Increasing the Reliability of Reliability Diagrams

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ABSTRACT

The reliability diagram is a common diagnostic graph used to summarize and evaluate probabilistic forecasts. Its strengths lie in the ease with which it is produced and the transparency of its definition. While visually appealing, major long-noted shortcomings lie in the difficulty of interpreting the graph visually; for the most part, ambiguities arise from variations in the distributions of forecast probabilities and from various binning procedures. A resampling method for assigning *consistency bars* to the observed frequencies is introduced that allows for immediate visual evaluation as to just how likely the observed relative frequencies are under the assumption that the predicted probabilities are reliable. Further, an alternative presentation of the same information on probability paper eases quantitative evaluation and comparison. Both presentations can easily be employed for any method of binning.

1. Introduction

Reliability diagrams are common aids for illustrating the properties of probabilistic forecast systems. They consist of a plot of the observed relative frequency against the predicted probability, providing a quick visual intercomparison when tuning probabilistic forecast systems, as well as documenting the performance of the final product; see, for example, Murphy and Winkler (1977, 1987), Atger (2004, 2003), Jolliffe and Stephenson (2003), and Wilks (1995). Yet the visual impression of the reliability diagram can be misleading. Even a perfectly reliable forecast system is not expected to have an exactly diagonal reliability diagram because of limited counting statistics (Jolliffe and Stephenson 2003). To evaluate a forecast system requires some idea as to how far the observed relative frequencies of that forecast system are expected to be from the diagonal if it *was* reliable. This paper provides two methods to visualize this expected deviation from the diagonal, thereby allowing the forecaster to see directly whether the observed relative frequencies fall within the variations to be expected even from a perfectly reliable forecast system.

In the first section, we revisit how reliability diagrams are constructed and explain in detail why limited counting statistics cause even perfectly reliable forecast systems to exhibit deviations from the diagonal. The next two subsections present two alternative approaches toward visualizing this information: the first is a revised set of consistency bars (Smith 1997) computed through a consistency resampling technique, and the second is a replotting of the same information in reliability diagrams on probability paper, providing a rather blunt presentation of the quality of the forecast system. Both methods aim to increase the reliability of interpretations of reliability diagrams. (The code to implement both approaches is available online at http://www.lse.ac.uk/collections/cats/.) We demonstrate the benefit of both approaches with synthetic datasets and show an application to London's Heathrow Airport temperature anomaly forecasts.

2. How to make a reliability diagram

This section explains briefly how reliability diagrams are computed [for an excellent explanation and connec-

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tions to various other statistics, see Wilks (1995)]. The main aim is to introduce the necessary terms and notation in order to facilitate the later discussion on shortcomings of a simple reliability diagram.

The reliability diagram is a diagnostic for probabilistic forecasts. In this paper, we will describe the occurrence or nonoccurrence of the event under concern by a variable Y that is equal to one (event does happen) or to zero (event does not happen). The variable Y is called the *verification.* Let Y_i, $i = 1, \ldots, N$, be a dataset of verifications. For each i we also have a *forecast value* X_i, a number between zero and one, representing a forecast probability that the corresponding Y_i will be equal to one. The forecast value X_i need *not* be assumed to be a probability in a frequentist sense [see Wilks (1995), pp. 9 for a discussion].

Reliability diagrams provide a diagnostic to check whether the forecast value X_i is *reliable.* Roughly speaking, a probability forecast is reliable[1] if the event actually happens with an observed relative frequency consistent with the forecast value. More specifically, considering only instances i for which $X_i = x$ for a certain value x, the event happens with an observed relative frequency equal to x. This definition implicitly assumes that the forecast values X_i can assume only a finite number of values, for example [0, 0.1, 0.2, . . . , 1], but there is an obvious problem with this definition when X_i can assume any value between zero and one. In that case, the event $\{X_i = x\}$ is unlikely to happen more than once for any x, rendering the computation of observed relative frequencies impossible. To be able to compute any nontrivial observed relative frequencies, the forecast values X_i are collected into a number of representative bins. The above definitions are slightly altered thusly: A forecast is reliable if the relative frequency of the event $Y_i = 1$, when computed over all i for which X_i falls into a small interval B, must be equal to the mean of X_i over that interval.

Reliability diagrams reveal reliability by plotting the observed relative frequencies versus the forecast values. If the bins are small, then in the limit of infinitely many forecast values these observed relative frequencies would fall along the diagonal for a reliable forecast. The remainder of the present subsection explains how basic reliability diagrams are computed.

First, the forecast values are partitioned into *bins* B_k, $k = 1, \ldots, K$ (which form a partition of the unit interval into nonoverlapping exhaustive subintervals). The B_k are often taken to be of equal width, but if the distribution of the forecast values is nonuniform, then choosing the bins so that they are equally populated is an attractive alternative.

Next, for each i, it is established which of the K bins the forecast value X_i falls into. For each bin B_k, let I_k be the collection of all indices i for which X_i falls into bin B_k; that is,

$$I_k := \{i; X_i \in B_k\}. \tag{1}$$

The corresponding *observed relative frequency* f_k is the number of times the event happens, given that $X_i \in B_k$, divided by the total number of forecast values $X_i \in B_k$. This can be expressed as

$$f_k = \frac{\sum_{i \in I_k} Y_i}{\#I_k}, \tag{2}$$

where $\#I_k$ denotes the number of elements in I_k.

Each bin B_k is represented by a single "typical" forecast probability r_k. Although the arithmetic center of the bin is often used to represent the forecast values in that bin, this method has a clear disadvantage: If the forecast is reliable, the observed relative frequency for a given bin B_k is expected to coincide with the average of the forecast values over that bin B_k, rather than with the arithmetic center of the bin. Plotting the observed relative frequency over the arithmetic center can cause even a perfect reliability diagram to be off the diagonal by up to half the width of a bin. In this paper, observed relative frequencies for a bin B_k are plotted versus the average of the forecast values over bin B_k. This average, denoted by r_k, is

$$r_k := \frac{\sum_{i \in I_k} X_i}{\#I_k}. \tag{3}$$

The reliability diagram comprises a plot of f_k versus r_k for all bins B_k.

a. Reliable reliability diagrams

The observed relative frequencies f_k for a given bin B_k fluctuate for several reasons. First, if we fix the forecast values falling into bin B_k, then under the hypothesis of reliability, the observed frequencies follow a binomial distribution with parameters I_k and r_k [i.e., the number of forecast values falling into bin B_k and the average of the forecast values over bin B_k, respectively; see Eqs. (1) and (3)]. Second, these two parameters fluctuate as well, with I_k being of larger impact than r_k, especially in bins already containing relatively few samples.

Several approaches to visualizing these effects quantitatively have been suggested. Commonly, a small viewgraph is plotted overlaying the reliability diagram,

[1] Sometimes the expression "calibrated" is used instead of "reliable."

showing the distribution of the forecast values X_i. This kind of plot is also known as a calibration diagram. Although this pair of plots conveys all relevant information, it is difficult to mentally integrate the two graphs to estimate possible variations of the observed relative frequencies; no direct quantitative consistency check is available. In the reliability diagrams of the European Centre for Medium-Range Weather Forecasts (ECMWF) (e.g., Hagedorn et al. 2005), the size of the symbol is often used to reflect I_k, the population of the bin. This is similar to the approach taken by Murphy and Winkler (1977), where the value of I_k is printed.

Although the information is visually displayed in these approaches, neither provides a measure of quantitative agreement with the hypothesis of reliability. In Smith (1997), the expected fluctuations in the observed frequency f_k for each bin are computed using the binomial density, but the r_k [see Eq. (3)] as well as the bin population I_k are assumed to be fixed. This is obviously an idealization, especially if the number of forecast values falling into bin B_k is small or if they are not uniformly distributed.

Our approach, which can be seen as an extension to Smith (1997), is simply to compute the variations of the observed relative frequencies over a set of reliable forecasts generated by a resampling technique referred to as *consistency resampling.* This method computes the fluctuations of the observed relative frequencies f_k taking into account uncertainties arising due to varying bin means r_k as well as bin populations I_k. Let (X_i, Y_i), $i = 1, \ldots, N$, be the dataset consisting of forecast–verification pairs. A single resampling cycle consists of the following steps. We draw N times[2] with replacement from the set X_i, $i = 1, \ldots, N$, obtaining a set of surrogate forecasts $\hat{X}_i$, $i = 1, \ldots, N$. We then create surrogate observations $\hat{Y}_i$, $i = 1, \ldots, N$, by means of

$$\hat{Y}_i = 1 \quad \text{if} \quad Z_i < \hat{X}_i \quad \text{and} \quad 0 \quad \text{else},$$

where Z_i is a series of independent uniformly distributed random variables.

Note that $\hat{X}_i$ is by construction a *reliable forecast* for $\hat{Y}_i$ (see appendix A). A reliability diagram is computed using the surrogate dataset comprising $\hat{X}$ and $\hat{Y}$. The resulting vector of the surrogate observed relative frequencies is recorded. This completes a single resampling step.

The resampling step is repeated N_{boot} times, yielding N_{boot} surrogate observed relative frequencies $\hat{f}_B$ for each bin B_k. We plot the range of the surrogates for each bin as a vertical bar over r_k (the average of the forecast values over bin B_k) in the reliability diagram (see Fig. 1). The bars extend from the 5% to the 95% quantiles, indicated by dashes. Henceforth, these bars will be referred to as *consistency bars*. Consistency bars, along with the observed relative frequencies of the original dataset, allow an immediate visual interpretation of the quality of the probabilistic forecast system. The extent to which the system is calibrated is reflected by where the observed relative frequencies fall within the consistency bars, not their "distance from the diagonal." In a bin with a large number of forecast values, the observed relative frequency may be quite close to the diagonal in terms of linear distance, but quite far in terms of probability. In this case, the consistency bars reflect the expected distances and will clearly indicate the failure of the forecast system. The benefit of the method is illustrated by comparing two synthetically generated datasets. These two datasets were constructed to illustrate a case where the closeness of the observed frequencies to the diagonal does *not* necessarily mean greater consistency with the null hypothesis of the data being reliable; both datasets are slightly unreliable by design. In Fig. 1, a reliability diagram with consistency bars for dataset I is shown. All observed relative frequencies (marked with a plus sign) fall

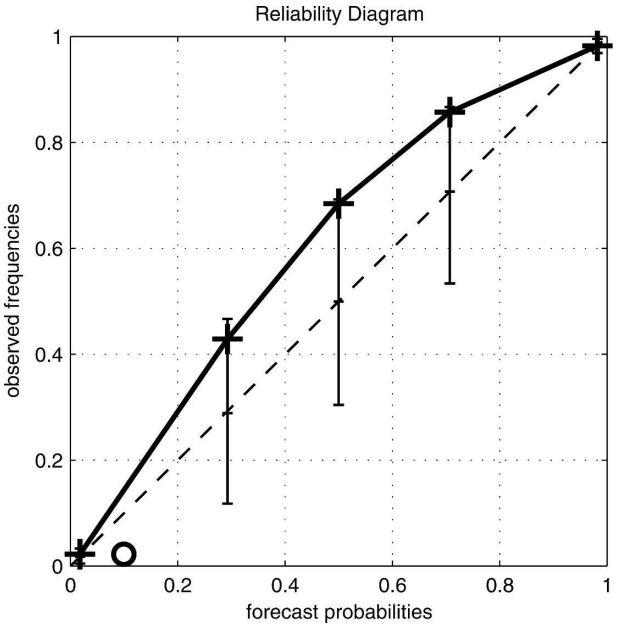


FIG. 1. Reliability diagram for dataset I using consistency bars. The observed relative frequencies all fall within the 5%–95% quantiles (indicated by vertical bars). Although the observed relative frequencies do not fall onto the diagonal, the deviation is still consistent with reliability. The bin boundaries were taken as [0, 0.2, 0.4, 0.6, 0.8, 1]. The observed frequencies are plotted vs r_k [as defined in Eq. (3)]. Plotting versus the bin centers would have caused substantial deviations from the diagonal (circle).

[2] Recall that N is the total length of the dataset.

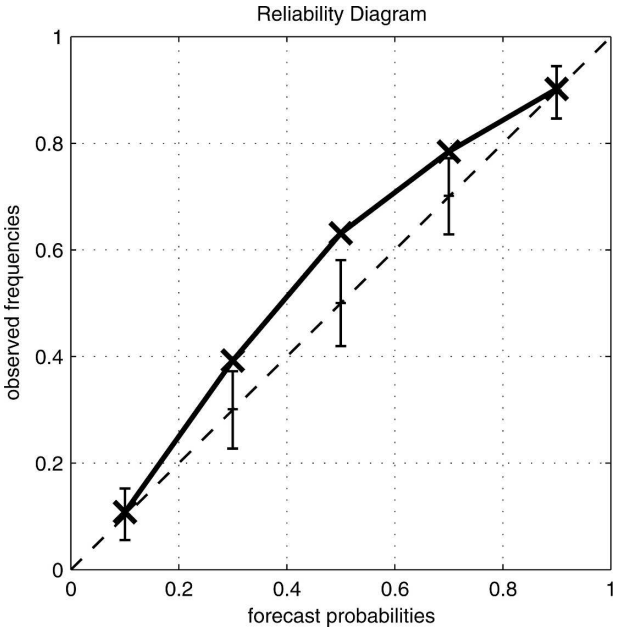


FIG. 2. Reliability diagram for dataset II using consistency bars. Although the observed relative frequencies are closer to the diagonal than in Fig. 1, there are observed relative frequencies that do *not* fall within the 5%–95% bootstrap limits. The bin boundaries were taken as [0, 0.2, 0.4, 0.6, 0.8, 1]. The observed relative frequencies are plotted vs r_k [as defined in Eq. (3)].

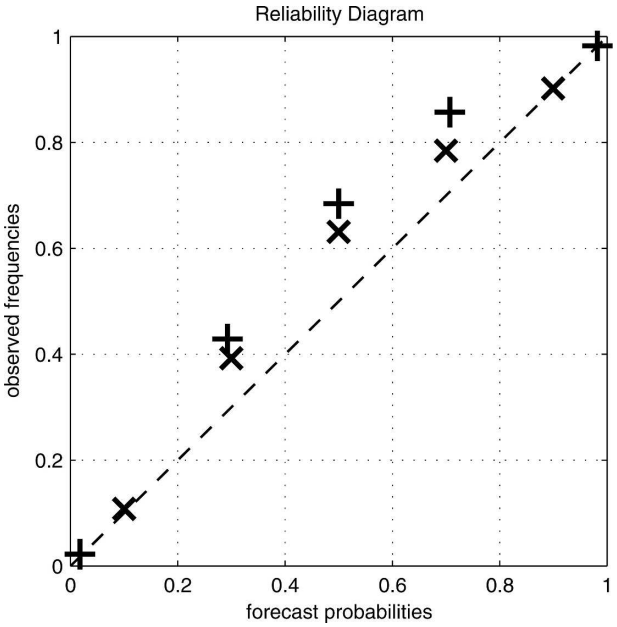


FIG. 3. Reliability diagram without consistency resampling bars for datasets I (plus signs) and II (times signs) used in Figs. 1 and 2, respectively. This plot gives the wrong impression that dataset II represents a more reliable forecast than dataset I.

within the 5%–95% consistency bars. Although the observed relative frequencies are obviously not on the diagonal, the deviation is not inconsistent with what would be expected if the forecast was reliable. Figure 2 shows a reliability diagram with consistency bars for dataset II. The observed relative frequencies (marked with a times sign) are closer to the diagonal than in Fig. 1, but the observed relative frequencies lie further outside the 5%–95% consistency bars. Figure 3 shows again the reliability diagrams for both dataset I (plus sign) and dataset II (times sign), now overlaid in one viewgraph and without consistency bars. Because the observed relative frequencies of dataset II do indeed lie closer to the diagonal, Fig. 3 alone might lead to the false conclusion that dataset II is a more reliable forecast. Another way to see this is by looking at the variance of the observed frequencies for the individual bins. For dataset I, this variance is larger; thus, the deviations from the diagonal are consistent with the sampling errors. For dataset II though, the deviations, albeit smaller than for dataset I, are not consistent with sampling errors, since the variance of the observed frequencies is smaller as well.

Note also that in Figs. 1–3 the observed relative frequencies have been plotted versus r_k [see Eq. (3)] rather than the arithmetic centers of the bins. As is explained in section 2, the observed relative frequencies of a reliable forecast system are expected to be equal to r_k, the average over the forecast values in the bins, not the arithmetic center. The impact of this effect is demonstrated in Fig. 1 for the lowest forecast bin (stretching from 0 to 0.2). By design, the forecast values for this bin are reliable. The distribution of forecast values in this bin is, however, very uneven. Plotting the observed relative frequency versus the arithmetic center of the bin would have caused the observed relative frequency to be off the diagonal (indicated by a circle), giving the false impression that the forecast is unreliable. As an alternative, the consistency bars could be plotted at the arithmetic center of the bin as well. In this case, both the consistency bar and the observed relative frequency for the lowest forecast bin would be off the diagonal, but the observed relative frequency would again fall into the consistency bar, thereby correctly indicating reliability.

b. Reliability diagrams on probability paper

Employing the consistency bars to indicate the distance of the observed relative frequencies from the diagonal *in probability* suggests a new graph, containing essentially the same information as the reliability diagram but plotted differently. In this graph, the x axis still represents the forecast values. The y axis, however, instead of showing the observed relative frequency directly, represents the probability that the observed relative frequency would have been closer to the diagonal

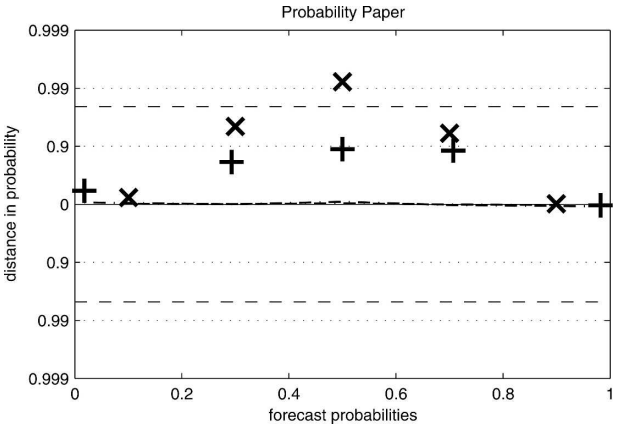


FIG. 4. Reliability diagrams on probability paper for datasets I (plus signs) and II (times signs). Some observed relative frequencies that seemed to be close to the diagonal in Fig. 3 are clearly farther away in probability. The dash–dotted line represents the exact position of the diagonal, which usually falls close to the zero line. For a reliable forecast, we would expect the entire diagram with a 90% chance to fall within the dashed lines.

than the actual observed relative frequency if the forecast was reliable (see Fig. 4). In other words, the y axis gives the likelihood of the observed frequency if the forecast was reliable, rather than the actual value of the observed frequency. This graph, showing the distance in probability of the observed relative frequencies from that expected for a reliable forecast system, will be referred to as a reliability diagram on *probability paper*. If the observed relative frequency fell exactly on top of the consistency bar, for example, its value on probability paper would be 0.9, because there is a 90% chance of the observed relative frequency to be within the range of the consistency bar if the forecast was reliable. The reliability diagrams on probability paper are mirrored vertically along the diagonal. Observed frequencies falling above the diagonal are plotted onto the upper panel, observed relative frequencies falling below the diagonal are plotted onto the lower panel.

Strictly speaking, the y axis in these plots represents the distance in probability from the 50% quantile rather than from the diagonal. These two would coincide if the chance of the observed relative frequency falling either above or below the diagonal were exactly 50%. Although this is not quite the case, we found the difference to be very small (the true position of the diagonal is indicated on probability paper by a dash–dotted line).

In principle, reliability diagrams on probability paper could be computed by the same resampling technique used to compute reliability diagrams with consistency bars (see section 2a), but plotting the fraction of *surrogate* observed frequencies closer to the diagonal than the *actual* observed frequency, rather than plotting the observed frequencies directly. Because reliability diagrams on probability paper require a high resolution at quantiles close to zero and close to one because of the logarithmic y axis, the consistency resampling as presented in section 2a was slightly modified: we used the same resampling to obtain surrogate bin populations, but rather than creating surrogate observed frequencies directly, we determined the value on the y axis by employing the binomial distribution. In detail, this is done as follows. As in section 2a, surrogate forecasts are created, from which we obtain surrogate bin populations $\hat{I}_k$ and surrogate representative forecasts $\hat{r}_k$. If the forecast were reliable, the number of surrogate events in each bin would follow a binomial distribution with parameters $\hat{I}_k$ and $\hat{r}_k$. Consequently, the fraction z_k of surrogate observed frequencies smaller than the actual observed frequency f_k is given by

$$z_k = \mathcal{B}([f_k\hat{I}_k]; \hat{I}_k, \hat{r}_k),$$

where $\mathcal{B}$ is the cumulative binomial distribution and $[f_k\hat{I}_k]$ is $f_k\hat{I}_k$, rounded to the nearest integer. The reliability diagram on probability paper comprises a plot of z_k versus f_k.

Figure 4 shows reliability diagrams for the two synthetic datasets considered in the previous section, but plotted on probability paper. Again, dataset I is plotted with plus signs, the dataset II is plotted using times signs. Dataset II, seemingly closer to the diagonal in the standard reliability diagram Fig. 3, is clearly farther away in probability for three out of five bins, with another bin effectively being a tie. This indicates that dataset II is actually less reliable than dataset I. The dash–dotted line represents the exact position of the diagonal, which usually falls close to the zero line.

Some general hints as to how reliability diagrams on probability paper should be read seem to be in order. For a given bin, there is a 90% chance the observed relative frequency falls within the range labeled 0.9 on the y axis. Likewise, there is a 99% chance the observed relative frequency falls within the range labeled 0.99, etc. Thus, the chance of being outside the 0.9 band in any one bin is 0.1. Of course the chance of seeing *some* points of the entire plot falling outside the 0.9 band is larger. For example, the chance of all points on a reliability diagram with six bins falling inside the 0.9 band is only $0.9^6 \approx 0.53$. A band that would encompass *all points* with a 90% chance is indicated by the dashed line.[3] In other words, if the forecast is reliable, then

[3] This line indicates the Bonferroni corrected 90% level (see Bonferroni 1936). Independence of the individual bins was assumed for this calculation.

there is a 90% chance that the entire diagram falls within the *dashed* line.

3. Relation of consistency resampling and the consistency bars with the common bootstrap

Consistency resampling is related to, but distinct from, bootstrap resampling in statistics (Efron and Tibshirani 1993). Both are resampling techniques, but the consistency resampling differs from the common bootstrap of statistics in that the latter bootstrap resamples the data to extract an estimate of the uncertainty in the statistic of interest (in this case this would be the observed relative frequencies), while the consistency resampling quantifies the range of results expected if the forecast values were in fact correct probabilities. While the traditional bootstrap would resample the forecast values, thereby quantifying the expected variation in the observed relative frequencies, the consistency resampling quantifies the range of relative frequencies that would be expected if the predicted probabilities were, in fact, reliable. This also differs from the common use of surrogate data in geophysics, where a null test is set up in the hope that it will be rejected (for a discussion, see Smith and Allen 1996). Both consistency resampling and bootstrap resampling are common in statistics (see Efron and Tibshirani 1993), and both are confusingly referred to as the "bootstrap." The two techniques address different questions and yield different information, so distinguishing them is important.

Two very different approaches to bootstrap resampling can be applied to the reliability diagram: one resampling by bin, the other resampling the entire diagram. Resampling within each bin yields the sampling uncertainty in the relative frequency of that bin; this is easily displayed by either form of the diagram and the resample realizations in different bins are independent. Alternatively, resampling from the entire diagram will alter the number of forecasts in each bin (or even the bins themselves) and introduces an interdependency between the bootstrap realizations across the diagram. This interdependency makes the entire diagram alternative more difficult to evaluate visually, suggesting that it is best displayed in a manner that evaluates the diagram as a whole, and while both alternatives have their uses we consider only the by-bin method in the remainder of this paper.

The presentation of the reliability diagram on probability paper offers the possibility of using both the consistency resampling and the common bootstrap in parallel. After plotting the reliability diagram on probability paper, common bootstrap resamples for each bin can be added to the plot to give an immediate visual impression of the sampling uncertainty in each frequency. Examples including this addition are discussed at the end of the following section.

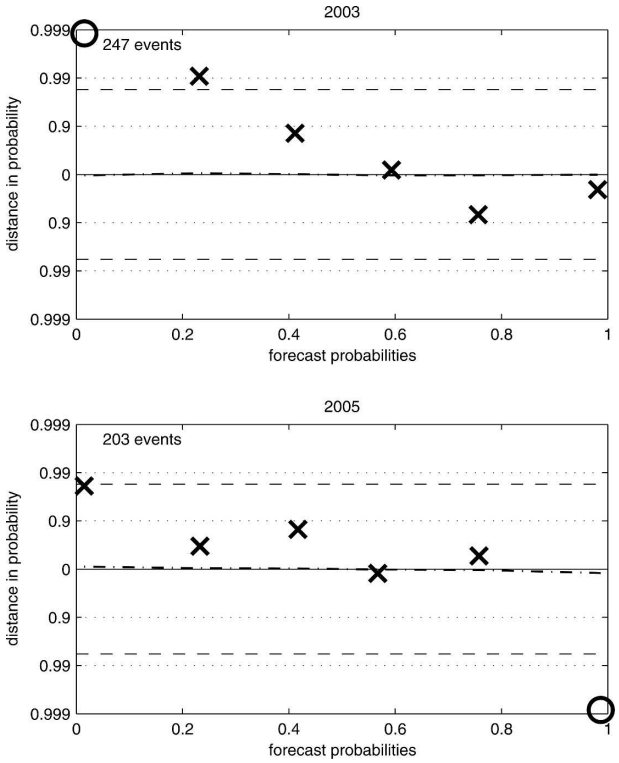
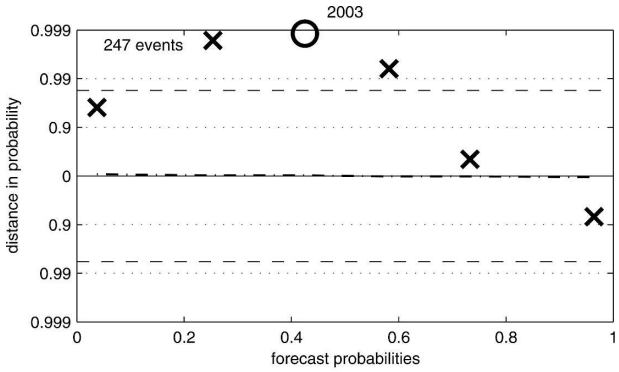


FIG. 5. Reliability diagrams on probability paper for 2-m temperature forecasts (above or below monthly average) at Heathrow Airport. Forecasts were provided by ECMWF's ensemble forecasts. The lead time is 1 day. The top (bottom) panel shows the performance for 2003 (2005). A large circle indicates an observed frequency outside the range of the y axis.

4. Numerical examples

Consider daily forecasts of the 2-m temperature at London's Heathrow Airport weather station taken at 1200 LT, specifically whether this value exceeded the monthly average computed for the previous 21 yr of data (1980–2000). The forecast was a 51-member ensemble, provided by downscaling output from ECMWF's global ensemble forecasting system (see appendix B for details of this procedure). The forecast value X_i was taken as the fraction of ensemble members exceeding the current monthly average.

Reliability diagrams where plotted for Heathrow Airport, contrasting the years 2003 and 2005 (containing 365 forecast instances each) for lead times of 1, 3, and 10 days. Figures 5–7 show reliability diagrams on probability paper, while Figs. 8–10 show conventional reliability diagrams with consistency bars. The overall



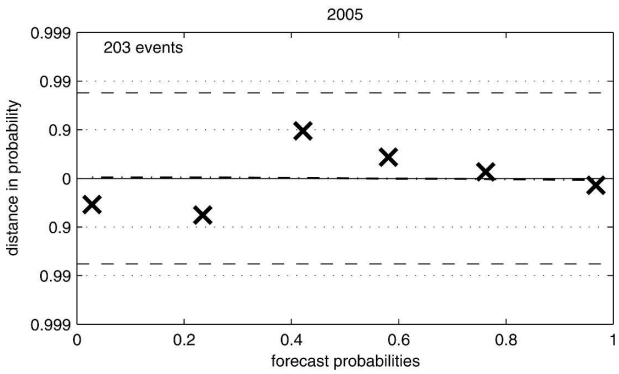
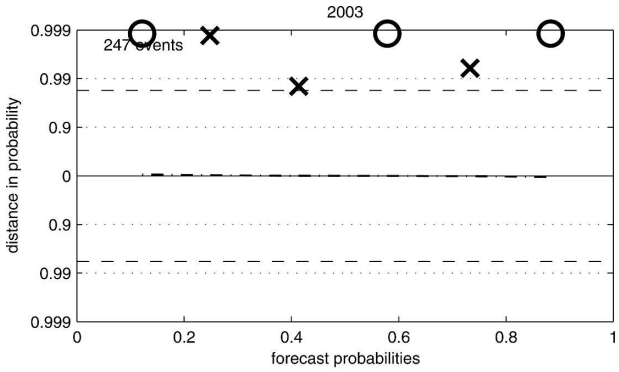


FIG. 6. Reliability diagrams on probability paper for 2-m temperature forecasts (above or below monthly average) at Heathrow. Forecasts were provided by ECMWF's ensemble forecasts. The lead time is 3 days. The top (bottom) panel shows the performance for 2003 (2005). A large circle indicates an observed frequency outside the range of the *y* axis.



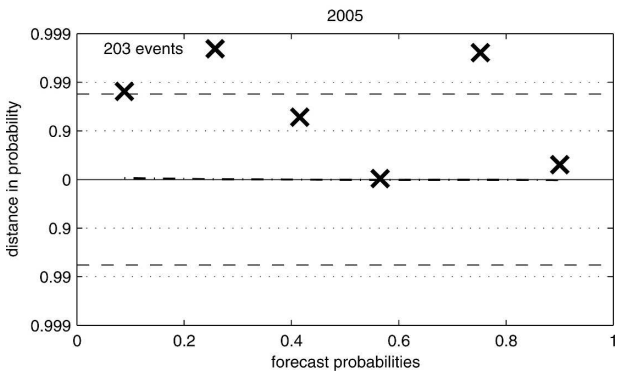


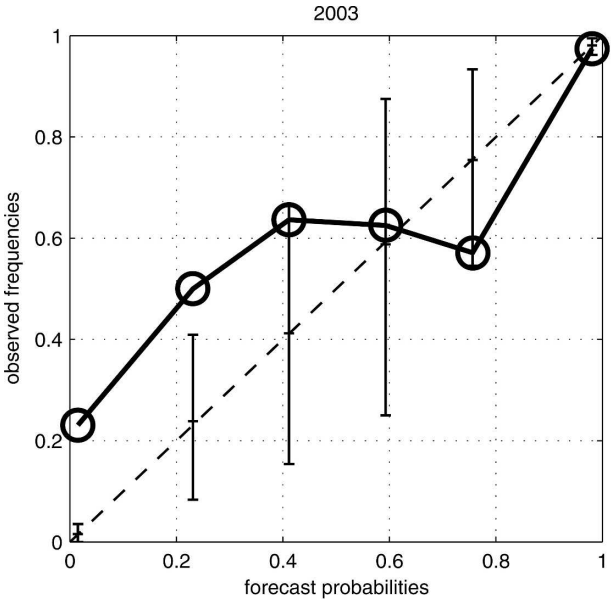
FIG. 7. Reliability diagrams on probability paper for 2-m temperature forecasts (above or below monthly average) at Heathrow. Forecasts were provided by ECMWF's ensemble forecasts. The lead time is 10 days. The top (bottom) panel shows the performance for 2003 (2005). A large circle indicates an observed frequency outside the range of the *y* axis.

impression is that, as far as reliability is concerned, the forecasts have improved.

For 1 day lead time (Figs. 5 and 8), the reliability of 2003 and 2005 is generally comparable. For forecast probabilities between 0.4 and 0.8, the observed relative frequencies fall within the 90% range for both years. It seems though that in 2003 the forecast system struggled to get the lower probabilities right (there were generally more events than the forecast probabilities would suggest), while in 2005 the system forecasts the high probabilities poorly. The confidence bars for the extreme events are very tight. This is a typical situation where the forecast is shown to be unreliable, although the observed relative frequencies appear "close to the diagonal" in terms of usual distance.

In 2003, there were considerably more days where the temperature exceeded the monthly average than in 2005 (247 versus 203), the year 2003 being one of the hottest on record in Europe. Therefore, a system usually overestimating the frequency of days hotter than normal would have scored better in 2003 than in 2005. It is not clear though whether high or low forecast probabilities would have been affected more. For lead time 3 days (Figs. 6 and 9), the forecast was better in 2005 in every single bin and is accepted as reliable at a 0.9 significance level. In fact, it is accepted even at a significance level of $0.9^6 \approx 0.53$, because all points fall within the 0.9 confidence band (see discussion at the end of section 2b). For lead time 10 days (Figs. 6 and 9), the forecast system seems to have given probabilities that were systematically too low in 2003. Since this bias is fairly uniform over different bins, simply subtracting a constant offset would have improved the performance. The forecast system obviously improved in 2005, although the overall reliability appears to be not as good as for lead time 3 days.

Using the same 3-day lead time data as in Figs. 6 and 9, Fig. 11 shows the traditional reliability diagram with by-bin bootstrap-resampled observed frequencies in addition to the consistency bars. Bootstrap resamples are shown as small circles while the consistency bars for the original forecasts is shown as before. Technically, each bootstrap resample has its own corresponding consistency bar; the dots here only reflect sampling un-



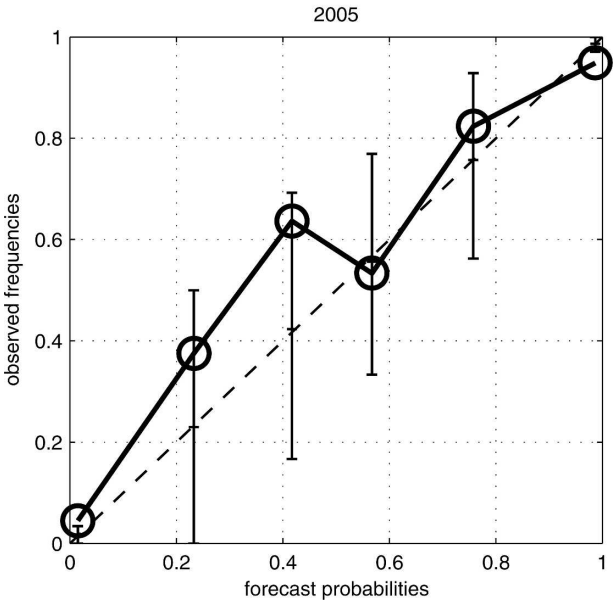
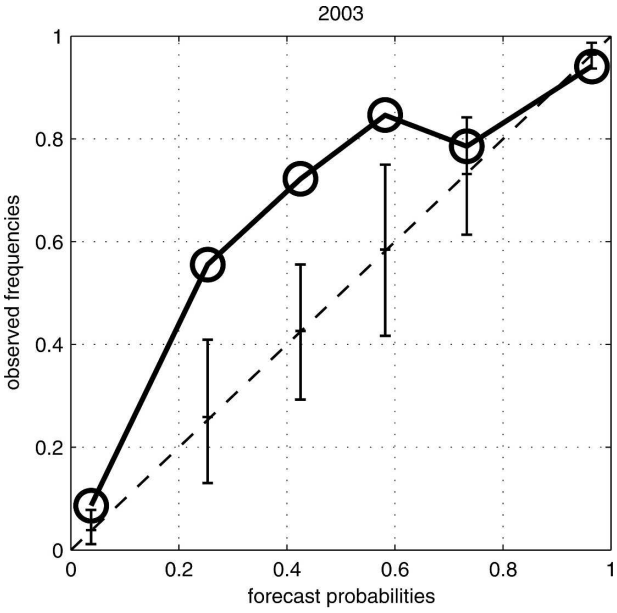


FIG. 8. Reliability diagrams with consistency bars for 2-m temperature forecasts (above or below monthly average) at Heathrow. Forecasts were provided by ECMWF's ensemble forecasts. The lead time is 1 day. The top (bottom) panel shows the performance for 2003 (2005).



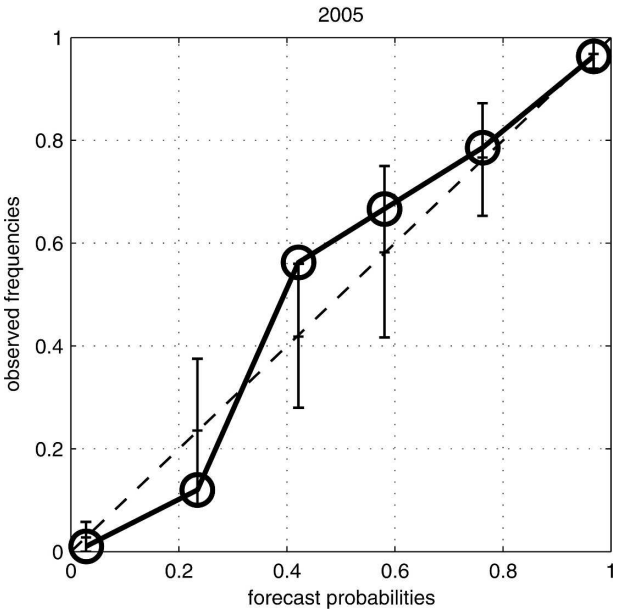


FIG. 9. Reliability diagrams with consistency bars for 2-m temperature forecasts (above or below monthly average) at Heathrow. Forecasts were provided by ECMWF's ensemble forecasts. The lead time is 3 days. The top (bottom) panel shows the performance for 2003 (2005).

certainty for each bin: for instance, is resampling likely to yield a value "near" the diagonal? This is, in fact, much more common in the 2005 forecasts than the 2003 forecasts. Note that variations in the variance of the dots indicate bins with small populations.

Figure 12 shows the same analysis on probability paper, in effect adding the bootstrap resample dots to Fig. 6. This figure is constructed to be consistent with Fig. 11: each dot here is the resampled frequency in terms of the probability defined by the true-sample frequency; an interesting alternative (not shown) is to place each dot at the individual probability implied by that bootstrap resample. The choice of which resampling technique to apply is determined by the question to be resolved; to avoid confusion, the details of the bootstrap scheme should be stated in every case.

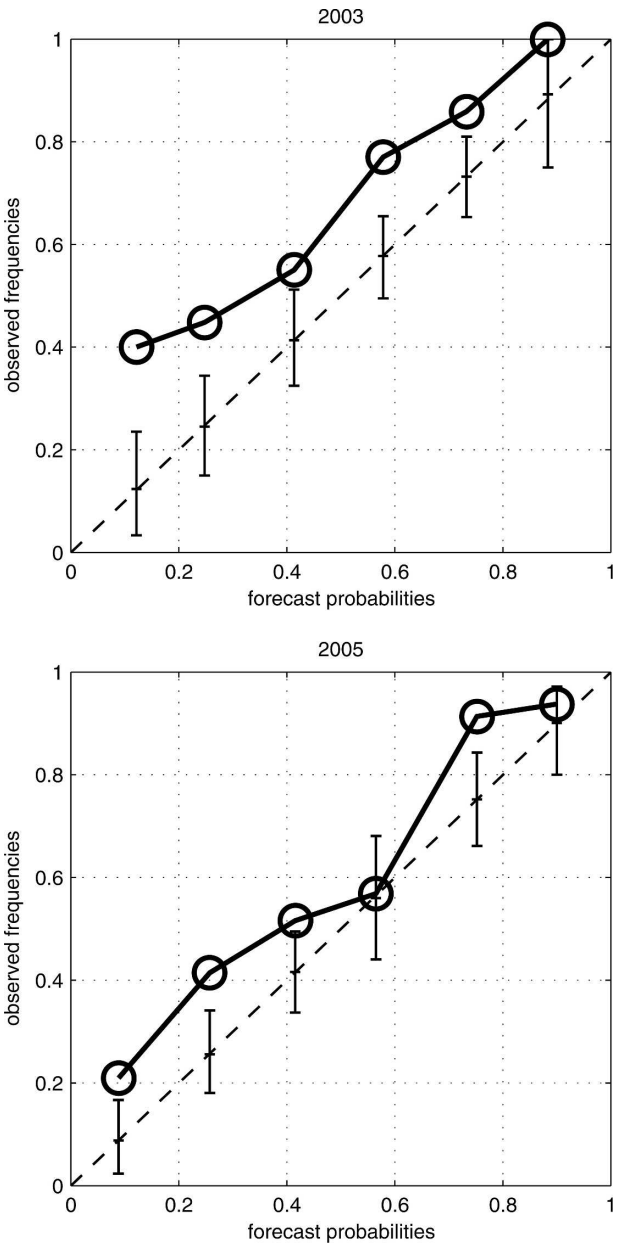


FIG. 10. Reliability diagrams with consistency bars for 2-m temperature forecasts (above or below monthly average) at Heathrow. Forecasts were provided by ECMWF's ensemble forecasts. The lead time is 10 days. The top (bottom) panel shows the performance for 2003 (2005).

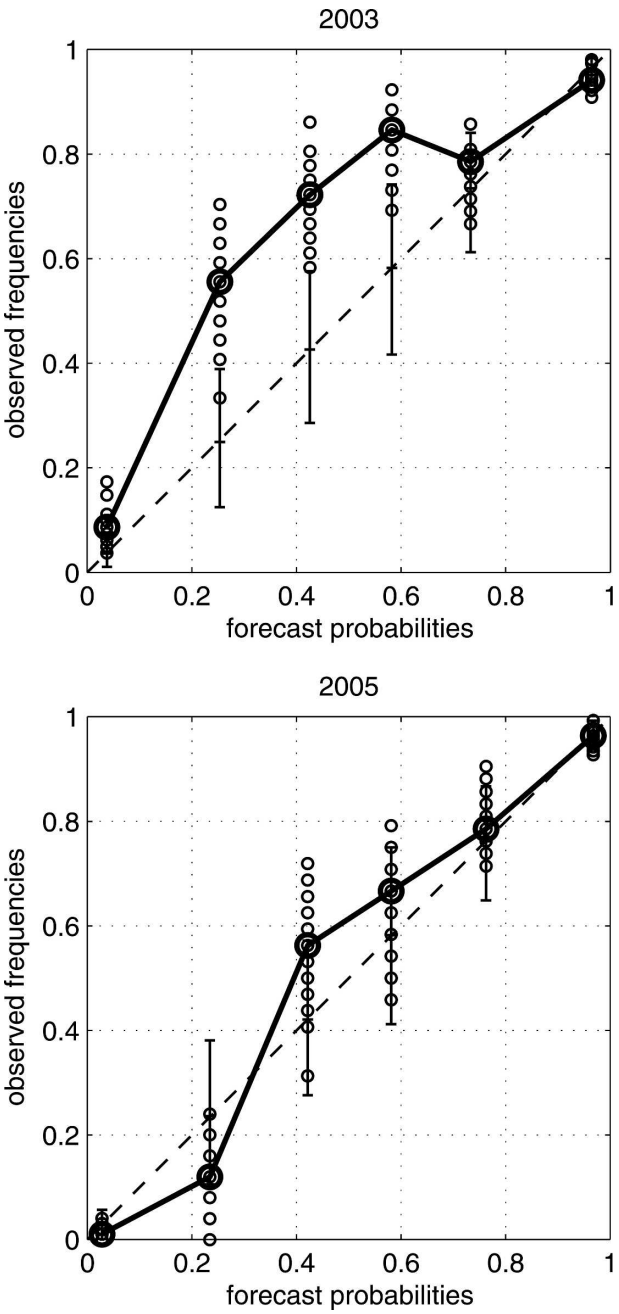


FIG. 11. Reliability diagrams with consistency bars and resampled observed frequencies (small circles). The data are the same as for Fig. 9 and were resampled 20 times. The actual observed frequencies are indicated with a large circle.

Figures 11 and 12 add confidence to the conclusions drawn from Figs. 6 and 9, namely that the forecast system is more reliable in 2005 than in 2003. This is clear after taking into account uncertainties in the observed frequencies (to the extent that this is simulated by resampling the original data). The resampled observed frequencies are much more often within the confidence bars for the 2005 data than for the 2003 data (see Fig. 11). Similarly on probability paper, the resampled observed frequencies fall within higher probability regions for the 2005 data than for the 2003 data (note that in the 2003 data, dots even fall beyond the range of the probability graph in three of the bins).

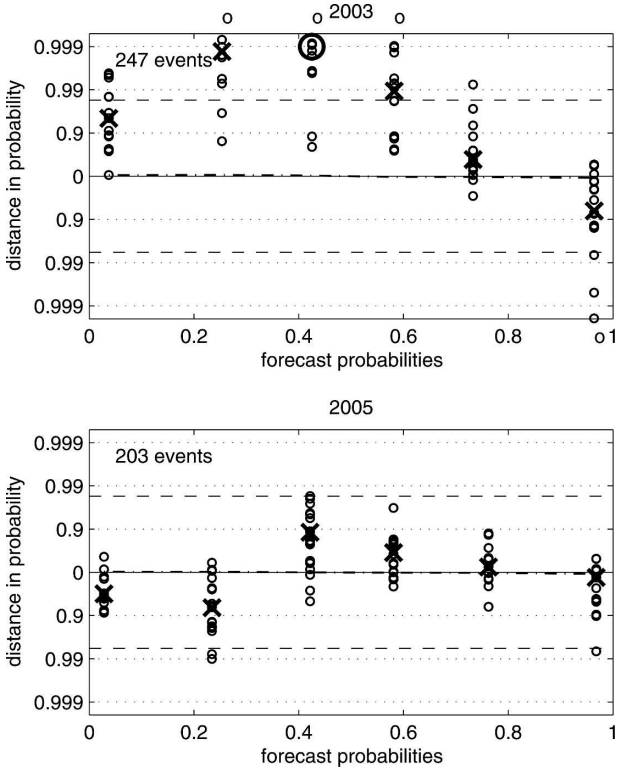


FIG. 12. Reliability diagram on probability paper with resampled observed frequencies (small circles). The data are the same as for Fig. 6 and were resampled 20 times. The actual observed frequencies are indicated with an X, as before. The large (little) circles either above or below the panel indicate that some actual (resampled) observed frequencies are outside the range of the *y* axis.

5. Conclusions

The reliability diagram is a common diagnostic aid for quickly evaluating the reliability of probabilistic forecast systems. A reliable forecast should have a reliability diagram close to the diagonal, but how close exactly? Both forecasts and observed relative frequencies fluctuate, and to interpret a reliability diagram correctly, these deviations have to be taken into account. We have introduced a consistency resampling method for assigning consistency bars to the diagonal of the reliability diagram, indicating the region where a reliable forecast would fall into, or, in other words, how likely the observed relative frequencies are under the assumption that the predicted probabilities are accurate.

We have shown a numerical example using synthetic data where two forecasts are compared using the proposed technique. One of the forecasts, although seemingly closer to the diagonal than the other, turns out to be farther away in probability and therefore constitutes a less reliable forecasts. The method was also applied to anomaly data for Heathrow Airport temperature and ECMWF ensemble forecasts from 2003 and 2005. The overall reliability appears to have improved. We argue that the more reliable reliability diagrams as introduced in this paper add more credibility to this finding than conventional reliability diagrams.

Two approaches to make the quantification of just how reliable the forecast system is more visually accessible have been suggested; each contains the same reliability information as the traditional diagram. Examining a variety of meteorological forecast systems suggests that in practice diagrams that look "good" in the traditional presentation often have many points *well beyond* the 0.90 probability threshold (suggesting the forecast system is not reliable). We hope these new diagrams make reliability diagrams even more valuable in practice.

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APPENDIX A

Generating Data with a Specified Reliability Diagram

This appendix discusses how to create artificial forecast verification pairs with a given reliability diagram and Brier skill score. Let $p_i \in [0, 1]$, $i = 1, \ldots, N$ be identically distributed (iid) random variables with a probability density function $g(p)$. To generate a random variable $Y_i \in [0, 1]$, $i = 1, \ldots, N$ for which p_i is a reliable forecast, draw another random variable R_i, independent from p_i, from a uniform distribution on the unit interval and then set

$$Y_i = \begin{cases} 1 & \text{if} \quad R_i < p_i \\ 0 & \text{else} \end{cases}. \tag{A1}$$

To see that this forecast is reliable, note that

$$\begin{aligned} P(Y_i = 1 | p_i = z) &= P(R_i < p_i | p_i = z) \\ &= P(R_i < z | p_i = z) \\ &= P(R_i < z) = z, \end{aligned}$$

where the equality signs follow (in that order) from the definition of Y_i, the properties of conditional probabilities, the fact that R_i is independent of p_i, and from its

uniform distribution. This technique is used to generate "fake" verifications consistent with a given set of forecast values to draw consistency resampling bars (see section 2a). Draws from an unreliable forecasts with a specified reliability graph (i.e., with a specified reliability diagram in the large sample limit) can be generated as well. If $r(p)$ is the desired reliability graph, generate Y_i by applying Eq. (A1) but using $r(p_i)$ instead of p_i. Then the limiting observed relative frequencies corresponding to p_i are $r(p_i)$, as desired.

APPENDIX B

Downscaling the ECMWF Data

This appendix explains the method used to downscale the ECMWF data. ECMWF publishes the output of their global model on a certain grid. This grid lacks a point *exactly* at Heathrow Airport, but even if it did include such a point, we would not expect the forecast at that grid point to be all the model has to say about temperatures around Heathrow. Furthermore, ECMWF also publishes a forecast for Heathrow explicitly, which is interpolated from several neighboring grid points. This interpolation is done according to an interpolation scheme that does not involve any fitting to actual station data (this is why we use the word "interpolation" rather than "fitting"). Using these five ensemble forecasts (the four neighboring grid points and ECMWF's interpolated forecast), featuring 51 ensemble members each, we produce a forecast for Heathrow using a linear fit as follows. Letting $x_1, \ldots, x_5$ denote the mean values of the five ensemble forecasts, we find coefficients $a_0, \ldots, a_5$ by fitting

$$z = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5 \quad \text{(B1)}$$

to the observations at Heathrow using a least squares error criterion. Then Eq. (B1) is applied to the entire ensembles (rather than just the means) to find the final ensemble. The linear fit is carried out using 1 yr worth of data (2001). The same procedure is applied to each lead time individually. Note that the rest of the analysis described in this paper is carried out on different data, namely on the years 2002–05.

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