On Entrainment and Diffusion in a Gulf of Mexico Anticyclonic Ring

Joyce E. Schmitz and Andrew C. Vastano

Department of Oceanography, Texas A & M University, College Station 77843

27 May 1975 and 1 December 1975

ABSTRACT

The parametric model used by Schmitz and Vastano (1975) to investigate a Gulf Stream cyclonic ring has been applied to successive observations of a Gulf of Mexico anticyclonic ring. Coefficients for a polynomial representation of the transverse streamfunction $\psi(r,z)$ were determined for pairs of eddy diffusivity coefficients. Using the minimum least-squares error residual as a criterion, the best flow pattern for the ring occurred for $(K_h,K_z)=(10^6,0)$ cm² s⁻¹. The total transverse transport through the ring was found to be an order of magnitude larger than that found in the Gulf Stream cyclonic ring.

---

1. Introduction

Anticyclonic current rings have been found in all parts of the Gulf of Mexico and, prior to 1969, were assumed to be formed by the Gulf Loop Current. This was confirmed in May 1969, when the detachment and movement of an anticyclonic current ring was surveyed during three cruises of the Texas A & M University research vessel *Alaminos*.

Circulation in the Gulf is dominated by the Loop Current which joins inflow at Yucatan Straits with outflow at Florida Straits. The Yucatan Current lying close along Campeche Bank extends northward into the Gulf of Mexico approximately 300 n mi before turning in an anticyclonic arc to become the West Florida Current flowing south along the Florida shelf. Just as meanders in the Gulf Stream of the North Atlantic are necessary for ring generation, meanders which develop in the Gulf Loop Current can result in the shedding of an anticyclonic eddy. In years when meanders develop simultaneously in the western and eastern sides of the loop a shear zone forms, cutting off an anticyclonic ring. Cochrane (1969) discusses this separation process in greater detail.

The physical model developed by Schmitz and Vastano (1975) was applied to an anticyclonic ring in the Gulf of Mexico to investigate the processes of entrainment and diffusion. For each pair of eddy coefficients, the parametric study determined a mean velocity field for the ring, and the streamline pattern with the minimum least-squares error residual was selected as best indicating entrainment and diffusion in the ring.

2. The physical model

As a brief review, the diffusion and entrainment model assumes an axially symmetric ring with temperature governed by

$$\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(rK_h\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial T}{\partial z}\right)+\frac{1}{r}J(\psi,T), \qquad (1)$$

where K_h and K_z are the horizontal and vertical eddy diffusivity coefficients and $J(\psi,T)$ is the (r,z) Jacobian of the transverse streamfunction ψ and the temperature T. Observations made during cruises 66-A-11 from 4–18 August 1966, and 66-A-15 from 27 October to 4 November 1966, aboard the *Alaminos* supplied the serial data necessary to estimate temporal and spatial derivatives of the temperature.

Contouring horizontal sections of temperature revealed the ring was not axially symmetric. Since the model assumes a cylindrical ring, it was necessary to modify this aspect of the data analysis. Patzert (1969), confronted with a similar problem in his study of eddies in Hawaiian waters, indirectly used the areal extent of these elliptical eddies to synthesize symmetric current rings. This concept of constructing a ring of equal area was applied in a more direct manner to the 1966 cruise data. The area contained in a closed isotherm was measured with a planimeter and a circle of equal area was drawn. By repeating this process for all closed contours on the horizontal sections, axially symmetric rings were produced for the 66-A-11 and 66-A-15 observations.

Using these constructed rings, the remainder of the investigation was carried out in the same manner as the Gulf Stream cyclonic ring study. Temporal derivatives of temperature were estimated for the 77-day interval between completion of each cruise, while spatial derivatives were obtained from a two-dimensional (r,z) polynomial representation of the mean temperature field. A comparison of the mean and approximated temperature fields is presented in Fig. 1. The diffusivity parametric study was then continued by introducing a streamfunction

$$\psi=r^2z(A_0+A_1r+A_2z+A_3r^2+A_4rz+A_5z^2) \qquad (2)$$

which satisfies the condition that the radial velocity at the center of the ring and vertical velocity at the surface be zero. Eq. (1) together with (2) was solved for the coefficients $A_0, A_1, \ldots, A_5$; and the least-

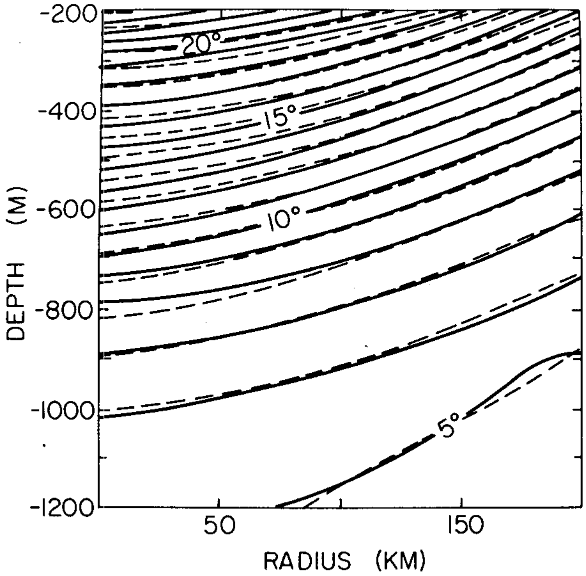

FIG. 1. Comparison of radial sections of temperature. The solid lines are the average temperature of constructed rings 66-A-11 and 66-A-15; the dashed lines are the temperature field generated by the approximating polynomial.

squares error residual

$$E=\int\!\!\int\left[r\frac{\partial T}{\partial t}-r(\text{diffusive terms})-J(\psi,T)\right]^2 drdz \tag{3}$$

was computed to indicate which (K_h,K_z) pair best satisfied (1).

3. Model results

Of the cases including diffusive and advective terms, $(K_h,K_z)=(10^6,0)$ cm² s⁻¹ resulted in the smallest residual. Table 1 presents (K_h,K_z) pairs with associated variances normalized by the least-squares error 0.3982×10^{12} °C² cm⁴ s⁻² for the advective case $(K_h,K_z)=(0,0)$. The integral

$$I=\int\!\!\int\left[r\frac{\partial T}{\partial t}\right]^2 drdz \tag{4}$$

evaluated for the ring is 0.7479×10^{13} °C² cm⁴ s⁻².

TABLE 1. Normalized least-squares error variances.

Diffusivity ($\mathrm{cm}^2\ \mathrm{s}^{-1}$)		Normalized variance	$\psi_{\max}$ ($10^{10}\ \mathrm{cm}^3\ \mathrm{s}^{-1}\ \mathrm{rad}^{-1}$)
K_h	K_z		
0	0	1	−15.05
10^4	0	0.9975	−15.07
10^5	0	0.9764	−15.18
10^6	0	0.9204	−16.35
10^4	1	1.0234	−15.53
10^5	1	1.0025	−15.65
10^6	1	0.9493	−16.81

The streamfunction for the optimum case suggests that water at 1200 m is transported to upper levels in the ring, or it exits the ring at depths greater than 800 m. Water also enters the ring between 400 and 800 m and has associated with it the only perturbation to an otherwise uniform velocity field. For these depths the radial component is directed toward the center over a portion of the ring, whereas all other regions of the ring have a horizontal velocity away from the center. Vertical velocity is toward the surface at all points in the radial section. The fields of streamlines and isotachs of radial and vertical velocities for the (K_h,K_z) pair resulting in the minimum least-squares error residual are shown in Figs. 2, 3 and 4 respectively; Table 2 lists the coefficients of ψ which generate these fields. The remaining sets of eddy diffusivities produced flow patterns not too different from the optimum case. In general, an increase in horizontal eddy diffusivity decreases the amount of water entering the ring above 800 m but increases flow through the 1200 m level. A larger vertical eddy coefficient increases the amount of water transported through the ring at both levels.

4. Discussion

Since the analysis of entrainment and diffusion in an anticyclonic ring of the Gulf of Mexico has been accomplished using a model developed for Gulf Stream cyclonic rings of the North Atlantic, it might be of interest to compare the results of the two studies.

As expected, the streamlines for the two rings exhibit major differences, the most obvious of which is the direction of flow. The anticyclonic ring moves water up from 1200 m and out above 400 m or below 800 m, while water enters the cyclonic ring above 500 m and

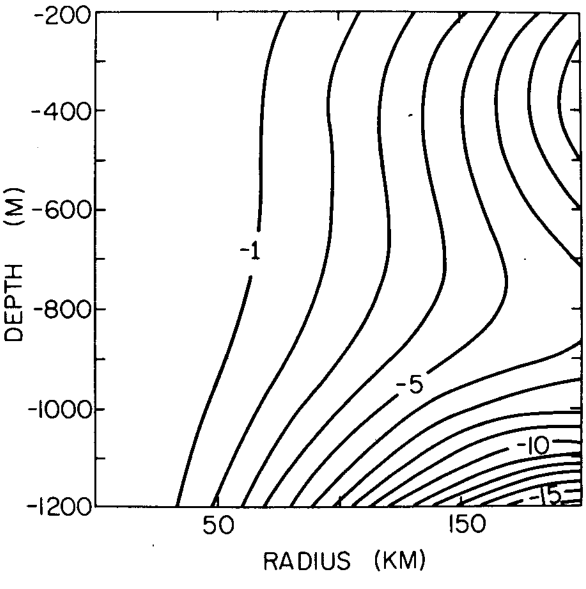

FIG. 2. Field of streamlines (10^{10} cm³ s⁻¹ rad⁻¹) for $K_h=10^6$ cm² s⁻¹ and $K_z=0$.

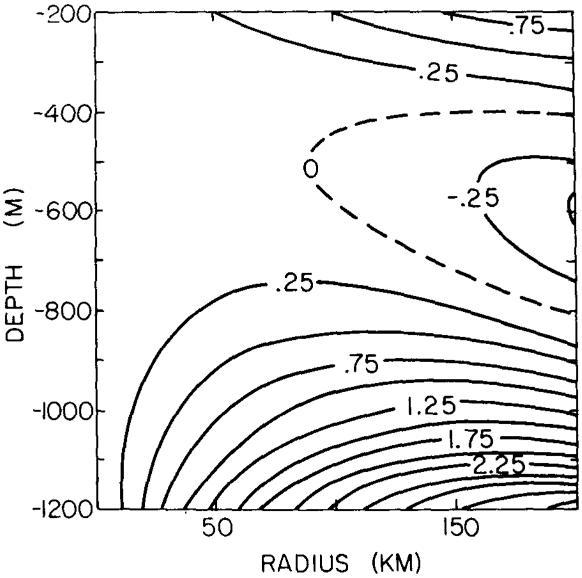

FIG. 3. Isotachs of the radial velocity component $u(10^{-3}$ cm s$^{-1})$ for $K_h=10^6$ cm^2 s^{-1} and $K_z=0$.

exits at depths greater than 800 m. Another difference in the field of streamlines is a second region of inflow between 400 and 800 m at radii greater than 150 km in the anticyclonic ring. Both eddies have a uni-directional vertical velocity field although opposite in direction and a bi-directional radial velocity field. In the cyclonic ring, there is an upper layer of inward directed velocity and a lower layer of outward directed velocity, whereas the anticyclonic ring of the Gulf of Mexico has a wedge-like layer of inward directed velocity which separates the outward directed radial velocity field between 400 and 800 m for radii greater than 100 km.

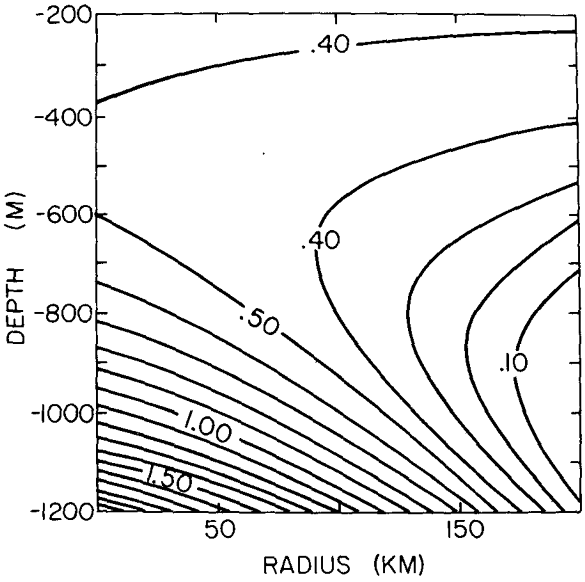

FIG. 4. Isotachs of the vertical velocity component $w(10^{-3}$ cm s$^{-1})$ for $K_h=10^6$ cm^2 s^{-1} and $K_z=0$.

TABLE 2. Coefficients of $\psi=r^2z(A_0+A_1r+A_2z+A_3r^2+A_4rz+A_5z^2)$ for the optimum case.

Coefficient	Value
A_0	0.1155×10^{-07}
A_1	0.1773×10^{-15}
A_2	0.2253×10^{-12}
A_3	-0.1695×10^{-23}
A_4	0.3316×10^{-20}
A_5	0.1666×10^{-17}

In the parametric study one noticeable difference occurs. The advective case $(K_h,K_z)=(0,0)$ resulted in the second smallest residual for the cyclonic ring. However, for this study, the advective case had the sixth largest error function for the cases considered. Thus, in the anticyclonic ring, diffusion seems to have a more important role than in the cyclonic ring. Perhaps this can be related to the environment of the ring. The cyclonic ring translates in a relatively uniform environment, the Sargasso Sea, free of interference from boundaries except for times when the ring coalesces with the Gulf Stream or another ring. In the Gulf of Mexico, the eddies which break off from the Loop Current are subjected to geographic and oceanographic barriers which could influence their life cycle. Frictional dissipation occurs as they come in contact with the Florida Escarpment to the east, Campeche Bank to the southwest, and the Gulf Loop Current to the south. In addition, the surrounding environment is less uniform than that of cyclonic rings, as fresher water from the Mississippi River lies along the northern edge and more saline water from the Caribbean Sea lies to the south.

With these outside influences, the model results represent a greater abstraction of the dynamic processes in the anticyclonic ring than in the cyclonic ring for which it was developed. In the interest of a more complete comparison of the results for the two rings, rate of isotherm movement, transport and lifetime have been calculated. The rate at which the 17°C isotherm migrates upward is 0.8 m per day while the net transport into the ring is 1027×10^3 m^3 s^{-1}. This transport is about 30 times as large as the entrainment calculated for the cyclonic ring in the North Atlantic (Schmitz and Vastano, 1975).

Acknowledgments. This work was supported by the Office of Naval Research under Contract Nonr N00014-68-0308-0002. The authors thank Dr. Worth Nowlin and Mr. Brady Elliott for ideas and suggestions which encouraged the study. Mr. Elliott continued to be of assistance by sharing information obtained through his own research efforts.

REFERENCES

Cochrane, John D., 1969: Separation of an anticyclone and subsequent developments in the Loop Current (1969). *Contributions on the Physical Oceanography of the Gulf of Mexico, Texas A&M University Oceanographic Studies*, Vol. 2, 91–106.

Patzert, William C., 1969: Eddies in Hawaiian waters. Rept. HIG-69-8, Hawaiian Institute of Geophysics, 7–8.

Schmitz, Joyce E., and Andrew C. Vastano., 1975: Entrainment and diffusion in a Gulf Stream cyclonic ring. *J. Phys. Oceanogr.*, **5**, 93–97.