CORRIGENDA

Corrigendum

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The purpose of this corrigendum is to address errors in several equations that were discovered in Mahalik et al. (2019, hereafter M19), which presented the derivations for both the azimuthal and divergent shear linear least squares derivative (LLSD) equations that serve as the foundation for the publication. While two of the errors are minor and typographical in nature, additional errors in two of the matrices, the coefficient matrix [M19's Eq. (8)] and the adjugate matrix [M19's Eq. (10)], introduce incorrect components within the final versions of the LLSD horizontal shear equations provided in M19's Eqs. (12a) and (12b), in addition to their fully expanded forms in M19's appendixes A and B. The errors and their subsequent corrections are addressed below in the order that they are encountered within M19.

The first two equation errors in M19 are both typographical in nature. The Δ (delta) modifier was incorrectly excluded from the range component of the two-dimensional radar variable u_{ij}, shown in M19's Eq. (5). This delta is needed as it describes the offset in range from the center of the kernel in which the derivatives are being calculated. The updated M19's Eq. (5) is as follows:

$$R = \sum_{k=0}^{m\times n} w_k[u(\Delta r_k, \Delta\theta_k) - (u_0 + u_r\Delta r_k + u_\theta\Delta\theta_k)]^2. \tag{1}$$

Similarly, the Δ (delta) modifier was incorrectly excluded from the azimuth offset in M19's Eq. (6c):

$$\frac{\partial R}{\partial u_0} = 0 = \sum_{k=0}^{m\times n} 2(-u_k + u_0 + u_r\Delta r_k + u_\theta\Delta\theta_k)w_k\,. \tag{2}$$

The remainder of the equation errors are attributed to an incorrect setup of both the coefficient matrix used to express the complete system of LLSD equations and its associated adjugate matrix. M19's Eq. (8), which outlines the complete system of LLSD equations expressed in matrix form, contains three matrices, the details of which are outlined in M19. Out of the three matrices, only the coefficient matrix contains errors. These errors relate only to the cross-diagonal coefficients, which are incorrectly switched across the diagonal, not allowing the original LLSD equations in M19's Eqs. (7a)–(7c) to be generated when the coefficient and variable matrices are multiplied. To correct this, these cross-diagonal coefficients are held in their original element positions, producing the corrected M19's Eq. (8) below:

$$\begin{bmatrix} \Sigma w_k\Delta r_k\Delta\theta_k & \Sigma w_k\Delta r_k^2 & \Sigma w_k\Delta r_k \\ \Sigma w_k\Delta\theta_k^2 & \Sigma w_k\Delta r_k\Delta\theta_k & \Sigma w_k\Delta\theta_k \\ \Sigma w_k\Delta\theta_k & \Sigma w_k\Delta r_k & \Sigma w_k \end{bmatrix} \begin{bmatrix} u_\theta \\ u_r \\ u_0 \end{bmatrix} = \begin{bmatrix} \Sigma w_k\Delta r_k u_k \\ \Sigma w_k\Delta\theta_k u_k \\ \Sigma w_k u_k \end{bmatrix}. \tag{3}$$

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As discussed in M19 and shown by M19's Eq. (9), the derivatives of the individual LLSD azimuthal and divergent shear components are found by multiplying the answer matrix by the inverse of the coefficient matrix. This inverse is found by calculating both the determinate and adjugate of the coefficient matrix. The adjugate matrix shown in M19's Eq. (10), however, is not set up correctly. Part of the process of creating the adjugate matrix involves switching the cross-diagonal elements. In the case of M19's Eq. (10), only the a_{23} and a_{32} elements were switched. To correctly create the adjugate matrix, the a_{12} element must be switched with the a_{21} and the a_{13} element switched with the a_{31}. Adjusting M19's Eq. (10) to account for these errors produces the following corrected equation:

$$\mathbf{M}^{-1} = \frac{1}{D}\mathrm{adj}(\mathbf{M}) = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \left(\frac{1}{D}\right). \tag{4}$$

Since the adjugate matrix shown in M19's Eq. (10) had to be corrected, all downstream equations have been adjusted using the corrected adjugate matrix. The corrected versions of these downstream equations are displayed below where Eq. (5) is the corrected form of M19's Eq. (11), Eq. (6) is the corrected form of M19's Eq. (12a), and Eq. (7) is the corrected form of M19's Eq. (12b):

$$\begin{bmatrix} u_\theta \\ u_r \\ u_0 \end{bmatrix} = \begin{bmatrix} a_{11}/D & a_{21}/D & a_{31}/D \\ a_{12}/D & a_{22}/D & a_{32}/D \\ a_{13}/D & a_{23}/D & a_{33}/D \end{bmatrix} \begin{bmatrix} \Sigma w_k \Delta r_k u_k \\ \Sigma w_k \Delta\theta_k u_k \\ \Sigma w_k u_k \end{bmatrix}, \tag{5}$$

$$u_\theta = \sum_{k=0}^{m\times n} w_k \Delta r_k u_k \left(\frac{a_{11}}{D}\right) + \sum_{k=0}^{m\times n} w_k \Delta\theta_k u_k \left(\frac{a_{21}}{D}\right) + \sum_{k=0}^{m\times n} w_k u_k \left(\frac{a_{31}}{D}\right), \tag{6}$$

$$u_r = \sum_{k=0}^{m\times n} w_k \Delta r_k u_k \left(\frac{a_{12}}{D}\right) + \sum_{k=0}^{m\times n} w_k \Delta\theta_k u_k \left(\frac{a_{22}}{D}\right) + \sum_{k=0}^{m\times n} w_k u_k \left(\frac{a_{32}}{D}\right). \tag{7}$$

The corrected versions of M19's appendixes A and B, the expanded forms of M19's Eqs. (12a) and (12b) [Eqs. (6) and (7) above, respectively] are shown, respectively, in Eqs. (8) and (9) below. Using Eq. (3), the original determinate in M19's appendixes A and B was also corrected and is also shown in Eqs. (8) and (9):

$$\begin{aligned} u_\theta = {} & \Bigg(\sum_{k=0}^{m\times n} \Delta r_k \Delta\theta_k \sum_{k=0}^{m\times n} \Delta r_k \Delta\theta_k \sum_{k=0}^{m\times n} w_k - 2\sum_{k=0}^{m\times n} \Delta r_k \Delta\theta_k \sum_{k=0}^{m\times n} \Delta\theta_k \sum_{k=0}^{m\times n} \Delta r_k \\ & + \sum_{k=0}^{m\times n} \Delta r_k^2 \sum_{k=0}^{m\times n} \Delta\theta_k \sum_{k=0}^{m\times n} \Delta\theta_k - \sum_{k=0}^{m\times n} \Delta r_k^2 \sum_{k=0}^{m\times n} \Delta\theta_k^2 \sum_{k=0}^{m\times n} w_k + \sum_{k=0}^{m\times n} \Delta r_k \sum_{k=0}^{m\times n} \Delta\theta_k^2 \sum_{k=0}^{m\times n} \Delta r_k \Bigg)^{-1} \\ & \Bigg[\sum_{k=0}^{m\times n} \Delta r_k u_k \left(\sum_{k=0}^{m\times n} \Delta r_k \Delta\theta_k \sum_{k=0}^{m\times n} w_k - \sum_{k=0}^{m\times n} \Delta\theta_k \sum_{k=0}^{m\times n} \Delta r_k\right) \\ & + \sum_{k=0}^{m\times n} \Delta\theta_k u_k \left(\sum_{k=0}^{m\times n} \Delta r_k \sum_{k=0}^{m\times n} \Delta r_k - \sum_{k=0}^{m\times n} r_k^2 \sum_{k=0}^{m\times n} w_k\right) \\ & + \sum_{k=0}^{m\times n} u_k \left(\sum_{k=0}^{m\times n} \Delta r_k^2 \sum_{k=0}^{m\times n} \Delta\theta_k - \sum_{k=0}^{m\times n} \Delta r_k \sum_{k=0}^{m\times n} \Delta r_k \Delta\theta_k\right)\Bigg], \end{aligned} \tag{8}$$

$$\begin{aligned} u_r = {} & \Bigg(\sum_{k=0}^{m\times n} \Delta r_k \Delta\theta_k \sum_{k=0}^{m\times n} \Delta r_k \Delta\theta_k \sum_{k=0}^{m\times n} w_k - 2\sum_{k=0}^{m\times n} \Delta r_k \Delta\theta_k \sum_{k=0}^{m\times n} \Delta\theta_k \sum_{k=0}^{m\times n} \Delta r_k \\ & + \sum_{k=0}^{m\times n} \Delta r_k^2 \sum_{k=0}^{m\times n} \Delta\theta_k \sum_{k=0}^{m\times n} \Delta\theta_k - \sum_{k=0}^{m\times n} \Delta r_k^2 \sum_{k=0}^{m\times n} \Delta\theta_k^2 \sum_{k=0}^{m\times n} w_k + \sum_{k=0}^{m\times n} \Delta r_k \sum_{k=0}^{m\times n} \Delta\theta_k^2 \sum_{k=0}^{m\times n} \Delta r_k \Bigg)^{-1} \\ & \Bigg[\sum_{k=0}^{m\times n} \Delta r_k u_k \left(\sum_{k=0}^{m\times n} \Delta\theta_k \sum_{k=0}^{m\times n} \Delta\theta_k - \sum_{k=0}^{m\times n} \theta_k^2 \sum_{k=0}^{m\times n} w_k\right) + \sum_{k=0}^{m\times n} \Delta\theta_k u_k \left(\sum_{k=0}^{m\times n} \Delta r_k \Delta\theta_k \sum_{k=0}^{m\times n} w_k\right. \\ & \left. - \sum_{k=0}^{m\times n} \Delta r_k \sum_{k=0}^{m\times n} \Delta\theta_k\right) + \sum_{k=0}^{m\times n} u_k \left(\sum_{k=0}^{m\times n} \Delta r_k \sum_{k=0}^{m\times n} \theta_k^2 - \sum_{k=0}^{m\times n} \Delta r_k \Delta\theta_k \sum_{k=0}^{m\times n} \Delta\theta_k\right)\Bigg]. \end{aligned} \tag{9}$$

It should be noted that while M19 did not include the correct final version of the LLSD azimuthal shear (AzShear) equation, the correct version is employed in the operational code set that is utilized in the workflow to generate the rotation track products within the current Multi-Radar Multi-Sensor (MRMS) system. In addition, the figures in M19 are unaffected by these equation errors.

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