VISUALIZING VAPOR PRESSURE

A Mechanical Demonstration of Liquid–Vapor Phase Equilibrium

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BUILDING A VIBRATING-PLATE DEMONSTRATION. The mechanical demonstration described in this paper is relatively straightforward and inexpensive to build, assuming the user has access to a signal generator (or similar). If it is to be used for quantitative use—say, in a physical meteorology laboratory course—then a little more expense and effort is required. We outline the components we used for the system below; in many cases, components could be substituted:

- mechanical wave driver, or vertically oriented speaker with mounting bar (PASCO SF-9324);
- function generator (we used Pasco PI-9587C, now available as PASCO PI-8127; any typical function generator will work);
- aluminum plate with recessed well and slight taper, anodized or painted black for viewing (custom built);
- mounting bracket to attach plate to speaker (custom built);
- small ball bearings or BBs (commonly available, e.g., hobby shop or McMaster-Carr); and
- leveling base (optional, custom built).

For quantitative measurements:

- laser pointer (commonly available, or Edmund Optics),
- small mirror (commonly available, or Edmund Optics),
- spherical mirror for amplification (Edmund Optics), and
- measuring scale (commonly available).

UNCERTAINTY CALCULATIONS FOR THE TWO-LEVEL VIBRATING PLATE. Given measures of the natural variability in the number of balls counted on the upper level N_2, compared with the number on the lower level N_1 of the vibrating plate, we wish to calculate the overall uncertainty in the "normalized" ratio, $r_k = (N_2/N_1)_k/(N_2/N_1)_0$, for amplitude category k (as given in Table ES1). First, we note that the number in the lower level is calculated from the total number N_{tot} of balls in the system as $N_1 = N_{tot} - N_2$. Thus,

$$r_k = \frac{N_2/(N_{tot}-N_2)}{N_{20}/(N_{tot}-N_{20})} = \frac{N_2}{N_{20}}\left(\frac{1-N_{20}/N_{tot}}{1-N_2/N_{tot}}\right), \qquad \text{(ES1)}$$

where N_{20} is the number in level 2 at the chosen point of normalization. Uncertainties in the individual ratios depend on the uncertainties δN_2 in the counts of N_2 alone. Second, we recognize that uncertainty δN_{20} also exists in the normalization ratio. Both N_2

Table ES1. Data and statistics related to the vibrating-plate experiment (using N_{tot} = 300 balls and a frequency f = 40 Hz). "Energy" is the relative kinetic energy E_{rel} calculated as $(A\omega)^2$, where A is the plate amplitude and $\omega = 2\pi f$ is the angular frequency. The normalized ratio r_k is computed using amplitude category k = 9 as the reference (bold N_2 value).

Amplitude category k	Applied voltage (V)	Plate amplitude (mm)	Energy ($m^2\ s^{-2}$)	I/E_{rel} ($m^{-2}\ s^2$)	Mean count N_2	Standard deviation δN_2	Relative deviation $\delta N_2/N_2$	Normalized ratio r_k	Relative fluctuation $\delta r_k/r_k$	Absolute fluctuation δr_k
1	3.52	0.194	0.00135	742	0.40	0.60	1.510	0.0084	1.52	0.0129
2	3.63	0.200	0.00142	706	0.97	0.96	0.984	0.0206	1.00	0.0206
3	3.67	0.202	0.00144	693	1.35	1.34	0.990	0.0286	1.01	0.0289
4	3.79	0.208	0.00152	658	2.60	1.78	0.684	0.0553	0.71	0.0394
5	3.94	0.215	0.00162	618	5.11	1.88	0.369	0.1096	0.41	0.0452
6	4.10	0.223	0.00173	579	9.30	3.12	0.335	0.2023	0.39	0.0783
7	4.23	0.229	0.00182	550	14.46	3.84	0.266	0.3204	0.33	0.1052
8	4.39	0.236	0.00193	517	26.49	8.15	0.308	0.6127	0.38	0.2323
9	4.50	0.241	0.00202	496	**40.95**	6.11	0.149	1.0000	0.24	0.2446

and N_{20} vary naturally, and their separate variations (δN_2 and δN_{20}, respectively) affect the variations in r_k.

The overall variation in the normalized ratio r_k is calculated with the assumption that the variations in N_2 and N_{20} are independent of each other. Thus, the final uncertainty is calculated in quadrature:

$$\delta r_k = \left[\left(\frac{\partial r_k}{\partial N_2}\delta N_2\right)^2 + \left(\frac{\partial r_k}{\partial N_{20}}\delta N_{20}\right)^2\right]^{\frac{1}{2}}. \quad \text{(ES2)}$$

The partial derivatives are determined from Eq. (ES1) to be

$$\frac{\partial r_k}{\partial N_2} = \frac{1}{N_{20}}\frac{\left(1-N_{20}/N_{tot}\right)}{\left(1-N_2/N_{tot}\right)^2} \quad \text{and (ES3)}$$

$$\frac{\partial r_k}{\partial N_{20}} = \frac{-N_2}{N_{20}^2\left(1-N_2/N_{tot}\right)}. \quad \text{(ES4)}$$

Substitution of Eqs. (ES3) and (ES4) into Eq. (ES2) yields, after algebraic manipulation,

$$\delta r_k = \frac{N_2}{N_{20}}\left(\frac{1-N_{20}/N_{tot}}{1-N_2/N_{tot}}\right)\left[\left(\frac{\delta N_2/N_2}{1-N_2/N_{tot}}\right)^2 + \left(\frac{\delta N_{20}/N_{20}}{1-N_{20}/N_{tot}}\right)^2\right]^{\frac{1}{2}}$$

(ES5)

Alternatively, we may wish to determine the relative uncertainty, which is achieved by combining Eqs. (ES1) and (ES5) to obtain the relatively simple expression

$$\frac{\delta r_k}{r_k} = \left[\left(\frac{\delta N_2/N_2}{1-N_2/N_{tot}}\right)^2 + \left(\frac{\delta N_{20}/N_{20}}{1-N_{20}/N_{tot}}\right)^2\right]^{\frac{1}{2}}. \quad \text{(ES6)}$$

Note that the effect of a limited number of balls is to magnify the relative uncertainties in N_2 and N_{20}. As $N_{tot} \to \infty$, the overall relative uncertainty in r_k becomes

$$\frac{\delta r_k}{r_k} = \left[\left(\frac{\delta N_2}{N_2}\right)^2 + \left(\frac{\delta N_{20}}{N_{20}}\right)^2\right]^{\frac{1}{2}}. \quad \text{(ES7)}$$

Equation (ES6) is used to calculate the actual uncertainties about each data point.

Theoretically, a system composed of independent and random realizations of a variable are described by Poisson statistics (Taylor 1997, chapter 11). The Poisson standard deviation is systematically related to a variable x as $\delta x = x^{1/2}$. With this principle applied to Eq. (ES7),

$$\left.\frac{\delta r_k}{r_k}\right|_{\text{Poisson}} = \left(\frac{1}{N_2} + \frac{1}{N_{20}}\right)^{\frac{1}{2}}. \quad \text{(ES8)}$$

Equation (ES8), when applied incrementally over the full range of N_2, gives the spread ($r_k \pm \delta r_k$) expected in the limit of many particles obeying Poisson statistics. Note the similarity to the expectation of inverse proportionality to the square root of the number of particles being counted.

The treatment so far has focused on the fluctuations that naturally arise during the counting of balls or particles in a system. However, we are also interested in how closely the mean count, calculated from a number of determinations, agrees with the population mean and with theory. If the number of trials (determinations) is n_{trials} and the standard deviation is σ_r ($=\delta r_k$), then the standard error of the mean is (Taylor 1997, p. 102)

$$\sigma_{\bar{r}} = \sigma_r \,/\, n_{\text{trials}}^{\frac{1}{2}}. \quad \text{(ES9)}$$

Uncertainty in the mean decreases with the number of determinations, but the standard deviation, representing the statistical fluctuations inherent in random variables, remains the same. The best way to reduce the variability from determination to determination is to increase the number of particles in the system.

REFERENCES

Taylor, J. R., 1997: *An Introduction to Error Analysis.* University Science Books, 327 pp.