Edge Waves in the Presence of an Irregular Coastline

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ABSTRACT

We examine the generation of trapped edge waves on a continental shelf when a long wave from the deep ocean reaches an irregular coast, and alterations in the propagation characteristics of trapped edge waves due to the coastal irregularities. The continental shelf is modeled by a single flat-step model, and the coast is straight except for irregularities represented as a centered stationary random function of distance along the coast. The relevant boundary value problems are thus stochastic, with the randomness introduced through the boundary condition at the coast. We find the power flux into trapped edge waves and into a continuous spectrum of leaky modes, both generated by the scattering of an incident wave from the deep ocean. Numerical results, assuming a Gaussian spectrum for coastal irregularities, indicate that there is less power transferred to the forward traveling trapped wave than the backward one, and less power to the scattered leaky modes than to either the forward or backward traveling trapped modes. We obtain the attenuation coefficient of a trapped edge wave, the "tilting" of the wave toward the coast, and the correction to the dispersion relation due to the coastal irregularities. The results are valid for wave periods much shorter than the period associated with the Coriolis parameter f and for wavelengths much greater than the average size of the coastal irregularities.

1. Introduction

Recently a considerable amount of attention has been devoted to studying the effects of longshore variations on long barotropic coastal waves. The types of waves considered include Kelvin and Poincaré waves (Pinsent, 1972; Miles, 1972; Howe and Mysak, 1973; Mysak and Tang, 1974; Mysak and Howe, 1977), and continental shelf waves (Allen, 1976; Grimshaw, 1977). In this paper we study the behavior of the remaining class of coastal waves—edge waves—in the presence of longshore coastal variations. As in the work of Mysak and collaborators, we consider here a coastline which is straight except for small irregularities, treated as a stationary random function of position along the coast. The continental shelf/slope region is modeled by a flat shelf that drops off to a deep-sea region of uniform depth.

In this paper, two basic problems are considered, which are similar to the two considered by Howe and Mysak (1973) and Mysak and Tang (1974). First, we consider the problem of a wave incident upon the shelf originating in the deep ocean. We calculate the reflection coefficient and the fraction of the energy scattered into various "trapped" and "leaky" edge wave modes. Second, we look at the effects of coastal irregularities on the propagation of a coherent trapped edge wave. In addition to filling an obvious gap in the literature, this work was motivated by a need to fully understand the behavior of tsunamis reaching the continental shelf of Japan. Numerical examples presented in this paper use the parameters corresponding to the northeast coast of Japan.

In Section 3, an expression is obtained for the reflection coefficient which depends on the coastal irregularities. Section 4 deals with the transfer of energy from the incident wave to the scattered field; an expression is derived for the power flux from the incident wave into the trapped edge waves and into a continuous spectrum of long-wave radiation to the deep ocean. These quantities are plotted as a function of incidence angle for a Gaussian coastline. Section 5 deals with the alteration in the dispersion relation of trapped edge waves. Finally, in Section 6 expressions are obtained for the altered phase speed, for the e-fold decay length of the coherent wave field and for the wave "tilt" toward the coast in the deep-ocean region.

2. Waves on a continental shelf with a straight coast

The mathematical analysis of coastal waves in the presence of an irregular coast is best understood if compared to the analysis for waves along a straight coast. Therefore it is convenient to give a brief outline of the latter case first.

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Assuming a time dependence of the form $\exp(-i\omega' t')$ for the free surface elevation ϕ', the familiar shallow water equations for a constant depth h imply that ϕ' satisfies Helmholtz's equation

$$\nabla'^2\phi'+(\omega'^2/gh)\phi'=0. \tag{2.1}$$

The horizontal velocity components in the x' and y' directions (see Fig. 1) are given by

$$u'=-(ig/\omega')\partial\phi'/\partial x', \tag{2.2a}$$

$$v'=-(ig/\omega')\partial\phi'/\partial y'. \tag{2.2b}$$

As mentioned in the Introduction, the model of the ocean bottom used in this paper is a single-step topography, illustrated in Fig. 1. Thus,

$$h(x)=\begin{cases} h_1, & 0\leqslant x'<W \\ h_2, & W<x'<\infty \end{cases}$$

where h_1 and h_2 are constants.

We now nondimensionalize the variables. Lengths in the horizontal plane will be divided by the shelf width W, and time will be nondimensionalized with respect to $W/(gh_1)^{\frac{1}{2}}$, the time of travel of a long wave across the shelf. Lengths in the vertical direction—the vertical displacements of the waves—are nondimensionalized with respect to the shallow depth h_1. Dimensional variables are denoted by a prime throughout this paper, and nondimensionalized variables are unprimed.

In nondimensional form the equations for shallow water waves in the shelf and deep-sea regions are, from (2.1),

$$\nabla^2\phi_1+\omega^2\phi_1=0, \qquad 0\leqslant x<1,\ -\infty<y<\infty, \tag{2.3a}$$

$$\nabla^2\phi_2+(\omega^2/\gamma^2)\phi_2=0, \quad 1<x<+\infty,\ -\infty<y<+\infty, \tag{2.3b}$$

where $\gamma^2=h_2/h_1>1$.

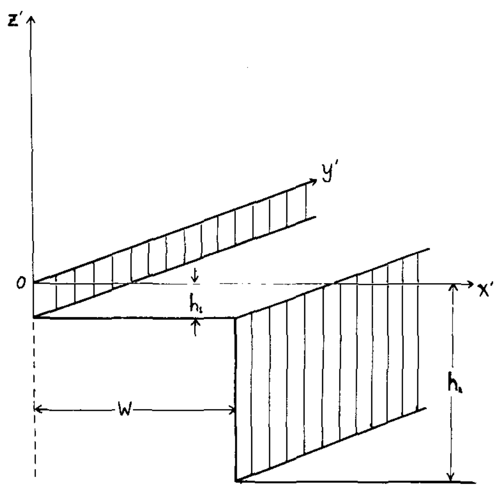

FIG. 1. Single-step model of ocean bottom.

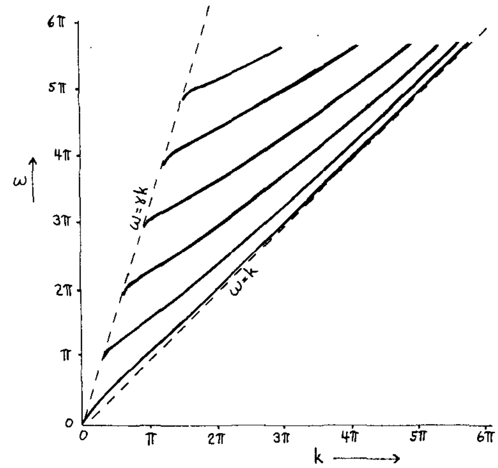

FIG. 2. Dispersion relation for trapped edge waves, $1<c<\gamma$, $\gamma^2=10$.

From the solutions of (2.3), together with the boundary conditions of zero horizontal velocity normal to the coast, continuity of displacement and mass transport at the shelf edge and finite displacement at $x=+\infty$, Buchwald and de Szoeke (1973) obtained the following results.

For wave solutions ϕ_i $(i=1,2)$ proportional to $\exp(iky)$, there are two cases of interest, corresponding to two ranges of the phase speed $c=\omega/k$: $c>\gamma$ and $\gamma>c>1$.

The case $c>\gamma$ is called the "leaky mode." Plane waves incident upon the shelf from the deep ocean are refracted at the shelf edge in accordance with Snell's law; perfect reflection takes place at the coast; and the wave is refracted again at the shelf edge and returns to the deep ocean. Because of perfect reflection, the energy flux onto the shelf region is equal to the energy flux back to the deep ocean from the shelf region.

For the case $\gamma>c>1$, the energy flux is only along the coast, and the waves are exponentially trapped against the shelf. These waves, known as "trapped mode" edge waves obey the following dispersion relation which allows only a finite number of modes for waves of a given frequency:

$$m\tan m=\gamma^2 l, \tag{2.5}$$

where

$$\left.\begin{array}{l} m=(\omega^2-k^2)^{\frac{1}{2}} \\ l=(k^2-\omega^2/^2)^{\frac{1}{2}}\gamma \end{array}\right\}.$$

The graph of the dispersion relation for $\gamma^2=10$ is presented in Fig. 2.

3. Scattering by an irregular coastline of an incident wave from the deep ocean—Determination of the reflection coefficient

It is assumed that the coast has bays, peninsulas and inlets which are deviations from an otherwise straight coast, parallel to the shelf edge, which is assumed to be straight. The deviations are assumed to be small compared to the shelf width. The irregular coastline is specified by

$$x=s(y), \tag{3.1}$$

where $s(y)$ is a stationary random function of y with zero mean. When y is fixed, a random function is a random variable, with a probability distribution, over an ensemble of coastlines. Averages of quantities are taken over an ensemble of statistically equivalent coastlines. This may appear to be confusing, since there is in reality only one coast under consideration. But if it is assumed that the random function is ergodic, then ensemble averages are equal to spatial averages over the length of the one real coast. Ensemble averages are used to simplify computations of averages over the length of the coast. To be stationary, a random function must have statistical properties independent of position y. For example, the mean over the ensemble, denoted by $\langle s(y)\rangle$, must be a constant, independent of y, and the autocovariance

$$R(y,Y)=\langle s(y)s(y+Y)\rangle \tag{3.2}$$

must be a function of only the lag Y. In the present case it is assumed that $s(y)$ is centered, that is, $\langle s(y)\rangle=0$. The small size of the coastal irregularities is expressed by requiring

$$\epsilon\equiv[\langle s^2(y)\rangle]^{\frac{1}{2}}\ll 1,$$

that is, the average size of the irregularities should be small compared to 1, the nondimensional shelf width.

The boundary condition of zero velocity normal to the coast becomes

$$u=v\partial s/\partial y \quad \text{on } x=s(y). \tag{3.3}$$

Since s is small, the boundary condition (3.3) may be expanded about the mean $s=0$, stopping at terms involving ϵ^2, and evaluating at $x=0$:

$$u=vs_y-u_xs-\tfrac{1}{2}u_{xx}s^2+v_xss_y \quad \text{on } x=0. \tag{3.4}$$

It should be noted that (3.4) is not valid if u_x, v_x and u_{xx} are large, offsetting the smallness of s. If λ is the wavelength of the incident wave, then u_x will be of order $1/\lambda$. Thus for very short wavelengths (3.4) is invalid. This is a shortcoming common to Pinsent (1972), Howe and Mysak (1973) and Mysak and Tang (1974). The approximation (3.4), which is a key one in this paper, is valid only for incident waves of wavelength much greater than the average size of the coastal irregularities.

In a particular realization of the wave field ϕ, corresponding to a given coastline, the scattered wave field $\hat{\phi}$ is a correction to the mean field $\langle\phi\rangle$, so that

$$\phi=\langle\phi\rangle+\hat{\phi}. \tag{3.5}$$

By taking the ensemble average of the wave equations (2.3a,b), it follows that

$$\nabla^2\langle\phi_1\rangle+\omega^2\langle\phi_1\rangle=0,\quad \nabla^2\hat{\phi}_1+\omega^2\hat{\phi}_1=0,\quad 0\leqslant x<1, \tag{3.6a,b}$$

$$\nabla^2\langle\phi_2\rangle+(\omega^2/\gamma^2)\langle\phi_2\rangle=0,\ \nabla^2\hat{\phi}_2+(\omega^2/\gamma^2)\hat{\phi}_2=0,\ x>1. \tag{3.7a,b}$$

Similarly, the two matching conditions separate:

Continuity of displacement

$$\langle\phi_1\rangle=\langle\phi_2\rangle,\ \hat{\phi}_1=\hat{\phi}_2 \quad \text{at } x=1. \tag{3.8a,b}$$

Continuity of mass transport

$$\partial\langle\phi_1\rangle/\partial x=\gamma^2\partial\langle\phi_2\rangle/\partial x,\ \partial\hat{\phi}_1/\partial x=\gamma^2\partial\hat{\phi}_2/\partial x \quad \text{at } x=1. \tag{3.9a,b}$$

The condition of finiteness at $x=+\infty$ applies separately to the mean and scattered fields.

The coupling between $\langle\phi\rangle$ and $\hat{\phi}$ is introduced into the problem by the boundary condition (3.4) at $x=0$. Using the relations (2.2a,b) in nondimensional form, the boundary condition (3.4) may be expressed in terms of s and ϕ_1 alone:

$$\phi_{1x}=\phi_{1y}s_y-\phi_{1xx}s-\tfrac{1}{2}\phi_{1xxx}s^2+\phi_{1yx}ss_y \quad \text{on } x=0. \tag{3.10}$$

Following Howe and Mysak (1973), this condition is represented formally by

$$L\phi_1=G_1\phi_1+G_2\phi_1, \tag{3.11}$$

where L, G_1, G_2 are linear operators. The operator L is nonrandom, so that $L\phi_1=0$ is the boundary condition for a straight coast. The operators G_1 and G_2 involve the random function s linearly and quadratically, respectively. So in the present case,

$$L=\partial/\partial x, \tag{3.12a}$$

$$G_1=s_y\partial/\partial y-s\partial^2/\partial x^2, \tag{3.12b}$$

$$G_2=ss_y\partial^2/\partial x\partial y-\tfrac{1}{2}s^2\partial^3/\partial x^3. \tag{3.12c}$$

Since $\langle G_1\rangle=0$, taking the ensemble average of (3.11) gives

$$L\langle\phi_1\rangle=\langle G_1\hat{\phi}_1\rangle+\langle G_2\rangle\langle\phi_1\rangle+\langle G_2\hat{\phi}_1\rangle. \tag{3.13}$$

Expanding (3.11) we have

$$L\langle\phi_1\rangle+L\hat{\phi}_1=G_1\langle\phi_1\rangle+G_1\hat{\phi}_1+\langle G_2\rangle\langle\phi_1\rangle+\hat{G}_2\langle\phi_1\rangle+G_2\hat{\phi}_1,$$

and subtracting (3.13) from it yields

$$L\hat{\phi}_1=G_1\langle\phi_1\rangle+\hat{G}_2\langle\phi_1\rangle+\{G_1\hat{\phi}_1-\langle G_1\hat{\phi}_1\rangle\}+\{G_2\hat{\phi}_1-\langle G_2\hat{\phi}_1\rangle\}. \tag{3.14}$$

Some approximations can be made, keeping in mind that the object is to use $\hat{\phi}$ to calculate $\langle\phi\rangle$ correct to $O(\epsilon^2)$, not to calculate $\hat{\phi}$ itself. In keeping with this, only terms up to $O(\epsilon)$ need be kept in the boundary condition (3.14) for $\hat{\phi}$, since higher order terms will influence $\langle\phi\rangle$, through (3.13), to $O(\epsilon^3)$ or higher—because G_1 is $O(\epsilon)$ and G_2 is $O(\epsilon^2)$. This means that

(3.13) and (3.14) may be approximated by

$$L\langle\phi_1\rangle=\langle G_1\hat{\phi}_1\rangle+\langle G_2\rangle\langle\phi_1\rangle, \quad (3.15a)$$

$$L\hat{\phi}_1=G_1\langle\phi_1\rangle. \quad (3.15b)$$

Eq. (3.15b) is known as the Born approximation. Inserting the expressions (3.12a,b,c) into (3.15a,b), and noting that $\langle G_2\rangle=-\frac{1}{2}\langle s^2\rangle\partial^3/\partial x^3$, we obtain

$$\langle\phi_{1x}\rangle=\langle s_y\hat{\phi}_{1y}-s\hat{\phi}_{1xx}\rangle-\tfrac{1}{2}\langle s^2\rangle\langle\phi_{1xxx}\rangle \quad \text{at } x=0, \quad (3.16a)$$

$$\hat{\phi}_{1x}=s_y\langle\phi_{1y}\rangle-s\langle\phi_{1xx}\rangle \quad \text{at } x=0. \quad (3.16b)$$

It is now assumed that a wave is incident on the shelf from the deep ocean; it is refracted at the shelf and partial reflection takes place at the coast, which introduces a reflection coefficient into the analysis. Finally, the partially reflected wave is refracted out to the deep ocean. So in mathematical terms we assume that

$$\langle\phi_1\rangle=AT\exp i(-m_1x+ky-\omega t)+AT\,R_1\exp i(m_1x+ky-\omega t), \quad (3.17a)$$

$$\langle\phi_2\rangle=A\exp i[-m_2(x-1)+ky-\omega t-\theta]+AR_2\exp i[m_2(x-1)+ky-\omega t+\theta], \quad (3.17b)$$

where A is the amplitude of the incident wave, T a transmission coefficient, R_1 the reflection coefficient as observed on the shelf, R_2 the reflection coefficient as observed in the deep ocean, k the longshore wavenumber, m_1 and m_2 the wavenumbers perpendicular to the coast on the shelf and in the deep ocean, respectively, and θ a parameter arising from the matching conditions at $x=1$, satisfying $\tan\theta=(m_1\tan m_1)/\gamma^2 m_2$.

The two matching conditions at $x=1$ are used to express R_2 and T in terms of R_1. The boundary condition (3.16b) is used to find $\hat{\phi}$ in terms of the unknown R_1; then (3.16a) is used with the expression for $\hat{\phi}$ to find R_1.

Substituting the expression for $\langle\phi_1\rangle$ into the boundary condition (3.16b), and dropping the amplitude A and the time dependence, we find

$$\hat{\phi}_{1x}=T[ik(1+R_1)s_y+m_1^2(1+R_1)s]\exp(iky) \quad \text{at } x=0. \quad (3.18)$$

It is assumed that $\hat{\phi}_1$ and $\hat{\phi}_2$ have a y-dependence of the form $\exp(i\eta y)$; then since $\hat{\phi}_1$ and $\hat{\phi}_2$ satisfy the differential equations (3.6b) and (3.7b), the following Fourier integral representations hold:

$$\hat{\phi}_1=\int_{-\infty}^{+\infty}\{A_{11}(\eta)\exp[(\eta^2-\omega^2)^{\frac{1}{2}}x]+A_{12}(\eta)\exp[-(\eta^2-\omega^2)^{\frac{1}{2}}x]\}\exp(i\eta y)d\eta, \quad 0\leqslant x<1, \quad (3.19a)$$

$$\hat{\phi}_2=\int_{-\infty}^{+\infty}\{A_{21}(\eta)\exp[(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}(x-1)]+A_{22}(\eta)\exp[-(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}(x-1)]\}\exp(i\eta y)d\eta, \quad 1<x<\infty, \quad (3.19b)$$

where the positive square roots are taken in the exponentials.

The branch points at $\eta=\pm\omega$ and at $\eta=\pm\omega/\gamma$ and the poles of the integrand must be avoided by indentation of the path of integration in the complex η-plane above or below these singularities in accordance with the radiation condition. The latter requires that waves generated in a particular region travel away and not toward that region. The appropriate path of integration is shown in Fig. 3. The boundedness condition on $\hat{\phi}_2$ implies that $A_{21}(\eta)=0$ for $|\eta|>\omega/\gamma$. In the region $|\eta|<\omega/\gamma$, $\exp[(\eta^2-\omega^2)^{\frac{1}{2}}]=\exp[-i(\omega^2-\eta^2)^{\frac{1}{2}}]$, representing component waves of the scattered field traveling toward the coast, where the scattering occurs. This is a violation of the radiation condition unless $A_{21}(\eta)\equiv 0$ for $|\eta|<\omega/\gamma$. So $A_{21}(\eta)=0$ for all η.

The two matching conditions (3.8b) and (3.9b) at $x=1$ may now be Fourier transformed (with respect to y) and used to solve for $A_{11}(\eta)$ and $A_{12}(\eta)$ in terms of $A_{22}(\eta)$. Doing this, we obtain

$$\hat{\phi}_{1x}|_{x=0}=\int_{-\infty}^{+\infty}\tfrac{1}{2}\{[(\eta^2-\omega^2)^{\frac{1}{2}}-\gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}]\times\exp[-(\eta^2-\omega^2)^{\frac{1}{2}}]-[(\eta^2-\omega^2)^{\frac{1}{2}}+\gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}]\times\exp[(\eta^2-\omega^2)^{\frac{1}{2}}]\}A_{22}(\eta)\exp(i\eta y)d\eta. \quad (3.20)$$

The Fourier transform of (3.18) with respect to y

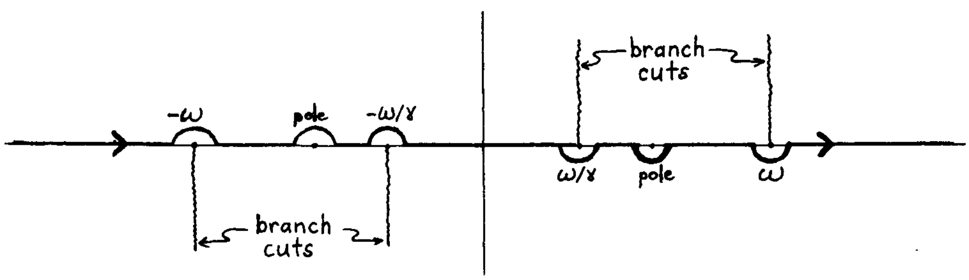

FIG. 3. Indentation of the path of integration in the complex η plane.

must be equal to the integrand of (3.20), which gives an expression for $A_{22}(\eta)$. Substituting these expressions for the A_{ij}'s into (3.19a,b), we obtain $\hat{\phi}_1$ and $\hat{\phi}_2$ in terms of R_1, correct to $O(\epsilon)$:

$$\hat{\phi}_1=(1/2\pi)T(1+R_1)\exp(iky)\int\int_{-\infty}^{+\infty} a(\eta)(\omega^2-k\eta)s \times\exp[-i(\eta-k)(Y-y)]d\eta dY, \quad 0\leqslant x<1, \quad (3.21a)$$

$$\hat{\phi}_2=(1/\pi)T(1+R_1)\exp(iky) \times\int\int_{-\infty}^{+\infty}\{\exp[-(\eta^2-\omega^2)^{\frac{1}{2}}(x-1)](\omega^2-k\eta)s \times\exp[-i(\eta-k)(Y-y)]/F(\eta)\}d\eta dY, \quad 1<x<\infty, \quad (3.21b)$$

where

$$a(\eta)=\{[(\eta^2-\omega^2)^{\frac{1}{2}}-\gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}] \times\exp[(\eta^2-\omega^2)^{\frac{1}{2}}(x-1)] +[(\eta^2-\omega^2)^{\frac{1}{2}}+\gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}] \times\exp[-(\eta^2-\omega^2)^{\frac{1}{2}}(x-1)]\}/F(\eta)(\eta^2-\omega^2)^{\frac{1}{2}},$$

$$F(\eta)=[(\eta^2-\omega^2)^{\frac{1}{2}}-\gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}]\exp[-(\eta^2-\omega^2)^{\frac{1}{2}}] -[(\eta^2-\omega^2)^{\frac{1}{2}}+\gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}]\exp[(\eta^2-\omega^2)^{\frac{1}{2}}].$$

If there is a pole at $\eta=\eta_0$, say, then the expressions for $\hat{\phi}$ evaluated at the pole include a factor

$$\exp(iky)\exp[-i(\eta_0-k)(-y)]=\exp(i\eta_0 y);$$

that is, the pole at η_0 corresponds to a traveling wave with longshore wavenumber η_0. The poles are located where $F(\eta)=0$, i.e.,

$$(\omega^2-\eta_0^2)^{\frac{1}{2}}\tan(\omega^2-\eta_0^2)^{\frac{1}{2}}=\gamma^2(\eta_0^2-\omega^2/\gamma^2)^{\frac{1}{2}}, \quad (3.22)$$

which is the trapped mode dispersion relation (2.5). Thus (3.21a,b) include trapped mode edge waves as part of the scattered radiation.

Now expression (3.21a) for $\hat{\phi}_1$, in terms of R_1, may be used in the boundary condition (3.16a) for $\langle\phi_1\rangle$.

All the terms of (3.16a) may now be evaluated. Performing straightforward algebra, using the definition of the power spectral density Φ of an ergodic stationary random function s, *viz.*,

$$\Phi(\zeta)=(1/2\pi)\int_{-\infty}^{+\infty}R(v)e^{-i\zeta v}dv,$$

where $R(v)=\langle s(z)s(z+v)\rangle$ is the autocovariance of s, using the fact that

$$\langle s(Y)s_y(y)\rangle=\frac{\partial}{\partial y}\langle s(Y)s(y)\rangle=\frac{\partial}{\partial y}R(Y-y)=-R'(Y-y),$$

and keeping terms only to $O(\epsilon^2)$, we obtain

$$R_1=1-(2i/m_1)\int_{-\infty}^{\infty}S(\eta;\omega,\gamma)(\omega^2-k\eta)^2 \times\Phi(\eta-k)d\eta, \quad (3.23)$$

where

$$S(\eta;\omega,\gamma)=\{[(\eta^2-\omega^2)^{\frac{1}{2}}-\gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}] \times\exp[-(\eta^2-\omega^2)^{\frac{1}{2}}] +[(\eta^2-\omega^2)^{\frac{1}{2}}+\gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}] \times\exp[(\eta^2-\omega^2)^{\frac{1}{2}}]\}/F(\eta)(\eta^2-\omega^2)^{\frac{1}{2}}. \quad (3.24)$$

It was noted, after the boundary condition expansion (3.4), that the computations in this section are invalid for waves of a very short wavelength. This difficulty appears in (3.23), for although $\Phi\propto\epsilon^2$, we have $(\omega^2-\eta k)^2\propto\omega^4$, $S(\eta;\omega,\gamma)\propto 1/\omega$ and $1/m_1\propto 1/\omega$, making the correction term proportional to $\epsilon^2\omega^2$. If ω is too large, then the condition $\epsilon^2\omega^2\ll 1$, required for the boundary condition (3.4), will be violated. In addition, the magnitude of the reflection coefficient, $|R_1|$, will become greater than 1, which is physically impossible.

4. Scattering by an irregular coastline of an incident wave from the deep ocean—Power fluxes into different modes of the scattered wave

The expression $-\langle p'\rangle\langle u'\rangle$ (p' and u' are the dimensional pressure and x-velocity) is the power flux of the mean field $\langle\phi'\rangle$ in the negative x direction. When evaluated at the coast, $x=0$, the expression is the work per unit time per unit cross-sectional area done by the mean wave field on the coast. Since the coast does not move, all of this energy goes into the scattered waves. Thus, the power flux of the scattered waves is $-\langle p'\rangle\langle u'\rangle$. A more useful quantity is the power flux averaged over one cycle of the mean field, represented by $-\overline{\langle p'\rangle\langle u'\rangle}$. First p' and u' are evaluated in terms of ϕ', i.e.,

$$p'=\rho g\phi', \quad u'=-(ig/\omega')\phi'_{x'}.$$

However, since these are real quantities, it is necessary to take only the real parts

$$p'=\tfrac{1}{2}(\rho g\phi')+\text{cc}, \quad u'=-(ig/\omega')\phi'_{x'}+\text{cc},$$

where cc stands for the complex conjugate. Now using (3.17a) for the mean field $\langle\phi_1\rangle$, it follows that

$$\langle p'\rangle=\tfrac{1}{2}\rho g h_1 AT(1+R_1)\exp(ik'y')+\text{cc} \quad \text{at} \quad x=0, \quad (4.1a)$$

$$\langle u'\rangle=\tfrac{1}{2}\rho g h_1(m_1'/\omega')AT(1-R)\exp(ik'y')+\text{cc} \quad \text{at} \quad x=0. \quad (4.1b)$$

Multiplying and averaging over one cycle of the mean field yields

$$-\overline{\langle p'\rangle\langle u'\rangle}$$
$$=\tfrac{1}{2}\rho g^2 h_1^2 A^2(m_1'/\omega')|T|^2(1-|R_1|^2) \quad \text{at} \quad x=0. \quad (4.2)$$

It is now a simple matter to show that the power flux of the incident wave at the coast $x=0$, averaged over one cycle, is $\frac{1}{2}\rho g^2 h_1^2 A^2(m_1'/\omega')|T|^2$—simply set $R_1=0$ in (4.2) to focus attention on only the incident wave. Therefore, the fraction of the incident power flux which goes into the scattered field is $(1-|R_1|^2)$. Now, putting

$$I=\int_{-\infty}^{+\infty} S(\eta;\omega,\gamma)(\omega^2-k\eta)^2\Phi(\eta-k)d\eta,$$

it is simple to show that

$$1-|R_1|^2=-(4/m_1)\,\mathrm{Im}(I)+O(\epsilon^4), \quad (4.3)$$

where $\mathrm{Im}(I)$ is the imaginary part of I. In order to continue, it is necessary to determine $\mathrm{Im}(I)$.

First of all, it is clear that $S(\eta;\omega,\gamma)$ is real in the regions where $\eta^2>\omega^2$, since all of the square roots have positive arguments. In the regions where $\omega^2/\gamma^2<\eta^2<\omega^2$, and for $\eta\neq\eta_j$ where η_j is a pole of S, S reduces to

$$S(\eta;\omega,\gamma)=\left[\frac{(\omega^2-\eta^2)^{\frac{1}{2}}+\gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}\tan(\omega^2-\eta^2)^{\frac{1}{2}}}{(\omega^2-\eta^2)^{\frac{1}{2}}\tan(\omega^2-\eta^2)^{\frac{1}{2}}-\gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}}\right]\times\frac{1}{(\omega^2-\eta^2)^{\frac{1}{2}}} \quad (4.4)$$

which is obviously a real quantity, in the ranges where $\omega^2/\gamma^2<\eta^2<\omega^2$. However, there is a contribution to the imaginary part of I in this range from the poles, where $(\omega^2-\eta^2)^{\frac{1}{2}}\tan(\omega^2-\eta^2)^{\frac{1}{2}}=\gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}$:

$$iP(\eta_j)=-i\pi\frac{1}{\eta_j}\left[1+\frac{1}{(\eta_j^2-\omega^2/\gamma^2)^{\frac{1}{2}}}Q(\eta_j)\right]^{-1}\times(\omega^2-k\eta_j)^2\Phi(\eta_j-k), \quad (4.5)$$

where

$$Q(\eta_j)=\frac{\gamma^2(\omega^2-\eta_j^2)^{\frac{1}{2}}+(\eta_j^2-\omega^2/\gamma^2)^{\frac{1}{2}}\tan(\omega^2-\eta_j^2)^{\frac{1}{2}}}{(\omega^2-\eta_j^2)^{\frac{1}{2}}+\gamma^2(\eta_j^2-\omega^2/\gamma^2)^{\frac{1}{2}}\tan(\omega^2-\eta_j^2)^{\frac{1}{2}}}.$$

For poles in the range $-\omega<\eta_j<-\omega/\gamma$, the path indentations are above the real line, and the expression for $P(\eta_j)$ is identical, except for a positive sign replacing the negative sign in front. For a given ω, there is a finite number of poles, as can be seen in Fig. 2 for the dispersion relation. So the total contribution to the scattered fraction of the power flux from the poles is $-(4/m_1)\sum_j P(\eta_j)$, which is simply the sum of the power fluxes of all the trapped edge waves that can exist for a given frequency.

It is worthwhile to check that the sign of $-(4/m_1)P(\eta_j)$ is positive. Referring to (4.5), the complicated factor in square brackets is obviously positive, provided $\tan(\omega^2-\eta_j^2)^{\frac{1}{2}}>0$—but $\tan(\omega^2-\eta_j^2)^{\frac{1}{2}}=\gamma^2(\eta_j^2-\omega^2/\gamma^2)^{\frac{1}{2}}/(\omega^2-\eta_j^2)^{\frac{1}{2}}$ by the dispersion relation and this is positive. The power spectral density is a positive quantity, and the factor on the left of the square brackets is $-\pi/\eta_j$, when $\eta_j>0$ and $+\pi/\eta_j$, when $\eta_j<0$. Thus this factor is negative. The remaining factor $(\omega^2-k\eta_j)^2$ is positive, obviously. So, altogether, $P(\eta_j)$ is negative, which makes

$$-(4/m_1)\sum_j P(\eta_j)>0.$$

Another contribution to $\mathrm{Im}(I)$ comes from the regions where $\eta^2<\omega^2/\gamma^2$. In this region, it is a simple matter to show that

$$\mathrm{Im}[S(\eta;\omega,\gamma)]=-\frac{\gamma^2(\omega^2/\gamma^2-\eta^2)^{\frac{1}{2}}}{(\omega^2-\eta^2)\sin^2(\omega^2-\eta^2)^{\frac{1}{2}}+\gamma^4(\omega^2/\gamma^2-\eta^2)\cos^2(\omega^2-\eta^2)^{\frac{1}{2}}} \quad (4.6)$$

which is a negative quantity. The contribution to the power flux is

$$D=(4/m_1)\int_{-\omega/\gamma}^{\omega/\gamma}\mathrm{Im}[S(\eta;\omega,\gamma)]\times(\omega^2-k\eta)^2\Phi(\eta-k)d\eta. \quad (4.7)$$

The quantity D is a sum over a continuum of longshore wavenumbers $|\eta|<\omega/\gamma$ of waves which escape from the shelf to the deep ocean.

Summarizing, the portion of the incident power which is scattered by the coastline is made up of two parts: waves trapped on the shelf and waves escaping to the deep ocean. Represented as a fraction of the incident power flux, this is

$$1-|R_1|^2=-(4/m_1)\sum_j P(\eta_j)+D. \quad (4.8)$$

We have calculated numerical examples, using the Gaussian spectrum

$$\Phi(\zeta)=2(\pi)^{\frac{1}{2}}L\epsilon^2\exp(-\zeta^2L^2/4), \quad (4.9)$$

where L is the nondimensional correlation length. Fig. 4 shows the scattered power fluxes as a function of the angle of incidence of the incoming wave at the shelf edge for a typical case representing the situation for a shelf of width 21 km and depth 200 m, depth beyond the shelf of 2000 m, and a coastline whose power spectrum is this Gaussian with correlation length 3 km and standard deviation of the coastal irregularities from the zero mean of 8 km. This roughly represents the situation for the northeast coast of Japan between Miyako and Sendai Bay.

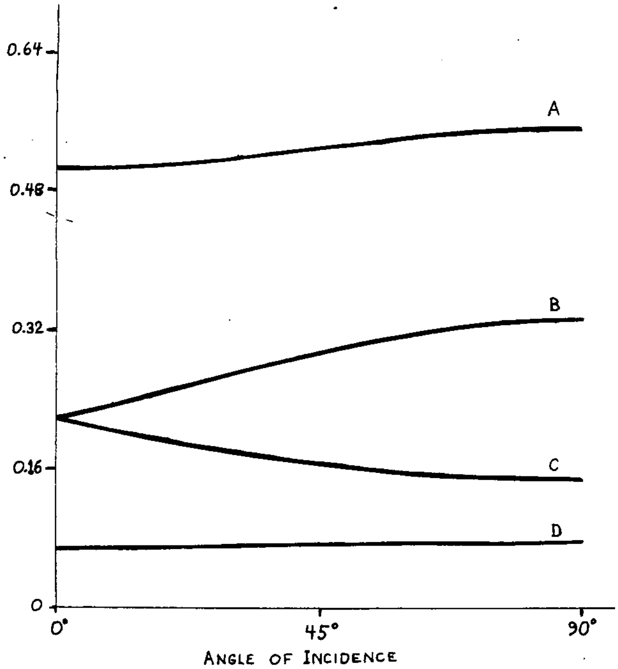

FIG. 4. Scattered power fluxes versus angle of incidence at the shelf, using example with $W=21$ km, $h_1=200$ m, $h_2=2000$ m, Gaussian coastal spectrum (4.9) having $L=0.143$, $\epsilon=0.38$, and period of wave = 1 h (corresponding to a nondimensional $\omega=0.826$). Curve A is the total scattered power flux $1-|R_1|^2$; curve B is the backward-scattered flux $-(4/m_1)P(-k)$; curve C is the forward-scattered flux $-(4/m_1)P(+k)$; and curve D is the leaky scattered flux D, all as in (4.8).

Our calculations suggest certain basic results: more energy is transferred into the backward-scattered trapped mode than into the forward-scattered mode; a coast of greater correlation length brings about a greater transfer of energy into the scattered modes than does a coast of shorter correlation length; and an incident wave of greater frequency transfers more of its energy to the scattered modes than an incident wave of lower frequency. In fact, the expression for the energy transfer is unbounded for large incident frequencies. This is due to a violation of the restriction mentioned earlier, that the incident wavelength must be much greater than the average size of the coastal irregularities.

5. Influence of coastal irregularities on edge waves trapped on the shelf—Altered dispersion relation

In this section it is assumed that a trapped mode wave has been generated on the shelf. In the absence of coastal irregularities, the dispersion relation for such waves is (2.5), which will be referred to as the zeroth-order dispersion relation.

It is shown that coastal irregularities will cause some scattering and hence attenuation of the trapped wave. Other alterations to physical quantities, such as an altered phase speed, will also be discussed. The first step is to determine the alteration of the dispersion relation due to the coastal irregularities. A different method from that used in the reflection coefficient problem is employed here, namely, the method of Mysak and Tang (1974).

The mean field $\langle\phi\rangle$ again satisfies the differential equations (2.3a) and (2.3b) and the matching conditions (3.8a) and (3.9a) at $x=1$. In addition, the solution $\langle\phi\rangle$ must be finite for all x and y. The boundary condition at the coast is the same condition as before—zero normal velocity—but now expressed as

$$(B+C)\phi_1=0 \quad \text{on } x=0, \tag{5.1}$$

where B is a deterministic linear operator and C a random linear operator. Note that in the absence of coastal irregularities, $C=0$, and so $B\langle\phi_1\rangle=0$. This is the case of zero x-velocity at the coast, so $B=\partial/\partial x$ and

$$\langle\phi_1\rangle=\cos(mx+\delta)\exp(iky), \qquad 0\leqslant x<1, \tag{5.2a}$$

$$\langle\phi_2\rangle=A\exp[-l(x-1)+iky], \qquad x>1. \tag{5.2b}$$

With $\delta=0$ and m, k, l real, Eq. (5.2) represents the solution for a straight coast, $C=0$. For the irregular coast, δ is assumed small and nonzero; also the wavenumbers may be complex. We assume that $\mathrm{Re}(l)>0$, so that the wave is trapped.

The matching conditions and the differential equations applied to (5.2a,b) give

$$m\tan(m+\delta)=\gamma^2 l, \tag{5.3a}$$

$$m^2+k^2=\omega^2, \tag{5.3b}$$

$$-l^2+k^2=\omega^2/\gamma^2. \tag{5.3c}$$

If δ were zero, Eqs. (5.3) would be the dispersion relation (2.5) for unattenuated trapped modes with a straight coast would determine a dispersion relation $\omega=\omega(k)$ if δ were known as a function of ω and k. The boundary condition (5.1) at $x=0$ will be used to determine δ.

Upon expressing u and v in terms of ϕ and expanding the boundary condition (3.3) at $x=0$ to $O(\epsilon^2)$, the following expressions for the linear operators B and C are obtained:

$$B=\partial/\partial x \quad \text{at } x=0, \tag{5.4a}$$

$$C=(s\partial_x{}^2-s_y\partial_y)+(\tfrac{1}{2}s^2\partial_x{}^3-s_ys\partial_y\partial_x)+O(\epsilon^3) \quad \text{at } x=0. \tag{5.4b}$$

Note that once again this is only valid for long wavelengths, i.e., $\epsilon\omega\ll 1$.

Using the operator expansion and averaging technique of Mysak and Tang (1974, Section 2), it follows from (5.1) that, to $O(\epsilon^2)$

$$(B+\langle C\rangle+\langle C\rangle B^{-1}\langle C\rangle-\langle CB^{-1}C\rangle)\langle\phi_1\rangle=0 \text{ on } x=0. \tag{5.5}$$

Since $2\langle s_y s\rangle = \partial R(0)/\partial y = 0$ for a stationary random function, and since $\langle s\rangle = 0$ and $\langle s_y\rangle = 0$, it follows from (5.4b) that

$$\langle C\rangle = \tfrac{1}{2}\langle s^2\rangle \partial_x{}^3. \tag{5.6}$$

Now, B^{-1} is an integral operator which maps a function, say $f(y)$, defined on the boundary $x=0$ onto a function defined on the whole region $x \geqslant 0$. In terms of a Green's function G,

$$B^{-1}f(y) = \int_{-\infty}^{+\infty} G(x,y;z)f(z)dz, \tag{5.7}$$

where

$$G = \begin{cases} G_1, & 0 \leqslant x < 1 \\ G_2, & 1 < x < \infty \end{cases}$$

is the solution of the following boundary value problem:

$$\partial^2 G_1/\partial x^2 + \partial^2 G_1/\partial y^2 + \omega^2 G_1 = 0, \quad 0 \leqslant x < 1, \tag{5.8a}$$

$$\partial^2 G_2/\partial x^2 + \partial^2 G_2/\partial y^2 + (\omega^2/\gamma^2) G_2 = 0, \quad 1 < x < \infty, \tag{5.8b}$$

$$BG_1 = \partial G_1/\partial x = \delta(y-z), \quad x=0, \tag{5.8c}$$

$$G_1 = G_2, \quad x=1, \tag{5.8d}$$

$$\partial G_1/\partial x = \gamma^2 \partial G_2/\partial x, \quad x=1. \tag{5.8e}$$

Also, there is the boundedness condition at $x=+\infty$, and the radiation condition applies. The δ function represents a source of waves at $y=z$ and $x=0$.

We now introduce the Fourier transform $\tilde{G}$ of G with respect to y:

$$\tilde{G}(x,\eta;z) = \int_{-\infty}^{+\infty} G(x,y;z)e^{-i\eta y}dy. \tag{5.9}$$

Application of (5.9) to (5.8) gives rise to a simple boundary value problem for $\tilde{G}$ involving only derivatives with respect to x. Solving this problem for $\tilde{G}$ is relatively straightforward and the details will be omitted. The solution for G, upon using the Fourier inversion theorem, is given by

$$G_1(x, y-z) = (1/2\pi)\int_{-\infty}^{+\infty} \{d_{11}(\eta)\exp[(\eta^2-\omega^2)^{\frac{1}{2}}x] + d_{12}(\eta)\exp[-(\eta^2-\omega^2)^{\frac{1}{2}}x]\} \times \exp[i\eta(y-z)]d\eta, \quad 0 \leqslant x < 1, \tag{5.10a}$$

$$G_2(x, y-z) = (1/2\pi)\int_{-\infty}^{+\infty} d_{22}(\eta) \times \exp[-(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}(x-1) + i\eta(y-z)]d\eta, \quad 1 < x < \infty, \tag{5.10b}$$

where

$$d_{11}(\eta) = \exp[-(\eta^2-\omega^2)^{\frac{1}{2}}] \times [(\eta^2-\omega^2)^{\frac{1}{2}} - \gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}]/F(\eta)(\eta^2-\omega^2)^{\frac{1}{2}},$$

$$d_{12}(\eta) = \exp(\eta^2-\omega^2)^{\frac{1}{2}} \times [(\eta^2-\omega^2)^{\frac{1}{2}} + \gamma^2(\eta^2-\omega^2/\gamma^2)^{\frac{1}{2}}]/F(\eta)(\eta^2-\omega^2)^{\frac{1}{2}},$$

$$d_{22}(\eta) = 2/F(\eta),$$

and where, for the same reasons given in the reflection coefficient problem, indentations of the path of integration around poles and the branch points $\eta = \pm\omega/\gamma$ are made below the real axis for $\eta > 0$ and above the real axis for $\eta < 0$.

From (5.6) we see that $\langle C\rangle$ is $O(\epsilon^2)$. Therefore, $\langle C\rangle B^{-1}\langle C\rangle$ is $O(\epsilon^4)$, and so it may be dropped from the calculations. The first two terms of (5.5) are

$$(B+\langle C\rangle)\langle \phi_1\rangle = (-m\sin\delta + \tfrac{1}{2}m^3\langle s^2\rangle\sin\delta)\exp(iky). \tag{5.11}$$

It should be remembered that in evaluating $CB^{-1}Cg(x,y)$, C takes $g(x,y)$ and converts it to a function of y only—$(Cg)(y)$; then B^{-1} applied to $(Cg)(y)$ produces a function of both x and y—$(B^{-1}Cg)(x,y)$; and C applied to this produces a function of y only—$(CB^{-1}Cg)(y)$. Carefully following this step-by-step procedure, retaining terms to $O(\epsilon^2)$, using

$$\left.\begin{aligned} \langle s(y)s(z)\rangle &= R(y-z) \\ \langle s(y)s_z(z)\rangle &= \partial R(y-z)/\partial z = -R'(y-z) \\ \langle s_y(y)s(z)\rangle &= \partial R(y-z)/\partial y = +R'(y-z) \\ \langle s_y(y)s_z(z)\rangle &= \partial[R'(y-z)]/\partial z = -R''(y-z) \end{aligned}\right\}$$

and using the property of the power spectral density

$$F\{R^{(n)}(Y)\} = (-i\zeta)^n\Phi(\zeta)$$

(F denotes the Fourier transform), it can be shown that (5.5) becomes, to $O(\epsilon^2)$,

$$m\tan\delta(1-\tfrac{1}{2}m^2\langle s^2\rangle) = -\int_{-\infty}^{+\infty} S(\eta;\omega,\gamma)(\omega^2-k\eta)^2\Phi(\eta-k)d\eta. \tag{5.12}$$

Since δ is small, the approximation $\tan\delta \approx \delta$ may be made. Keeping terms only to $O(\epsilon^2)$ in (5.12) and solving for δ yields, finally,

$$\delta = -(1/m)\int_{-\infty}^{+\infty} S(\eta;\omega,\gamma)(\omega^2-k\eta)^2\Phi(\eta-k)d\eta, \tag{5.13}$$

where S is defined in (3.24).

Note that the integral of (5.13) and the integral of (3.23) are identical except that the parameter k is in the range $\omega/\gamma < k < \omega$ in (5.13), but in the range $0 < k < \omega/\gamma$ in (3.23).

6. Consequences of the altered dispersion relation

We now insert the expression for δ into the dispersion relation (5.3a). Since δ is $O(\epsilon^2)$,

$$m\tan(m+\delta)=m\tan m+\delta m\sec^2 m\delta+O(\epsilon^4). \quad (6.1)$$

It is assumed that m, k and l may be written as

$$m=m_0+m_1, \quad (6.2a)$$

$$k=k_0+k_1, \quad (6.2b)$$

$$l=l_0+l_1, \quad (6.2c)$$

where m, k, l are $O(\epsilon^2)$. Inserting (6.2a,b,c) into the expressions (5.3a,b,c) and equating terms of $O(1)$ gives (2.5), the zeroth-order dispersion relation $\omega=\omega(k_0)$ for trapped edge waves. Equating terms of $O(\epsilon^2)$ gives

$$m_1=-(k_0/m_0)k_1, \quad (6.3a)$$

$$l_1=+(k_0/l_0)k_1, \quad (6.3b)$$

$$m_1\tan m_0+m_1m_0\sec^2 m_0+\delta m_0\sec^2 m_0=\gamma^2 l_1. \quad (6.3c)$$

Using (6.3a) and (6.3b) and the expression (5.13) for δ, (6.3c) becomes

$$k_1=\frac{-I\sec^2 m_0}{k_0(\tan m_0/m_0+\sec^2 m_0+\gamma^2/l_0)}, \quad (6.4)$$

where

$$I=\int_{-\infty}^{+\infty} S(\eta;\omega,\gamma)(\omega^2-\eta k)^2\Phi(\eta-k)d\eta. \quad (6.5)$$

From the earlier work, it was shown that

$$\mathrm{Im}(I)=-(m_1/4)D+\sum_j P(\eta_j).$$

Hence as in the reflection coefficient problem $\mathrm{Im}(I)<0$.

How is the mean field $\langle\phi\rangle$ affected by these alterations to m, k, l due to the coastal irregularities? To investigate this let $k_1=\alpha+i\beta$. From (6.4) we then have

$$\begin{Bmatrix}\alpha\\ \beta\end{Bmatrix}=-K\begin{Bmatrix}\mathrm{Re}(I)\\ \mathrm{Im}(I)\end{Bmatrix}, \quad (6.6)$$

where

$$K=\frac{\sec^2 m_0}{k_0(\tan m_0/m_0+\sec^2 m_0+\gamma^2/l_0)}.$$

Note that $\beta>0$, for $k_0>0$. Hence the $\exp(iky)$ factor in $\langle\phi\rangle$ becomes

$$\exp[i(k_0+\alpha)y-\beta y]. \quad (6.7)$$

Since $k_0>0$ implies the wave is traveling in the $+y$ direction, it is apparent that the $\exp(-\beta y)$ factor represents a decay in the direction of propagation. The e-fold decay length d—the distance over which the wave amplitude decays to $1/e$ of its value at $y=0$—is, from (6.6),

$$d=1/\beta=\frac{k_0(\tan m_0/m_0+\sec^2 m_0+\gamma^2/l_0)}{-\mathrm{Im}(I)\sec^2 m_0}. \quad (6.8)$$

From (6.3b), it can be seen that

$$l_1=(k_0/l_0)(\alpha+i\beta).$$

Therefore, in the deep ocean region, the coherent wave takes the form

$$\langle\phi_2\rangle=\exp\{-(l_0+\alpha k_0/l_0)(x-1)-\beta y+i[(k_0+\alpha)y-(k_0/l_0)\beta(x-1)-\omega t]\}. \quad (6.9)$$

So in the deep region, instead of just decay away from the shelf, there is a small wavenumber component in the x direction, with the net result being a wave component toward the coast, i.e.,

$$\exp[-i(k_0/l_0)\beta(x-1)-i\omega t].$$

This represents a "tilting" of the wave toward the coast.

Corresponding to this "tilting" effect, the phase and group velocities are affected by the coastal irregularities. For example, from (6.9) we see that the phase velocity takes the form

$$\mathbf{C}_p=\frac{\omega}{|\mathbf{k}|^2}(k_x,k_y)=\frac{\omega}{(k_0^2/l_0^2)\beta^2+(\alpha+k_0)^2}\times[-(k_0/l_0)\beta,\ \alpha+k_0]. \quad (6.10)$$

It is not possible to say in general whether the phase speed is diminished or augmented by the presence of coastal irregularities, because of the uncertainty in the sign of α, which has the complicated factor $\mathrm{Re}(I)$. Calculations using parameter values corresponding to the northeast coast of Japan give only tiny corrections to the first mode of the dispersion relation. The range of validity is so narrow that corrections to the second mode cannot even be considered.

Summarizing, we have seen that when the coherent, incident wave is an $O(1)$ trapped edge wave, there are a number of effects. There is attenuation in the direction of motion, as wave energy is scattered into incoherent trapped and leaky modes of power flux $O(\epsilon^2)$. The coherent wave in the deep ocean also "tilts" toward the shelf.

7. Summary and conclusions

We have calculated various quantities of interest concerning long gravity waves on a continental shelf in the presence of a coastline which is straight except for small irregularities. An expression for the reflection coefficient of the coast is found in Section 3. The fluxes of power from a wave incident from the deep ocean into edge waves trapped on the shelf and into a continuous spectrum of long-wave radiation back to the deep ocean are calculated in Section 4. However, it was noted that the calculations are valid only for incident waves of wavelength considerably greater than the average size of the coastal irregularities, i.e., for $\epsilon\omega \ll 1$. This limitation applies to the papers by Pinsent (1972), Howe and Mysak (1973) and Mysak and Tang (1974) as well.

Edge waves trapped on the shelf have their propagation characteristics altered by the presence of the coastal irregularities. By calculating the altered dispersion relation, the following things are determined: the altered phase speed, the e-fold decay length and the "tilting" of the deep ocean waves toward the coast.

Trapped edge waves can be generated by a tsunami striking an irregular coast. The results shown in Fig. 4 indicate that the trapped waves can be quite significant. The scattered fraction of the incident power flux is about 50%, so the amplitudes of the forward- and backward-scattered trapped waves are about 70% of the amplitude of the incident wave, since power flux is proportional to the square of the amplitude. Our value of 1 h for the period was chosen to ensure that ω was within the range of validity prescribed by $\epsilon\omega \ll 1$. A tsunami typically has a period of 10–20 min, which puts ω outside the range of validity when we nondimensionalize with respect to the parameters for the northeast coast of Japan. So Fig. 4 should be taken only as an indication that the scattering can be quite significant, and in particular that the backward-scattered trapped mode may be quite large.

Once a trapped wave has been generated, either by the scattering of a tsunami originating in the deep ocean or by a disturbance on the continental shelf, it decays in amplitude due to scattering. The e-fold decay length (6.8) tells us how rapidly the trapped wave decays. For a first mode trapped wave of period 1 h, as considered above, the amplitude decays to one-third of its original size within 435 km on the northeast Japanese continental shelf. The wave thus travels quite far before it is significantly attenuated by scattering.

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