Variability and Sources of the Internal Wave Continuum Examined from Global Moored Velocity Records

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Abstract: Energy for ocean turbulence is thought to be transferred from its presumed sources (viz., the mesoscale eddy field, near-inertial internal waves, and internal tides) to the internal wave continuum, and through the continuum via resonant triad interactions to breaking scales. To test these ideas, the level and variability of the oceanic internal gravity wave continuum spectrum are examined by computing time-dependent rotary spectra from a global database of 2260 current meter records deployed on 1362 separate moorings. Time series of energy in the continuum and the three “source bands” (near-inertial, tidal, and mesoscale) are computed, and their variability and covariability examined. Seasonal modulation of the continuum by factors of up to 5 is seen in the upper ocean, implicating wind-driven near-inertial waves as an important source. The time series of the continuum is found to correlate more strongly with the near-inertial peak than with the semidiurnal or mesoscale. The use of moored internal-wave kinetic energy frequency spectra as an alternate input to the traditional shear or strain wavenumber spectra in the Gregg–Henyey–Polzin finescale parameterization is explored and compared to traditional strain-based estimates.

Keywords: Inertia-gravity waves; Internal waves

1. Introduction

Internal gravity waves, which occupy frequencies from the local inertial frequency \( f \) to the buoyancy frequency \( N \), provide a pathway to turbulent dissipation of the ocean’s mechanical energy (Frankignoul and Joyce 1979; Watson 1985; Heney et al. 1986; Gregg 1989; Polzin et al. 1995; Wunsch and Ferrari 2004). Away from energy sources and sinks, the energy distribution within this frequency range is commonly referred to as the continuum of internal gravity waves. The “universal” Garrett and Munk spectrum (hereafter GM; Garrett and Munk 1972, 1975; Cairns and Williams 1976) predicts most of the observed spectral distribution (level and slope; Polzin and Lvov 2011); namely, an \( \omega^{-2} \) frequency dependence and a white wavenumber spectrum at wavelengths larger than 10 m, with a “saturation” range with \( k^{-1/3} \) slope before reaching the \( k^{+1/3} \) turbulent spectrum. Though the GM spectrum has a “peak” (really a singularity) at \( f \), it cannot rise and fall independently from the rest of the spectrum, and does not contain tidal peaks. In this study, we wish to study the covariation of the continuum with the near-inertial, tidal, and mesoscale bands, we define the internal gravity wave continuum as the energy distribution with frequencies between \( f \) and \( N \) excluding the near-inertial and tidal peaks. The procedure for separating these will be given in detail below.

A well-known theoretical framework describes the relevant processes creating the continuum; namely, weakly nonlinear resonant triad interactions transferring energy from large vertical scales to smaller, breaking, scales (Olbers 1974, 1976; McComas and Müller 1981). However, a number of questions remain. Within this weakly nonlinear framework, Müller et al. (1986) and Polzin et al. (2014) show that energy in the internal wavefield will typically be transferred from large vertical scales to small at constant horizontal wavenumber and consequently from high frequency to low. With such transfer, a source of internal wave energy at high frequency is required for a stationary balance. Such high-frequency energy sources have not yet been observed (Polzin and Lvov 2011).

In spite of these caveats, the rise and fall of the low-wavenumber portion of the spectrum, a measure of the energy in the internal wave field, has been demonstrated to be correlated with turbulence (Gregg 1989), resulting in the so-called Gregg–Henyey–Polzin parameterization (hereafter GHP). Wijesekera et al. (1993) and Whalen et al. (2015) have demonstrated the same for strain rather than shear spectra. Simply, in steady state, the dissipation must equal the transfer through the spectrum, which in turn is proportional to the internal wave spectral density. Many studies, starting with Hasselmann (1966), have modeled these processes in terms of a kinetic equation. Lvov et al. (2012) offers a summary of previous related work.

The GHP parameterization relates the transfer rate through the continuum to dissipation, but neither the means nor the rate at which energy is supplied to the internal wave continuum, which must also balance dissipation in steady state, is known. Only a small number of source mechanisms exist, including resonant triad interactions from (i) near-inertial waves (NIWs) and (ii) internal tides [see Olbers et al. (2020) for a summary of related studies], and (iii) generation by mesoscale currents directly by diffusion (Watson 1985), wave–current interaction (Bühler and McIntyre 2005; Polzin 2010), or lee wave generation as they flow over topography (Nikurashin and Ferrari 2010; Clément et al. 2014).

(i) NIWs lie at the low-frequency end of the internal gravity wave continuum and are generated by several mechanisms (Alford et al. 2015). The most common is the...
generation of NIWs through wind-work at the surface of the ocean. As a consequence, these waves are intermittent, following the passage of storms. Alford and Whitmont (2007) highlight the seasonal variability of the near-inertial (NI) peak in a previous version of the dataset used in our study (section 2), showing that the kinetic energy (KE) of a near-inertial peak can vary by more than an order of magnitude between seasons (mainly summer–winter differences).

(ii) The internal tides are generated continuously as the surface tide flows over seafloor topography. Some of the wave energy dissipates over complex topography during generation, leading to local turbulent mixing, while the remainder radiates away providing a reservoir for remote mixing during wave propagation, interaction with ocean currents, eddies, and nonuniform stratification, as well as reflection at other topographic features. Sarkar and Scotti (2017) and de Lavergne et al. (2019) provide useful reviews of the mechanisms leading to tidal wave generation and dissipation.

(iii) Mesoscale flow and internal gravity waves (IGWs) can interact through numerous mechanisms (Polzin 2008, 2010). As an example, the random distribution of small-scale IGWs within larger-scale mesoscale shear is known to diffuse the wave action at larger wavenumbers (Watson 1985). Near the bottom, the turbulence can be sustained by internal waves generated by geostrophic motions flowing over bottom topography (Nikurashin and Ferrari 2010). Clément et al. (2016) recently observed that IGW energy increases with mesoscale activity via eddy–topography interaction. Nikurashin et al. (2014), using a model with a filtered 3D topography, found that about 0.15 TW is globally transferred from geostrophic motions into IGW generation. This number is to be compared with the lee wave generation estimates from Scott et al. (2011) who found $0.34 \pm 0.49$ TW and Wright et al. (2014) who found $0.75 \pm 0.19$ or $0.57 \pm 0.16$ TW depending on the climatology used.

A fourth source mechanism is the generation of IGW by surface gravity waves through a process called spontaneous creation because it operates without internal waves being initiated by remote forcing. This number is to be compared with the lee wave generation estimates from Scott et al. (2011) who found $0.34 \pm 0.49$ TW and Wright et al. (2014) who found $0.75 \pm 0.19$ or $0.57 \pm 0.16$ TW depending on the climatology used.

The current work complements the pioneering of Webster (1969) who used current meters from one location and obtained an estimate for the internal wave dissipation time scale of 3 weeks—not far from the 50–100 days we understand now—as well as the insightful work of Polzin and Lvov (2011), who carefully examined some of these hypotheses at a small number of moorings. Here, we take a more adynamic approach, but use a much larger database in order to establish patterns that will inspire more theoretical work.

The paper is organized as follows. Methods are first presented, then several case studies documenting the rise and fall of the continuum and peaks are described in order to give the reader familiarity and confidence in the techniques. Next, spatial and temporal global patterns are presented, followed by calculations of the correlation coefficient between the continuum and each candidate source peak. Finally, GHP parameterized turbulence is traditionally computed from internal wave energy estimated via the level of the strain or shear wavenumber spectrum; an alternate method employing instead the rotary frequency spectrum of velocity is presented in section 5, followed by caveats and implications.

2. Data

a. Global Multi-Archive Current Meter Database

Our work follows Alford (2003) and Alford and Whitmont (2007), which used data from the Global Multi-Archive Current Meter Database (GMACMD, http://stockage.univ-brest.fr/~scott/GMACMD/gmacmd.html; Scott et al. 2011). The database is a global collection of tens of thousands of physical oceanographic time series derived from the research programs of many countries, collected across several decades. This study uses the 2351 time series of velocity, spanning 1974–2011.

b. Data selection

To focus on internal waves away from the margins, only moorings in water deeper than 1000 m are used. Further requiring records longer than 6 months with a sampling frequency faster than 3 h$^{-1}$ and a buoyancy frequency $N > 5$ cpd reduces the dataset to 2260 current meters on 1362 moorings (Fig. 1). The instruments’ depths range from 100 m below the surface to 200 m above the seafloor with a maximum of 6431 m depth. Finally, data within $5^\circ$ of the equator were discarded to avoid complications with the longer inertial periods at very low latitude (overlap with the mesoscale field and poor spectral precision).

3. Methods

a. Initial processing

Before any processing, each time series is scanned with a sliding 1-day window. A small number (<100) of physically
unreasonable measurement spikes were removed. Similarly, old current meters that stalled in very weak currents (<0.02 m s\(^{-1}\)) are interpolated through. Linear interpolation is used to fill time series gaps shorter than 6 h. Otherwise, we split the time series and only use the ones longer than six months. Altogether, we interpolate less than 0.01% of the total data.

b. Sensitivity to barotropic tides

Since our focus is on baroclinic motions, we compared the variability of kinetic energy in the IGW continuum computed with and without the barotropic tidal currents. The barotropic tides are computed with the TPXO7.2 model (Egbert and Erofeeva 2002) using 13 tidal components (M2, S2, N2, K2, O1, P1, Q1, MF, MN, M4, N4, MS4). The internal motions generally exceed barotropic motions and the barotropic tides have no impact on our time-dependent conclusions. Consequently, we chose to keep the barotropic tide in our time series and not apply the TPXO correction.

c. WKB scaling

To account for the propagation of internal waves through the ocean’s depth-varying stratification (Leaman and Sanford 1975), all velocities are WKB scaled using an estimate of \(N\) given by the Levitus and Boyer (1994) climatology:

\[
\tilde{\phi}_{\text{WKB}}(\omega, z_i) = \tilde{\phi}(\omega, z_i) \left[ \frac{N_{\text{GM}}}{N(z_i)} \right],
\]

where \(\tilde{\phi}\) is the rotary spectrum of kinetic energy, \(\omega\) is the frequency, \(z_i\) is the depth of the instrument, \(N(z_i)\) is the local stratification, and \(N_{\text{GM}} = 5.2 \times 10^{-3}\) rad s\(^{-1}\) is the stratification used to define the GM spectrum (Cairns and Williams 1976). Differences between the real and climatological stratification are generally only a few percent below a few hundred meters (Levitus and Boyer 1994). Since the kinetic energy is proportional to \(N\) in a WKB scaling, our estimates of the continuum’s energy level have corresponding errors of a few percent.

d. Calculation of rotary spectra

The time evolution of the Fourier coefficients is obtained by processing multitaper spectra (Riedel and Sidorenko 1995) over a window sliding along the complex velocities. The window length is a balance between spectral resolution needed to resolve adjacent tidal constituents such as M2/S2 and temporal resolution and is chosen to be the integer number of inertial periods nearest to 30 days. Following Alford and Whitmont (2007), each 30-day record is demeaned, detrended, and windowed with a Hanning filter. We set the number of tapers \(K = 3\), again a balance between spectral precision and frequency resolution. The spectra are finally smoothed over even increments of \(\log_{10}(\omega)\), where \(\omega\) is the cyclic frequency. Acknowledging a large amount of overlap between adjacent windows, the window is slid forward one day over each time series. As can be seen from a sample calculation (Fig. 2), frequency resolution with these choices is (barely) sufficient to resolve M2 and S2 separately, with a fundamental frequency of \(\Delta \omega = 1/30\) days and a resolution of \(K \Delta \omega = 1/10\) days.

The redness of the spectrum introduces concerns of spectral leakage. To ensure that this was not an issue, spectra of pre-whitened data were also computed by first differencing the data and then dividing the spectra by \((2\pi \omega)^2\) (Fig. 2, gray line). Differences are less than 0.1% within the continuum. The bulk of the difference lies entirely in the low-frequency range and does not impact the continuum.
e. Determination of continuum and peaks

The spectra are then used to make independent estimates of the energy in a set of peaks as well as the internal wave continuum. Since the peaks rise above and overlap with the continuum, care is needed to ensure they are not included in the continuum estimate. First, energy in the inertial and tidal peaks and their linear combinations was computed as shown in appendix A. Then, continuum energy $E_{\text{cont}}$ was computed as an offset of the GM spectrum $E_{\text{cont}} = aE_{\text{GM}}$ where $E_{\text{GM}}$ refers to the KE energy density in the GM frequency spectrum and $a$ is the ratio of the observed energy content in the continuum (minus the peaks) and the GM’s continuum energy content:

$$
a = \frac{\int f_{WKB}(\omega) d\omega}{\int f E_{\text{GM}}(\omega) d\omega}. \quad (2)
$$

In the following, we define the GM spectrum with the GM parameters from Cairns and Williams (1976) only allowing for the coefficients $a$ to vary in time and $f$ and $N$ to vary in space (i.e., instrument location). The other GM parameters such as the slope $s = -2$ or the wavenumber cutoff $m_0 = 2\pi/10 \text{ m}^{-1}$ are left constant since our main goal is to observe the variability of the continuum, $E_{\text{GM}}$ being a convenient reference to which to compare our observations. Using the GM parameters from Cairns and Williams (1976), $\int f E_{\text{GM}} \sim 5 \times 10^{-3} \text{ m}^2 \text{s}^{-2}$ with a kinetic energy part of about $3.7 \times 10^{-3} \text{ m}^2 \text{s}^{-2}$.

To confirm that energy in the peaks does not contaminate our continuum estimates, we also compute a second estimate by simply estimating the level and slope of the spectrum from 5 to 10 cpd, beyond the main peaks, and integrating after adjusting for the bandwidth difference. In nearly all cases, the differences between the two methods are negligible.

A second concern regarding the calculation of the continuum is contamination of its slope and level by harmonics of nonsinusoidal aspects of the time series. That is, could harmonics of the near-inertial peak masquerade as continuum energy? In appendix B, we show that sharp inertial
oscillations can indeed generate harmonics, but they do not impact the continuum even in the extreme case of a pure square wave.

4. Results

a. Variability in the IGW continuum and its relationship with kinetic energy sources

CASE STUDIES

Before moving to the aggregate statistics, several case studies are presented to familiarize the reader with the data and methods. One example has been selected showing apparent covariations between the continuum and each of the source peaks.

(i) Continuum and near-inertial peak

A current meter located at 1006-m depth, with a seafloor at 5258 m, recorded 600 days of ocean velocities between the years 1987 and 1988 in the northwest Atlantic (Fig. 3a). The mooring is located at 39.48°N, so the inertial frequency is \( f \approx 1.27 \) cpd. The “fuzz” apparent on top of the mesoscale-frequency modulation is the internal wave band—and is quite variable, as seen in zoom-ins from the periods indicated in pink and purple lines, respectively (Fig. 3d). Internal wave activity is visually stronger in winter than in summer.

Rotary spectra computed for the first period (Fig. 3f) indicate a near absence of an inertial peak and a continuum level below GM (Fig. 3f), pink). Near day 750, an increase in RMS velocity is reflected in a 50-fold increase in the near-inertial peak, and a factor of 10 increase in the level of the continuum. Both periods show strong polarization of motions near \( f \), decreasing toward rectilinear motions at high frequency as expected from the internal-wave polarization relations.

The time evolution of the rise and fall of the continuum and each peak is seen by plotting the rotary frequency spectrum versus time (Fig. 3b). As mentioned above, we can see the decrease of KE for frequencies higher than \( \pm 2 \) between days 500 and 700. This decrease is followed by an intensification of the high-frequency KE associated with an increase of KE in the near-inertial band.

Continuum energy \( E_{\text{cont}} \) and the energy in the mesoscale, tidal, and near-inertial peaks are plotted in Fig. 3c. A correlation is visibly apparent between the near-inertial peak and the continuum level, as found by Frankignoul and Joyce (1979) and Polzin and Lvov (2011). The correlation
coefficient between the NI peak and the variability of the continuum is $R = 0.8$ for this time series, with no apparent lag (±2.8 days).

(ii) Continuum and mesoscale

In this case, a current meter halfway between the Azores and the Irish shelf break located at 552-m depth with a seafloor at 4839 m recorded 300 days of ocean velocities during the year 2000 (Fig. 4a). The mooring is located at 45.0°N, so the inertial frequency is $\sim 1.41$ cpd. The mesoscale-frequency modulations are weak over most of the record except between days 400–450 (period 1) where the instrument experiences a strong reversal in the current direction (Fig. 4d).

On top of these low-frequency currents, both the tides and the NIWs are strong in the area (Figs. 4b,f). They increase the high-frequency variability of the raw zonal velocities, driving oscillations with an amplitude of almost 40 cm s$^{-1}$ around day 435. Later in the record, around day 550 (period 2), the mesoscale activity is weaker, and the IGW oscillations decrease to 10 cm s$^{-1}$ (Fig. 4e).

During the first period, the spectrum is 5–10 times higher than in the second period. The broad increase is collocated in time with a low-frequency feature lasting about 20 days (Figs. 4a,b). They increase the high-frequency variability of the raw zonal velocities, driving oscillations with an amplitude of almost 40 cm s$^{-1}$ around day 435. Later in the record, around day 550 (period 2), the mesoscale activity is weaker, and the IGW oscillations decrease to 10 cm s$^{-1}$ (Fig. 4e).

The variations of $E_{\text{cont}}$, the mesoscale, tidal and near-inertial peaks are plotted in Fig. 5c. Despite the small variations of $E_{\text{cont}}$, a correlation is visibly apparent between the semidiurnal peaks (Kunze 1985), possibly smoothing and widening the NIW peak.

(iii) Continuum and tidal peak

In the northwest Atlantic, a current meter located on the continental slope south of Woods Hole at 949-m depth with a seafloor at 1150 m recorded 200 days of ocean velocities during the year 1984 (Fig. 5a). The mooring is located at 39.8°N, so the inertial frequency is $\sim 1.28$ cpd. Large mesoscale modulations up to 40 cm s$^{-1}$ regularly occur about every $\sim 50$ days.

As with the other current meters, IGW activity is evident as high-frequency oscillations on top of the low-frequency modulations. For this record, the periods of high and low IW activity are around day 115 and day 219, respectively (Figs. 5d,e). During these periods, the tidal and mesoscale bands are the most energetic. Around day 115, the mesoscale and semidiurnal KE are about 3 times higher than around day 219, and the NI peaks differ by an order of magnitude (Fig. 5f). The continuum for both periods remains close to GM, showing only small variations.
tide and the continuum level with a correlation coefficient about \( R = 0.7 \), and no lag (\( \pm 1.65 \) day). The observed variability in the semidiurnal peak differs from the expected spring/neap cycle of the surface tides, possibly owing to the influence of the continental slope on the internal wave field where scattering and wave–wave interaction are likely to occur.

b. Correlations

We next move from the specific examples shown previously to statistical patterns in the entire database by examining the histogram of the best lagged cross-correlation between the continuum and each source peak (Fig. 6a). Most time series are only weakly correlated, but approximately 40% show correlation coefficients \( R > 0.5 \). Of these 40%, the correlation with the near-inertial frequency band is most common with 25.6% of correlation coefficient \( R > 0.5 \), followed by the semidiurnal peak with 17.8% and the mesoscale with 10.3%. The associated time lags for the time series with \( R > 0.5 \) sharply decrease after 10–20 days but a sparse number of moorings show significant correlations with longer lags (Fig. 6b).

It is generally assumed the characteristic time scale to dissipate the energy resident in the internal wavefield is about 50 days (Polzin and Lvov 2011). Consequently, the variability of the IW continuum should also match this time scale with implied buoyancy scaling (i.e., part of the variability matching \( N \) variations).

However, energy transfer within the continuum can occur on the order of just a few days if the source of energy is localized in frequency (i.e., frequency peak; Müller et al. 1986). Among other resonant interactions, the parametric subharmonic instability can transfer energy between two so-called daughter waves of both near half the frequency of a primary wave in less than 5 days (MacKinnon et al. 2013).

Our method does not allow to conclude on the nature of the energy transfers nor on the impact of a variable buoyancy. The latter could be seen as a consequence of the IW energy cascade—through increased mixing—or as a bias for the level of our observations relative to GM level. This bias could impact our current results and should be the object of a future study. Another limitation of our method is the large confidence interval around the 30 day \(^{-1} \) frequency which can impact the variability of our mesoscale time series.

c. Distribution of the continuum level and slope

The ratio \( E_{\text{cont}}/E_{\text{GM}} \) and continuum slope are then divided into geographic areas. The probability density function of these parameters for each ocean basin is shown in Fig. 7. In all ocean basins, the majority of the \( E_{\text{cont}} \) values are within 0.1 \( E_{\text{GM}} < E_{\text{cont}} < 5 E_{\text{GM}} \) (Fig. 7a) with an average value around \( E_{\text{GM}}/2 \). This result is comparable to Pollmann (2020) which obtains similar results on the IW energy density level using profiles from Argo floats. The mean IGW continuum slope is centered around the GM value (\( \omega^{-2} \)) in the Atlantic, the Caribbean Sea, the Agulhas Current, and the Atlantic Bight where the GM spectrum describes previous observations. In these regions, our estimated slopes compare well with the slopes (\( -\omega^{-2} \))...
found in Polzin and Lvov (2011). Slopes in the Pacific and the Indian Oceans are slightly steeper than the GM spectrum.

The variations around the average are next examined for clues regarding the dynamics of the continuum and the potential driving mechanisms. In the following sections, we present the $E_{\text{cont}}$ variability as a function of seasons, depth, and latitude to attribute the observed variations to a specific source of kinetic energy.

FIG. 6. Histogram of the best lagged correlation coefficients between (a) the time series of the continuum and the mesoscale (red), the semidiurnal tides (green), and the NI peaks (gray) and (b) their time lags. Number of time series (y axis) as a function of the correlation coefficient (x axis) is shown in (a). Number of time series with a correlation coefficient higher than 0.5 (y axis) as a function of the time lag is shown in (b).

FIG. 7. (a) Probability distribution of $E_{\text{cont}}$ and (b) continuum spectral slope in each oceanic basin. The dashed magenta lines represent the GM values. Samples refers to the number of time series.

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d. Depth dependence

All estimates of the spectral level in the continuum, the mesoscale, near-inertial and semidiurnal frequency band are plotted as a function of the instrument depth normalized by the seafloor depth and averaged per month (Fig. 8, gray dots and colored lines). Near-inertial KE (Fig. 8b) is centered around $10^{-2} \text{ m}^2 \text{ s}^{-2}$, and varies by an order of magnitude over the whole water column with a maximum in winter and a minimum in summer, as found by Alford and Whitmont (2007). By contrast, the semidiurnal tide (Fig. 8c) does not have a seasonal cycle, while the mesoscale KE (Fig. 8d) increases in summer in the upper third of the water column and close to the bottom.

In the upper ocean, a seasonal cycle of similar magnitude as that in NI KE is seen in the continuum variability (Fig. 8a). Together with the correlations presented above, the presence of a seasonal cycle in the continuum suggests a dynamical linkage. Continuum energy increases toward the bottom, and the seasonal cycle reduces—both possibly suggesting internal tides play a greater role than NIW at depth in supplying energy to the continuum.

e. Latitudinal variations

Kinetic energy is next averaged over the upper half of the water column and plotted versus latitude. We selected this part of the water column to capture the $E_{\text{cont}}$ seasonal signal observed in Fig. 8a. Continuum energy $E_{\text{cont}}$ and the NI peaks present a latitude-dependent seasonal cycle. The latitudinal variations match the expected storm-track pattern with an increase of KE at midlatitude in the winter period (Figs. 9a,b). The amplitude of the monthly averaged NI peaks increases with each hemisphere’s respective winter periods (January–March for the Northern Hemisphere and June–August for the Southern Hemisphere).

In the Northern Hemisphere, near-inertial KE presents a clear seasonal cycle south of 35°N, but only the NI peak shows a seasonal cycle similar to $E_{\text{cont}}$ with increased activity in winter while the mesoscale activity is slightly stronger in summer (Fig. 9d).

In the Southern Hemisphere, near-inertial KE presents a clear seasonal cycle south of 35°S. Continuum energy $E_{\text{cont}}$ also increases in winter but only within 30–40°S latitude. At higher latitudes, continuum energy is stronger from September to...
February. This intensification is comparable to the mesoscale activity in this region (Fig. 9d). We note that the results in the Southern Hemisphere are more speculative owing to the lower number of moorings available.

Close to the equator, within ±20°, $E_{cont}$ increases. When compared to the other frequency bands, only the semidiurnal KE also increases toward the equator (Fig. 9c). However, at these low latitudes, $E_{cont}$ is higher from September to
November and does not reflect the weak monthly variability observed for the semidiurnal kinetic energy. As mentioned earlier, one of the reasons that could explain the apparent weaker variability of the semidiurnal KE is the lack of frequency resolutions around 15 days. Comparatively, the mesoscale KE is variable, spanning over three to four orders of magnitude.

5. Discussion

a. Caveats

Our methods clearly show variations of the KE in the continuum following independent proxies of KE sources but the observed seasonal variability of the IGW continuum is compared to a temporally constant GM spectrum. Previous studies (Polzin et al. 2014; Pollmann 2020) have shown that seasonal cycles in the bandwidth and shear/strain ratio can reduce perceived cycles of $E_{\text{cont}}/E_{\text{GM}}$ in wavenumber spectra. Since our single-depth frequency spectra contain all wavenumbers, the effect should be reduced for our results compared to those using wavenumber spectra. Indeed, the 30-day window we are using impacts the confidence intervals at low frequencies and, consequently, may mask longer-time scale energy transfers to the continuum.

b. $\epsilon$ parameterization based on kinetic energy

The GHP parameterizations discussed in the introduction rely on the observed correlation between the shear associated within the internal wave field and the local turbulence to estimate the turbulent KE dissipation rate. Gregg (1989) introduced a scaling formula for $\epsilon$ in the midlatitude thermocline using in situ measurements and the GM framework:

$$e_{\text{G89}} = e_0 \frac{\langle N^2 \rangle \langle S_{40}^4 \rangle}{N_0^2 S_{40}^2_{\text{GM}}}$$

where $S_{40}^4$ the fourth moment of the observed shear at all vertical wavenumbers lower than $m_0 = 2\pi/10$ m$^{-1}$, $S_{40}^2_{\text{GM}}$ is the same quantity computed from the GM spectrum, $N$ is the local buoyancy frequency, and $N_0 = 5.24 \times 10^{-3}$ s$^{-1}$ is the reference GM buoyancy frequency. The term $e_0 = 7 \times 10^{-10}$ m$^2$ s$^{-3}$ is a constant used to match the estimated $\epsilon$ to in situ data. Gargett (1990) compares $e_{\text{G89}}$ to similar parameterizations $e_{\text{HWF}}$ (Henyey et al. 1986) and $e_{\text{MM}}$ (McComas and Müller 1981) and finds $e_{\text{G89}} \sim e_{\text{HWF}}$ and $\gamma e_{\text{G89}} \sim e_{\text{MM}}$.

Gregg (1989), and later Wijesekera et al. (1993), argue that $\langle S_{40}^4 \rangle/S_{40}^2_{\text{GM}} \sim (E_{\text{cont}}/E_{\text{GM}})^2$, allowing for an estimate of $\epsilon$ using the energy content in the continuum. Here, we follow their method and replace these two parameters in Eq. (3) in order to estimate $\epsilon$ from KE time series (hereafter $\epsilon_{\text{KE}}$):
\[ \varepsilon_{\text{KE}} = \varepsilon_0 \left( \frac{N^2}{N^2_0} \right) \left( \frac{E_{\text{cont}}}{E_{\text{GM}}} \right)^2. \] (4)

A similar approach has already been attempted by Silverthorne and Toole (2009) using the ratio of the near-inertial kinetic energy and the average summertime near-inertial kinetic energy. They find a significant seasonal cycle in the turbulent dissipation rate, similar to our results. However, Polzin et al. (2014) shows that the \( \varepsilon_{\text{KE}} \) seasonal cycle disappears after adjusting for the seasonal variation of \( m_0 \). Using a large amount of data from Argo floats, Pollmann (2020) also find that taking the variability of parameters like \( m_0 \) into account instead of using constant GM model values, removes the seasonality of the ratio \( E_{\text{cont}}/E_{\text{GM}} \).

The approach presented in this manuscript also complements the one by Thurnherr et al. (2015) which uses wavenumber spectra of kinetic energy as a proxy to estimate turbulent kinetic energy dissipation. Being aware that the use of constant values for the GM description of the IW field can significantly impact the variability of the \( \varepsilon_{\text{KE}} \), we nonetheless forge ahead to compute the monthly average of \( \varepsilon \) derived from KE frequency spectra (Fig. 10). To do so, we use the average value of the in situ turbulence measurements over smooth topography from the microstructure database presented in Waterhouse et al. (2014) in order to calibrate \( \varepsilon_{\text{KE}} \). Using this dataset, we find \( \varepsilon_0 = 6.5 \times 10^{-9} \text{m}^2 \text{s}^{-3} \) (close to the value found for \( \varepsilon_{\text{GM}} \)). The obtained values are centered around \( 10^{-10} \text{Wkg}^{-1} \). By construction, the monthly estimates of \( \varepsilon \) as a function of depth show a seasonal cycle similar to the one observed for \( E_{\text{cont}}/E_{\text{GM}} \) estimates.

The \( \varepsilon_{\text{KE}} \) increases toward the surface similarly to direct \( \varepsilon \) observations from microstructure profilers over smooth topography (Waterhouse et al. 2014). Near the bottom, \( \varepsilon_{\text{KE}} \) increases within the last 10% of the water column. This increase matches the direct microstructure observations over smooth topography (Waterhouse et al. 2014).

**Comparison with strain-based parameterization**

In an attempt to qualitatively evaluate the performance of the parameterization from velocity frequency spectra, we compare \( \varepsilon_{\text{KE}} \) to the strain-based \( \varepsilon_{\text{strain}} \) estimations obtained from Argo floats (Whalen et al. 2015). We chose to compare these two estimates because the global coverage of \( \varepsilon_{\text{strain}} \) allows for geographical comparison. The sparsity of our data makes the comparison with similarly sparse direct measurements of \( \varepsilon \) from microstructure profilers more difficult (Waterhouse et al. 2014).

Whalen et al. (2015) used the following parameterization:

\[ \varepsilon_{\text{strain}} = \varepsilon_0 \left( \frac{\langle \xi_z^2 \rangle}{\langle \xi_z^2 \rangle_{\text{GM}}} \right)^2 h(R_w) L(f, N), \] (5)

where \( \langle \xi_z \rangle \) and \( \langle \xi_z \rangle_{\text{GM}} \) are the observed and GM strain variances, respectively. \( h(R_w) \) describes the dependence on the ratio between shear and strain \( R_w \). They used \( R_w = 3 \) [i.e., \( h(R_w) = 1 \)]. The term \( L(f, N) \) is a latitudinal structure function. More details on these parameters can be found in the literature (Gregg et al. 2003; Kunze et al. 2006; Whalen et al. 2015).

Whalen et al. (2015) presents global estimates of \( \varepsilon_{\text{strain}} \) at three different depth ranges, namely, 250–500, 500–1000, and 1000–2000 m. Consequently, we averaged our \( \varepsilon_{\text{KE}} \) estimates over the same depth range for the comparison. In Fig. 11, we compare \( \varepsilon_{\text{KE}} \) to \( \varepsilon_{\text{strain}} \). We first average the strain-based estimates within a 5° radius circle around the mooring locations. Then, we bin \( \varepsilon_{\text{KE}} \) around discrete values of \( \varepsilon_{\text{strain}} \). The KE parameterization matches \( \varepsilon_{\text{strain}} \) within a factor of 3 for most of our observations, with a tendency to overestimate the strain estimate for low values. Our study simply aims to examine the possibility that frequency spectra of velocity can be used to estimate wave-driven turbulence; careful examination of the performance of the \( \varepsilon_{\text{KE}} \) presented here will require another study dedicated to this topic.

**c. Energy fluxes: Input and sinks**

The correlated time series and the similar seasonal cycle and latitude dependence of NIW and continuum KE suggest that
the increase in storm-induced wind stress during the winter periods injects energy at the NI frequency. This energy would subsequently cascade in the IGW continuum (Figs. 8 and 9).

Once generated, the wind-generated NIWs propagate away and downward from their generation area, transferring their energy to smaller scales.

The seasonality observed for both the NI peaks and $E_{\text{cont}}$ provides an opportunity to estimate 1) the flux of KE from the wind going into the deep ocean and 2) the flux of KE dissipated through turbulence in the context of the GHP parameterization.

1) WIND-DRIVEN KINETIC ENERGY LOSS

The winter–summer differences of the NI peaks observed in Fig. 8 provide a time scale $\tau$ (~180 days) that can be used to estimate the loss rate of NI kinetic energy, using

$$F_{\text{NI}} = \rho \int_{z_1}^{z_2} \Delta KE_{\text{NI}}(z) \, dz \, (\text{W m}^{-2}).$$

where $\Delta KE_{\text{NI}}$ is the winter–summer difference of the measured NI kinetic energy within a $z_1$, $z_2$ depth range and $\rho$ is the density of seawater. We define our $F_{\text{NI}}$ estimates over three depth ranges: 0–1000, 1000–3000, and 3000–5000 m.

Parameter $F_{\text{NI}}$ is close to 0.07 mW m$^{-2}$ over the whole water column (Fig. 12b), compared to a global-mean wind work of ~1.1 mW m$^{-2}$ (Alford 2001; Watanabe and Hibiya 2002). However, substantially less than this may be available for propagating NIW energy owing to shallow turbulence losses (Furuichi et al. 2008; Alford 2020).

2) KINETIC ENERGY DISSIPATION

We can also use the KE-based $e_{\text{KE}}$ to estimate the power dissipation, using

$$F_e = \rho \int_{z_1}^{z_2} e_{\text{KE}}(z) \, dz \, (\text{W m}^{-2}).$$

Consequently, the winter–summer differences in the $e_{\text{KE}}$ profiles observed in Fig. 8 can be used to estimate the amount of KE dissipated through turbulence associated with the forcing mechanisms varying on a seasonal time scale like the NIW. We find that the seasonal variations in $e_{\text{KE}}$ are stronger in the upper 2000 m of our data with a seasonal difference of KE flux reaching almost 0.65 mW m$^{-2}$ (Fig. 12a), or about 50% of the wind work. This is about 10 times higher than our estimates of the wind energy input estimated from the NI peak, suggesting NIWs lose energy to wave–wave interactions faster than to propagation. All of these calculations are rough and based on not enough data.

6. Conclusions

The energy transfers within the continuum are often described as a pathway from source to turbulent dissipation scales: The wind, the tides and mesoscale currents inject energy in the continuum; this energy interacts with the background circulation (e.g., mesoscale, internal wave field) and is ultimately transferred to turbulent dissipation scales where it undergoes a different type of downward energy cascade before being converted into heat through viscosity.

According to this description, the continuum’s energy content is a function of the energy sources and sinks, which operate at widely different spatial–time scales. This broad range of scales implies a residence time for the energy within the continuum and, consequently, drives a time variability of the continuum’s energy content. Previous studies (e.g., Polzin and Lvov 2011) based on a small number of observations showed the internal wave spectrum varies only modestly in shape across the global ocean. Explaining this weak...
variability is one of the challenges the community needs to overcome in order to close the global ocean energy budget (Whalen et al. 2020).

We used a global mooring dataset with 2260 time series of ocean velocities to study the variability of the KE content within the IGW continuum ($E_{\text{cont}}$). Continuum energy $E_{\text{cont}}$ is computed while sliding 30-day spectral windows along the time series. Along with the time-dependent estimates of KE in the continuum, this method also provides estimates of the KE content within the mesoscale, near-inertial and semidiurnal frequency bands. The variability of these three sources of KE is compared with the continuum’s variations to establish potential relationships. To make sure not to include any shared variability between the sources and the continuum, all significant peaks found in the spectra are removed from the time variability of $E_{\text{cont}}$.

The global and time average of $E_{\text{cont}}$ is about a factor 2 lower than the universal $E_{\text{GM}}$ value but can vary up to 5 times $E_{\text{GM}}$. The slope of the observed IGW continuum also matches the GM framework in the Atlantic Ocean, showing slightly steeper slopes in the Pacific Ocean. Continuum energy $E_{\text{cont}}$ tends to increase toward the equator and follows a seasonal cycle. The monthly average of $E_{\text{cont}}$ varies in like manner as the near-inertial KE over most of the water column. The correlation is enhanced at midlatitude with a maximum in the respective winters of both hemispheres, possibly implying a direct path between the IGW activity and the near-inertial waves. 25% of our time series are correlated ($R > 0.5$) with the near-inertial peaks, while 17% and 10% are correlated with the semidiurnal tide and the mesoscale, respectively.

In other words, it is possible to observe the variability of the continuum’s KE following the fluctuations of the main energy sources in the ocean. Our study emphasizes the similar variability between the continuum and the NIWs without excluding the possibility of interaction between continuum and the mesoscale and the tides. The seasonal cycle in the continuum’s variability suggests increased energy transfer through wave–wave interaction of the NI KE in winter.

In an attempt to provide information on the turbulent KE dissipation rate driven by wave–wave interaction, we advance the hypothesis that the ratio $E_{\text{cont}}/E_{\text{GM}}$ estimated from KE frequency spectra can be used in GHP parameterizations. We use this observation to get a KE-based estimate of $\varepsilon$ from velocity time series. We acknowledge that the validity of this method needs to be established with a dedicated study, but our estimates agree within a factor of 3 with a traditional strain-based estimation of $\varepsilon$. Using these dissipation estimates and the

FIG. B1. (a) Time series of a purely GM internal wave field (yellow line), a near-inertial sinusoidal wave on top of the same GM wave field (red line), and a near-inertial square wave on top of the same GM wave field (blue line). (b) Frequency spectra of the previous. The black line shows the theoretical GM slope.
observed near-inertial KE, we find winter–summer differences in the near-inertial and turbulence-driven energy fluxes on the order of 0.07 and 0.65 mW m−2, respectively.

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Amy Waterhouse provided the microstructure database of direct measurements of ε from microstructure profilers (https://microstructure.ucsd.edu/, Waterhouse et al. 2014). This database is hosted by the Clivar and Carbon Hydrographic Data Office (CCHDO) and is updated since 2014 to provide the ocean mixing community access to most of the turbulence observations in a single location. In our study, we only used a subset of database only using profiles located in areas where the bottom is equal or greater to 1000-m depth. This includes all the following campaigns: BBTRE96, BBTRE97, DIMES, DIMES PACIFIC, FIEBERLING, FLUX STATS, GRAVILUCK, NATRE, SPAM1, and SPAM2.

APPENDIX A

Peak Detection

We select the major peaks of a single time series from its average rotary frequency spectrum of velocity (Fig. A1). The average rotary spectrum is the time average of the spectra resulting from the 30-day window sliding FFTs over a time series. We add to this time-averaged spectrum the standard deviation resulting from the 30-day window sliding FFTs over a time series. The average rotary spectrum is the time average of the spectra resulting from the 30-day window sliding FFTs over a time series.

Once selected, these peaks are removed from the estimation of $E_{cont}$.

APPENDIX B

Square Wave Influence on the Spectrum

To test whether nonsinusoidal near-inertial motions could contaminate the continuum and thereby create spurious correlations, we generated synthetic realizations of a GM wavefield. As a worst-case scenario, we superimposed a near-inertial square-wave. The associated spectra (Fig. B1) show the harmonics of the square wave at multiples of $f$, with amplitude decreasing with the same power law as the GM spectrum. Importantly, no contamination is seen between the peaks.

REFERENCES


