Weakly Nonlinear Ekman Pumping in the Sri Lanka Dome and Southwest Monsoon Current

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ABSTRACT: The Sri Lanka Dome is a cyclonic recirculation feature in the Southwest Monsoon Current system in the southern Bay of Bengal. Cooler sea surface temperature (SST) in the vicinity of this system is often denoted as the Bay of Bengal “Cold Pool.” Although the wind shadow of Sri Lanka creates a region of strong positive wind stress curl, both sea level height dynamics and the distribution of cool SST cannot be explained by wind stress curl alone via traditional Ekman pumping. Moreover, the Cold Pool region is often aligned with the eastern portion of the Sri Lanka Dome, as defined by sea surface height. Previous work has attributed the spatial SST pattern to lateral advection. In this analysis, we explore whether low-latitude weakly nonlinear “vorticity” Ekman pumping could be an explanation for both cooling and observed changes in sea level height in the southwest Bay of Bengal. We show that weakly nonlinear upwelling, calculated from ERA5 and AVISO geostrophic currents, collocates with changes in sea level height (and presumably isopycnals). While the SST signal is sensitive to several factors including the net surface flux, regional upwelling explains changes in AVISO sea level height if the nonlinear terms are included, in both the Sri Lanka Dome and the region of the Southwest Monsoon Current.

KEYWORDS: Ekman pumping/transport; Nonlinear dynamics; Wind stress

1. Introduction

The Sri Lanka Dome (SLD) forms each year in the Southwest Monsoon Current (SMC) as a tropical thermal dome and is the largest cyclonic feature in the southern Bay of Bengal (Vinayachandran and Yamagata 1998; de Vos et al. 2014). Previous work indicates that the elevated thermoclines within the SLD affect both ocean temperature (de Vos et al. 2014; Burns et al. 2017) and the mixed layer depth (Vinayachandran et al. 2018; Cullen and Shroyer 2019; Lozovatsky et al. 2019), both of which can modulate monsoon activity (Gadgil 2003; Keerthi et al. 2016) and intraseasonal oscillations within the Indian summer (southwest) monsoon (Sengupta et al. 2001; Girishkumar et al. 2017; Li et al. 2017). The SLD forms in a region of positive wind stress curl between the wind jet south of Sri Lanka and the wind shadow of the island (de Vos et al. 2014; Burns et al. 2017). The highest chlorophyll concentrations in the southern Bay of Bengal occur in the Southwest Monsoon Current system in the same location as the eastern flank of the SLD (de Vos et al. 2014; Sarangi and Devi 2017), away from the region of strongest wind stress curl that occurs near the coast of Sri Lanka. The broad region of low sea surface temperature (SST) often encompassing the SLD is commonly referred to as the Bay of Bengal “Cold Pool” and has been studied independent of the Sri Lanka Dome (Nair et al. 2011; Das et al. 2016; George et al. 2017; Vinayachandran et al. 2020). The cool anomalies of the Cold Pool are often aligned with the eastern periphery of the SLD, as defined in sea surface height. In this paper, we examine if weakly nonlinear Ekman pumping (Stern 1965), modified for low latitude, can explain changes in sea level height in SLD and larger Cold Pool region and spatial variation of SST anomalies in the Sri Lanka Dome and Southwest Monsoon Current System.

a. The Southwest Monsoon Current and the Bay of Bengal Cold Pool

The Southwest Monsoon Current system (comprising the eastward flowing SMC, the SLD, and a persistent anticyclonic eddy) forms annually from May to September due to the southwesterly monsoon winds (Gadgil 2003). As a conduit between the high-salinity Arabian Sea and the low-salinity Bay of Bengal, the SMC is an important transport pathway of salt into the Bay of Bengal (Murty et al. 1992; Vinayachandran et al. 1999). After passing south of Sri Lanka, the SMC bifurcates with one branch forming the interior extension of the SMC (Schott and McCreary 2001; Anutaliya et al. 2017) and the second branch feeding into the cyclonic circulation of the SLD (Burns et al. 2017; Cullen and Shroyer 2019; Pirro et al. 2020). Both wind stress forced eddies and planetary waves create variability in transport and position of the SMC (Webber et al. 2018). The SMC system transports cool, nutrient-rich waters into the Bay of Bengal, and its hydrographic properties are influenced by the strong coastal upwelling offshore of southern Sri Lanka (Yapa 2000; Vinayachandran et al. 2004; Jothibabu et al. 2015) and enhanced wind mixing (Rao et al. 2006a,b). Cool SST anomalies in this region have a strong
influence on regional weather patterns through air–sea interaction. When SST is less than 27.5°C in the southern Bay of Bengal, atmospheric deep convection, and hence monsoon activity, is inhibited (Gadgil 2003). Nair et al. (2011) observed this suppression of atmospheric deep convection over the cooler SMC system during the southwest monsoon. Suppression is accompanied by a decrease in precipitation and a reduction of low- and midlevel clouds (Nair et al. 2011; George et al. 2017). The onset of atmospheric convection events during the southwest monsoon is associated with the strength of the north–south SST gradient in the Bay of Bengal that forms between the warm northern bay and the Cold Pool (Shankar et al. 2007). Other work has linked the strength of the SLD (and presumed cooling from Ekman pumping) to atmospheric cyclone formation over the Bay of Bengal (Burns et al. 2017). Cool anomalies within the SMC system have been attributed to surface heat fluxes, horizontal advection of upwelled coastal waters, enhanced vertical exchange associated with a weakened barrier layer, and wind-curl-driven Ekman pumping within the SLD (Vinayachandran et al. 2020). The Cold Pool region spans roughly 3°–13°N, 77°–90°E during the southwest monsoon (Nair et al. 2011; Das et al. 2016; George et al. 2017). Das et al. (2016) suggests the Cold Pool forms during the onset of the southwest monsoon when cloud cover leads to a decrease in the net surface flux and ocean cooling. In contrast, George et al. (2017) note that cooling in the SMC tends to happen during the “break phase” of the monsoon, as indicated by a southern movement of the low-level jet over India, with accompanying high surface wind speeds and positive net surface heat fluxes (i.e., a warming of the ocean). Using in situ data, Vijith et al. (2020) attributed horizontal advection as the dominant forcing mechanism of SST. Vinayachandran et al. (2020) also explored the role of horizontal advection by the SMC in maintaining the Cold Pool after formation. These authors also concluded that increased winds during the break phase of the monsoon were associated with cooling in the SMC.

b. Weakly nonlinear Ekman pumping

Multiple studies have described the importance of relative vorticity on wind-driven divergence in the surface layer (Stern 1965; Niiler 1969; Pedlosky 2008; Gaube et al. 2015; Wenegrat and Thomas 2017). Stern (1965) considered the problem of an idealized vortex in steady winds, while Niiler (1969) investigated the wind-driven divergence within a midlatitude jet in the Gulf Stream. Other studies have examined so-called weakly nonlinear Ekman pumping in simulated mesoscale/submesoscale eddies (Mahadevan and Tandon 2006; Ratheesh et al. 2019) and balanced currents with curvature (Wenegrat and Thomas 2017). Gaube et al. (2015) considered the contribution of this effect relative to upwelling by surface wind stress curl driven by SST gradients within midlatitude eddies. As weakly nonlinear effects depend on scale (in particular length, velocity, and planetary vorticity), many studies have explored the general applicability of relative vorticity-driven Ekman pumping using both theoretical and numerical methods (Stern 1965; Thomas and Rhines 2002; Pedlosky 2008; Wenegrat and Thomas 2017). Of particular relevance to our region of interest is the modeling study of Ratheesh et al. (2019), who considered weakly nonlinear Ekman pumping in the Bay of Bengal north of 9°N. These authors found that Ekman pumping due to relative vorticity contributed significantly to upwelling in the northern Bay at horizontal scales finer than 30 km, but was negligible at larger spatial scales. To date, no observation-based studies have considered weakly nonlinear Ekman pumping in the Bay of Bengal. And, in general, few such studies exist globally. Souza et al. (2014) noted that the spatial SST and chlorophyll signatures in Agulhas rings matched the distribution from upwelling from Stern’s theory (Stern 1965), which included contributions from relative vorticity. Other studies have compared vorticity-driven Ekman pumping to observed distributions of chlorophyll in midlatitude mesoscale eddies, although this effect was weak relative to other terms (Gaube et al. 2015; He et al. 2016; Park et al. 2019). As theorized by Stern (1965), the Ekman pumping velocity (w) incorporating surface current relative vorticity can be expressed as

\[ w = \nabla \times \left( \frac{\tau}{\rho_0(f + \zeta)} \right), \]

where \( \tau = (\tau_x, \tau_y) \) is the vector wind stress, \( \rho_0 \) density, \( f \) the Coriolis parameter, and \( \zeta \) the geostrophic relative vorticity. Previous literature often separates the above into two distinct forcing contributions, the first a “classic” Ekman pumping using an \( f \)-plane approximation with fixed Coriolis parameter \( f_0 \),

\[ w_c = \frac{1}{\rho_0(f_0 + \zeta)} \nabla \times \tau, \]

and the second contribution from “vorticity” Ekman pumping,

\[ w_V = \frac{1}{\rho_0(f_0 + \zeta)} \left( \frac{\partial \zeta}{\partial y} - \frac{\tau_y}{\rho_0 f_0} \right), \]

where \( w_c \) is analogous to linear Ekman pumping on an \( f \)-plane with the planetary vorticity modified by the relative vorticity. Parameter \( w_V \) is dependent on the strength of \( \tau \) and lateral vorticity gradients. While Stern (1965) considered only small Rossby numbers \([i.e., U/(fL) \ll 1]\), both Niiler (1969) and Wenegrat and Thomas (2017) suggested that weakly nonlinear Ekman pumping also applies for larger Rossby numbers \([i.e., U/(fL) < 1]\) as long as the Ekman Rossby number remains small \([U_e/(fL) \ll 1]\), where \( U_e \) and \( U_L \) are scalings for the velocity and Ekman velocity, respectively, and \( L \) is the horizontal length scale. This modification expands the applicability of weakly nonlinear Ekman pumping to lower latitudes than those considered previously, as noted in Wenegrat and Thomas (2017).

In this article, we evaluate the application of weakly nonlinear Ekman pumping theory to the SLD and SMC system in the southwest Bay of Bengal (5°–15°N) using remotely sensed sea surface temperature (SST) and sea level height, combined with reanalysis vector wind stress (section 2). Due to proximity to the equator, we extended the Stern (1965) Ekman pumping to include latitudinal gradients in the planetary vorticity (i.e., the \( \beta \) effect in section 2; to our knowledge this is
the first application of Stern’s theory at low latitudes. Observations of cool anomalies in the SLD are quantified in section 3, which are then compared to the spatial distribution of weakly nonlinear Ekman pumping (see section 4a) in section 4b. Section 5 presents observational evidence that the weakly nonlinear Ekman pumping also contributes to changes in sea surface height in the SLD and the SMC system. Collectively, this work suggests that weakly nonlinear Ekman pumping is crucial to the dynamics of the SMC system, and, along with other mechanisms such as horizontal advection and the net surface heat flux, may be important to cool anomalies in the Bay of Bengal Cold Pool. Since a full heat budget analysis is not possible with the data at hand, the latter suggestion relies on the observed collocation of cool anomalies and regions of strong upwelling along with the long recognized role for Ekman induced entrainment within mixed layer budgets of heat and salt over local to global scales (Wyrtki 1981; Williams 1989; Qu 2001, 2003; Seo et al. 2016; Buckley et al. 2015; Xu et al. 2020).

2. Materials and methods

a. Theory

We applied weakly nonlinear Ekman pumping following Stern (1965) with constraints on Rossby number relaxed to $U/(fL) < 1$ and $U_e/(fL) \ll 1$, as suggested by Wenegart and Thomas (2017). Assuming a minimum length scale set by the resolution of remotely sensed products (0.25°) and using the typical magnitude of the regional geostrophic currents, the weaker constraint on Rossby number translates to a minimum latitude of applicability of the theory of 5°N. As the latitudinal gradient of $f$ is the same order of magnitude as $\nabla \zeta$, we also assume the $\beta$-plane approximation. This yields an additional term of $\tau \beta$ in the expression for weakly nonlinear Ekman pumping. Consequently, Eq. (3) is modified to reflect planetary vorticity gradients:

$$w_\zeta = \frac{1}{\rho_0(f + \zeta)} \left[ \tau_x \frac{\partial \zeta}{\partial y} + \beta \right] - \tau_y \frac{\partial \zeta}{\partial x}. \tag{4}$$

Hereafter, we assume a constant surface ocean density $\rho_0 = 1025 \text{ kg m}^{-3}$ throughout. Classic Ekman pumping ($w_\zeta$) is calculated using Eq. (2), and the vorticity Ekman pumping is calculated from Eq. (4). Total Ekman pumping is still described by Eq. (1). Note that in this paper, SST induced Ekman pumping is included within $w_\zeta$. As described below, the wind stress includes the contribution from the relative ocean currents.

b. Data products

We used a combination of remote sensing datasets, merged products, and atmospheric and ocean reanalyses to calculate weakly nonlinear Ekman pumping [Eqs. (2) and (4)], and assess observed upwelling. All products were subset to the region bounded by 5°–5°N, 80°–93°E to capture the southern Bay of Bengal (Fig. 3a).

1) OCEANIC FIELDS

Sea Level Heights, or Absolute dynamic topography (ADT) and geostrophic velocities were taken from AVISO’s SSALTO/DUACS L4 global merged product at 0.25° resolution. ADT was used to track the SLD from 1993 to 2018, using a threshold criteria of 0.135 m deformation as outlined in Cullen and Shroyer (2019). Geostrophic surface current velocity data were used to calculate the current-relative wind stress (as noted above) and the vertical component of the relative vorticity ($\zeta$), using first differences of the geostrophic currents. ADT was also used to assess observed upwelling within the SLD, using the rate of change in time of unfiltered surface deformation ($\eta$).

2) ATMOSPHERIC FIELDS

Daily atmospheric conditions (surface winds, SST, and air–sea heat fluxes) were obtained from European Center for Medium-Range Weather Forecasts’ fifth generation reanalysis (ERAS) dataset (Hersbach et al. 2018). The surface wind stress ($\tau$) was calculated from ERAS winds relative to AVISO geostrophic surface current speeds using Large and Pond (1981), as recommended by Gaube et al. (2015). Seo et al. (2019) suggests that consideration of the true wind stress, in contrast to stress calculated under the assumption of negligible ocean currents, is important for the Bay of Bengal eddy field. Accounting for the relative wind speed leads to corrections of 12% within the Sri Lankan wind shadow where ocean currents are strong relative to the winds.

3. Observed cool signals in the Sri Lanka dome

Distinct cool SSTs are often observed within the SLD and the adjacent SMC, such as the signal shown from 15 July 2006 (Fig. 1b). An imprint of the cool SST from the SLD on the atmosphere is also evident as enhanced outgoing longwave radiation (OLR), which is consistent with reduced atmospheric convection (Fig. 1c), suggesting that cool SST inhibits atmospheric deep convection and disrupts cloud formation. The coldest SSTs are offset to the east and south, away from the region of strongest $V \times \tau$ on the coast of Sri Lanka as observed by Cullen and Shroyer (2019). This SST signal is also offset to the southeast from the SSH low (Figs. 1a,b), and thus off set from the doming of the thermocline. This eastern offset in cool SST is a persistent feature of the SLD (Fig. 2b). From 2006 to 2018, the spatial mean SST within the SLD described by (Cullen and Shroyer 2019) ranged over 3°C from 27.7° to 30.9°C (Fig. 2a). The minimum SST measured inside the SLD spanned a similar range from 26.7° to 30.6°C, and the average difference between the mean SST and the minimum SST within the SLD was 0.44°C (Fig. 2a). Of the minimum SST observations, only 4% of days were less than 27.5°C, but 12% of daily observations fell below 28.5°C (Fig. 2a). The year 2006 was the coolest in the 2006–18 record with 26 observations of daily mean SST less than 27.5°C occurring during that season (Fig. 2a). However, despite cool signals associated with the southeast region the SLD (Figs. 1a,b), the mean SST inside the SLD is often warmer than the mean SST over the
rest of the Bay of Bengal at the same latitude (76% of the observations, Fig. 2b). The coldest SST in the SLD occurs within the eastern half of the SLD in 83% of observed days (Fig. 2c). The difference between the mean SST in the eastern and western halves of the SLD can reach up to 0.90°C, as observed, for instance, on 21 June 2007. On average, the eastern half of the SLD is colder than the western half of the SLD by 0.20°C. Periods when the SLD is much warmer (>0.4°C) over its eastern half tend to occur during the formation or dissipation of the SLD (e.g., in 2007, 2012, 2016, 2017, and 2018 Fig. 2c). The east–west temperature difference is weakly correlated with the area of the dome (correlation coefficient, \( r_x \), is 0.31 with \( p < 10^{-4} \)). However, even in years when the SLD is small and intermittent (e.g., 2008), the tendency for the eastern side to be cooler than the western side is still apparent. Although still weak, the east–west temperature difference has a slightly higher correlation with the vorticity (see Cullen and Shroyer 2019) within the SLD (\( r_x \), 0.36, \( p < 10^{-5} \)) (Figs. 2c,d). This correlation is suggestive of a link between vorticity (which is important to weakly nonlinear Ekman pumping) and east–west SST gradients across the SLD.

The persistence of cool anomalies to the east of the ADT minimum offers context for the previously presented difference between the SLD mean temperature (within the interior of the ADT contour) and the mean temperature outside the SLD over the same latitude range. The ADT low is rarely collocated with the SST minima. Given its persistence, the spatial offset relative to both the ADT and the region of positive wind stress curl provides a clue into the interior dynamics.

![Fig. 1. Maps of (a) AVISO ADT, (b) SST, (c) \( \tau \) (shading and white vectors), and (d) NOAA’s Outgoing Longwave Radiation (OLR) Daily Climate Data Record at 1° over the SMC and SLD (black contour) on 15 Jul 2006. Black arrows in all panels show AVISO geostrophic currents. White arrows in (b) indicate winds.](image)

![Fig. 2. (a) Mean (black) and minimum (gray) SST calculated within the SLD contour, (b) the mean SST within the SLD minus the mean SST outside the SLD in the Bay of Bengal over the same latitude band (negative indicates a cooler SLD), (c) difference between the mean SST over the eastern and western halves of the SLD (negative indicates cooler water to the east), and (d) vorticity of the SLD. Note that SST is often warmer within the SLD contour than outside, but the eastern half of the SLD is almost always cooler than the western half. Values are shown for periods of time when the SLD contour is reliably tracked.](image)
supporting the cool anomaly. This east–west gradient is consistent across the 14-yr time series, even when the SLD is small in area. The location where the SLD is most likely to be cold is away from the region of strongest $V \times \tau$. Instead, the region of strongest $V \times \tau$ is often observed to coincide with relatively warm SST. As noted above, this cold signal overlaps with the BoB Cold Pool, and possible explanations for the cool anomaly include enhanced wind mixing, lateral advection in the SMC of upwelled coastal waters, and interaction between the wind stress and vorticity within the SLD and SMC system. In the following, we explore the latter through consideration of the role of weakly nonlinear Ekman pumping may also contribute to the dynamics in this region, and thus (along with lateral advection) setting the spatial distribution of cool SSTs.

### 4. Weakly nonlinear Ekman pumping

#### a. Comparison to traditional Ekman pumping in the SMC system

The vorticity Ekman pumping correction changes the expected spatial pattern of upwelling within the SLD from that based on wind stress curl alone. Consider a “typical” day during the Southwest Monsoon with steady southwesternly winds over the Bay of Bengal and the SLD forming in between 6° and 9°N, such as 15 July 2006 (Fig. 3a). The wind stress curl, $V \times \tau$, sets the spatial structure of classic Ekman pumping, $w_c$; thus, $w_c$ is determined by the topography of Sri Lanka and the wind shadow it creates within the relatively steady southwest monsoon winds. The region of strong $V \times \tau$ is confined close to shore between the wind jet and the wind shadow (Fig. 3b). Note that $w_c$ is proportional to $1/(f + \zeta)$, but otherwise does not include any adjustment associated with relative vorticity (Fig. 3b). For comparison, Ekman pumping on an $f$ plane ($w_{f0}$) is shown in Fig. 3e. For this day, the maximum $\zeta$ is $10^{-5}$ s$^{-1}$, and at most is approximately half of $f$ at 6°N. Relative to $w_{f0}$, the magnitude of $w_c$ decreases (increases) by up to ~20% in regions of positive (negative) vorticity (Figs. 3b,e).

The nonlinear component of the vorticity correction, $w_z$, is dependent on both the magnitude of wind stress and the relative vorticity gradients [Eq. (3)], both of which are strong in the SMC and SLD. Over this region, southwesterly winds (with positive $t_x$ and $t_y$) prevail during the summer monsoon (e.g., Fig. 3a). The meridional vorticity gradient term ($w_z \propto -t_x \partial \zeta/\partial y$) is negative (positive) in the northern (southern) portion of the SLD (Fig. 3g). The zonal vorticity gradient term ($w_y \propto -t_y \partial \zeta/\partial x$) is negative on the western side of the SLD near the coast, and positive extending into the anticyclonic eddy to the east of the SLD into the SMC (Fig. 3f). We also group the effect of planetary vorticity ($w_{p} \propto \tau_x \beta$), which is always associated with upwelling (Fig. 3h), into the expression for $w_z$. Together, the vorticity gradients are such that upwelling occurs along the southern half of the SMC and the southeast SLD, extending past the SLD into the interior Bay of Bengal (Fig. 3e). Negative $w_c$ compensates for a positive $w_z$ on the western side of the SLD. When $w_z$ is combined with $w_c$, the total Ekman pumping, $w_{tot}$, indicates the strongest

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**Fig. 3.** (a) Components used in the calculation of Ekman pumping include $\zeta$ (shading) calculated from AVISO geostrophic currents (gray arrows), $\tau$ (blue arrows), and contours of $f$, which is comparable to $\zeta$ within the region. Decomposition of low-latitude weakly nonlinear Ekman pumping into (b) classic, (c) vorticity, and (d) total components over the SMC region and SLD (black contour). (e) For comparison, Ekman pumping on an $f$ plane ($w_{f0}$) is shown, and the components within $w_c$ are decomposed into (f) $w_{c0}$, (g) $w_{cx}$, and (h) $w_{p}$.**
upwelling is offset to the eastern flank of the SLD, extending into the SMC and the Bay of Bengal Cold Pool (Fig. 3d).

b. Comparison of Ekman pumping to observed cooling

Given the strength of the SST signal in 2006, we consider the SST evolution of this year in detail (see online supplemental material for examples of other years). During 2006, the SLD first appeared (based on the ADT tracking algorithm) on 27 May and was not reliably tracked after 30 September (Fig. 4). Cool events occurred along the eastern side of the SLD and 100–200 km into the eastern Bay of Bengal. These “events” often lasted ~10 days (Fig. 4a). Cooling is first observed in late June and is most pronounced during the month of July (Fig. 4a). Three cool events reached temperatures less than 27.5°C: 26 July, 27 July, and 11 August. As with other years, the west side of the SLD was warmer than the east by 0.5–1°C. Cool events occur during periods of increased wind stress and variable air–sea heat flux ($Q_{\text{net}}$) (Figs. 4c,d). Note, however, the lower-frequency SST signal, which is delineated by the cooling in late June/early July followed by warming in late August/early September. This lower-frequency signal is consistent with the onset and maintenance of strong upwelling.

After the SLD develops in 2006, $w_{\text{tot}}$ has a distinct spatial pattern relative to the SLD’s ADT contour (Fig. 4b). While weak Ekman Pumping ($w_{\text{tot}} \leq 0.2$ m day$^{-1}$) occurs on the western side of the SLD during formation (early to mid-June, Fig. 4b), strong positive Ekman pumping ($w_{\text{tot}} \geq 0.2$ m day$^{-1}$) occurs on the east side of the SLD for most of the season (starting from late June, Fig. 4b). Consistent with the single realization shown in Fig. 3, the region of upwelling extends ~200 km into the interior Bay of Bengal. Ekman pumping is strongest between 2 July and 25 August along the eastern edge of the SLD (0.4–0.8 m day$^{-1}$, Fig. 4b). As one measure of the relative contributions to the total Ekman pumping, we find that $w_c$ was typically equal to or greater than $w_t$ along the eastern contour of the SLD (85% of days, Fig. 4c). Cool SSTs and weakly nonlinear Ekman pumping collocate in space along the eastern half of the SLD and extend eastward (Figs. 4a,b). Episodic cooling events lasting ~10 days are evident in both $w_{\text{tot}}$ and SST with large drops in SST often occurring during strong Ekman pumping (e.g., 15 July, Figs. 4a,b). However, the strongest correlation ($r \geq 0.65$) between $(d/dt)$ SST and $w_{\text{tot}}$ occurs at a lag of 2–4 weeks. This suggests that Ekman pumping may be more important on weekly to monthly time scales relative to other forcing like wind mixing and air–sea heat fluxes. Also of note is that the region where Ekman pumping is the strongest experiences the most variation in SST (Figs. 4a,b), possibly an indication that Ekman pumping may be shoaling the mixed layer, leading to a more rapid mixed layer response to surface forcing.
5. Comparison to observed surface deformation

a. The SLD sea level height deformation

Inside the SLD contour, the mean rate of change in sea level height ($d\eta/dt$) shows the evolution of the SLD with negative (positive) values indicating growth (relaxation) of the SLD low. In an example time series from 2006 to 2018, $d\eta/dt$ varies from $-0.017$ to $0.018$ m day$^{-1}$ (Fig. 5d). Large negative values of $d\eta/dt$ typically occur when the SLD first forms (although sometimes occurs in the middle to late season such as 2003), and positive values of $d\eta/dt$ typically occur when the SLD is dissipating (Fig. 5d). Classic Ekman pumping $w_c$ varies from $-0.51$ to $1.01$ m day$^{-1}$ with a mean of $0.13$ m day$^{-1}$ and a standard deviation of $0.18$ m day$^{-1}$ (Fig. 5a). The distribution of $w_c$ ranges from $-0.27$ to $0.66$ m day$^{-1}$ with a mean and standard deviation of $0.08$ and $0.13$ m day$^{-1}$, respectively (Fig. 5b). When the two components of Ekman pumping are combined, $w_{tot}$ with in the SLD varies between $-0.59$ and $1.04$ m day$^{-1}$ with a mean of $0.21$ m day$^{-1}$ and a standard deviation of $0.21$ m day$^{-1}$ (Fig. 5c). In contrast to $w_c$, $w_c$ often peaks early in the season, suggestive that $w_c$ may be key to growth of the SLD (Figs. 5a,b). The magnitudes of $w_c$ and $w_z$ are similar within the SLD contour, with $w_z$ exhibiting larger variability than $w_c$.

As surface deformation should compensate for vertical isopycnal displacement, we expect a negative correlation between $d\eta/dt$ and $w$. Both $w_c$ and $w_z$ are negatively correlated with $d\eta/dt$ and ($r_{xy} = -0.18$ and $-0.27$, respectively, with $p < 10^{-6}$). In contrast, the correlation between $w_{tot}$ and $d\eta/dt$ is $-0.32$, ($p < 10^{-5}$). While weak, the combination of classic and vorticity terms into $w_{tot}$ better explains variability in $d\eta/dt$ than either alone. Previous literature has noted strong correlations between wind forcing and the SLD strength on seasonal scales, but to our knowledge no paper has yet explained changes in $\eta$ on daily time scales. While the correlation between $w_{tot}$ and $d\eta/dt$ only partially explains daily $\eta$ changes within the SLD, it is strong considering other the variability of the subsurface density structure, the strong background eddy/Rossby wave field in the southern BoB, and the temporal resolution of the AVISO product.

b. Climatological observations

The 1993–2018 climatology of the total monthly change in AVISO ADT ($\Delta \eta$) shows the typical development of the SMC system (Figs. 6a–c), while reducing noise of the background eddy field present in each year. The formation of the SMC is visible in June with $\eta$ decreasing by $0.05$ m over this month along the northern edge of the current (Fig. 6a). During July, $\Delta \eta$ decreases by $0.08$ m near $7.7^\circ$N, $84.5^\circ$E, marking the strengthening of the recirculating SLD (Fig. 6b). As the SLD stretches northeast in August, $\Delta \eta$ decreases by $0.05$ m as far north as $10.8^\circ$N (Fig. 6c).
One way to assess the impact of $w_z$ is to compare the rate of change in the observed $h$ to expected changes from upwelling. If the predicted lifting of isotherms below the Ekman layer over a month from classic Ekman pumping ($\int w_c$) cannot predict $\Delta \eta$, more complex dynamics (such as weakly nonlinear Ekman pumping) may be important. For June, July, and August, maps of $\int w_c$ (where here and elsewhere $\int$ represents the integral in time over a month) are nearly identical (Figs. 6d-f, shading), emphasizing the persistence of monsoon winds, with the largest integrated upwelling (23 m month$^{-1}$) occurring across the wind shadow close to Sri Lanka at 82.2°E. Weak regions of downwelling (negative values) are offset to the northeast and southwest of this location. The calculated $\int w_c$, does not well with regions of sea level deformation (negative $\Delta \eta$) for any month (cf. contours and color shading in Figs. 6d-f).

In contrast to $\int w_c$, $\int w_{tot}$ occupies a broader region with an evolving structure over the season (Figs. 6g-i). In June, $\int w_{tot}$ of roughly 20 m month$^{-1}$ extends over a band stretching from the eastern shoreline of Sri Lanka out to 86°E south of 8°N (Fig. 6g). Over July, $\int w_{tot}$ approaches 35 m over the region of the SLD, aligning with the change in $h$ (Fig. 6h). $\int w_{tot}$ intensifies near Sri Lanka during August, with values exceeding 40 m (Fig. 6i). While weaker over the $\Delta \eta$ minimum (<20 m), $\int w_{tot}$ is positive along the region of decreasing $\eta$, and negative in regions of increasing $\eta$, such as at 5°N, 86°E (Fig. 6i). Over all three months, the spatial structure of $\Delta \eta$ better compares to the structure of $\int w_{tot}$ than $\int w_c$ (Figs. 6a-i). The shape of the maximum $\int w_{tot}$ and $\Delta \eta$ aligns particularly well in July when growth in the SLD occurs (Fig. 6h), as documented in Cullen and Shroyer (2019). Within the SLD contour, the maximum change in $\Delta \eta$ is 0.15 m, while $\int w_{tot}$ is 36 m. $\int w_c$ is much smaller at 5 m outside of the wind shadow region. The magnitude of $\int w_{tot}$ is consistent with previous analysis of isopycnal displacements from ship measurements (Vinayachandran et al. 2018), Argo profiles (Cullen and Shroyer 2019) and moored.

FIG. 6. Climatological maps of (a)–(c) the monthly change in AVISO ADT ($\Delta \eta$; in m month$^{-1}$), (d)–(f) monthly integrated $\int w_c$, and (g)–(i) monthly integrated $\int w_{tot}$. Contours in (d)–(i) show regions of negative $\Delta \eta$ (solid) and positive $\Delta \eta$ (dashed).
observations (Lozovatsky et al. 2019) that show a measured 20–50 m raising of the thermocline associated with 0.14–0.17 m change in η within the SLD. Parameter Δη is more strongly correlated to \( \int w_c \) than \( \int w_t \) (Fig. 7). Observations of negative Δη are more often found in regions of positive \( \int w_t \) compared to regions of positive \( \int w_c \) (as indicated by the distributions by bin in Fig. 7). While a negative trend between \( \int w_c \) and Δη is apparent (e.g., associated with instances when \( w_c \gg w_c \) close to Sri Lanka), the trend with the addition of the vorticity correction is stronger, particularly under strong upwelling conditions (Figs. 7a,b).

6. Summary and discussion

The above analysis illustrates the importance of weakly nonlinear Ekman pumping both for the SLD and the larger SMC systems. Strong wind stress and vorticity gradients (both of which are roughly steady during the southwest monsoon) exist across the region of cooler SST, denoted as the Bay of Bengal Cold Pool. Vorticity Ekman pumping may be yet another explanation, along with previously suggested wind mixing and lateral advection, as to why this region is cool and biologically productive relative to the rest of the Bay of Bengal. To our knowledge, previous studies have not considered the role of upwelling in the northern SMC, as classic Ekman pumping contributions are too small with the wrong spatial distribution. Inclusion of the weakly nonlinear correction to Ekman pumping significantly alters both the magnitude and spatial distribution of upwelling, moving the anticipated contribution of upwelling closer to the observed pattern of cooling and changes in sea level height. Vorticity Ekman pumping may impact horizontal advective heat tendency, a key contributor to SST dynamics in the region. Previous work identified lateral advection as key to maintaining the Bay of Bengal Cold Pool, and highlighted coastal upwelling as the source of cool SST upstream. We suggest that in addition to this source, cooling distributed along the SMC will also contribute to lateral advective heat tendency measured downstream. Upwelling, along with mixing and surface fluxes heat tendency may contribute to cooling the SMC, and remotely impact horizontal heat transport. Vorticity-driven upwelling and divergence within the Southwest Monsoon Current system should inform future analysis and interpretation of the Bay of Bengal Cold Pool dynamics. While horizontal components of vorticity Ekman transport were not calculated in this paper, the strength of vertical Ekman transport indicates that weakly nonlinear terms are important to the dynamics of the Southwest Monsoon Current system.

Observed cooling is offset to the east of the SLD, extending into the SMC, in a region where vorticity Ekman pumping is the strongest. The east–west gradient in SST within the SLD is constant across the time series, no matter the size, shape, or position of the SLD with occasional exceptions during the early and late southwest monsoon. During these transitional periods, which roughly correspond to the start and end of the SLD’s lifespan, winds tend to be less steady and a relatively weak SMC system has associated weak vorticity gradients. Vorticity Ekman pumping is equal in magnitude of classic Ekman pumping within the eastern SLD so that total upwelling is enhanced on the eastern half of the SLD, consistent with the spatial patterns of SST cooling. Ekman pumping may also shock the MLD, which could result in associated changes of the upper-ocean heat content, surface proximity of cool thermocline waters, and time scale of the ocean’s response to air–sea fluxes. While many other processes, such as lateral advection and surface fluxes drive SST in this region, the spatial overlap between SST and Ekman pumping is suggestive of a role for weakly nonlinear Ekman pumping in the mixed layer heat budget.

Temporal change in η correlates with weakly nonlinear Ekman pumping, matching expected patterns due to upwelled isopycnals. The SLD grows in extent and amplitude and sometimes stretches to the northeast consistent with increased vorticity Ekman pumping along its eastern side. As the SLD evolves, its evolving vorticity leads to further modification of Ekman pumping; inclusion of the weakly nonlinear correction to the Ekman pumping thus acts as a feedback mechanism on the SLD sea level height. This feedback may explain why previous studies (de Vos et al. 2014; Burns et al. 2017) correlated the eastward position of the SLD to increased wind stress over the Bay of Bengal. Increased wind enhances vorticity Ekman pumping both directly by increasing \( \tau \), but also indirectly by strengthening current velocities and thus vorticity gradients across the SMC. Previous studies debated whether the SLD formed from classic Ekman pumping (assumed proportional to \( V \times \tau \)) or vorticity in the Southwest Monsoon Current (Vinayachandran and Yamagata 1998; de Vos et al. 2014); consideration of weakly nonlinear Ekman pumping offers the framework to consider the interaction between Ekman pumping and geostrophic vorticity. These two fields cannot be separated, i.e., the formation, growth, and propagation of the SLD may depend on the nonlinear feedback between Ekman pumping and vorticity. Weakly nonlinear
Ekman pumping provides a simple way to describe this interaction consistent with the observed evolution of the SLD.

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Data availability statement. Data used in this paper include the European Centre for Medium-Range Weather Forecasts’ (ECMWF) ERA5 wind speeds, heat flux, and SST, obtained from Copernicus Climate Change Service (cds.climate.copernicus.eu). AVISO ADT and geostrophic velocities are distributed by Copernicus Marine Environmental Monitoring Service (marine.copernicus.eu).

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