Salinity Mixing and Diahaline Exchange Flow in a Large Multi-Outlet Estuary with Islands

XIANGYU LI, a,b,c MARVIN LORENZ, e KNUT KLINGBEIL, e EVRIDIKI CHRYSAGI, e ULF GRAWE, c JIAXUE WU, a,b AND HANS BURCHARD c

a School of Marine Sciences, Sun Yat-sen University, Zhuhai, China
b Southern Marine Science and Engineering Guangdong Laboratory (Zhuhai), Zhuhai, China
c Leibniz Institute for Baltic Sea Research Warnemünde, Rostock, Germany

(Manuscript received 7 December 2021, in final form 14 April 2022)

ABSTRACT: The relationship between the salinity mixing, the diffusive salt transport, and the diahaline exchange flow is examined using salinity coordinates. The diahaline inflow and outflow volume transports are defined in this study as the integral of positive and negative values of the diahaline velocity. A numerical model of the Pearl River Estuary (PRE) shows that this diahaline exchange flow is analogous to the classical concept of estuarine exchange flow with inflow in the bottom layers and outflow at the surface. The inflow and outflow magnitudes increase with salinity, while the net transport equals the freshwater discharge \( Q_r \) after sufficiently long temporal averaging. In summer, intensified salinity mixing mainly occurs in the surface layers and around the islands. The patchy distribution of intensified diahaline velocity suggests that the water exchange through an isohaline surface can be highly variable in space. In winter, the zones of intensification of salinity mixing occur mainly in deep channels. Apart from the impact of freshwater transport from rivers, the transient mixing is also controlled by an unsteadiness term due to estuarine storage of salt and water volume. In the PRE, the salinity mixing and exchange flow show substantial spring–neap variation, while the universal law of estuarine mixing \( m = 2SQ_r \) (with \( m \) being the sum of physical and numerical mixing per salinity class \( S \)) holds over longer averaging period (spring–neap cycle). The correlation between the patterns of surface mixing, the vorticity, and the salinity gradients indicates a substantial influence of islands on estuarine mixing in the PRE.

KEYWORDS: Estuaries; Mixing; Numerical analysis/modeling

1. Introduction

Classical estuaries are coastal water bodies partially enclosed by land and subject to terrestrial freshwater runoff. The effect of salinity gradients dominates their density gradients. Therefore, it is helpful to analyze estuarine dynamics in an isohaline framework, using salinity coordinates instead of three-dimensional Cartesian coordinates. Such a procedure reduces the dimensionality of the problem and can lead to new insights that might be overlooked in the complexity of three-dimensional space. Moreover, it can provide a unified framework to classify and compare the wide variety of existing estuaries. It was Walin (1977) who first proposed such an isohaline analysis framework for estuaries, given the essentially nontidal Baltic Sea. This framework considers transports of volume and tracers for surfaces of constant salinity, the so-called isohalines. Between isohalines of different salinity, discrete isohaline volumes can be defined for which budgets of volume, tracers, and other properties such as mixing can be analyzed. Independently of Walin (1977), the application of the isohaline framework to tidal estuaries has first been proposed by MacCready and Geyer (2001), who characterized the tidally averaged motion of the isohalines as a competition between seaward advection with the mean flow and landward motion due to turbulent mixing. They concluded that the long-term averaged volume transport across an isohaline must equal the river runoff and the long-term averaged salt transport across an isohaline surface must be equal to the isohaline salinity times the river runoff. MacCready et al. (2002) extended this theory to allow isohalines to partially leave the estuary, which implies the definition of a seaward estuarine limit in the sense of an open boundary of a numerical model. In that framework, an estuarine volume is bounded by a fixed section toward the freshwater runoff, a fixed section toward the open ocean and a movable isohaline that may intersect with the open ocean section and the bottom of the estuary. This view of a truncated estuary had been extended by Hetland (2005), who included the entire river plume into the estuarine volume bounded toward the ocean by a moving isohaline.

Wang et al. (2017) showed that the long-term averaged diahaline volume transport is not evenly distributed over the two-dimensional isohaline surface as a seaward-oriented entrainment velocity. Instead, a distinct diahaline exchange flow exists with an upward (toward lower salinities) contribution in the up-estuarine region of the isohaline and a downward contribution in the seaward region. This can be seen from their Figs. 5 and 6. Indeed, after sufficiently long temporal averaging, the downward contribution needs to dominate since the isohaline area integral of the diahaline velocity must give the (seaward-oriented) river runoff \( Q_r \). This concept of a diahaline

Denotes content that is immediately available upon publication as open access.

Corresponding author: Xiangyu Li, lixiangyu@mail.sysu.edu.cn

DOI: 10.1175/JPO-D-21-0292.1
© 2022 American Meteorological Society. For information regarding reuse of this content and general copyright information, consult the AMS Copyright Policy (www.ametsoc.org/PUBSReuseLicenses).
exchange flow can be formulated within the framework of the total exchange flow (TEF) developed by MacCready (2011), who defines the time-averaged volume transports per salinity class in isohaline coordinates on fixed vertical transects across the estuary. By a separate integration over all incoming and outgoing volume fluxes per salinity class, the well-known Knudsen (1900) relations can be consistently reproduced (MacCready 2011; Burchard et al. 2018).

The quantification of salinity mixing as the (local) destruction of salinity variance per unit volume has been proposed by Burchard and Rennau (2008) and became a useful tool in analyzing estuarine dynamics (Burchard et al. 2009; Ralston et al. 2010, 2017; Wang et al. 2017; Wang and Geyer 2018; Li et al. 2018; Lange et al. 2020). In numerical models, the mixing analysis method proposed by Burchard and Rennau (2008) and later improved by Klingbeil et al. (2014) allows for the accurate separation of explicit mixing due to the parameterization of turbulence and implicit (or spurious) mixing due to numerical truncation errors of monotone tracer advection schemes. For an estuarine volume bounded toward the ocean using a fixed transect, MacCready et al. (2018) and Burchard et al. (2019) could link the estuarine circulation quantified by the Knudsen relations to the long-term mean volume-integrated salinity mixing $M = S_0 S_0 Q_r$, where $S_0$ and $S_0$ are representative inflow and outflow salinities, respectively. This theory was extended by Burchard (2020) for estuarine volumes bounded by a moving isohaline of salinity $S$, such that the volume-integrated mixing is exactly calculated as $M = \bar{S} Q_r$, (for long-term averaging and no air–sea exchange of freshwater). The mixing per salinity class is then obtained by differentiating this relation with respect to $S$, resulting in $m = \frac{\partial M}{\partial S} = 2S Q_r$, which is the universal law of estuarine mixing postulated by Burchard (2008). As shown by Burchard et al. (2021), the mixing per salinity class equals twice the diahaline salt transport, a finding that quantitatively links salinity mixing to diahaline transport.

Most of the estuarine mixing and exchange flow studies have been carried out for idealized configurations (Hetland 2005; MacCready et al. 2018; Burchard et al. 2019; Burchard 2020; Burchard et al. 2021) or realistic estuaries with a fairly simple single-channel bathymetry such as the Hudson River Estuary (Wang et al. 2017; Wang and Geyer 2018), the Connecticut River Estuary (Ralston et al. 2017), or the small nontidal Warnow Estuary (Lange et al. 2020). Such low-complexity topographies simplify the analysis of estuarine mixing and exchange flow which largely resemble the predictions from models with idealized topographies. There is, however, a variety of more complex estuarine topographies, where mixing and exchange flow are spatially and temporally highly variable. Trivially, also for those nonstandard topographies, the fundamental estuarine relations (Knudsen relation, Knudsen mixing relation, universal law of estuarine mixing) must hold. Besides the temporal variability (e.g., spring–neap cycle) a major question for such estuaries is the spatial distribution of mixing and exchange flow and their correlation. An isohaline analysis of these processes would greatly help to understand the functioning of such complex estuarine systems.

Therefore, in the present study, we are analyzing the estuarine dynamics in the Pearl River Estuary (PRE, South China), a tropical estuary characterized by a complex multichannel river network leading to a highly distributed freshwater discharge. Furthermore, a large number of islands of various sizes is located in the region of strong salinity gradients where it impacts on local mixing. The central research question to be answered is the role of these topographic features in shaping the mixing and exchange flow. Still, we will focus on idealized scenarios representing two seasons, i.e., high runoff and upwelling favorable wind (summer season) and low runoff and downwelling-favorable wind (winter season).

Section 2 presents the mathematical derivation of the salinity mixing relations and the entrainment velocity using salinity coordinates. In section 3, we briefly introduce the study area and the numerical model adopted in this study. We present a detailed description of the spatial and temporal patterns of the salinity mixing and exchange flow in section 4. Finally in section 5, we draw our conclusions.

2. Theory

a. Mixing relation in an estuary bounded by isohalines

Here, we will derive the equation of estuarine mixing in a volume bounded by an isohaline surface, where we impose a temporal change of the water and the salt content within this estuarine volume.

Let $V(S)$ be an estuarine volume bounded by an isohaline surface of salinity $S$, the sea surface, the sea bottom, and a river boundary, where a freshwater runoff is prescribed (see an illustration in Fig. 1a). Averaged over a prescribed time interval $\Delta t$, the budgets of water volume and salt within these boundaries are closed as follows (Hieronymus et al. 2014; Burchard 2020):

$$ V_{\text{stor}} = -Q + Q_r; \quad S_{\text{stor}} = -SQ - F_s, \quad (1) $$

where

$$ V_{\text{stor}}(S) = \frac{1}{\Delta t} \left\{ V(S, t + \Delta t) - V(S, t) \right\} \quad \text{and} \quad S_{\text{stor}}(S) = \frac{1}{\Delta t} \left[ \int_{V(S, t)} s \, dV - \int_{V(S, t + \Delta t)} s \, dV \right] \quad (2) $$

are the storage terms for water volume and salt (change in volume and volume-integrated salinity during the time interval $\Delta t$), and

$$ Q(S) = \left( \int_{A(S)} u^{\text{dia}} \, dA \right) \quad \text{and} \quad F_s(S) = \left( \int_{A(S)} -K_n \frac{\partial s}{\partial n} \, dA \right) \quad (3) $$

are the diahaline water transport and the diahaline turbulent salinity transport across the isohaline $S$, respectively, with the outward-pointing diahaline velocity $u^{\text{dia}} = (u - u^*) \cdot n$, the velocity vector $u$, the velocity of a point moving with the isohaline surface $u^*$, the unit vector $n$, which is orthogonal to the isohaline and pointing toward higher salinity, the diahaline diffusivity $K_n$ (which is a weighted average of the horizontal diffusivity $K_h$ and the vertical diffusivity $K_v$; see Burchard et al. 2021), the diahaline salinity gradient $\partial_s$, the isohaline area $A(S)$, and the time averaging operator $\langle \cdot \rangle$. 

Unauthenticated | Downloaded 03/31/24 04:10 AM UTC
For each isohaline surface of salinity $S$, the integrated mixing per salinity class is defined as the $S$ gradient of the volume-integrated and time-averaged mixing $M(S)$ (Burchard et al. 2020; Burchard et al. 2021):

$$m = \partial_S M(S) = \left( \int_{A_x(S)} x^i \left( \partial_S x^i \right)^{-1} dA \right)_{\text{with}}$$

$$M(S) = \left( \int_{V(S)} x^i dV \right)$$

where

$$x^i = 2K_{ps}(\nabla s)^2 + 2K_{ps}(\partial_s s)^2 = 2K_{ps}(\partial_s s)^2$$

is the salt mixing per unit volume, with $\nabla s$ being the horizontal salinity gradient. It had been shown by Burchard et al. (2021) that the mixing per salinity class is directly related to the diahaline turbulent transport $F^v$:

$$m = -2F^v.$$  

Combining relations (1) and (6) gives the transient salinity mixing relation

$$m(S) = \frac{2SQ_x}{m_x} + \frac{2(S - SV_{\text{stor}})}{\dot{m}}.$$  

In (7), $m_x$ is the impact of freshwater transport from rivers, and $\dot{m}$ is the unsteadiness due to estuarine storage of salt and water volume. For sufficiently long temporal averaging (vanishing storage term), the universal law of estuarine mixing $m = 2SQ_x$ is retrieved (Burchard 2020). However, the averaging time may differ between estuaries according to their external forcings and internal time scales. The tidal variation of $\dot{m}(S)$ might be significant compared to $m_x$ if a large amount of water and salt exchange occur on the isohaline surface. Nevertheless, previous studies have shown that $\dot{m}$ converges to zero when averaged over several spring–neap cycles in relatively simple estuaries (Burchard 2020; Burchard et al. 2021). In this study, we will explore the convergence properties of $\dot{m}$ in an estuary with more complex geometry.

The long-term averaged mixing per salinity class is calculated by two methods in this study. The first method calculates the mixing per salinity class $m$ as

$$m = m^{\text{phy}} + m^{\text{num}},$$

where $m^{\text{phy}}$ is the physical mixing (caused by turbulent diffusion) and $m^{\text{num}}$ is the numerical mixing (caused by discretization errors of the numerical advection scheme). The latter two terms are obtained by applying the analysis approach proposed by Klingbeil et al. (2014). The second method calculates $m$ from $m(S) = m_x + \dot{m}$ as described in (7). In (6), both $m$ and $F^v$ are located at the isohaline between adjacent salinity bins. However, the mixing per salinity class $m$ (including physical and numerical contributions) in (8) is associated with the center of one salinity bin. For these reasons, the results from Eqs. (7) and (8) will deviate by a numerical truncation error related to the binning process.

**b. Water-column consideration**

Apart from the temporal variation of salinity mixing, salinity mixing and diahaline transport also show high spatial variations on the isohaline surface.

Let us denote for stable salinity stratification the part of an estuarine water column with salinities $s > S$ (i.e., the distance between the isohaline and the bottom) as $l^r(S) = z(S) + H$. Here, $z(S)$ is the vertical position of the isohaline with salinity $S$ (see an illustration in Fig. 1b). Expression (6) has been
proved to be valid for any water column [Eq. (19) in Burchard et al. 2021]:

\[ m_{\text{io}} = 2f_{\text{diff}}^n / (\mathbf{n} \cdot \mathbf{k}), \]

where \( m_{\text{io}} = d m / d A \), is the local mixing per salinity class within this water column, \( d A_z = -\mathbf{n} \cdot \mathbf{k} dA \) is the projection of isohaline surface on the \( z \) plane, \( \mathbf{k} \) is the vertical unit vector and pointing upward, and \( f_{\text{diff}}^n \) is the diahaline diffusive flux.

Next, we will present the derivation of the expression for the local diahaline transport (entrainment) velocity. The vertical velocity at \( z = z(S) \) and at the stationary bottom \( z = -H \) is then following these kinematic conditions:

\[
\begin{align*}
    w(S) &= \frac{\partial z(S)}{\partial y} + w_e(S) + u(S)\frac{\partial z(S)}{\partial x} + v(S)\frac{\partial z(S)}{\partial y}, \\
    w(-H) &= -u(-H)\frac{\partial z}{\partial y} - v(-H)\frac{\partial z}{\partial y},
\end{align*}
\]

with the entrainment velocity \( w_e \) being the vertical projection of the diahaline velocity \( w_e(S) = u^\text{dia}(S)[u(S) \cdot \mathbf{k}] \). Note that due to the much smaller vertical scales than the horizontal scales in estuaries, the isohaline surface is substantially flat. Thus, the magnitudes of \( u^\text{dia} \) and \( w_e \) are very close to each other.

We further define the horizontal transports below the isohaline as

\[
p^{p>}(S) = \int_{F(S)} u dS \quad \text{and} \quad p^{p>}(S) = \int_{F(S)} S dS
\]

and obtain

\[
\frac{\partial z(S)}{\partial t} = -u \cdot \nabla p^{p>} - \nabla p^{p>} - w_e.
\]

We would obtain

\[
Q = \left( \int_{A(S)} -w_e(S) dA \right) = \left( \int_{A(S)} u^\text{dia}(S) dA \right)
\]

by comparison with (1). Note that positive \( w_e \) means upward diahaline velocity, while positive \( u^\text{dia} \) points to higher salinity. After averaging (12), we can get an equation for the time-averaged entrainment velocity

\[
\langle w_e \rangle = -\frac{\partial}{\partial t} \langle p^{p>} \rangle - \frac{\partial}{\partial y} \langle p^{p>} \rangle - z_{\text{store}}.
\]

with the average vertical motion of the isohaline surface during the averaging period,

\[
z_{\text{store}} = \frac{1}{\Delta t} [z(S, t + \Delta t) - z(S, t)].
\]

The rigorous numerical treatments of \( F(S) \), \( p^{p>}(S) \), and \( p^{p<}(S) \) can be found in Klingbeil et al. 2019.

Moreover, consider the local salt budget below the isohaline:

\[
\frac{\partial}{\partial t} \int_{F(S)} S dS = -\frac{\partial}{\partial x} \int_{F(S)} u S dS - \frac{\partial}{\partial y} \int_{F(S)} v S dS - w_e S + f_{\text{diff}} / (-\mathbf{n} \cdot \mathbf{k}).
\]

Then combining (16) with (12), we can get

\[
f_{\text{diff}} / (\mathbf{n} \cdot \mathbf{k}) = \int_{F(S)} \left( \frac{\partial}{\partial z} (-\partial_x p^{p>} - \partial_y p^{p>} - \partial_y z) \right) S dS - w_e S = \int_{F(S)} \left( \frac{\partial}{\partial z} w_e \right) S dS - w_e S.
\]

This relation has been derived by Wang et al. (2017). In this way, expressions (9) and (18) relate the local salinity mixing to the entrainment velocity through the diffusive salt flux.

3. Study area and methods

a. Pearl River Estuary

The Pearl River Estuary (PRE) is a multioutlet estuary located at the northern reach of the South China Sea (Figs. 2a,b). Freshwater from three main tributaries (West River, North River, and East River) passes through a highly complex river network and then discharges into the coastal ocean through eight outlets (short red lines in Fig. 2b). A large amount of freshwater, sediments, and nutrients are thus discharged to the northern shelf of the South China Sea, affecting the dynamics of the shelf and the various biogeochemical processes in the region (Harrison et al. 2008).

The river discharge shows substantial seasonal variations, with \( \sim 80\% \) of the river discharge occurring in the wet season (April–September). The total river discharge of all the tributaries reaches 20000 m3 s\(^{-1}\) in summer and drops to 4000 m3 s\(^{-1}\) in winter, roughly half of which enters the PRE (Dong et al. 2004). The wind in the PRE is dominated by two main patterns: northeasterly monsoon in winter and southwesterly monsoon in summer. The dynamics in the PRE exhibit a strong seasonal variation due to the large differences of runoff and wind direction between the wet season and the dry season (Dong et al. 2004). During summer, the plume is spreading cross shelf due to the upwelling-favorable wind as well as the high runoff (Pan et al. 2014). During winter, the plume is confined along the coast due to the downwelling-favorable wind and the low runoff (Lai et al. 2016). M2 and K1 are the two dominant tidal constituents in the PRE, making the PRE a mixed semidiurnal tidal regime (Mao et al. 2004). The tidal range is \( \sim 1 \) m at the mouth of the PRE (near the red box in Fig. 2b) and is amplified to 1.7 m at the head of the PRE (near the green box in Fig. 2b). The observed water level at Makou station (for location, see Fig. 2a) shows that the tidal waves can progress far upstream into the river network (Lai et al. 2015). In the PRE, there are two deep zonal-oriented channels (the West Channel and the East
Channel, see Fig. 2b), while the rest of the estuarine area consists of shoals and islands. In the shallow regions, the water depth varies from 2 to 10 m. Around the islands, the depth increases, reaching up to 30 or 40 m at the outer islands.

The presence of the islands disturbs the tidal flow and leads to the generation of strong horizontal density gradients in their vicinity. Additionally, vortex-like structures can emerge at the lee side of the islands. Such structures are a quite common feature in the PRE, as can be seen by available satellite images (Fig. 2c). Island wakes have been extensively studied due to the so-called island mass effect—the enhancement of biological productivity in the vicinity of the islands. Besides, island wakes play a crucial role in the cascade of mesoscale energy toward dissipation and turbulent mixing (Heywood et al. 1990; Hasegawa et al. 2004; Chang et al. 2013; Liu and Chang 2018; Chang et al. 2019). We observe a similar feature: an enhanced near-surface mixing around the islands in the PRE.

b. Model description and simulation setup

In this study, we performed the high-resolution simulations of the PRE using the General Estuarine and Transport Model (GETM; see Burchard and Bolding 2002). GETM is a coastal ocean model supporting curvilinear horizontal and vertically boundary-following coordinates. It has a nonlinear free surface with robust drying-and-flooding capability and an efficient sub-domain decomposition for massively parallel computations on high performance computing (HPC) systems. More details and the governing equations can be found in Klingbeil and Burchard (2013) and Klingbeil et al. (2018).

A curvilinear grid with 918 cells along the estuary and 742 cells across the estuary (Fig. 2a) was constructed using the Delft3D grid generation tool (https://oss.deltares.nl/web/delft3d/get-started) and then applied to this study. One challenge of building a structured numerical grid for the PRE is to resolve the highly complex river network. Based on the analysis of model results from the unstructured-grid Finite-Volume Community Ocean Model (FVCOM) PRE circulation model, Lai et al. (2015) found that the amount of freshwater discharged through each of the eight outlets in the PRE is controlled by the upstream river runoff, the tidal energy entering the river network, and the complex water exchange at the intersections inside the river network. Therefore, it is crucial to resolve the geometry of the river network to reproduce the freshwater dynamics. In contrast to previous estuarine studies on salinity mixing (e.g., Burchard 2020; Burchard et al. 2021)
that have used either idealized or low-complexity topographies, we use a realistic topography. This enables us to also investigate the potential effects of the deep channels and the multichannel river network on the spatial distribution of estuarine mixing. The model domain starts from the hydrologic stations (Gaoyao, Shijiao, and Boluo) of the three major tributaries, resolves the complex river network and the downstream estuaries, and then fans out to cover the shelf of northern South China Sea to the 4000-m isobath (Fig. 2a).

The horizontal resolution varies from 60 to 800 m in the river network (Fig. 2d), from 150 to 300 m inside the estuary (Fig. 2c), from 300 to 800 m in the coastal ocean area, and decreases to 9000 m at the southern open boundary. The horizontal grid size around the islands is approximately one-tenth of the scale of the eddies, suggesting the ability to resolve the eddies around the islands. In the vertical, we applied 40 terrain-following $\sigma$ layers. Since the depth inside the PRE is mostly below 10 m, this vertical resolution is fine enough to resolve the vertical structure of the river plume. We used a baroclinic time step of 10 s and a barotropic time step of 1 s for the temporal discretization.

As stated above, the dynamics in the PRE exhibit a strong seasonal variation due to the large differences in runoff and wind direction between the wet and dry seasons. Therefore, we conducted two idealized forcing simulation scenarios: the summer simulation (high runoff) and the winter simulation (low runoff). The configurations of these two simulations differ only in the river discharge and the wind; the summer simulation has a total steady runoff of 15 600 m$^3$ s$^{-1}$ and a constant southwesterly wind of 5 m s$^{-1}$. In contrast, the winter simulation has a total steady runoff of 3550 m$^3$ s$^{-1}$ and a constant northeasterly wind of 5 m s$^{-1}$. These two different regimes are typical for their seasons (Mao et al. 2004). According to their climatological mean, we specified the runoff data at three hydrologic stations (Gaoyao, Shijiao, and Boluo) of the three major tributaries. In both simulations, the open ocean boundary conditions are driven by simplified tidal water level from the TPXO tidal datasets (Egbert and Erofeeva 2002), with only four harmonic constituents ($M_2$, $S_2$, $K_1$, and $O_1$) to reflect the spring–neap variation. Since density is mainly controlled by salinity in the estuarine region of the PRE, we neglected temperature in the model simulations. Although evaporation and precipitation have an impact on estuarine mixing (Lorenz et al. 2021), we do not consider the air–sea exchange of heat or freshwater. These simplifications help to bind the salinity variance of an estuarine volume to an isohaline surface enclosed by the river mouth and the open ocean. The initial sea surface elevation and currents are zeros, while the initial salinity distribution is from the GLBv0.08 dataset of the HYCOM global ocean forecasting results (www.hycom.org/data/glbv0pt08/expt-57pt2). Salinities at the open boundaries are set to 35 g kg$^{-1}$. Both simulations were run for 12 months and started to exhibit a repeating spring–neap cycle after 5 months. To ensure the analysis of a quasi-steady state, we only analyzed the last month of the year-long simulation.

Vertical turbulent diffusion is calculated through the coupling with the turbulence module of the General Ocean Turbulence Model (GOTM; see Burchard and Bolding 2001) by solving the $k$–$\varepsilon$ two-equation turbulence closure model. Horizontal turbulent diffusion is parameterized following Smagorinsky (1963) with a Smagorinsky constant of 0.28 and a turbulent Prandtl number of 2.0 for salinity. For advection of velocities and salinity, the TVD-Superbee scheme is employed in directional-split mode due to its minimal numerical mixing (Klingbeil et al. 2014). For the isohaline analysis, a discrete salinity increment of $\Delta S = 0.1$ g kg$^{-1}$ is adopted. The salinity binning is computed during each baroclinic time step of the model as proposed by Lorenz et al. (2019). Sensitivity tests show that the results are only weakly dependent on $\Delta S$.

4. Results

We show the modeled surface salinity and surface current averaged over the spring tide as well as over the neap tide from both the summer (high runoff) simulation and the winter (low runoff) simulation in Fig. 3. In summer, the combination of the upwelling-favorable winds and the strong runoff induces a wide spreading of the river plume into a southeastward direction. In contrast, in winter, the plume turns southwestward along the coast with a narrower width under the control of the downwelling-favorable wind. The spatial distributions of the simulated salinity and current matches previous studies well, such as Pan et al. (2014) (for summer condition) and Lai et al. (2016) (for winter condition). During neap tide, accompanied by the weakened effect of tidal forcing, the surface plume extends more seaward than during spring tide, especially in winter. Inside the PRE, surface currents are stronger in the two deep channels than on the shoals. In summer, the deep channel westward of the Lantau Island (for location, see Fig. 3a) serves as an important passage for discharging water.

a. Mixing

The analysis of mixing per salinity class shows that the universal law holds for both the high-runoff simulation and the low-runoff simulation when averaged over 14 days (Fig. 4). Due to the numerical discretization and the binning procedure for calculating properties per salinity class, the numerical representations of the relations (7) and (8) are not identical (e.g., Fig. 4a). However, both methods still closely follow the theoretical prediction of $m(S) = 2SQ_{s}$. Note here that for water with higher salinities (e.g., $S > 22$ g kg$^{-1}$ in Fig. 4a), the mixing per salinity class $m$ deviates from the theoretical linear curve since the isohaline volume interacts with the numerical open boundary.

For both simulations, the physical mixing $m_{\text{phy}}$ dominates the total mixing $m$, whereas the numerical mixing $m_{\text{num}}$ contributes at most one-third of the total mixing. The higher contributions of the numerical mixing to the total mixing in summer could be related to the seaward spreading of the river plume pushed by the stronger runoff into the region with coarse resolution.

b. Diaphaline exchange flow

The long-term averaged diahaline water transport $Q(S)$ is balanced by the runoff $Q_{r}$ for almost all salinities $S$ (Figs. 4c,d).
This finding indicates that the net outflow across any isohaline $S$ is compensated by the freshwater discharge under steady-state conditions. Actually, after integrating relations (9) over all water columns, for a given isohaline surface with salinity $S$, we can get $m(S) = -2F_s(S)$. Then from (1), $F_s(S) = -Q(S)S$ is obtained when $S_{stor} = 0$. For long averaging periods the runoff has to pass through every isohaline surface since the estuarine volume $V(S)$ must stay constant. In this case, due to salt conservation, the outward (toward higher salinity) advective salt transport $SQ$ and the inward (toward lower salinity) diffusive salt transport $F_s$ balance each other on every isohaline surface. Since the diahaline velocity $u_{dia}$ can go in either direction orthogonal to the isohaline surface, its vertical projection $w_e$ can go in both upward and downward direction in the water column. Consequently there must be regions with positive and negative diahaline water transport on the isohaline surface. Then, following (13), the outflow and inflow of diahaline water transport can be defined as

$$Q_{out} = - \int_{A_s(S)} w_e|_{w_e < 0} dA \quad \text{and} \quad Q_{in} = - \int_{A_s(S)} w_e|_{w_e > 0} dA,$$

respectively, with $w_e > 0$ representing the upward (pointing to lower salinity) diahaline velocity in a single water column and $A_s$ representing the projection of the isohaline area to a geopotential area.

When horizontally integrated over all locations where a given isohaline surface with salinity $S$ occurs, $Q_{out}(S)$ and $Q_{in}(S)$ can be regarded as the exchange flow with respect to

---

**FIG. 3.** Horizontal distribution of surface salinity averaged over (a),(b) spring tide and (c),(d) neap tide from the high-runoff (summer) simulation and the low-runoff (winter) simulation, respectively. Surface current velocities greater than 0.1 m s$^{-1}$ are shown in red arrows in every 20th grid point.
this isohaline surface. The magnitude of the outflow of the di-ahaline water transport \( Q_{\text{out}} \) equals \( Q_r \) for \( S = 0 \), while the magnitude of the inflow of the dihaline water transport \( Q_{\text{in}} \) equals 0 for \( S = 0 \) (Figs. 4c,d). For \( S > 0 \) they both increase with salinity. Although the net dihaline water transport \( Q = Q_{\text{out}} + Q_{\text{in}} \) equals the constant runoff \( Q_r \) when averaged over sufficiently long time, the strength of dihaline exchange flow \( Q_{\text{st}} = (Q_{\text{in}} + Q_{\text{out}}) / 2 \) increases with salinity. For smaller salinities, \( Q_{\text{st}} \) is less than the runoff \( Q_r \). However, for larger salinities, the strength of dihaline exchange flow becomes larger than the runoff. Take the winter (low-runoff) scenario for example, the strength of dihaline exchange flow \( Q_{\text{st}} \) can be more than 3 times the runoff \( Q_r \) at high salinities (\( S > 25 \text{ g kg}^{-1} \) in Fig. 4d).

c. Vertical distribution of dihaline exchange flow

To analyze the vertical distribution of dihaline exchange flow, the local dihaline water transport \( Q(x, y, S) \) calculated as (13) was mapped into \( \sigma \) coordinates with \( \Delta \sigma = 0.025 \text{ g kg}^{-1} \) (Fig. 5). The mapping process from salinity space to \( \sigma \) space is presented in the appendix of this paper. For both seasons, the long-term averaged dihaline water transport \( Q \) is pointing to lower salinity in the bottom layers and pointing to higher salinity in the surface layers, which is very similar to the classical exchange flow. In general, the magnitude of the dihaline water transport inflow \( Q_{\text{in}} \) is zero at the bottom (\( \sigma = -1 \)), and then gradually increases to its maximum in the near-bottom layer, followed by the decrease to zero again at middle depth. Please note that we do not exactly have the bottom \( \sigma = -1 \) value. Since there is no water transport through the bottom and \( Q_{\text{in}} \) approaches zeros toward the bottom (e.g., \( \sigma \leq -0.95 \)), we set the bottom \( \sigma = -1 \) values to zero. Influenced by the buoyant runoff, the outflow of dihaline water transport \( Q_{\text{out}} \) generally is largest in surface layers and then decreases to zero at that middle depth. Because of the relations (9) and (18), the magnitudes of local mixing per salinity class \( m_x \) (or in other

![Fig. 4. Properties related to the salinity mixing and the dihaline transport as a function of salinity S calculated from long-term averaged model data from (left) the high-runoff simulation and (right) the low-runoff simulation, respectively. (a),(b) Total [red line, Eq. (8)], physical (green line), and numerical (blue line) mixing per salinity class \( m(S) \), in comparison to another method \( m = m_x + \tilde{m} \) [brown line, Eq. (7)] as well as to the universal law under equilibrium condition \( m = 2SQ_r \) (dashed gray line). (c),(d) The total dihaline transport (red line), the inflow (green line), and the outflow (blue line) of the dihaline transport as a function of salinity \( S \). The brown line represents the magnitude of the exchange flow expressed as \((-Q_{\text{in}} + Q_{\text{out}})/2\).](image-url)
FIG. 5. Vertical distribution of the (negative) diahaline transport \(\langle \int_{A_z(s)} w_z dA \rangle\) per \(\sigma\) (a),(b) averaged over 14 days and averaged over (c),(d) spring tide and (e),(f) neap tide. Left panels represent results from the high-runoff simulation, right panels represent the low-runoff simulation. In each panel, the (negative) diahaline transport at different salinities is shown in different gray levels. The blue and red shading in (b) helps to illustrate the definition of outflow \(Q_{out}\) and inflow \(Q_{in}\) of diahaline water transport. \(\sigma = -1\) means bottom, and \(\sigma = 0\) means surface.
words, the diffusive diahaline salinity flux \( f_{\text{dia}} \) will reach its maximum at this middle depth when \( u_{\text{dia}} = 0 \) for any single water column. The upper part of the isohaline surface would span a large estuarine area in the form of the surface plume when the runoff is large or when the tides are weak (especially during neap tide in summer). In this case, the domain-integrated diahaline water transport for high salinities may not be simply monotonously increasing or decreasing along with the \( \sigma \) coordinates due to the complexity of the geometry and dynamics in the PRE. An example for such an exception can be seen occurring around \( S = 20 \) in Figs. 5e and 5f, which will be further explained in section 4e.

d. Lateral distribution of mixing and entrainment

The spring–neap averaged spatial distributions of the mixing per salinity class and the entrainment velocity at \( S = 20 \) g kg\(^{-1}\) are presented in Fig. 6. The horizontal distribution of the salinity mixing on isohalines \( m_\sigma(S) \) are similar to the horizontal distribution of the depth-averaged mixing \( \chi \) presented in other studies (e.g., Warner et al. 2020). In this study, the \( S = 20 \) g kg\(^{-1}\)
isohaline is a good representative among different isohaline surfaces to show the spatial distribution of salinity mixing. In Figs. 6a and 6b, areas with nonzero $m$ are those where isohaline surface with $S = 20$ g kg$^{-1}$ occurs during the spring-neap cycle. Comparing Fig. 6a and Fig. 6b, the isohaline with salinity $S = 20$ g kg$^{-1}$ covers a much larger area in summer than in winter. The major difference occurs near the surface layers (e.g., $0.2 < < 0$). As mentioned above, the size and shape of the river plume are mainly controlled by the river discharge and the wind field. The isohaline areas seaward of the $\sigma = -0.2$ iso-sigma contour are the part of the isohaline area which are very close to the sea surface. For salinity $S = 20$ g kg$^{-1}$, this upper part of the isohaline surface spans in summer a major part of the isohaline area due to the combined effects of buoyancy and wind stress. In contrast, it only covers a minor part in winter.

In summer, intensified salinity mixing occurs in the surface layers, seaward the Lantau Island and around many other small islands (Fig. 6a). As we have shown in Figs. 5a and 5b, the entrainment velocities are generally upward in the bottom layers and downward in the surface layers (Figs. 6c,d). However, there are also upward entrainment velocities in the surface layers in summer, mainly seaward of Lantau Island (Fig. 6c). In summer, the islands in the PRE play an essential role in controlling the salinity mixing and entrainment velocity (see Fig. 3 for the spreading of the plume). At the same time, in winter, the zones of intensification correspond more to the two deep channels. Negative $w_e$ shows the areas where freshwater leaves the estuarine volume bounded by the isohaline surface. The patchy distribution of intensified $w_e$ in summer suggests that the water exchange through an isohaline surface can be spatially highly variable.

e. Detailed view along a transect

We analyzed a vertically resolved transect to examine the vertical structure of the salinity mixing and the entrainment velocity in the PRE under high-runoff conditions. We placed it in the deep channel to the southwest of the Lantau Island (thick green line in Fig. 6a), oriented along the main direction of surface currents in summer (see Figs. 3a,c). The water along this transect is strongly stratified during the entire spring-neap cycle in summer, with a thin plume covering roughly the upper 3 m of the water column (Fig. 7a). In the context of the following analysis, the slope of the isohalines does not need to be considered for the distinction between $w_e$ and $u_{\text{diff}}$, since the $n \cdot k$ in (9) can roughly be neglected here.

The relationship between entrainment velocity, salinity mixing, and diffusive salt flux can be inferred from Figs. 7b–d. For each single water column, saltwater from the bottom and freshwater from the surface are brought into the inner layers due to entrainment. Note that for each of the isohaline surfaces, there are both subareas with downward $w_e$ and other subareas with upward $w_e$ (see Fig. 5). The $S = 20$ g kg$^{-1}$ isohaline roughly separates the downward and upward $w_e$, especially in

---

**Fig. 7.** Vertical distribution of (a) salinity, (b) entrainment velocity (positive means upward), (c) mixing $\chi'$, and (d) local mixing per salinity class $m_{lo}$ along the transect shown in Fig. 6 averaged over 14 days from the high-runoff simulation. The thick green line in (a) represents the depth-averaged salinity $\bar{S}$. The thick blue line in (c) represents the depth-integrated mixing $\int \chi' dz$. The thick blue line in (d) represents the local salinity mixing $m_{lo}(S)$ at $S = 20$ g kg$^{-1}$. The dashed magenta, black, and blue lines in each panel correspond to the time-averaged positions of isohalines $S = 4, 20$, and 34.5 g kg$^{-1}$, respectively.
the outer plume region (seaward of 22 km in Fig. 7b). However, in the near-surface region ($s > 0.2$), this isohaline still crosses some nonzero regions of $w_s$, for example at 25 km along the transect. This explains the positive $w_s$ for $S = 20$ to the southwest of Lantau Island in Fig. 6c.

According to (4), $\chi^2$ is equal $m_{lo}$ times the salinity gradient $\partial_s S$. Equation (9) further shows that the local salinity mixing $m_{lo}$ is directly related to the diffusive salt flux $f_{\text{diff}}$. The vertical structure of local salinity mixing $m_{lo}$ (or in other words, $-2 f_{\text{diff}}$) shown in Fig. 7d is similar to the distribution of mixing $\chi^2$ shown in Fig. 7c. This pattern reflects the strong correlation between the salinity mixing $\chi^2$ and the diffusive salt flux $f_{\text{diff}}$.

When averaged over a sufficiently long time, the diffusive salinity transport $P$ through any isohaline surface must be balanced by the advective diahaline salinity transport $\overline{S_Q}$. However, due to the presence of the horizontal salinity flux divergence (the first two terms on the RHS of expression 16), the diffusive salt flux $f_{\text{diff}}$ in a single water column is not necessarily in local equilibrium with the advective diahaline salt flux $u^{\text{dia}} S$. Instead, the local diffusive salt flux $f_{\text{diff}}$ is balanced by $\int_{S > S_m} u^{\text{dia}} ds$ [Eq. (18)].

f. Spring–neap and intratidal variations

Whereas the strong dependence of salinity mixing on the runoff in a steady state can be illustrated by the universal law of estuarine mixing, drastic variations of estuarine mixing can occur during a spring–neap cycle or even within single tidal cycles. To illustrate this, Fig. 8 shows the temporal variation of salinity mixing $m$ (scaled by $2 S_Q$, such that the temporally averaged value should be unity) for salinities in the estuary within nine days from neap tide to spring tide. The resemblance between Figs. 8a and 8c as well as between Figs. 8b and 8d indicates that the results from the two methods [Eqs. (7) and (8)] calculating $m$ are consistent. The salinity mixing $m$ oscillates around its long-term averaged value $m = 2 S_Q$ throughout the flood–ebb tidal cycles. In the high-runoff simulation, the temporal variation of the normalized salinity mixing $m/2 S_Q$ does not show distinct differences in magnitude and phase between different salinities (Figs. 8a,b). Therefore, the total mixing $M(S) = \int_{S > S_m} m(S) ds$ within the estuarine volume $V(S)$ and the salinity mixing $m(S)$ on the isohaline surface of $S = 20$ g kg$^{-1}$ show very similar temporal variability (Fig. 9).
Nevertheless, the normalized magnitude $m/2Q_r$ indicates some discrepancy between high salinity and low salinity for low run-off (e.g., $\sim 343.5$ day in Fig. 8b).

The spring–neap variation of the low-pass-filtered $M$ shows that the estuarine mixing in the PRE is more vigorous during neap tide than during spring tide (Fig. 9a). The temporal oscillation of $M$ in a flood–ebb tidal cycle can deviate by a factor of 2 from the steady-state theoretical value $Q_rS^2$. The volume-integrated mixing $M$, the mixing per salinity class $m$, and the inflow $Q_{in}$ and outflow $Q_{out}$ of diahaline water transport are

---

**Fig. 9.** (a),(b) Time series of volume-integrated mixing $M(S)$ (thin black line) and its theoretical value $Q_rS^2$ under steady condition (dashed black line) from the high-runoff simulation (left) and low-runoff simulation (right), respectively. (c),(d) Time series of mixing per salinity class $m(S)$ (thin black line) and its theoretical value $2SQ_r$ under steady condition (dashed black line). (e),(f) Time series of diahaline inflow $Q_{in}$ (red line) and diahaline outflow $Q_{out}$ (blue line). Note that the negative $Q_{in}$ is shown for better interpretation. (g),(h) Time series of diahaline area $A(S)$ (thin black line). Thicker lines in all the panels represent the corresponding low-pass-filtered variables.
well in phase with each other (Figs. 9a,c,e). The isohaline is relatively flat during the neap tide, which leads to a larger isohaline area than during spring tide (Figs. 9g,h). As stated in MacCready and Geyer (2001), Hetland (2005), and Burchard (2020), reduced local mixing (e.g., when tidal forcing is weak) requires a larger isohaline surface to maintain the freshwater transport across the isohaline surface. Since $m = 2SQ$, high runoff $Q_r$ also results in an enhanced salinity mixing $m$, which require an increased isohaline area $A$ (comparing Figs. 9g and 9h).

g. Impact of islands on mixing

As discussed above, during the high-runoff scenario, when the plume-influenced area extends further to the south (Figs. 3a,c), the spring–neap averaged surface mixing increases substantially around the islands (Fig. 6). To further examine the role of the islands on mixing, instantaneous fields of surface physical mixing $x_{mix}^{phy}$ are contrasted with the lateral gradients of velocity $\nabla b = \sqrt{u_1^2 + v_1^2 + u_2^2 + v_2^2}$ (McWilliams et al. 2015; Wang et al. 2021), the lateral gradients of buoyancy $\nabla b$ as well as with the Rossby number $Ro = \epsilon f$ (Fig. 10). Here, $b = - \gamma (\rho - \rho_0) / \rho_0$ is the buoyancy, $\epsilon = v_2 - u_1$ is the vertical component of the relative vorticity, and $f$ is the Coriolis frequency. Note that we conducted idealized simulations where the temperature has been excluded. Therefore, the salinity controls the density field. Around the islands, alternating patterns of high positive and negative vorticities emerge. As expected, the cyclonic vorticities dominate at the western side of the islands and the anticyclonic ones at the eastern side (Fig. 10b). The mixing hotspots are highly correlated with these strong vorticity tails and the sharp lateral density and velocity gradients. This correlation points to a potentially important role of submesoscale island wakes on turbulent mixing. Surface numerical mixing $x_{out}^{num}$ is also strong in regions with flow separations (not shown here). However, physical mixing still dominates in the region with islands. Some preliminary results, from a new simulation using a horizontal grid resolution of ~70 m in the area of the islands, show that finer horizontal resolution seems to result in finer filamentary structures and relatively less numerical mixing. Although a comprehensive analysis of mixing and submesoscale island wakes is a potentially important role of submesoscale island wakes on horizontal density gradients and velocity (salinity). This quantitative relation points to a potentially important role of submesoscale island wakes in the PRE. Oceanic island wakes (or topographic wakes in general) act as sites of submesoscale generation. Many complex dynamical processes can arise around the islands, such as vorticity generation, wake instability, coherent vortex formation, intense lateral and even diapycnal mixing for the case of centrifugal instability (Liu and Chang 2018). Those processes depend on numerous factors such as the water depth, the shape and size of the island, and the existence of a mean flow or tides (Hasegawa et al. 2004). Therefore, the submesoscale island wakes in the PRE and the plume fronts will be investigated separately in a subsequent study. There, we will use realistic simulations that include both the temperature field and a realistic atmospheric forcing. This is essential for a comprehensive analysis since the wind field is instrumental not just in the dynamics of submesoscale motions (Chrysagis et al. 2021) but also for the wake structures.
However, for simplicity, we have neglected this explicit realism in the current study. Apart from the impact of freshwater transport from rivers $m_\infty = 2SQ_n$, the transient salinity mixing is also controlled by an unsteadiness term due to estuarine storage of salt and water volume $\dot{m} = 2(S_{\text{stor}} - SV_{\text{stor}})$. Due to the complex bathymetry and the energetic tidal flow around the islands and despite a low-diffusivity advection scheme, the contribution of the numerical mixing to the total mixing per salinity class amounts to up to 30% for the high-runoff simulation at a medium salinity of $20 \text{ g kg}^{-1}$. This contribution reduces to about only 15% for the low-runoff simulation. In the PRE, the maximum $m(S)$ within a spring–neap cycle reaches twice its long-term averaged value $m_\infty = 2SQ_n$, while the minimum $m(S)$ drops to only half of $m_\infty$. The isohaline area is larger during neap tide than during spring tide. This variation is caused by reduced local mixing (e.g., when tide forcing is weak), which requires a larger isohaline surface to maintain the freshwater transport across the isohaline surface (MacCready and Geyer 2001; Hetland 2005; Burchard 2020). Despite the

---

**FIG. 10.** Instantaneous distribution of (a) horizontal gradient of surface velocity $|\nabla U| = \sqrt{u_1^2 + u_2^2 + u_3^2}$, (b) Rossby number $Ro = \frac{\xi}{f}$, where $\xi = v_t - u_t$ is the vertical component of the relative vorticity, (c) horizontal gradient of surface buoyancy $|\nabla b|$, where $b = -g(\rho - \rho_0)/\rho_0$ is the buoyancy in the surface layer, and (d) surface physical mixing $\chi_{\text{phy}}$ taken on flood tide during the spring tide from the high-runoff simulation. Black arrows in (a) represent the surface velocities.
complex estuarine topographies in the PRE, the islands within the estuarine area, and the spring–neap variations, our model simulations confirm the universal law of estuarine mixing $m = 2SQ_i$. It holds for typical summer and winter scenarios, as long as the averaging period is long enough (one spring–neap cycle).

Acknowledgments. XL and JW are supported by the Natural National Science Foundation of China (41906142, U1901209) and the Innovation Group Project of Southern Marine Science and Engineering Guangdong Laboratory (Zuhai, China) (311021004). KK, EC, and HB acknowledge the support by the German Research Foundation for the Collaborative Research Center TRR181 on Energy Transfers in Atmosphere and Ocean (Project 274762653). UG and ML are supported by the project ECAS-BALTIC funded by the German Federal Ministry of Education and Research (FKZ: 03F0860D). XL would like to thank the International Postdoctoral Exchange Fellowship Program of Tianhe-2. We thank Erika Henell (IOW) for constructive comments.

Data availability statement. Data analyzed in this study were based on numerical model results. Datasets are available at http://doi.io-warnemuende.de/10.12754/data-2021-0008.

APPENDIX

Mapping from Salinity Space to $\sigma$ Space

The mapping process from salinity coordinates to $\sigma$ coordinates to produce Fig. 5 is described as follows. Note that this mapping process is done in the postprocessing, and it is irrelevant to the vertical coordinates used in the numerical simulation. First, the $\sigma$ range $-1 \leq \sigma \leq 0$ is discretized in $K - 1$ equidistant $\sigma$ intervals with $\Delta \sigma = 1/(K - 1)$, $\sigma_1 = -1$, and $\sigma_K = 0$. Next, the relative position $\sigma$ of a temporally averaged isohaline surface for each water column $i$ can be calculated as

$$\langle \sigma \rangle_{i,j} = \left( \langle \sigma_i \rangle \right)_{j} / (D_j) - 1, \quad \text{(A1)}$$

where $\langle \cdot \rangle$ denotes the time-averaging operator, $j$ is the index of salinity $S = s_j$ in salinity coordinates with $1 \leq j \leq J$, $D_j$ is the distance between the isohaline and the bottom, and $D$ is the total water depth of the water column. From (14), the temporally averaged diahaline water transport $Q = -\int_{A_i} \langle \omega \rangle w_e dA_z$ across this isohaline surface can be calculated as

$$\langle Q \rangle_{i,j} = \left[ \tilde{\alpha}_x \langle p^+_{i,j} \rangle + \tilde{\alpha}_y \langle p^+_{i,j} \rangle + \left( \langle z_{stol} \rangle \right)_i \right] A_i, \quad \text{(A2)}$$

where $A_i$ is the area of water column $i$. Note that the salinity mixing $m$ and the diahaline velocity $w_e$ can also be mapped from salinity space to $\sigma$ space in a similar way. Finally, the diahaline water transport in the $k$th $\sigma$ layer for each isohaline $S = s_j$ is calculated as

$$\langle Q \rangle_{i,k} = \sum_i \langle Q \rangle_{i,j} \text{ for } k \geq 2. \quad \text{(A3)}$$

Note that $\langle Q \rangle_{i,k} = 0$ when $k = 1$, since there is no water transport through the bottom ($\sigma = -1$).

REFERENCES


