Supplemental Material

Online Supplemental Material

Investigating the Roles of External Forcing and Ocean Circulation on Atlantic Multidecadal Variability in a Large Ensemble Climate Model Hierarchy

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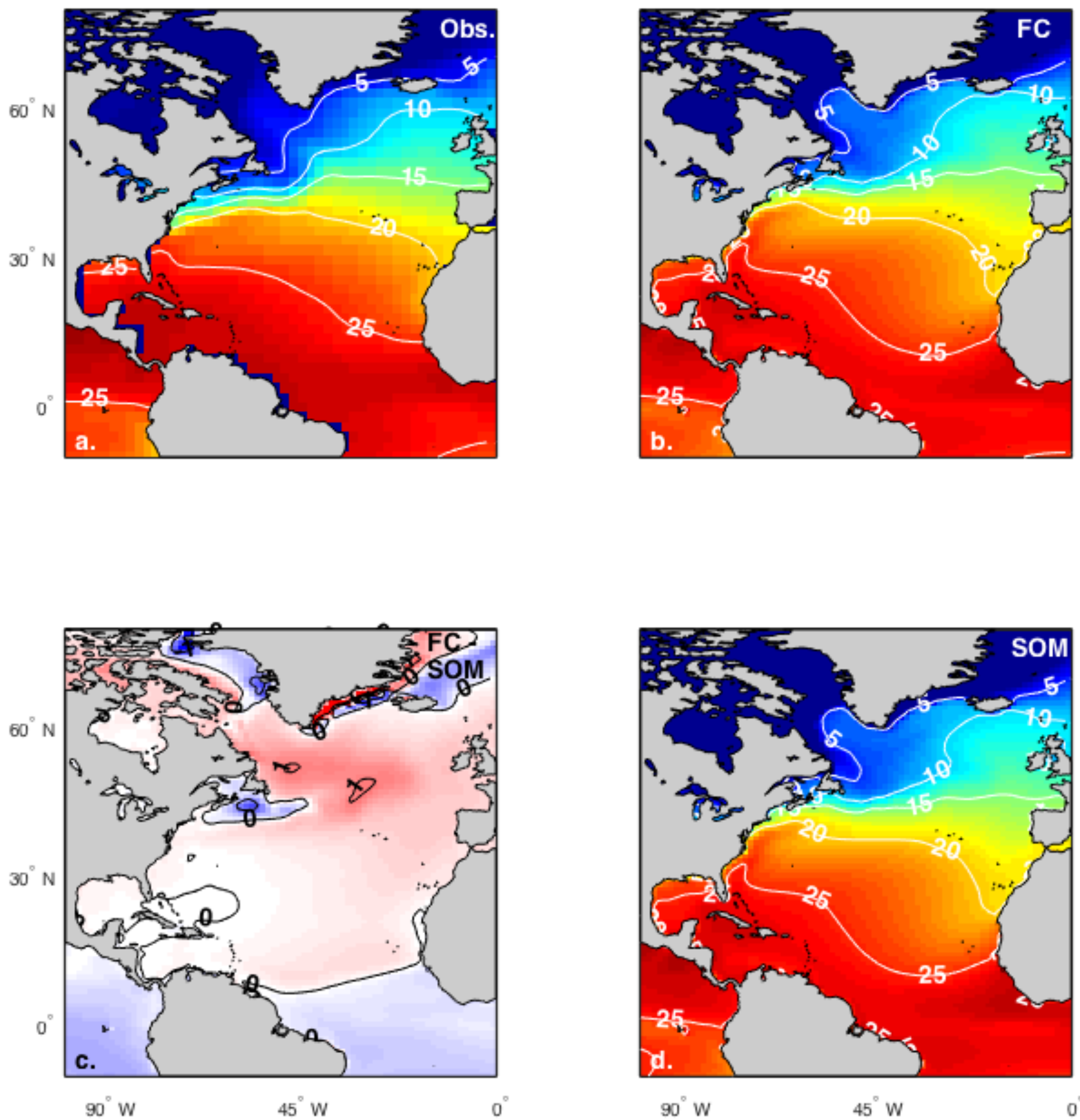

Figure S1: Average SST for observations (a.), LENS-FC (b.), LENS-SOM (d.) and the difference between model averages (c.). All values are in degrees C and are calculated over the years 1920 – 2005.

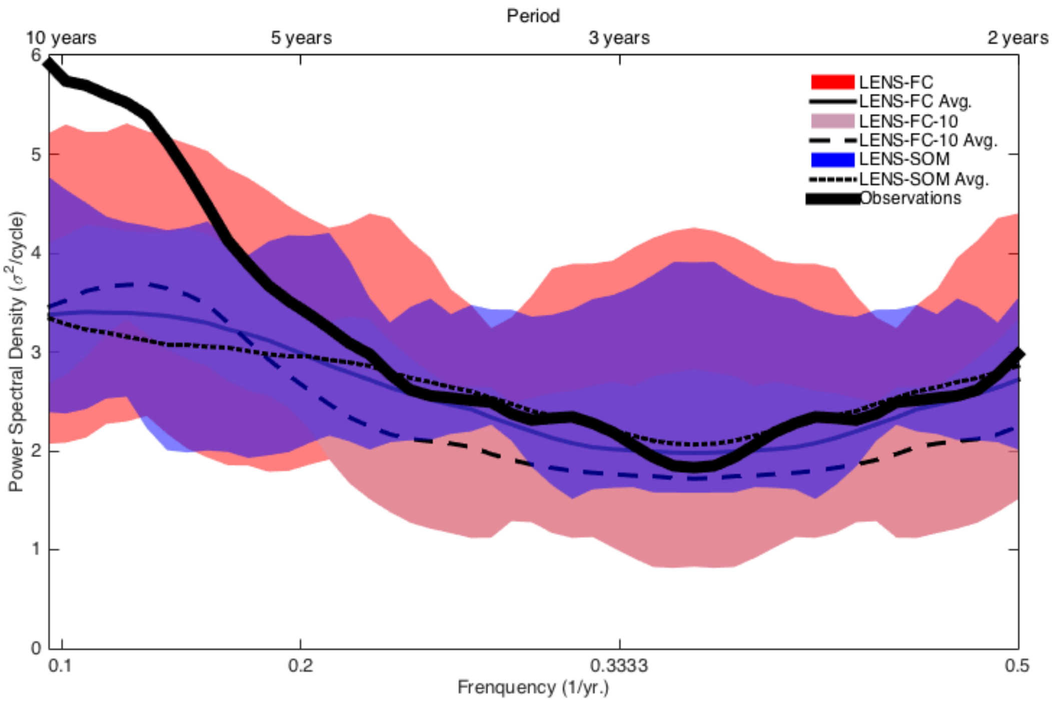

Figure S2: Spectra of the winter December-January-February (DJF) North Atlantic Oscillation index from LENS-FC, LENS-SOM, a randomly-subsetted 10 members of LENS-FC, and observations. The average spectra of the individual ensemble members of LENS-FC (solid), LENS-SOM (dotted), and a random set of 10 LENS-FC members (dashed). The 95% ensemble spread are reported in the colored regions: LENS (red), LENS-SOM (blue), and a random set of 10 LENS-FC members (pink). The spectra of the observed NAO index is plotted in the think black line, which comes from Climate Data Guide for 1920 – 2005 (Hurrell, James & National Center for Atmospheric Research Staff (Eds). Last modified 24 Apr 2020. **"The Climate Data Guide: Hurrell North Atlantic Oscillation (NAO) Index (station-based)."** Retrieved from https://climatedataguide.ucar.edu/climate-data/hurrell-north-atlantic-oscillation-nao-index-station-based.)

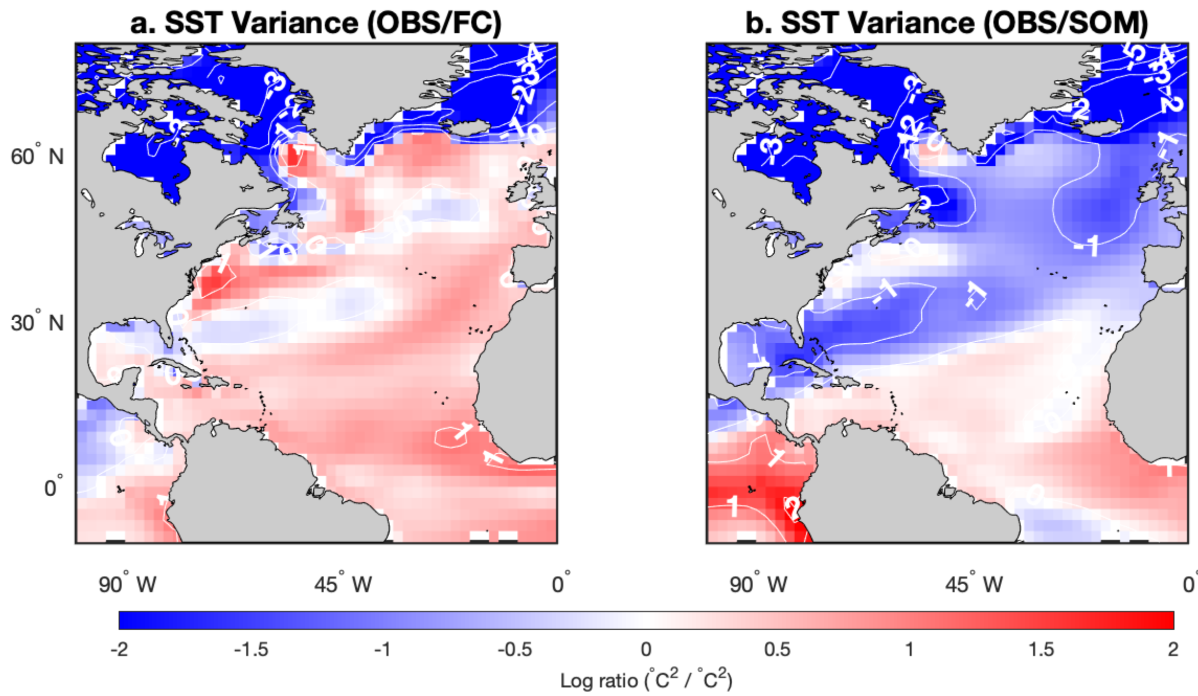

Figure S3: Ratio of the log of ERSSTv5 to LENS-FC (a.) and ERSSTv5 to LENS-SOM (b.) annual average (unfiltered) SST variance from 1930 – 2005. LENS-FC and LENS-SOM were interpolated onto the ERSSTv5 2º x 2º grid.

Better estimates of variance

It is common to estimate the forced variance, σ_F^2, as equal to σ_{EM}^2, the variance of the ensemble mean (EM), and the mean internal variance, σ_I^2, as equal to $\sigma_{MOE}^2 - \sigma_{EM}^2$, where σ_{MOE}^2 is the mean of the ensemble variance (MOE). These estimates are biased; in the main text we use unbiased estimators. The derivations here are altogether standard, though usually phrased in terms of signal and noise rather than forced and internal.

Suppose we have J ensemble members and write $T_j(t) = T_F(t) + T_{I,j}(t)$ where T_F is the forced component of T common to all ensemble members and $T_{I,j}$ is the internal variation of ensemble member j. Defining

$$\overline{T}(t) \equiv \frac{1}{J}\sum_{j=1}^{J} T_j(t) = T_F(t) + T_I(t); \quad T_I(t) \equiv \frac{1}{J}\sum_{j=1}^{J} T_{I,j}(t). \tag{A1}$$

The expected value of $T_I(t) = 0$ and we might formally define $T_F = \lim_{J\to\infty} \overline{T}$. Using brackets $<\cdots>$ to denote the expected value of the time mean, $\sigma_F^2 \equiv <T_F^2>$; $\sigma_I^2 \equiv \frac{1}{J}\sum_{j=1}^{J} <T_{I,j}^2>$. Also

$$\sigma_{MOE}^2 \equiv \frac{1}{J}\sum_{j=1}^{J} <T_j^2(t)> = \frac{1}{J}\sum_{j=1}^{J} <(T_F + T_{I,j})^2> = \sigma_F^2 + 2 <T_F T_I> + \sigma_I^2$$

and because the expected value of the covariance of T_F and T_I is zero,

$$\sigma_{MOE}^2 = \sigma_F^2 + \sigma_I^2. \tag{A2}$$

We need the intermediate result that

$$<T_I^2> = \frac{1}{J^2} <\Big(\sum_{j=1}^{J} T_{I,j}\Big)^2> = \frac{1}{J^2}\sum_{j=1}^{J} <T_{I,j}^2> + \frac{1}{J^2}\sum_{j=1}^{J}\sum_{k\neq j} <T_{I,j}T_{I,k}> = \frac{1}{J}\sigma_I^2,$$

where we use the fact that the expected value of the covariance of different ensemble members is zero. It now follows that

$$\sigma_{EM}^2 \equiv <\overline{T}^2> = <(T_F + T_I)^2> = \sigma_F^2 + \frac{2}{J} <T_F T_I> + <T_I^2> = \sigma_F^2 + \frac{1}{J}\sigma_I^2 \tag{A3}$$

Solving (A2), (A3) for σ_I^2, σ_F^2:

$$\sigma_I^2 = (1 + \frac{1}{J-1})(\sigma_{MOE}^2 - \sigma_{EM}^2) \tag{A4}$$

$$\sigma_F^2 = \sigma_{EM}^2 - \frac{1}{J-1}(\sigma_{MOE}^2 - \sigma_{EM}^2) \tag{A5}$$

We see that the common estimates $\sigma_F^2 = \sigma_{EM}^2$, and $\sigma_I^2 = \sigma_{MOE}^2 - \sigma_{EM}^2$ are biased low for σ_I^2 and biased high for σ_F^2. The bias is $O(1/(J-1))$ and is $\approx 2\%$ for the larger FC ensemble, while for the SOM ensemble with just 10 members it is about 5% for the forced variance σ_F^2 and about 11% for the internal variance σ_I^2..Using these cruder estimates would not have altered our conclusions.