



# AMS

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## Supplemental Material

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## **Online Supplemental Material**

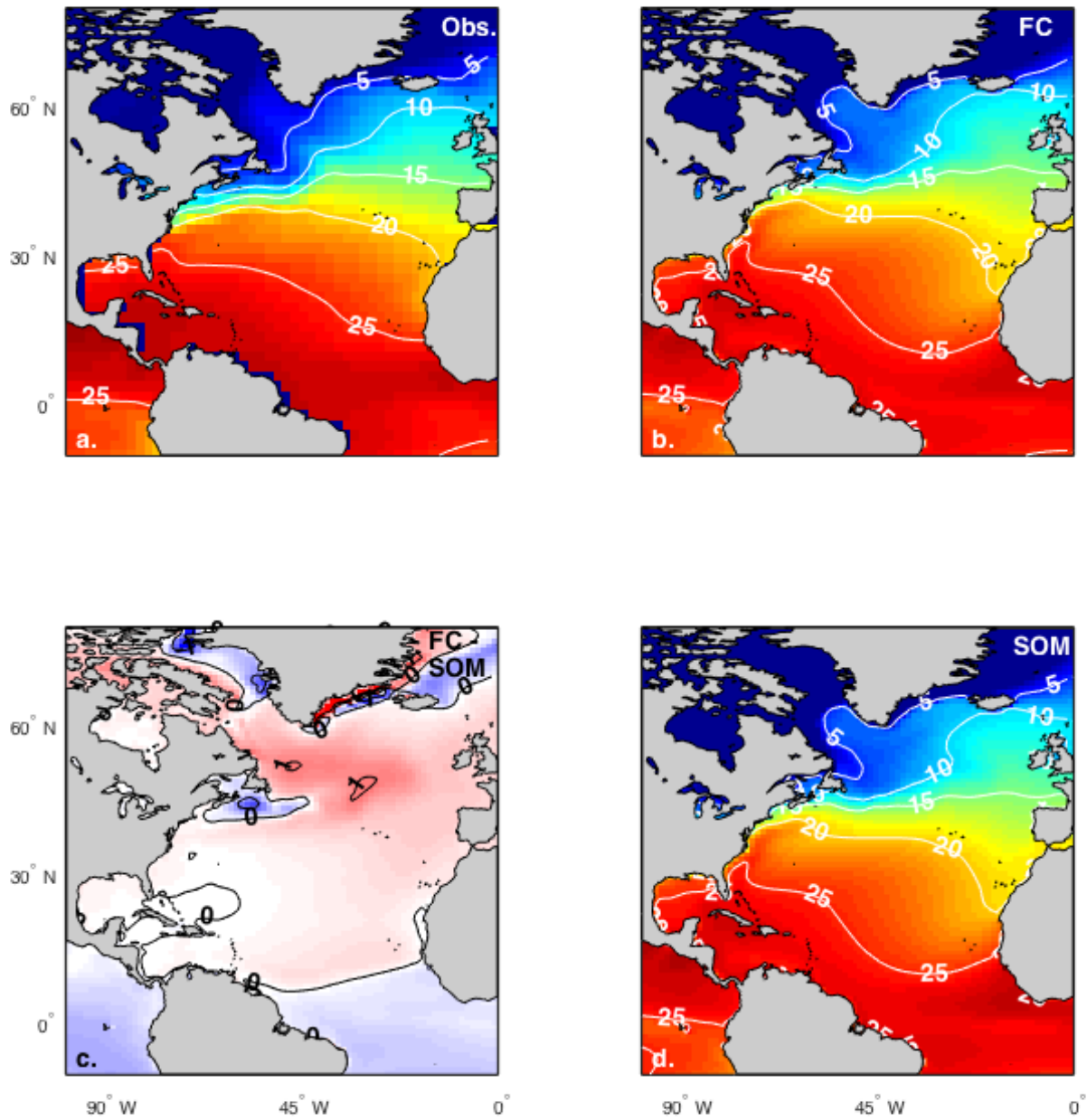
### **Investigating the Roles of External Forcing and Ocean Circulation on Atlantic Multidecadal Variability in a Large Ensemble Climate Model Hierarchy**

Lisa N. Murphy<sup>1</sup>, Jeremy M. Klavans<sup>1</sup>, Amy C. Clement<sup>1</sup>, Mark A. Cane<sup>2</sup>

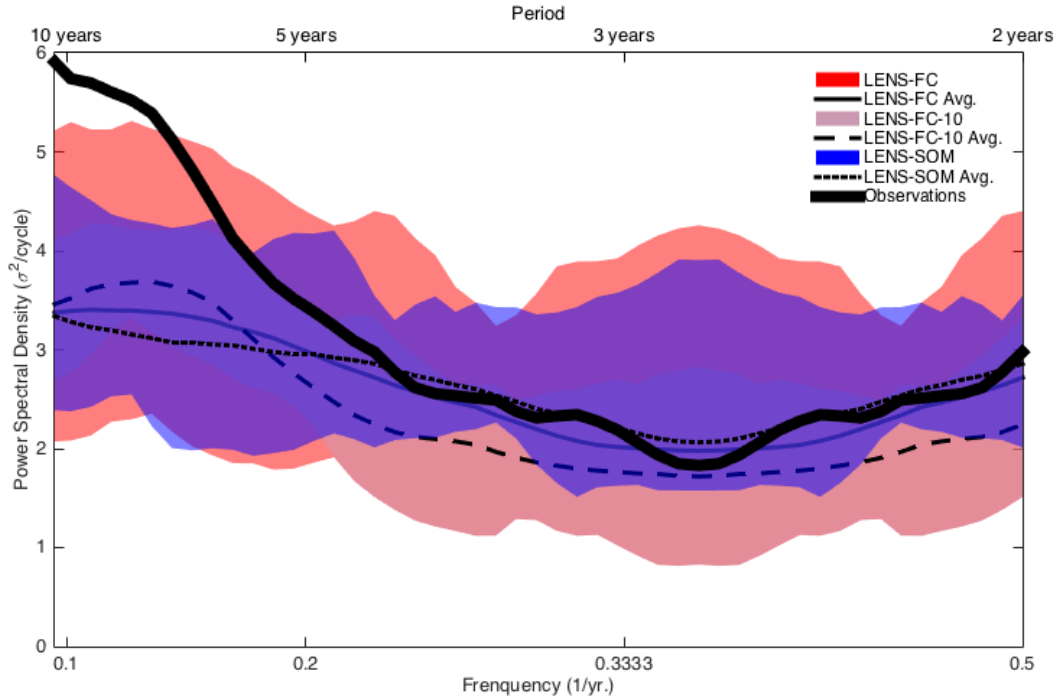
1. Rosenstiel School of Marine and Atmospheric Science, University of Miami, Miami, FL
2. Lamont-Doherty Earth Observatory, Columbia University, Palisades, NY

#### **Contents of this file**

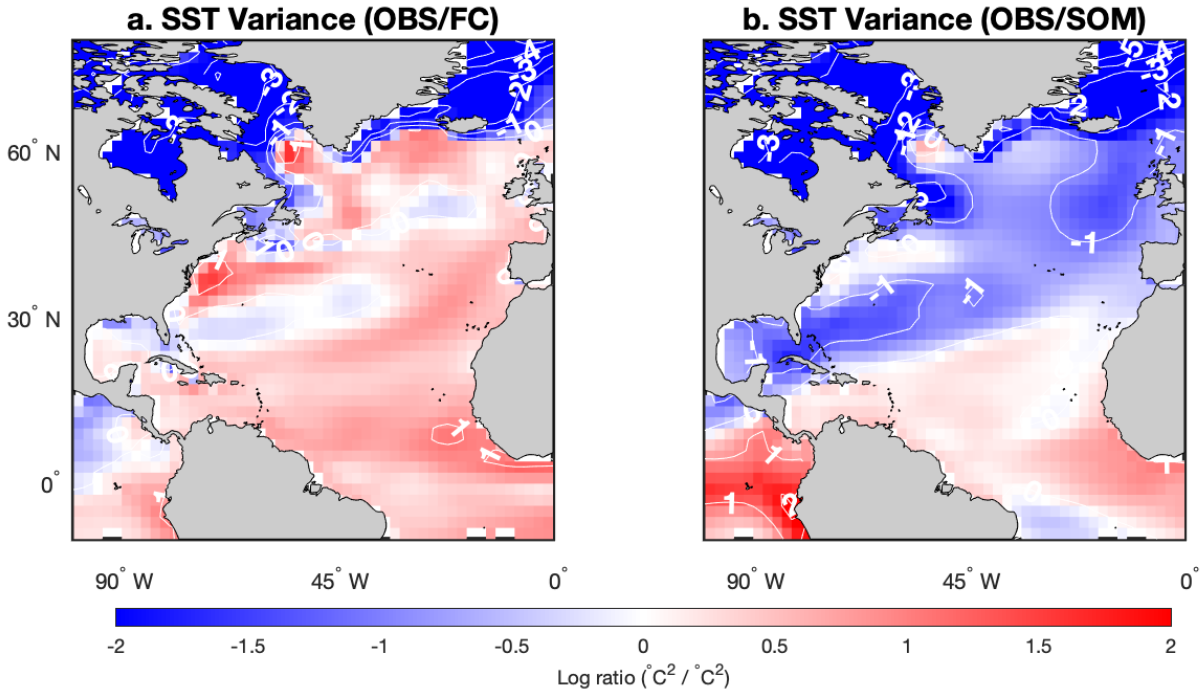
Figures S1 to S3  
Better estimates of variance



**Figure S1:** Average SST for observations (a.), LENS-FC (b.), LENS-SOM (d.) and the difference between model averages (c.). All values are in degrees C and are calculated over the years 1920 – 2005.



**Figure S2:** Spectra of the winter December-January-February (DJF) North Atlantic Oscillation index from LENS-FC, LENS-SOM, a randomly-subsetted 10 members of LENS-FC, and observations. The average spectra of the individual ensemble members of LENS-FC (solid), LENS-SOM (dotted), and a random set of 10 LENS-FC members (dashed). The 95% ensemble spread are reported in the colored regions: LENS (red), LENS-SOM (blue), and a random set of 10 LENS-FC members (pink). The spectra of the observed NAO index is plotted in the thick black line, which comes from [Climate Data Guide](https://climatedataguide.ucar.edu/climate-data/hurrell-north-atlantic-oscillation-nao-index-station-based) for 1920 – 2005 (Hurrell, James & National Center for Atmospheric Research Staff (Eds). Last modified 24 Apr 2020. **"The Climate Data Guide: Hurrell North Atlantic Oscillation (NAO) Index (station-based)."** Retrieved from [https://climatedataguide.ucar.edu/climate-data/hurrell-north-atlantic-oscillation-nao-index-station-based.](https://climatedataguide.ucar.edu/climate-data/hurrell-north-atlantic-oscillation-nao-index-station-based))



**Figure S3:** Ratio of the log of ERSSTv5 to LENS-FC (a.) and ERSSTv5 to LENS-SOM (b.) annual average (unfiltered) SST variance from 1930 – 2005. LENS-FC and LENS-SOM were interpolated onto the ERSSTv5 2° x 2° grid.

## Better estimates of variance

1

2 It is common to estimate the forced variance,  $\sigma_F^2$ , as equal to  $\sigma_{EM}^2$ , the variance of the  
 3 ensemble mean (EM), and the mean internal variance,  $\sigma_I^2$ , as equal to  $\sigma_{MOE}^2 - \sigma_{EM}^2$ , where  
 4  $\sigma_{MOE}^2$  is the mean of the ensemble variance (MOE). These estimates are biased; in the main  
 5 text we use unbiased estimators. The derivations here are altogether standard, though usually  
 6 phrased in terms of signal and noise rather than forced and internal.

7 Suppose we have  $J$  ensemble members and write  $T_j(t) = T_F(t) + T_{I,j}(t)$  where  $T_F$  is the  
 8 forced component of  $T$  common to all ensemble members and  $T_{I,j}$  is the internal variation of  
 9 ensemble member  $j$ . Defining

$$\bar{T}(t) \equiv \frac{1}{J} \sum_{j=1}^J T_j(t) = T_F(t) + T_I(t); \quad T_I(t) \equiv \frac{1}{J} \sum_{j=1}^J T_{I,j}(t). \quad (\text{A1})$$

The expected value of  $T_I(t) = 0$  and we might formally define  $T_F = \lim_{J \rightarrow \infty} \bar{T}$ . Using brackets  $\langle \dots \rangle$  to denote the expected value of the time mean,  $\sigma_F^2 \equiv \langle T_F^2 \rangle$ ;  $\sigma_I^2 \equiv \frac{1}{J} \sum_{j=1}^J \langle T_{I,j}^2 \rangle$ . Also

$$\sigma_{MOE}^2 \equiv \frac{1}{J} \sum_{j=1}^J \langle T_j^2(t) \rangle = \frac{1}{J} \sum_{j=1}^J \langle (T_F + T_{I,j})^2 \rangle = \sigma_F^2 + 2 \langle T_F T_I \rangle + \sigma_I^2$$

10 and because the expected value of the covariance of  $T_F$  and  $T_I$  is zero,

$$\sigma_{MOE}^2 = \sigma_F^2 + \sigma_I^2. \quad (\text{A2})$$

We need the intermediate result that

$$\langle T_I^2 \rangle = \frac{1}{J^2} \langle \left( \sum_{j=1}^J T_{I,j} \right)^2 \rangle = \frac{1}{J^2} \sum_{j=1}^J \langle T_{I,j}^2 \rangle + \frac{1}{J^2} \sum_{j=1}^J \sum_{k \neq j} \langle T_{I,j} T_{I,k} \rangle = \frac{1}{J} \sigma_I^2,$$

11 where we use the fact that the expected value of the covariance of different ensemble members  
 12 is zero. It now follows that

$$\sigma_{EM}^2 \equiv \langle \bar{T}^2 \rangle = \langle (T_F + T_I)^2 \rangle = \sigma_F^2 + \frac{2}{J} \langle T_F T_I \rangle + \langle T_I^2 \rangle = \sigma_F^2 + \frac{1}{J} \sigma_I^2 \quad (\text{A3})$$

13 Solving (A2), (A3) for  $\sigma_I^2, \sigma_F^2$ :

$$\sigma_I^2 = \left(1 + \frac{1}{J-1}\right) (\sigma_{MOE}^2 - \sigma_{EM}^2) \quad (\text{A4})$$

14

$$\sigma_F^2 = \sigma_{EM}^2 - \frac{1}{J-1} (\sigma_{MOE}^2 - \sigma_{EM}^2) \quad (\text{A5})$$

15 We see that the common estimates  $\sigma_F^2 = \sigma_{EM}^2$ , and  $\sigma_I^2 = \sigma_{MOE}^2 - \sigma_{EM}^2$  are biased low for  
 16  $\sigma_I^2$  and biased high for  $\sigma_F^2$ . The bias is  $O(1/(J-1))$  and is  $\approx 2\%$  for the larger FC ensemble,  
 17 while for the SOM ensemble with just 10 members it is about 5% for the forced variance  $\sigma_F^2$   
 18 and about 11% for the internal variance  $\sigma_I^2$ . Using these cruder estimates would not have altered  
 19 our conclusions.