On the Diffusion Parameterization of Mesoscale Eddy Effects from a Numerical Ocean Experiment

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ABSTRACT

A method for evaluating the utility of a given parameterization field in a spatially inhomogeneous circulation is described. This method is used to examine the mean field diffusion parameterization of the heat, momentum and vorticity time deviation eddy terms from a mesoscale resolution numerical ocean circulation experiment. The diffusion model does not satisfactorily describe the eddy terms for any field throughout any of the different regions of the model flow.

1. Introduction

The energetic mesoscale motions that have been found in many parts of the ocean are currently the subject of considerable research. One aspect of the mesoscale eddy problem that has received limited attention concerns that extent to which the characteristics of the long-time average eddy terms can be related to characteristics of the mean fields themselves. Studies into such mean field parameterizations of mesoscale eddy effects are of interest because they offer insight into the role(s) of eddies in the climatic-scale circulation.

Unfortunately, there exist insufficient data about the oceanic mean fields and their statistics for this type of work to be carried out at this time. However, there are numerical model oceanic flows with mesoscale motions (EGCM's) for which parameterization studies can be carried out. These mesoscale resolution numerical general ocean circulation experiments (e.g., Holland and Lin 1975; Robinson *et al.*, 1977) can use the computer to generate the data required to investigate relationships between the model mean fields and eddy terms.

From such model data sets it is in principle possible to empirically determine parameterizations of a particular flow as well as to investigate the applicability of proposed (e.g., Stone 1972; Green 1970) or traditional parameterizations. To date no systematic study of either of these types has been reported. This note describes an investigation into the applicability of the traditional diffusion parameterization to description of the mesoscale eddy terms of the Robinson *et al.* (1977, hereafter RHMS) EGCM experiment.

The traditional diffusion parameterization, in which eddy terms are modeled by a constant times the Laplacian of the appropriate mean field, assumes that fluxes are always along the mean gradient and proportional to the gradient. This model of eddy effects has its origin in ideas relevant to molecular-scale motions but has been used to describe the net effects of motions on many scales in analytical (e.g., Munk 1950; Pedlosky, 1968) and numerical (e.g., Bryan and Cox, 1968; Holland, 1971; Bryan *et al.*, 1975) models of ocean circulation, despite the absence of compelling arguments for its applicability to such purposes. EGCM data sets provide an opportunity to investigate its ability to describe mesoscale eddy effects.

Perhaps one reason that similar studies have not heretofore been carried out is that EGCM flows exhibit strong spatial inhomogeneity in both mean fields and eddy terms. The kinematics and dynamical balances of regions separated by as little as a few hundred kilometers can be very different (see, e.g., RHMS Sections 3 and 4).[1] Such inhomogeneity can easily obscure relationships between mean fields and eddy terms unless the examination method respects the considerable spatial structure of the fields under study. A simple method for determining the utility of a given parameterization in an inhomogeneous flow is described in the next section, and is then applied in Section 3 to the eddy diffusion model for RHMS eddy terms.

2. Method

Consider the general problem of determining whether a given field $G(x,y,z)$ satisfactorily describes

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[1] There exist data to suggest that this type of behavior may also be found in the ocean (e.g., Schmitz, 1977; Luyten, 1977).

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another field $F(x,y,z)$ over a volume R. A satisfactory description is here understood to mean that arbitrary integrals, within R, of G are consistent in sign and relative magnitude with the same integrals of F.[2] The method here proposed is to partition R into a set of subregions $\{r\}_R$ and then examine the subregional correlation coefficients

$$C_r \equiv \frac{\int_r F(x,y,z)G(x,y,z)}{\left[\int_r F^2(x,y,z)\int_r G^2(x,y,z)\right]^{1/2}}.$$

If the two fields have similar spatial structure the C_r will all be of the same sign and of large magnitude (say, >0.5) and G can provide a basis for a quantitative description of F in R. Should the C_r be consistent in sign but of small magnitude, or be inconsistent in sign, then G and F are sufficiently different in structure that G is not likely to be useful as a descriptor of F. It should be noted, however, that consistency of sign may indicate a physically interesting relationship between F and G even when the C_r magnitudes are small, but other means of comparing the fields would be needed to document any such relationship. In order for G to reproduce integrals, large magnitudes and consistent sign should be found for all C_r.

When G is sufficiently similar to F to provide a useful description, it remains only to introduce a normalization factor b, so that bG quantitatively as well as qualitatively describes F. In the simplest situation b will be a constant that can be determined by evaluating $\int_r F / \int_r G$ for any subregion of R, but in general it will be a smooth, slowly varying field that could be constructed in a number of ways.

The above is useful for the parameterization of spatially inhomogeneous fields so long as the volume over which a parameterization is sought can be partitioned into a set of regions $\{R\}$ over which the flow is quasi-homogeneous. Given this set of regions, the utility of G as parameterization of the eddy term F in each region R is examined by the method discussed above. Many criteria can be imagined for selecting the analysis regions of a given domain, but a study of the kinematics and dynamical balances of the flow can generally be expected to identify distinct regions. If G is found to be spatially satisfactory in a particular region and if b is either constant or its variation can be related to mean field characteristics, then a reasonable parameterization has been found for that region.

This method provides a relatively strong test of a parameterization, as it requires a good local correspondence between F and G. It is felt that this degree of similarity is needed in the spatially complex EGCM systems. For other fluid systems it may be sufficient for a parameterization simply to reproduce large-scale transports, e.g., net meridional transports, and less demanding tests of a parameterization could then be used.

The eddy terms that enter the time averaged equations for heat, zonal and meridional momentum, and vorticity in the primitive equation fluid of RHMS are

$$\frac{\partial}{\partial x_j}\overline{u_j'T'},\quad \frac{\partial}{\partial x_j}\overline{u_j'u'},\quad \frac{\partial}{\partial x_j}\overline{u_j'v'}$$

and

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x_j}\overline{u_j'v'}\right) - \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x_j}\overline{u_j'u'}\right)$$

respectively, where $(u_1,u_2,u_3) = (u,v,w)$, $(x_1,x_2,x_3) = (x,y,z)$ and u, v, w, x, y, z have their traditional definitions, and the summation convention is invoked for repeated subscripts ($j = 1, 2, 3$). In RHMS the contribution of the vertical derivative can be neglected compared to that of the horizontal derivative, so attention is here directed to the extent to which the horizontal parts of these terms resemble

$$\nabla_H{}^2\bar{T},\quad \nabla_H{}^2\bar{u},\quad \nabla_H{}^2\bar{v},\quad \text{and}\quad \nabla_H{}^2\left(\epsilon_{3jk}\frac{\partial\bar{u}_k}{\partial x_j}\right),$$

where

$$\nabla_H{}^2 = \frac{\partial^2}{\partial x_\lambda \partial x_\lambda}\ (\lambda = 1,\ 2).$$

Fig. 1a shows the horizontal boundaries of an appropriate set of distinct regions for RHMS, superimposed on the mean transport streamfunction field. The kinematical and dynamical properties of these regions have been described in detail in RHMS (Sections 3 and 4). Fig. 1b shows the horizontal boundaries of the subregions which will be used for each region. Data are available at five levels in the vertical from the numerical model, so that C_r values are evaluated over depth bands corresponding to each depth level as well as for the entire depth of the basin.

3. Results

Figs. 2–5 show C_r values for the mean heat, vorticity, zonal momentum and meridional momentum eddy terms at three different depths (40, 490 and 2690 m) and vertically integrated. Subregional correlation coefficients are rounded off to the nearest tenth.

Scrutiny of the C_r values reveals that for none of the eddy terms is it possible to find a region over

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[2] This criterion is motivated by the knowledge that, for a parameterization of eddy terms, the most important quantity to be reproduced is the transport due to eddy processes.

which all values are of the same sign. Although there can be relatively large C_r values for a particular subregion or two, or a relatively smooth distribution of C_r values over a few adjacent subregions at a particular depth, there is generally little vertical coherence or horizontal spatial constancy for any of the eddy terms.

Thus, while there are interesting special areas and depths for particular eddy terms, the criteria advanced above lead to the conclusion that neither a constant nor variable coefficient diffusion model is an appropriate description of the RHMS eddy terms.

4. Discussion

Although large correlation coefficients and b values greater than 10^7 cm^2 s^{-1} can be found for particular limited volumes of the domain, the eddy diffusion model does not satisfactorily describe the eddy heat, momentum or vorticity terms in any of the distinct regions of Fig. 1a. Even if the regional boundaries of Fig. 1a are eliminated and the parameterization is asked to provide a description only in the area in the NBC(W) and Near Field where the eddy terms are largest compared to mean terms, this conclusion remains unaltered. There is simply no general constancy of sign, much less magnitude, of correlation for adjacent subregions.

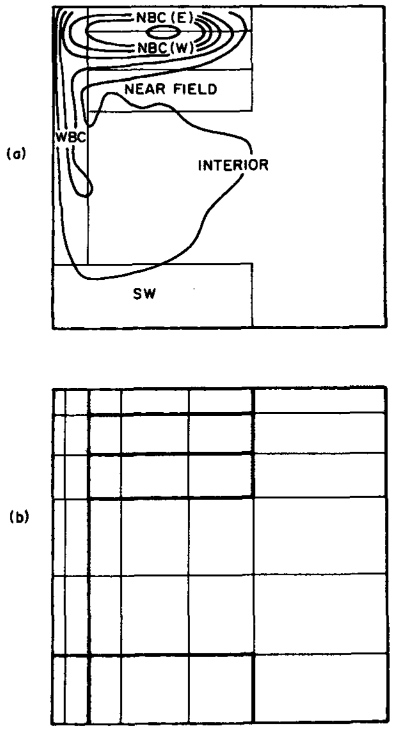
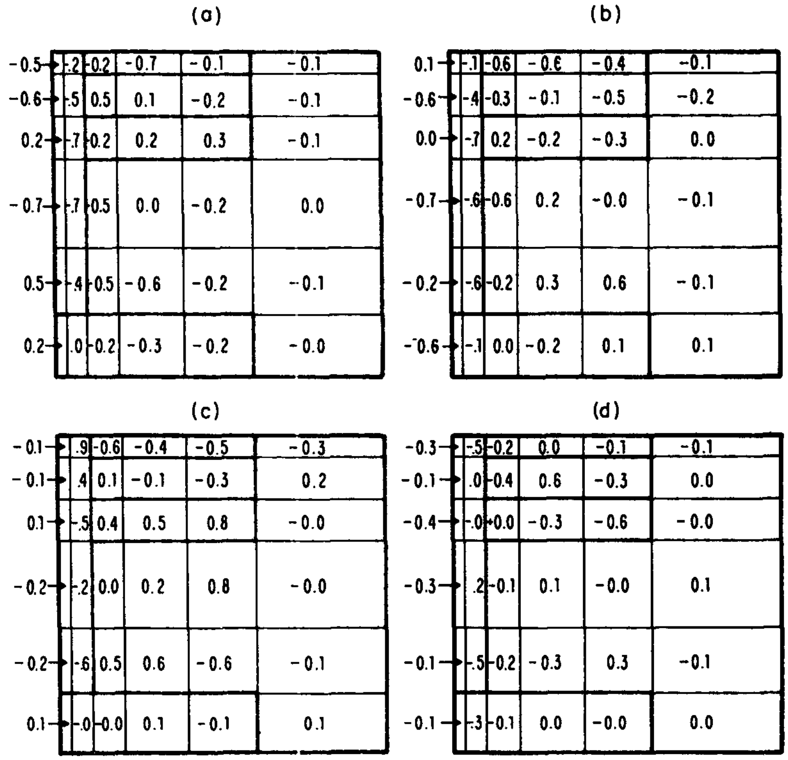


FIG. 1. Distinct regions of RHMS superimposed on the mean transport streamfunction field (a) and regional and subregional horizontal boundaries used in the analysis (b).



(a)

-0.5	-.2	-0.2	-0.7	-0.1	-0.1
-0.6	-.5	0.5	0.1	-0.2	-0.1
0.2	-.7	-0.2	0.2	0.3	-0.1
-0.7	-.7	-0.5	0.0	-0.2	0.0
0.5	-.4	-0.5	-0.6	-0.2	-0.1
0.2	.0	-0.2	-0.3	-0.2	-0.0

(b)

0.1	-.1	-0.6	-0.6	-0.4	-0.1
-0.6	-.4	-0.3	-0.1	-0.5	-0.2
0.0	-.7	0.2	-0.2	-0.3	0.0
-0.7	-.6	-0.6	0.2	-0.0	-0.1
-0.2	-.6	-0.2	0.3	0.6	-0.1
-0.6	-.1	0.0	-0.2	0.1	0.1

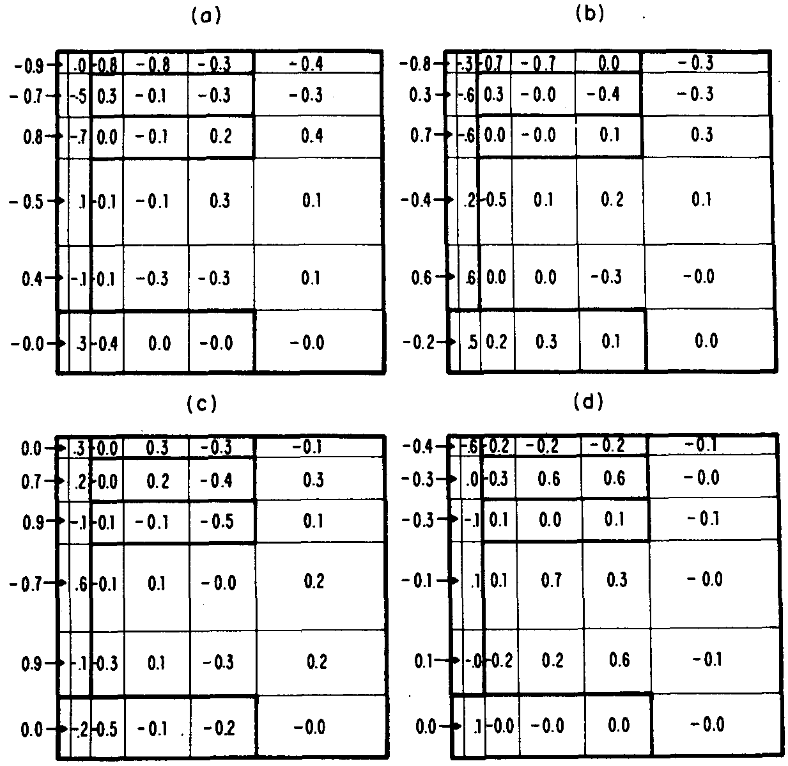
(c)

-0.1	.9	-0.6	-0.4	-0.5	-0.3
-0.1	.4	0.1	-0.1	-0.3	0.2
0.1	-.5	0.4	0.5	0.8	-0.0
-0.2	-.2	0.0	0.2	0.8	-0.0
-0.2	-.6	0.5	0.6	-0.6	-0.1
0.1	-.0	-0.0	0.1	-0.1	0.1

(d)

-0.3	-.5	-0.2	0.0	-0.1	-0.1
-0.1	.0	-0.4	0.6	-0.3	0.0
-0.4	-.0	-0.0	-0.3	-0.6	-0.0
-0.3	.2	-0.1	0.1	-0.0	0.1
-0.1	-.5	-0.2	-0.3	0.3	-0.1
-0.1	-.3	-0.1	0.0	-0.0	0.0

FIG. 2. Eddy heat term. Subregional correlation coefficients between $(\partial/\partial x_\lambda)\overline{u'T'}$ and $\nabla_H{}^2\overline{T}$ using (a) 40 m, (b) 490 m, (c) 2690 m and (d) basin-depth vertically integrated data.



(a)

-0.9	.0	-0.8	-0.8	-0.3	-0.4
-0.7	-.5	0.3	-0.1	-0.3	-0.3
0.8	-.7	0.0	-0.1	0.2	0.4
-0.5	.1	-0.1	-0.1	0.3	0.1
0.4	-.1	-0.1	-0.3	-0.3	0.1
-0.0	.3	-0.4	0.0	-0.0	-0.0

(b)

-0.8	-.3	-0.7	-0.7	0.0	-0.3
0.3	-.6	0.3	-0.0	-0.4	-0.3
0.7	-.6	0.0	-0.0	0.1	0.3
-0.4	.2	-0.5	0.1	0.2	0.1
0.6	.6	0.0	0.0	-0.3	-0.0
-0.2	.5	0.2	0.3	0.1	0.0

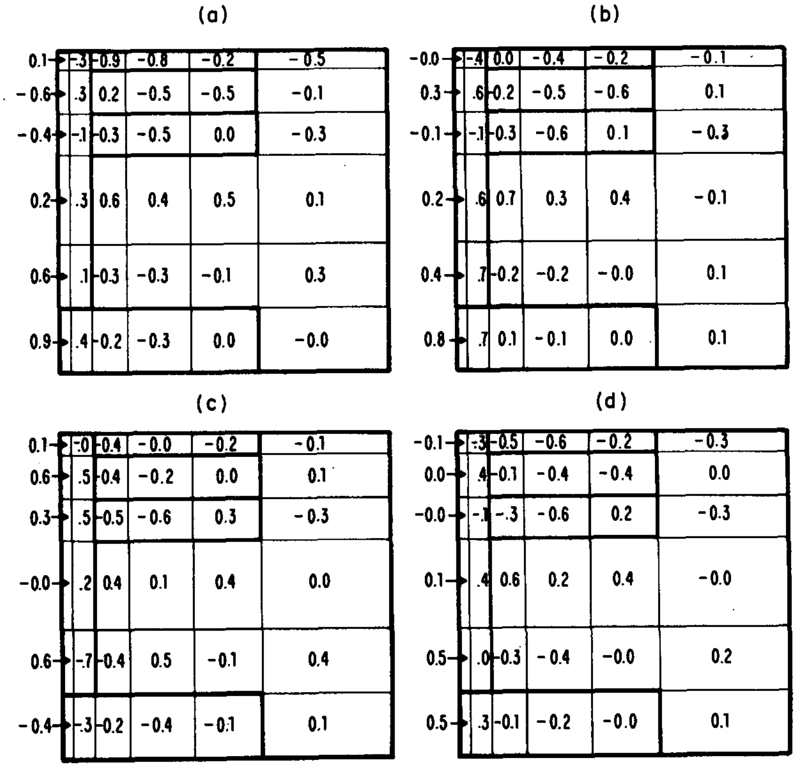
(c)

0.0	.3	-0.0	0.3	-0.3	-0.1
0.7	.2	-0.0	0.2	-0.4	0.3
0.9	-.1	-0.1	-0.1	-0.5	0.1
-0.7	.6	-0.1	0.1	-0.0	0.2
0.9	-.1	-0.3	0.1	-0.3	0.2
0.0	-.2	-0.5	-0.1	-0.2	-0.0

(d)

-0.4	-.6	-0.2	-0.2	-0.2	-0.1
-0.3	.0	-0.3	0.6	0.6	-0.0
-0.3	-.1	0.1	0.0	0.1	-0.1
-0.1	.1	0.1	0.7	0.3	-0.0
0.1	-.0	-0.2	0.2	0.6	-0.1
0.0	.1	-0.0	-0.0	0.0	-0.0

FIG. 3. As Fig. 2 except for eddy vorticity term. Correlation coefficients between $\epsilon_{3jk}\partial^2\overline{(u_j'u_k')}/\partial x_j\partial x_k$ and $\nabla_H{}^2\epsilon_{3jk}\partial\bar{u}_k/\partial x_j$.



(a)

0.1	-.3	-0.9	-0.8	-0.2	-0.5
-0.6	.3	0.2	-0.5	-0.5	-0.1
-0.4	-.1	-0.3	-0.5	0.0	-0.3
0.2	.3	0.6	0.4	0.5	0.1
0.6	.1	-0.3	-0.3	-0.1	0.3
0.9	.4	-0.2	-0.3	0.0	-0.0

(b)

-0.0	-.4	0.0	-0.4	-0.2	-0.1
0.3	.6	-0.2	-0.5	-0.6	0.1
-0.1	-.1	-0.3	-0.6	0.1	-0.3
0.2	.6	0.7	0.3	0.4	-0.1
0.4	.7	-0.2	-0.2	-0.0	0.1
0.8	.7	0.1	-0.1	0.0	0.1

(c)

0.1	-.0	-0.4	-0.0	-0.2	-0.1
0.6	.5	-0.4	-0.2	0.0	0.1
0.3	.5	-0.5	-0.6	0.3	-0.3
-0.0	.2	0.4	0.1	0.4	0.0
0.6	-.7	-0.4	0.5	-0.1	0.4
-0.4	-.3	-0.2	-0.4	-0.1	0.1

(d)

-0.1	-.3	-0.5	-0.6	-0.2	-0.3
0.0	.4	-0.1	-0.4	-0.4	0.0
-0.0	-.1	-.3	-0.6	0.2	-0.3
0.1	.4	0.6	0.2	0.4	-0.0
0.5	.0	-0.3	-0.4	-0.0	0.2
0.5	.3	-0.1	-0.2	-0.0	0.1

FIG. 4. As Fig. 2 except for eddy zonal momentum term. Correlation coefficients between $(\partial/\partial x_\lambda)\overline{u_\lambda'u'}$ and $\nabla_H{}^2\bar{u}$.

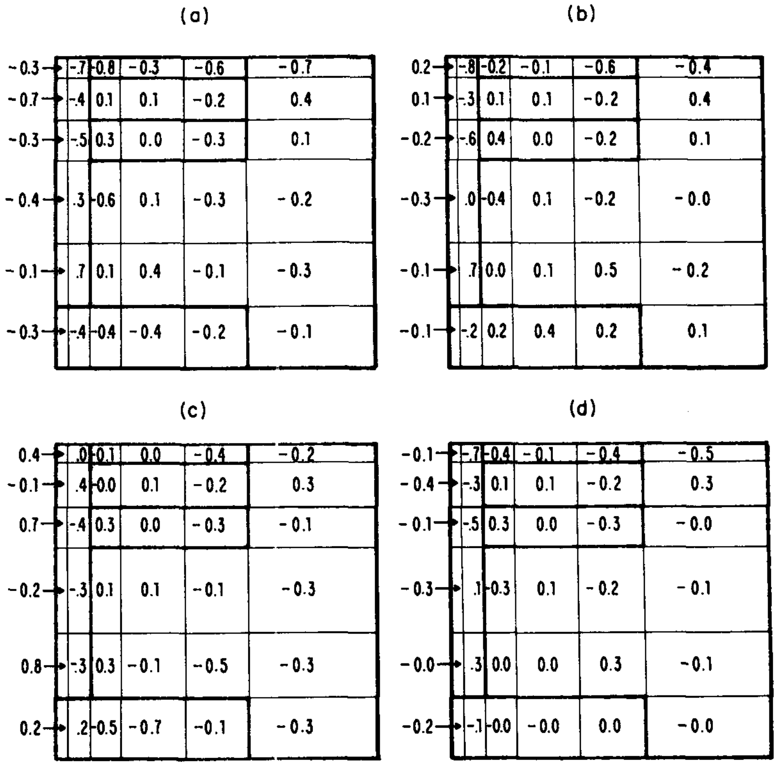


FIG. 5. As Fig. 2 except for meridional momentum term. Correlation coefficients between $(\partial/\partial x_\lambda)\overline{u_\lambda' v'}$ and $\nabla_H{}^2\bar{v}$.

Holland (personal communication) has reached a similar conclusion, by comparing various zonally integrated fields, for several of his EGCM experiments. However, Semtner and Mintz (1977, p. 228) describe their eddies as "explicit horizontal diffusers of heat" with a lateral diffusion coefficient of 1–3 × 10^7 cm^2 s^{-1} in their Gulf Stream region. This result is based on *b* values evaluated from vertically integrated fields at several points in the region of interest (Semtner, personal communication). Averaging methods like zonal integration, which combine data from dynamically distinct regions of flow, risk overlooking areas of significant correlation but are unlikely to lead to a false positive conclusion. Point methods require consideration of large numbers of randomly selected points before confident positive or negative conclusions can be drawn. The Semtner and Mintz result should be regarded as tentative until more systematic examination is complete.

The idea of eddy diffusion as a model of the net effect of motions on all scales from mesoscale and smaller is particularly prevalent in large-scale numerical general ocean circulation modeling. To the extent that effects of the energetically dominant mesoscale motions are believed to dominate effects of smaller scale motions, mesoscale eddy effects should simulate diffusion to justify continued use of the diffusion parameterization in these numerical models.

Several factors affect the impact of these remarks. The most important is that it is not possible at this time to determine the extent to which the details of the flow and statistics of EGCM's resemble those of the ocean. Further, the range of model flows and statistics, and their dependence upon the various EGCM model parameters, is not well known. The RHMS results may prove to be atypical of further EGCM results or of the ocean.

Despite these limitations it is only from studies like this that general eddy term/mean field relationships can presently be examined. Further scrutiny of other EGCM results, acquisition of new ocean data, and efforts to determine how EGCM results relate to ocean data are all needed if progress on parameterization is to be made.

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