Topographic Rossby Waves off East Australia: Identification and Role in Shelf Circulation

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ABSTRACT

Hamon *et al.* (1975) analyzed two years of surface current data from ships' set along two tracks parallel to 560 km of the east Australian shelf and found a phase lag of 10 days between long (120 day) period current fluctuations 19 km offshore and 6.5 km offshore. This is explained in terms of the propagation characteristics of topographic waves on the continental shelf. It is shown that these current fluctuations can lead to a significant amount of coastal upwelling as they are damped out by bottom friction. The average circulation on the east Australian shelf is discussed. The simplest response to the known longshore pressure gradient would be a steady longshore current and upwelling circulation, but it is shown that this is incompatible with a heat budget for the water on the shelf. Hence something other than bottom friction is required to balance most of the longshore pressure gradient; the onshore momentum flux of the topographic waves identified in the current data is shown to be a likely candidate. The implications of this interpretation for the circulation in deeper water off the shelf are discussed.

1. Introduction

A remarkable set of surface longshore current data based on the drift of merchant ships was reported by Hamon *et al.* (1975). Two years of data were obtained along 560 km of the east Australian coast for two separate tracks. The first track, taken by southbound ships exploiting the East Australian Current, followed the 200 m isobath at an average distance of 19 km offshore from 11 landmarks with respect to which the drift was estimated. The second track, approximately 6.5 km offshore in an average depth of about 70 m, was taken by northbound ships avoiding the main southward current. The data from these two tracks will be referred to as offshore and inshore, respectively.

After various corrections, described in detail by Greig (1974), the data for each of the 10 sections between landmarks were low-passed with a filter having a half-power point at 0.033 cycles per day (cpd) and used as a data set of mean currents at intervals of 5 days. Each data point thus obtained was estimated by Hamon *et al.* to have a random error with a standard deviation of 0.1 m s^{-1}. Contoured plots of the data as functions of time and longshore distance revealed current fluctuations as large as the mean current over large space and time scales, with the mean and fluctuations being considerably greater offshore than inshore. From the slope of ridges and troughs in the contoured data and from lagged correlations between different sections, Hamon *et al.* established that patterns of current moved south at 0.1 m s^{-1}. This is considerably less than the mean southward surface current of about 0.8 m s^{-1} offshore and 0.3 m s^{-1} inshore (Godfrey, 1973).

Frequency spectra for the northern five sections of data peaked at a period of 70 days offshore, but lacked any significant peak inshore. For the southern five sections the spectra, for both the offshore and inshore data (shown in Fig. 1), peak at a period of 117 days, the neighboring spectral estimates being at periods of 175 and 88 days. The spectra have been corrected for the effect of the low-pass filter, but only plotted out to 20 cycles/350 days rather than the Nyquist frequency of 35 cycles/350 days as the high-frequency parts of the spectra with a level of 0.003 m^2 s^{-2} $cycle^{-1}$/350 days appear to be white noise with an rms level for the 5-day means (after adding the filter response again) of 0.18 m s^{-1}, slightly greater than the estimate of Hamon *et al.* mentioned above.

Confidence limits are difficult to estimate. The number of degrees of freedom for the spectral estimates, from each section of coast, for the 50% overlapped Parzen window of length 350 days used, is 7.2 (Welch, 1967). Thus if the five sections of coast are all independent, the total number of degrees of freedom is 36. This may be true at frequencies above about 15 cycles/350 days, for which the computed coherence squared between inshore and offshore data is comparable with the expected value of 0.06 for 36 degrees of freedom (Haubrich, 1965) if the true coherence is zero.

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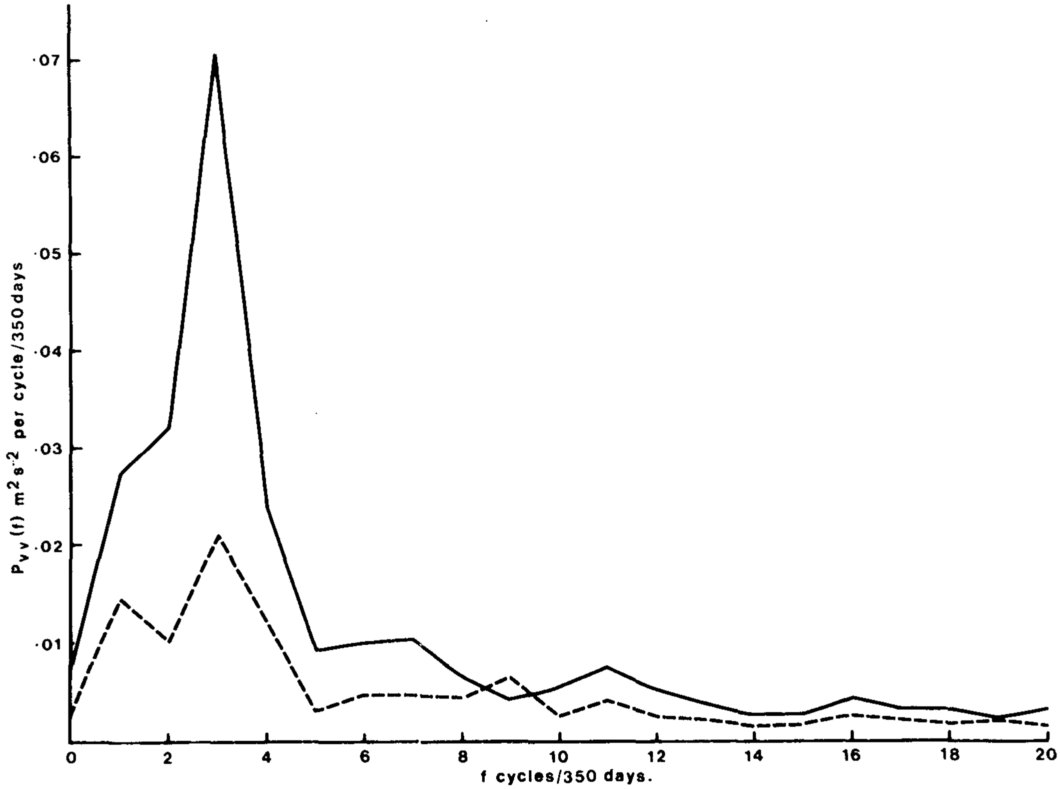

FIG. 1. Power spectra of longshore current fluctuations for southern five sections, redrawn from the data of Hamon *et al.* (1975) with the effect of their low-pass filter removed. Solid line, offshore currents; dashed line, inshore currents.

However, at lower frequencies the five sections of coast are not independent, so that the number of degrees of freedom is less than 36, but still more than 7.2. The 95% confidence limits on the spectral level correspond to factors (0.44, 4.0) for 7.2 and (0.65, 1.7) for 36 degrees of freedom (Jenkins and Watts, 1968).

Fig. 2 shows the squared coherence and phase between offshore and inshore data for the southern five sections. At the frequency of the spectral peak the squared coherence is 0.64 and the inshore current lags the offshore by 30° or 10 days. The 95% confidence limit on the phase at this coherence is ±24° for 7.2 degrees of freedom and ±11° for 36 degrees of freedom (Jenkins and Watts, 1968). Any squared coherence above about 0.7 for 7.2 and about 0.2 for 36 degrees of freedom is significant at the 95% level (Haubrich, 1965). Even if the coherence seems to be barely significant, the relative constancy and positive value of the phase lag over a wide range of frequencies indicates a physical effect.

The purpose of this paper is to suggest an interpretation of this phase lag and to discuss the role of the current fluctuations in the overall circulation of this part of the continental shelf off the east coast of Australia.

2. Topographic Rossby waves

The vertical extent of large-scale long-period currents in the deep ocean off east Australia is not precisely known, but it is thought (Boland and Hamon, 1970) that the currents decrease from a surface maximum to half this value at about 250 m and to zero at some depth greater than 2000 m. As deepwater fluctuations, meanders or eddies associated with the East Australian Current, encounter the bottom topography of the continental slope in depths of 2000 m or less (i.e., within ~60 km or less of the coast), restoring forces associated with the motion of vortex columns across isobaths come into play. This is to say topographic Rossby waves will be generated, with frequency and longshore wavenumber corresponding to those of the deepwater fluctuations, and a wavenumber normal to the shore given by the dispersion relation.

The probable existence of such waves in other oceans is well recognized. In a comprehensive study of deepwater current meter data on the continental rise north of the Gulf Stream, Thompson (1977) found convincing evidence for bottom-trapped topographic Rossby waves corresponding to the theory of Rhines (1970). In shallow or weakly strati-

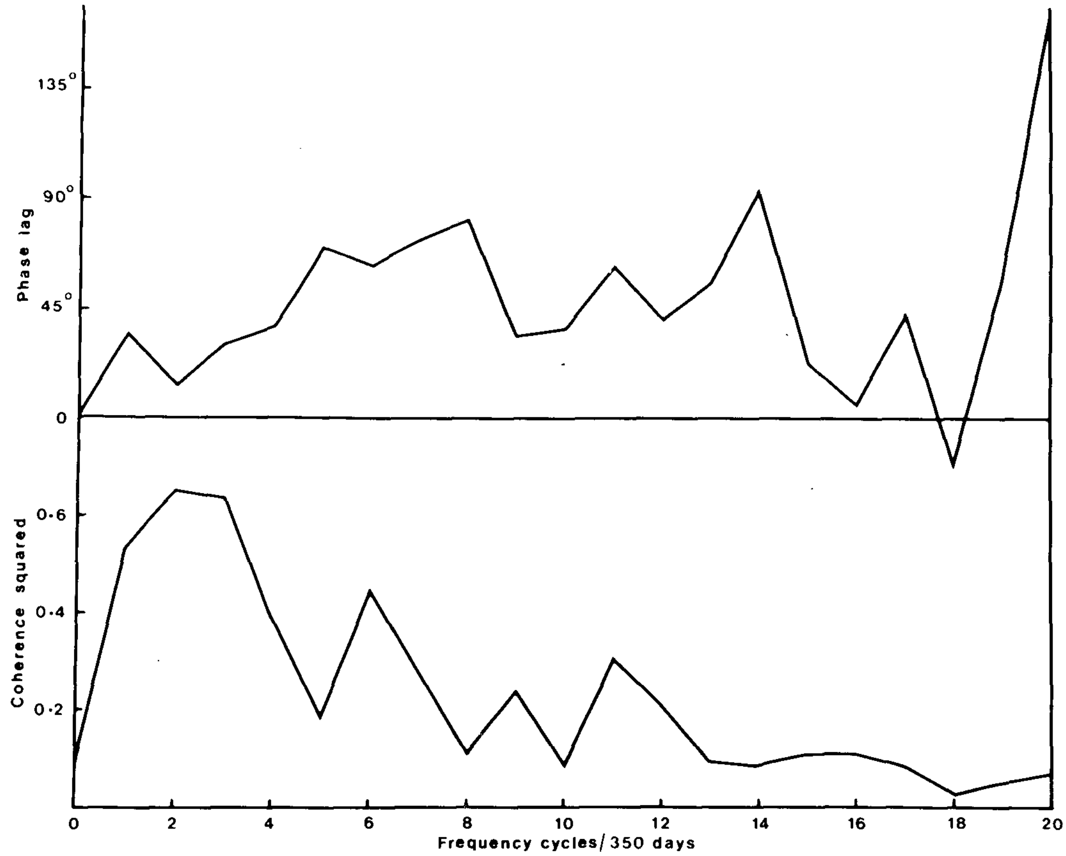

FIG. 2. Coherence squared and phase between the inshore and offshore current fluctuations corresponding to the power spectra of Fig. 1. Positive phase means inshore currents lag offshore currents.

fied water these waves extend to the surface. Kroll and Niiler (1976) have argued that the barotropic topographic Rossby waves appropriate to a homogeneous ocean can, in some cases, propagate as far as the 25 m isobath before being eliminated by bottom friction, and Petrie and Smith (1977) find some evidence for such waves propagating onshore on the Scotian Shelf.

3. A simple model

The essential features of an idealized model are shown in Fig. 3. The y axis is taken along the coastline, assumed straight, and the isobaths are taken to be parallel to the coast. The sloping lines represent constant phase lines of topographic Rossby waves, which propagate in a manner appropriate to their dispersion relation and are also advected parallel to the coast by a mean longshore current V, with $V < 0$ in practice. For the moment V is taken to be independent of x; the effect of $V(x)$ will be discussed later. Also ignoring for now the density stratification of the water, the dispersion relation for barotropic topographic Rossby waves with phase $kx + ly - \omega' t$ is (e.g., Kroll and Niiler, 1976)

$$\omega' = \frac{-fSl}{k^2 + l^2 + S^2/4}\,. \tag{3.1}$$

Here ω' is the frequency of the waves relative to the mean flow, k and l are the longshore and offshore wavenumbers, respectively, f is the Coriolis parameter and S the exponent in an assumed exponential depth profile $h(x) = h_0 \exp(Sx)$, so that $S = h^{-1}dh/dx$. The β effect is negligible compared with the effect of the slope in the present application, and is neglected.

We take $\omega' > 0$, without loss of generality, merely to ensure that the direction of the vector (k,l) is the same as the direction of phase propagation of the waves relative to the mean flow. Eq. (3.1) then requires $l > 0$, as $f < 0$; this implies northward phase propagation relative to the mean flow. The group velocity is

$$\mathbf{c}_g = (\partial\omega'/\partial k, \partial\omega'/\partial l) = fS(k^2 + l^2 + S^2/4)^{-2} \times (2lk,\ -k^2 + l^2 - S^2/4). \tag{3.2}$$

For onshore energy propagation we require $\partial\omega'/\partial k < 0$ and so $k > 0$ as $l > 0$ and $f < 0$. Thus the constant phase lines in Fig. 3, $kx + ly$ = constant, do indeed slope offshore from north to south (decreasing y) as shown.

We now suppose that in the presence of the mean current V the waves have a frequency ω as measured at a fixed point on the coast, and that the current patterns move in the y direction at speed

$$c_y = \omega/l = (\omega'/l) + V. \qquad (3.3)$$

This is the longshore trace speed rather than the y component of the phase velocity. Eq. (3.3) gives l if c_y and ω are known (and requires that we take $\omega < 0$ in our case as $c_y < 0$ and $l > 0$). Given V, Eq. (3.3) also implies a value for ω' and hence from the dispersion relation (3.1),

$$k = [fS/(V - c_y) - l^2 - S^2/4]^{1/2}. \qquad (3.4)$$

From this we may evaluate the offshore trace speed, that is, the apparent speed with which current patterns propagate offshore,

$$c_x = \omega/k = \omega'/k + Vl/k. \qquad (3.5)$$

We note that c_x is made up of two parts: the offshore trace speed relative to the water (ω'/k) and the apparent speed (Vl/k) due to the sloping phase lines being swept alongshore.

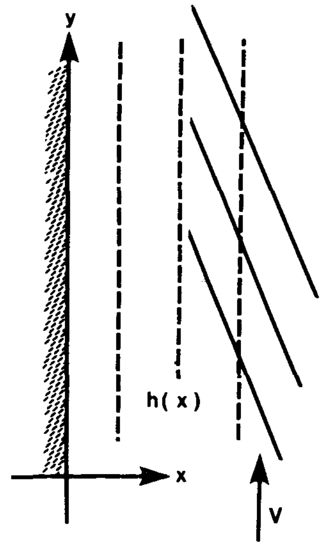

FIG. 3. Schematic diagram of topographic waves near a straight coast with isobaths (dashed lines) parallel to the coast. The sloping lines indicate lines of constant phase at a particular time, i.e. $kx + ly$ = constant, where (k,l) is the wavenumber. There is a mean longshore current V.

4. Application to the east Australian shelf

We now apply this simple model to the situation described in the Introduction, and reserve for later a discussion of the many over-simplifications and omissions. We take $f = -7.3 \times 10^{-5}\ \mathrm{s^{-1}}$ corresponding to latitude 30°S. We take $S = 5.4 \times 10^{-5}\ \mathrm{m^{-1}}$ from the exponential fit of Buchwald and Adams (1968) and Hamon (personal communication) to the topography out to depths of 4 km or so, although the approximate mean depths along the two tracks imply a slightly larger value. With $V = -0.6\ \mathrm{m\ s^{-1}}$, $c_y = -0.1\ \mathrm{m\ s^{-1}}$, $\omega = -2\pi/(117\ \mathrm{days}) = -6.2 \times 10^{-7}\ \mathrm{s^{-1}}$ we get $l = 6.2 \times 10^{-6}\ \mathrm{m^{-1}}$ from (3.3) and $k = 8.4 \times 10^{-5}\ \mathrm{m^{-1}}$ from (3.4). The corresponding wavelengths are 10^3 km longshore and 75 km in the x direction, and the lines of constant phase should slope offshore at an angle of $\tan^{-1} l/k = 4°$.

From (3.5) we obtain $c_x = -7.4 \times 10^{-3}\ \mathrm{m\ s^{-1}}$, and a time lag from 19 km offshore to 6.5 km offshore of 20 days, compared with the 10 days indicated by the data. The offshore group velocity implied by (3.2) is $c_{gx} = -6.7 \times 10^{-2}\ \mathrm{m\ s^{-1}}$, so that the time taken for the energy of a disturbance to travel over 12.5 km is only 2 days. This is of great importance in permitting the current fluctuations to reach inshore in spite of the damping effect of bottom friction, which will be discussed in detail later in the paper.

The frequency of the waves relative to the water is $\omega' = \omega - Vl = 3.1 \times 10^{-6}\ \mathrm{s^{-1}}$, with a corresponding period of 23 days rather than the 117 days observed at the coast.

a. Relationship to shelf waves

The shelf waves studied theoretically by Buchwald and Adams (1968) and others satisfy the same dynamical equations as barotropic topographic Rossby waves. Indeed, each shelf-wave mode is composed of a pair of topographic waves, one propagating offshore and the other shoreward, trapped between the coast and a flattening of the topography at some distance offshore. Hamon (1976) accounted for sea level fluctuations in the frequency range 0.04–0.25 cpd in terms of wind-generated first mode shelf waves, with a longshore propagation speed of about 3 m s^{-1} toward the north.

The topographic waves identified in this paper correspond roughly to the shoreward propagating part of the third mode shelf wave of Buchwald and Adams (1968), with a northward propagation speed of about 0.5 m s^{-1}, less than the southward speed of the mean current.

The currents on the shelf associated with the wind-generated shelf waves are probably about 0.1 m s^{-1} in strength [from the amplitude of coastal sea level changes (Hamon, 1966) and the eigenfunctions of Buchwald and Adams (1968)], rather less than the lower frequency currents of up to 1 m s^{-1} discussed in this paper.

b. Sensitivity

For the numbers we have used, the first term in (3.4) is dominant by an order of magnitude. Hence, with $|c_y| \ll |V|$, we have $k \approx (fS/V)^{1/2}$ independent of l and ω, so that $c_x \approx \omega(V/fS)^{1/2}$ and $c_{gx} \approx (2V/$

$c_y)c_x$. An increased c_x, and smaller time lag between offshore and inshore fluctuations, would occur for higher frequency ω, larger current V or smaller f or slope S. The main reason for the large ratio of c_{gx}/c_x is that $c_{gx} \approx -2\omega'/k$, but the ω'/k contribution to c_x in (3.5) is almost cancelled by the second term.

5. Omissions

a. Scale problems

A serious shortcoming of the above interpretation of the ship drift data is the assumption that the motions responsible for the fluctuations are wave-like. In the longshore direction, the section of coast with the spectral peak at a period of 117 days is only about 320 km, less than the estimated longshore wavelength of about 1000 km. Similarly, the calculated wavelength normal to the coast (75 km) is rather greater than the distance over which significant attenuation due to bottom friction (see later) and changes in the slope parameter S might occur. Clearly, we do not have long wavetrains varying only slowly in space, but we assume that the physical effects embodied in the free wave solutions are still applicable, and that the calculated offshore wavenumber and speed are a reasonable approximation.

b. Nonlinearity

The theory leading to the dispersion relation for topographic Rossby waves neglects the nonlinear terms in the momentum equations. In fact it is easily shown from the dynamical equations that the nonlinear terms are negligible compared with the Coriolis force only if (SV_0/f) is small for the longshore momentum equation, and $(SV_0/f)(l^2/k^2)$ is small (a weaker condition) for the momentum equation normal to the shore, where V_0 is a typical amplitude of the longshore velocity fluctuations. For the data under discussion V_0 may be as much as 1 m s^{-1}, giving $SV_0/f \approx 0.7$, so that nonlinear effects may be important but not dominant. However, it is conceivable that the current fluctuations under investigation are confined near the surface, and their apparent movement onshore is a consequence of nonlinear dynamics, independent of the effect of bottom slope investigated in this paper. Long time series of current meter data at several depths and sites are probably required to resolve this point.

c. Shear

The mean longshore current V has been assumed to be independent of offshore distance x. In fact $dV/dx \approx -4 \times 10^{-5}$ s^{-1}. The main effect of this on the dispersion relation for barotropic topographic Rossby waves is probably to change the effective f from -7.3×10^{-5} s^{-1} to -11.3×10^{-5} s^{-1}, with a consequent 20% decrease in c_x.

Vertical shear dV/dz of the mean current may reduce V to a small value near the bottom (see Section 7). This would reduce the appropriate value of V to use in (3.4) and could reduce c_x by about 40%.

A proper investigation of the effects of shear requires an analysis of topographic waves in a general longshore velocity field $V(x,z)$.

d. Baroclinicity

In a stratified ocean topographic Rossby waves tend to be trapped near the bottom (Rhines, 1970). For constant Brunt-Väisälä frequency N, the important parameter is $\epsilon = Nh\kappa/f$, where κ is the total wavenumber $(k^2 + l^2)^{1/2} \approx k$ in the present case. The surface current is reduced by a factor sechϵ relative to the bottom current, and the dispersion relation is approximately (for $k \gg l$)

$$\omega' = -(fSl/k^2)\epsilon \coth\epsilon. \qquad (5.1)$$

This tends to increase the value of k obtained for a given ω'/l compared to the value obtained without stratification. The increase is by a factor of about $(\epsilon \coth\epsilon)^{1/2}$, which equals 1.15 for $\epsilon = 1$, an appropriate value for the present situation. The trace speed c_x will be decreased by a similar small amount.

e. Bottom friction

The simplest view of the role of bottom friction, as taken by Kroll and Niiler (1976) in their theoretical study of the transmission of barotropic topographic Rossby waves onto the shelf, is that it acts as a body force distributed over the depth of the fluid. This is a rather oversimplified view, particularly for a stratified fluid, but for the moment we accept it and examine the consequences. Taking a quadratic drag law with drag coefficient C_D, the decay rate is C_Dh^{-1} times a typical near-bottom velocity. If we take this to be the mean surface current along each track (though this may be an overestimate), we get, with $C_D = 2 \times 10^{-3}$, a decay rate of about 8×10^{-6} s^{-1}. This corresponds to a decay to a fraction ¼ in the two days taken for energy to propagate from the offshore to the inshore track. Without bottom friction, and with exponential bottom topography, the amplitude of the current fluctuations should increase like $h^{-1/2}$, or 1.7 from 200 to 70 m, giving an expected net decrease to 0.4 in the amplitude, or 0.2 in the energy. This is consistent with the appearance of current fluctuations inshore, but with considerably reduced energy as shown in the spectra in Fig. 1.

The total longshore component of the group velocity is roughly equal to c_y, or 10 km day^{-1}. So with a damping time of a few days the currents

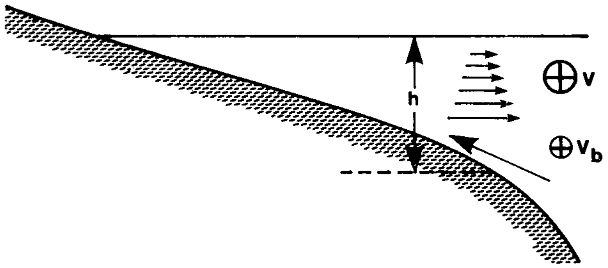

FIG. 4. Upwelling induced by a longshore current. The large arrow represents the bottom Ekman flux, and the small arrows the interior return flow. The sketch is appropriate looking north with $f > 0$, but applies for $f < 0$ if the currents are reversed.

found on the shelf should be a response to local offshore forcing rather than the consequence of longshore propagation of the topographic waves themselves. This is not incompatible with the small longshore damping, reported by Hamon (1976), of higher frequency wind-generated shelf waves. These travel much faster and can draw on a supply of energy in deeper water where the damping is less.

For topographic Rossby waves with a scale much less than the external Rossby radius of deformation $(gh)^{1/2}f^{-1}$ (a condition satisfied in the present example), the time-dependent term in the continuity equation may be neglected (Kroll and Niiler, 1976). This means that time-dependent terms occur only in the momentum equations along with the bottom friction terms; thus the friction only produces a damping rate and does not affect the real part of the frequency in the dispersion relation. This is important, given the shortness of the spindown time compared with the wave period.

f. Reflection

If partial reflection of the topographic waves occurs, either because of changes in bottom slope or at the coast, friction permitting, this would reduce the phase lag between offshore and inshore fluctuations.

g. Summary

The shortcomings and modifications of the simple model, while generally worsening the agreement between observed and predicted phase lag, probably do not invalidate the basic suggestion that the observed current fluctuations are topographic Rossby waves generated by meanders and eddies encountering the continental slope. Indeed, the discussion of the role of friction seems to add support to the interpretation, in that it shows that topographic waves can transmit energy onshore rapidly enough to escape complete damping, while still producing a much longer time lag for the current phase.

6. Current induced upwelling

Wave motions of the amplitude found may be expected to have significant secondary effects on the shelf circulation. We begin with a discussion of the induced flow in a plane normal to the coast.

Hsueh and O'Brien (1971) showed that a longshore current can produce coastal upwelling or downwelling. The basic physics of the situation is illustrated in Fig. 4. A longshore current V produces an onshore Ekman flux of τ_b/f in the bottom mixed layer, where the bottom stress τ_b is $C_D V_b^2$, with V_b the longshore current just above the mixed layer. If V is uniform longshore, this Ekman flux must return offshore in the interior, with an average offshore velocity τ_b/fh. The Coriolis force acting on this provides an average deceleration τ_b/h, as used in Section 5e.

If V is a steady current maintained against bottom friction by a longshore pressure gradient, this upwelling is persistent. However, if V is a transient, as produced by a topographic wave propagating onshore, V_b is finally damped out so that the upwelling motions cease, leaving behind a finite vertical displacement of the water shoreward of the initial current distribution.

A full treatment of the net effect of a topographic wave propagating onshore is difficult, but an exact solution can be obtained if the bottom is assumed to be flat, the Brunt-Väisälä frequency constant, and a longshore velocity V, independent of the longshore coordinate, is imposed at an initial instant. The solution, and details of the effect of longshore variations, will be submitted for publication separately, but the basic features of the solution, when V is confined outside some finite distance offshore, are shown in Fig. 5.

The offshore flow in the interior is somewhat stronger near the bottom than near the surface, but a net offshore volume displacement of approximately Vh/f is required to reduce the longshore bottom current to zero. This comes from upwelling, during the spindown time of order $h(C_D V)^{-1}$, over a horizontal distance of Nh/f, the internal Rossby radius of deformation, and is thus accompanied by a vertical displacement of isopycnals of approximately $(Vh/f)(f/Nh) = V/N$. This can be tens of meters if V is a fraction of a meter per second and $N \approx 10^{-2}\ s^{-1}$, and lends weight to Boland's (1978) suggestion that the upwelling events on the east Australian shelf studied by Rochford (1972, 1975) are indeed a consequence of bottom Ekman flux associated with an increase in southward flow on the continental slope. We note also the possibility of a reverse flow inshore of the initial current.

If the initial current has some longshore variation, so that on the basis of the above discussion one would expect alternate regions of upwelling and

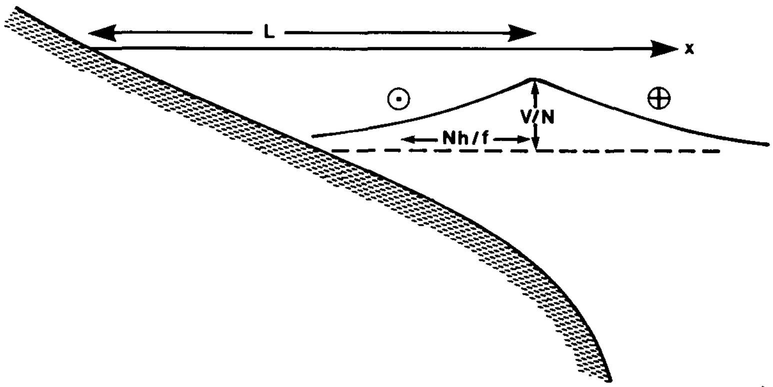

FIG. 5. Schematic of upwelling produced by a longshore current which is initially equal to V for $x > L$. The dashed line is the initial position of an isopycnal and the solid line its final position. As in Fig. 4 with $f > 0$.

downwelling, the upwelling and downwelling regions have a tendency to "communicate" via motions akin to internal Kelvin waves and the final vertical displacement of isopycnals is reduced. A theoretical treatment of this complicated problem will be presented elsewhere, but the conclusion for parameters typical of the east Australian shelf is that isopycnal displacements comparable to V/N are still to be expected.

7. The mean flow on the shelf

There is a substantial longshore gradient of mean dynamic height in the deep water off the east coast of Australia. In the region covered by the ship drift data under discussion, and with a depth of no motion at 1300 m in the deep water, the sea surface slopes down to the south with a gradient of approximately

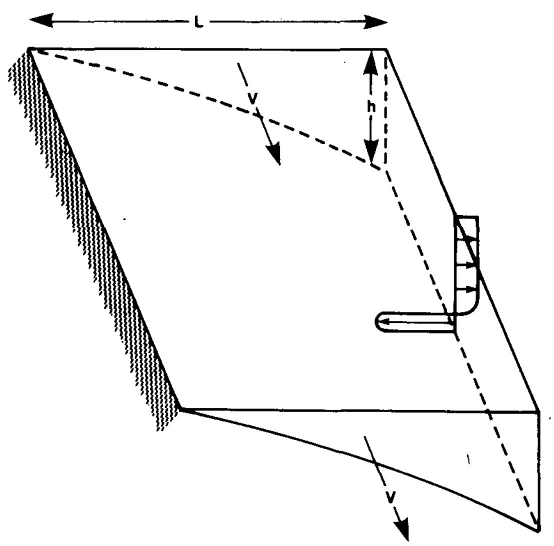

FIG. 6. A section of shelf water for which the heat budget is discussed in Section 7a.

5×10^{-7} (Hamon and Greig, 1972), corresponding to a height change of about 0.3 m along the section of coast covered by the ship drift data. This longshore slope, and corresponding pressure gradient, is likely to persist all the way in to the coast, as there is no southward acceleration of the mean current on the shelf comparable to it. If this longshore pressure gradient is geostrophically balanced, it implies an offshore flow at the surface of 0.07 m s^{-1} which is not greatly attenuated with depth for the small mean longshore density gradient observed. This situation requires a longshore bottom flow $V_b = (fhu/C_D)^{1/2} = 0.5$ m s^{-1} for $h = 100$ m. The bottom friction balances the pressure gradient integrated over depth, and the onshore Ekman flux in the bottom boundary layer balances the offshore geostrophic flow above it. The motion implied thus consists of a spiralling motion along the coast, which appears to be incompatible with the heat budget for the water on the shelf, as will be seen in the following rough calculation.

a. Shelf heat budget

We consider the heat balance of a wedge-shaped piece of water on the continental shelf, as illustrated in Fig. 6. Assume that the longshore current V is independent of the longshore coordinate y, as indicated by the data of Hamon *et al.* (1975), and has average $\bar{V}$ over the cross section. Then, if the longshore temperature gradient $\partial T/\partial y$ is independent of the x,z coordinates, the heat input per unit length of coastline is approximately $\frac{1}{2}Lh\bar{V}\rho c_p \partial T/\partial y$. Considering the shelf out to the offshore track, we take $L = 20$ km, $h = 200$ m and $\bar{V} = 0.6$ m s^{-1}. The water cools in the direction of the current, with $\partial T/\partial y \approx 5 \times 10^{-6}$ °C m^{-1} (Anonymous, 1949), so that the rate of heat input per unit distance alongshore, due to longshore advection, is 2.4×10^7 W m^{-1}. This is equivalent to a surface input 1200 W m^{-2}, so that the effect of air-sea exchange, typically of order 100 W m^{-2} in the region of a warm western boundary current (Bunker, 1976), is negligible by comparison.

We now consider the heat loss from the wedge due to an interior outflow at 0.07 m s^{-1} and a compensating shoreward mass flux of colder water in the bottom boundary layer. Vertical profiles of temperature in about 200 m of water on the relevant part of the east Australian shelf (B. V. Hamon, personal communication) show an annual mean difference between the depth-averaged temperature and the bottom temperature of about 3°C, so that the rate of heat loss per unit length of coastline is 1.7×10^8 W m^{-1}, about seven times the rate of heat input due to longshore advection.

This suggests that unless there is an implausibly large horizontal eddy flux of heat, the outflow can only be about 14% as great as that expected from a geostrophic balance of the longshore pressure gradient. Some other mechanism is then required to balance the remaining 86% of the mean longshore pressure gradient. This cannot be the wind stress, as the wind stress required to balance the presssure over the inner 20 km of the shelf is 0.5 Pa, which is at least an order of magnitude greater than the annual mean.

We note that if most of the longshore pressure gradient is balanced by some process other than bottom friction, the mean longshore bottom velocity must be close to zero, so that the average longshore current $\bar{V}$ over the cross section is closer to 0.3 m s^{-1} than 0.6 m s^{-1}. This reduces the longshore heat loss to 600 W m^{-2}, implying that 93% of the longshore pressure gradient is balanced by something other than bottom friction.

The final approximate interpretation of the heat budget, then, is of a mean offshore flow above the bottom Ekman layer at about 0.005 m s^{-1}, and a mean longshore bottom flow of about 0.1 m s^{-1}, or less if the fluctuations increase the effective friction. This is in response to 7% of the longshore pressure gradient, with the other 93% balanced by some other process.

b. Eddy momentum flux

Sturges (1974) used the Reynolds stress estimates of Schmitz and Niiler (1969) to argue that the longshore pressure gradient inshore of the Gulf Stream off the coast of Florida is balanced by the divergence of the eddy momentum flux. No direct Reynolds stress measurements have been made off east Australia (the cross track component of drift in the data under discussion being too uncertain) but we may estimate the stress by accepting the interpretation that the longshore current fluctuations are due to topographic Rossby waves.

For the barotropic model of Section 3, the Reynolds stress is $\overline{uv} = -(l/k)\overline{v^2}$, where u, v are the current components. If we take for $\overline{v^2}$ the variance at the spectral peak and at the two neighboring frequencies, we have $\overline{v^2} = 0.13$ m^2 s^{-2}, and with $l/k = 0.074$ from Section 4 we obtain $\overline{uv} = -9.4 \times 10^{-3}$ m^2 s^{-2}. This is at least of the right sign to oppose the longshore pressure gradient. The approximate cross-sectional area of the shelf between the offshore ship track and the coast is ½ × 200 m × 19 km = 1.9×10^6 m^2, so that the estimated momentum flux times the depth could, if its divergence over the cross section is uniform, balance a longshore pressure gradient of 1×10^6 m s^{-2}. This is only 20% of the observed 5×10^{-6} m s^{-2}.

c. The rest of the spectrum

So far we have just considered the onshore momentum flux associated with the spectral peak of the offshore current fluctuations. It is quite possible that the rest of the spectrum also represents topographic waves. We first remark, from the formula $c_x \approx \omega(V/fS)^{1/2}$ of Section 4a, that the onshore trace speed is proportional to the observed frequency ω. This implies that the angular phase lag of inshore currents behind offshore currents should be independent of frequency, and roughly equal to 60°. This is not inconsistent with the phase in the cross spectrum of Fig. 2 over the frequency band for which the coherence is significant.

The momentum flux depends on the value of $l/k \approx (\omega/c_y)(fS/V)^{1/2}$. Assuming that c_y is the same for all frequencies this shows that the higher frequency waves carry proportionately more momentum flux. [Also, the l^2 term in (3.4) remains negligible for ω up to nearly 10 times the peak frequency.] Fig. 7 shows the Reynolds stress spectrum estimated using the above value for l/k and with the spectrum now corrected for the low-pass filter used on the original data. The sum out to 10 cycles/350 days, above which the data is probably more noise than signal, is -1.8×10^{-2} m^2 s^{-2}. So it appears that even including the whole reliable part of the spectrum, the possible onshore momentum flux associated with topographic waves can only balance 40% of the longshore pressure gradient. However, given the uncertainty in the data and its interpretation, it is quite possible that the pressure gradient is in fact balanced by the momentum flux. The question would then arise as to whether this balance was coincidental or a configuration assumed by the system for some dynamical reason. In any case, direct Reynolds stress measurements from moored current meters seem very desirable.

8. Conclusions and discussions

It has been suggested that fluctuations with periods of order 100 days in surface longshore cur-

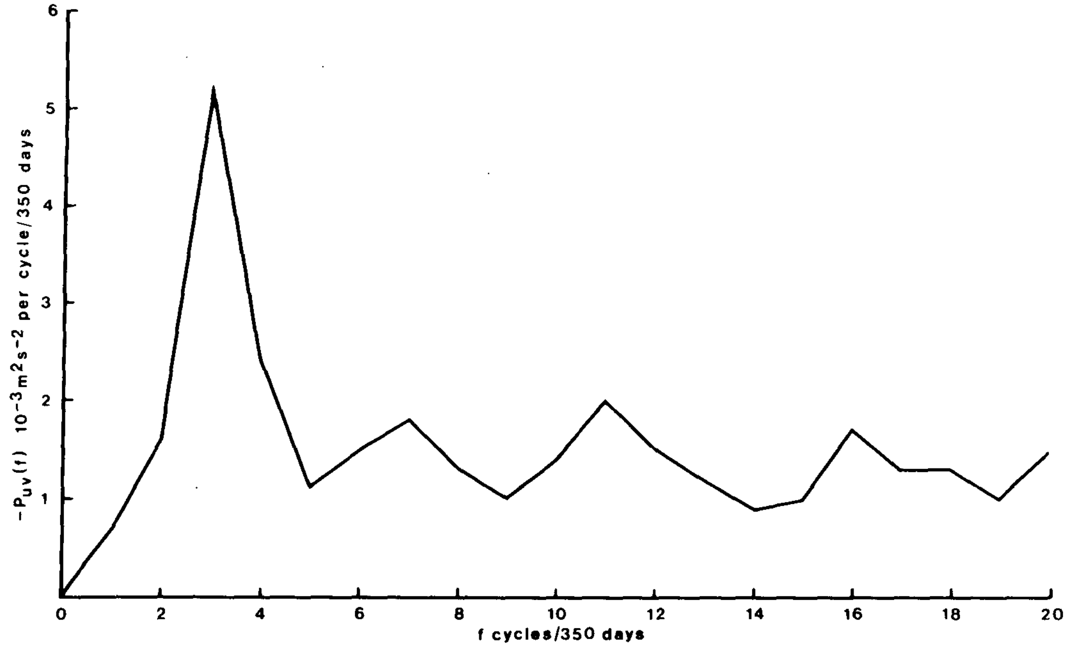

FIG. 7. Spectrum of Reynolds stress for the offshore track assuming that the fluctuations in V correspond to topographic Rossby waves.

rents, found by Hamon *et al.* (1975) in two years of ship drift data within 20 km of the east Australian coast, are due to topographic Rossby waves generated when eddies and meanders in the East Australian Current encounter the continental slope. The time lag between offshore and inshore current fluctuations in the southern sections of the data is within a factor of about 2 of that expected from this interpretation, and the existence of inshore fluctuations in spite of the damping effect of bottom friction is consistent with the relatively high onshore group velocity predicted.

The coastal upwelling generated by an increase in bottom current on the outer region of the shelf can be of considerable magnitude, supporting the qualitative suggestion of Rochford (1972, 1975) and Boland (1978) that upwelling events on the east Australian shelf are caused by offshore currents. Further work is required, however, to combine properly the propagation characteristics of topographic waves on a sloping bottom, and the upwelling or downwelling generated by bottom Ekman flux divergence.

The onshore momentum flux associated with the topographic waves which are held responsible for the current fluctuations appears to be only about 20–40% of that required to balance the estimated longshore pressure gradient, but with considerable room for error in both numbers, so that a balance is possible. If not, and given the comparatively small role of the mean longshore wind stress, the longshore pressure gradient must be balanced by bottom stress, implying a persistent coastal upwelling circulation, although this does not seen compatible with the heat budget of shelf waters. A moored current meter program is required to measure this steady circulation, to calculate the onshore eddy momentum flux at several depths, and to investigate the transient upwelling associated with increases in offshore current.

If the longshore pressure gradient on the shelf is indeed balanced by the eddy momentum flux from offshore, one wonders whether this is coincidental or not. Does an initially strong upwelling response on the shelf cause a large offshore temperature gradient? Does the associated current structure then go baroclinically unstable, and generate onshore momentum flux of just the right magnitude to oppose the pressure gradient? Is this situation a kind of equilibrium?

In any case, the momentum flux onshore must be balanced by a southward acceleration of the East Australian Current in deeper water. In a sense, the longshore pressure gradient on the shelf is not balanced there, but transferred by eddy fluxes to the deeper water offshore, providing a negative sidewall friction for the deepwater circulation!

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