 phase increases in E2 and E3, the reflection points of three incident rays ($I_1$–$I_3$) on the slope move shoreward (Figs. 8a,b). At the same time, $I_1/R_1$ is weakened but $I_2/R_2$ is enhanced. Since $I_2$ and $I_3$ pass through most depths at MB and $R_1$ is weakened, the upward traveling signals at this site become dominant in E2 and E3 (Figs. 8c,d). In E2, the vertical structure of internal tidal velocities at MB is similar to that at MA in E1 (cf. Fig. 6b and Fig. 8c) because $P_d(-45^\circ)$ at MA in E1 is close to that ($-30^\circ$) at MB in E2. The vertically standing signals are visible in the upper boundary because MB is still in the vicinity of the surface reflection region (Fig. 8a). As the phase of mode-2 further increases in E3, the reflection point is away from MB and the downward propagating signals become weaker therein (Figs. 8b,d).

The sensitive analysis shows that the model is relatively insensitive to $R$. When mode-1 and mode-2 have comparable amplitude ($1 < R < 3$), the model results do not show large
difference with three numerical experiments above. When mode-1 internal tides dominate over mode-2 (i.e., \( R/C^2 \approx 1 \)), the model shows that the reflection wave is largely weakened so that the upward propagating signals become dominant, as shown in the observations on 7–13 September during which mode-1 semidiurnal internal tides are much larger than mode-2.

4. Nonlinear instability of internal tides

a. Parametric subharmonic instability

Xie et al. (2013) have shown that the surface reflection of semidiurnal internal tides may enhance PSI which transfers M2 energy to two diurnal subharmonics M1. Our mooring observations also revealed that the nontidal M1 component has energy comparable to O1 and K1 near the surface reflection region (Fig. 2b). To highlight M1, the coherent O1 and K1 components are extracted from the raw velocity time series using Eq. (2). After removing these two diurnal tidal constituents, kinetic energy spectra show a dominant peak at M1 frequency in the near-surface reflection region (Figs. 2c,d), implying the potential PSI process from the M2 internal tides.

The ~14-day modulation of diurnal internal tides induced by interference of coherent O1 and K1 motions is observed in the upper layer but its phase is different from that of semidiurnal internal tides (Fig. 9a). The peak of the diurnal 14-day cycle lags the maximum diurnal barotropic tide amplitude for 3–4 days (cf. Figs. 9a,b). This time difference is close to the propagating time of diurnal internal tides from the wave source (Luzon Strait) to mooring site, similarly for semidiurnal internal tides (not shown). After removing O1 and K1 internal tides, their 14-day cycle disappears in the incoherent diurnal velocity but the enhanced M1 velocity is observed at all four semidiurnal springs, suggesting another potential diurnal 14-day cycle with a phase the same to that of semidiurnal internal tides. A significant peak around frequency of 0.07 cpd (~14 days) can be identified in the spectrum computing from semidiurnal and incoherent diurnal current variances, confirming the fortnightly modulation of M1 motions (Fig. 9c). The enhanced M1 velocity does not follow the barotropic fortnightly cycle (Figs. 9a,b), ruling out the possibility of M1 waves originating from the local tides. These observations provide evidence for energy transfer from semidiurnal internal tides to diurnal M1 waves.

A more quantitative approach for subharmonic transfer energy is the bicoherence analysis, which can be used to assess the significance of phase-locking between waves in a PSI triad (Carter and Gregg 2006; Sun and Pinkel 2013; MacKinnon et al. 2013). The bicoherence \( B(\omega_1, \omega_2) \), namely, the normalized bispectrum \( B(\omega_1, \omega_2) \), is defined as

\[
B^2(\omega_1, \omega_2) = \frac{|B(\omega_1, \omega_2)|^2}{E[U(\omega_1)U(\omega_2)^2]E[|U(\omega_1 + \omega_2)|^2]}
\]

\[
B(\omega_1, \omega_2) = E[U(\omega_1)U(\omega_2)U^*(\omega_1 + \omega_2)],
\]

where \( U(\omega) \) is the Fourier coefficient of horizontal velocities at \( \omega_* \) is the complex conjugate, and \( E[\cdot] \) represents an expected value. Theoretically, PSI requires two subharmonic waves to have opposite vertical propagation direction. The downward and upward traveling M1 waves with comparable amplitude...
can be observed above 200 m during the springs of semidiurnal tides in the incoherent diurnal velocity (Figs. 10a–c). Therefore, we use the following prefiltered bicoherence instead of Eq. (4).

\[
B^2(\omega_1, \omega_2, [\omega_1 + \omega_2]) = \frac{\langle B(\omega_1, \omega_2, [\omega_1 + \omega_2])^2 \rangle^2}{\langle |U(\omega_1)U(\omega_1)|^2 \rangle \langle |U(\omega_1 + \omega_2)|^2 \rangle^2}.
\]

where the upward (+) and downward (−) energy propagation components are prefiltered (Sun and Pinkel 2013).

Chou et al. (2014) suggested that the bispectral analysis may provide misleading results for the occurrence of PSI of semidiurnal internal tides in the presence of diurnal tides. To reduce the effects of diurnal waves, coherent K1 and O1 components are removed from the raw velocity record when computing Eq. (5). We select three 6-day-long velocity records from the first three semidiurnal springs. Data from the fourth semidiurnal spring are ruled out because of typhoon-induced modulations of semidiurnal internal tides in the presence of diurnal tides. To identify it, the signal at frequency and wavenumber resonance. The Eq. (5) with the same superscript (both upward or both downward) can be used to test this condition (Sun and Pinkel 2013). The results are given in Fig. 12. As expected, very small bicoherence (<0.4) is observed at triads \([M_1^+, M_1^+, M_2^-]\) (Fig. 12a) and \([M_1^-, M_1^-, M_2^-]\) (Fig. 12b) because inputs of Eq. (5) are dissonant in wavenumber.

The previous numerical studies have revealed that the primary wave undergoing PSI in the vicinity of the surface reflection region is associated with the incident wave (Gayen and Sarkar 2013). However, the Eq. (5) cannot distinguish which wave (incident or reflected waves) is the dominant contributor in the PSI triad. To identify it, the signal at frequency \((\omega_1 + \omega_2)^2\) is decomposed into upward \((\omega_1 + \omega_2)^2\) and downward \((\omega_1 + \omega_2)^2\) components in Eq. (5). For the PSI triad \([M_1^+, M_1^+, M_2^-]\) (Fig. 12c), its significant bicoherence value is larger than that at \([M_1^+, M_1^+, M_2^-]\) (Fig. 11a), while no significant bicoherence appears at \([M_1^+, M_1^-, M_2^-]\) (Fig. 12d). This suggests that the occurrence of internal tide PSI near the surface reflection region may be mainly caused by upward traveling incident waves, as shown in Gayen and Sarkar (2013).

b. Nonlinear interaction between incident and reflected waves

When two internal wave rays with the same frequency \(\omega\) meet, nonlinear (nonresonant) interaction of them may lead to generation of a mean flow \((=\omega - \omega)\) and higher-harmonic disturbances with frequency \(2\omega\) \((=\omega + \omega)\) (Tabaei et al. 2005).

In this section, we discuss the potential nonlinear interaction between incident and reflected waves.

1) Higher harmonics

Kinetic energy spectra at both mooring sites show a peak at semidiurnal higher harmonic \(M_4\) \((=2M_2)\) but its energy is
larger in the upper layer at MB than that at MA (cf. Figs. 2a,b). At MB, the largest bicoherence that is significant at the 95% level appears around the triad \([M^1_2, M^2_2, M^6_4]\) (Fig. 11a). Like the PSI triad, the significant results at \([M^1_2, M^2_2, M^6_4]\) are also confined to the vicinity of the surface reflection region. Meantime, the enhanced \(M^4\) waves are observed at all semidiurnal springs (Figs. 9a and 13a), suggesting that the enhanced \(M^4\) waves result from the nonlinear interaction between semidiurnal internal tides in the vicinity of the reflection region.

The \(M^4\) velocity field shows the vertically standing pattern during semidiurnal springs (Fig. 13a), implying two upward and downward traveling \(M^4\) waves with comparable amplitude, which could be confirmed by the separation of vertical propagation signals (Figs. 13b,c). However, the theoretical and numerical studies suggested that only downward propagating \(M^4\) waves radiate from the surface reflection boundary (Lamb 2004; Tabaei et al. 2005). The bicoherence analysis in the reflection region shows that the significant bicoherence (>0.65) at the 90% level appears at triads \([M^1_2, M^2_2, M^6_4]\) and \([M^1_2, M^2_2, M^6_4]\) (Figs. 12a,d), suggesting that the enhanced \(M^4\) waves may result from multiple interactions. The shelf interaction of incident waves may generate upward traveling \(M^4\) waves \([M^1_2, M^2_2, M^6_4]\), while the nonlinear interaction between incident and reflected waves generates the downward traveling \(M^4\) waves \([M^1_2, M^2_2, M^6_4]\), as shown in the theoretical and numerical studies (Lamb 2004; Tabaei et al. 2005).

2) MEAN FLOWS

The surface reflection of internal tides can also generate mean flows with direction the same to the wave propagation direction (Mercier et al. 2012; Xie et al. 2013). Since the semidiurnal internal tides in the mooring region propagate westward (Zhao 2014; Liu et al. 2019), a westward mean flow may be generated by nonlinear interaction between the incident and reflected waves in the vicinity of the surface reflection region. To explore this process, the subinertial velocities are plotted in Fig. 14. As expected, the westward mean flows are observed in the upper layer during semidiurnal springs at MB, except at the fourth spring during which strong northward–northeastward currents caused by Typhoon Fung-Wong dominate over the velocity field (Fig. 14a). At the first two springs, the enhanced westward flows are confined in the vicinity of the surface reflection region. On the contrary, the westward flows penetrate throughout the depth at the third spring. Current data from the CMEMS do not show the strong westward flow at the first spring, but they largely reproduce the zonal flows during the second and third springs (cf. Figs. 14b,c), suggesting that the wind-driven currents or mesoscale eddies can be important for generation of the westward flows during these two periods. However, the maximum geostrophic current speed appears at the sea surface (Fig. 14c), while the maximum westward flows observed at MB always occurs near the bottom of pycnocline (~100 m; Fig. 14b), where the maximum \(M^1\) and \(M^4\) amplitudes are also observed (inset panel in Fig. 14b). Therefore, nonlinear wave–wave interactions may partly contribute to the generation of the westward flows during semidiurnal springs.

5. Discussions

a. Estimation of the wave-induced mean flows

Theoretical studies of internal-wave boundary reflection have predicted generation of mean currents (Thorpe and Haines 1987; Tabaei et al. 2005). An inviscid nonlinear
Theoretical solution for the wave-induced mean flows is derived (see Appendix A in Zhou and Diamessis 2013),

\[ \tau = -2u_a k_z \cos(2k_z z)/N \]

where \( u_a \) and \( k_z \) are the maximum velocity amplitude and vertical wavenumber, respectively, of incident waves. At the first spring, \( u_a = 0.2 \text{ m s}^{-1} \) and \( k_z = 2\pi/300 \text{ m} \) above 150 m based on the mooring observations at MB. Taking \( N = 0.015 \text{ s}^{-1} (N_c \text{ at } 100 \text{ m}) \) and assuming \( \cos(2k_z z) = -1 \) (i.e., the flow direction toward the wave propagation direction), the maximum Eulerian current speed can be estimated as \( \sim 0.1 \text{ m s}^{-1} \), which underestimates the observations (\( \sim 0.2 \text{ m s}^{-1} \)). The theory also estimates that the flow direction is vertically variable with a

**FIG. 12.** Up–down \((\omega^+, \omega^-)\) prefiltered bicoherence averaged over depths above 120 m.

**FIG. 13.** (a) Time–depth maps of quarter-diurnal \((u^+, u^-)\), (b) \( u^+ \), and (c) \( u^- \) during four semidiurnal springs.
wavelength of $1/2k_z$, but the observed westward flows in a distance of $1/2k_z$ are unidirectional (i.e., toward the wave propagation direction), which is consistent with laboratory experiments of Mercier et al. (2012) and mooring observations of Xie et al. (2013). Comparing laboratory experiments and numerical simulations of the wave-induced mean flows, Grisouard et al. (2013) also found the same differences in the two-dimensional models but their three-dimensional numerical simulations largely offset these differences. These authors suggested that the wave-induced mean flows during the reflection not only depend on nonlinear effects but on dissipative effects, which are absent in the inviscid theory of Thorpe and Haines (1987) and Tabaei et al. (2005). In the three-dimensional system, nonlinear interactions, as well as dissipative effects, can generate vertical vorticity that makes the mean flows grow in time and reach an amplitude comparable to that of the incident waves (Bordes et al. 2012), as observed during the first semidiurnal spring and in laboratory experiments of Mercier et al. (2012).

It should be pointed out that the above estimation is based on a hypothesis that the effects of wind-driven currents or mesoscale eddies are ignorable at the first semidiurnal spring, during which the current estimation from the CMEMS product shows a similar background field with the observations (i.e., a westward zonal flow with speed smaller than 0.05 m s$^{-1}$; Figs. 14b,c). However, the modeled zonal flows before this period are opposite to the observations. Therefore, there are still some uncertainties in reanalysis data. To completely rule out the effects of other processes, more field data are necessary.

b. Schematic diagram

The mooring observations and numerical models have shown the near-surface reflection of low-mode internal tides on a subcritical continental slope. Internal tides with a single mode (e.g., mode-1) incident to the continental slope can also form vertically traveling waves, propagate upward and reflect from the upper boundary (Kelly et al. 2013b). Therefore, the evolution and energy decay of remotely generated low-mode M$_2$ internal tides on a subcritical continental slope can be illustrated in the schematic diagram of Fig. 15. On the slope, low-mode internal tides are scattered into higher modes, forming upslope propagating waves. When the waves propagate upward to the sea surface, they reflect from the boundary. On one hand, the reflection of internal tides enhances the surface energy density and causes resonant PSI, leading to generation of two subharmonic M$_1$ beams with opposite vertical propagation direction. On the other hand, nonlinear nonresonant interaction between incident and reflected waves in the surface reflection region induces a higher harmonic M$_4$ beam radiating downward away from the interacting region. The nonlinear shelf interaction of incident waves may cause the second M$_4$ beam, which is relevant to the wave–wave interactions reported in Wunsch (2017). In the interacting region, a mean flow toward the wave propagation direction may also be generated by nonlinear reflection of internal tides.

6. Summary and conclusions

Using both the mooring data and the numerical models, we have investigated evolution of low-mode internal tides on a subcritical continental slope. The mooring observations show the near-surface intensification and vertically standing pattern of semidiurnal internal tides, indicating the surface reflection of these waves on the slope. The numerical models reproduce the observations and suggest that the remotely generated low-mode internal tides are scattered to higher modes on the continental slope, forming vertically propagating waves which
The scattering of low-mode internal tides on the continental slope and their interactions with the pycnocline can reduce wave scale, while the near-surface reflection enhances the wave energy density. The reduction of wave scale and near-surface enhancement in the reflection region make internal tides more susceptible to subharmonic instability, leading to generation of smaller-scale subharmonics. Furthermore, nonlinear interaction between incident and reflected waves in the vicinity of the surface reflection region can generate higher harmonic beams. In the interacting region, energy of internal tides may also be transferred to a mean flow toward the wave propagation direction. Therefore, the surface reflection of internal waves may be a potential important mechanism for energy decay of low-mode internal tides impacting on the continental slope. Associated dissipation and mixing may also be enhanced in the upper layer on the continental slope. Using the CELT model, Kelly et al. (2013a) have shown that the boundary reflections of low-mode internal tides are common on the global continental margins. Therefore, the present mechanism may be applied to any continental slopes. It should be pointed out that the employed model is linear and it only reproduces the occurrence of the surface reflection near the mooring site. Future researches need consider a complete nonlinear model which can reproduce nonlinear instability of low-mode internal tides in the vicinity of the surface reflection region and quantify the effect of enhanced nonlinear interaction in the reflection region on dissipation and mixing of internal tides.

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