Supplemental Material

Journal of Climate
Optimal Fingerprinting with Estimating Equations
https://doi.org/10.1175/JCLI-D-22-0681.1

Supplementary Material to "Optimal Fingerprinting with Estimating Equations"

Sai Ma[1], Tianying Wang[2], Jun Yan[1], and Xuebin Zhang[3]

[1]Department of Statistics, University of Connecticut, Storrs, CT, USA.

[2]Department of Statistics, Colorado State University, Fort Collins, CO, USA.

[3]Climate Research Division, Environment and Climate Change Canada, Toronto, ON, Canada.

S-1 Confidence Interval of a

Based on Appendix C, the feasible EE of a is

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T} Q_t(a;\hat{\Psi}_+,\hat{\beta}_T)=0, \tag{1}$$

where

$$Q_t(a;\hat{\Psi}_+,\hat{\beta}_T)=a\left\{(Y_t-\tilde{X}_t\hat{\beta}_T)^\top\hat{\Psi}_+^{-1}(Y_t-\tilde{X}_t\hat{\beta}_T)-\left(S-\frac{1}{T}\right)\sum_{j=1}^{J}\frac{\hat{\beta}_{Tj}^2}{m_j}\right\}-\left(S-\frac{1}{T}\right).$$

To construct the confidence interval of a, we first state some notations as follows:

$$\begin{aligned}
C&=\lim_{T\to\infty}C_T,\\
C_T&=\begin{pmatrix}C_{1T}&0\\C_{2T}&C_{3T}\end{pmatrix},\\
C_{1T}&=A_T^{-1},\\
C_{2T}&=-2\hat{a}_T\left\{\frac{1}{T}\sum_{t=1}^{T}\tilde{X}_t\hat{\Psi}_+^{-1}(Y_t-\tilde{X}_t\hat{\beta}_T)+(S-\frac{1}{T})\,\mathrm{diag}\left(\frac{1}{m_1},\cdots,\frac{1}{m_p}\right)\hat{\beta}_T\right\},\\
C_{3T}&=\frac{1}{T}\left\{(Y_t-\tilde{X}_t\hat{\beta}_T)^\top\hat{\Psi}_+^{-1}(Y_t-\tilde{X}_t\hat{\beta}_T)-(S-\frac{1}{T})\sum_{j=1}^{p}\frac{\hat{\beta}_j^2}{m_j}\right\},\\
D&=\lim_{T\to\infty}D_T,\\
D_T&=\begin{pmatrix}D_{1T}&D_{2T}\\D_{2T}&D_{3T}\end{pmatrix},\\
D_{1T}&=B=\mathrm{cov}\left\{T^{-1/2}\sum_{t=1}^{T}G_t(\hat{\gamma};\hat{\Sigma}_+)\right\},\\
D_{2T}&=\frac{1}{T}\mathrm{cov}\left\{\sum_{t=1}^{T}G_t(\hat{\gamma};\hat{\Sigma}_+),\sum_{t=1}^{T}Q_t(\hat{\gamma};\hat{\Sigma}_+)\right\},\\
D_{3T}&=\mathrm{cov}\left\{T^{-1/2}\sum_{t=1}^{T}Q_t(\hat{\gamma};\hat{\Sigma}_+)\right\}.
\end{aligned}$$

We then stack the estimations of $\hat{\beta}$ and $\hat{a}$ together to get the joint asymptotic distribution of $\hat{\gamma} = (\hat{\beta}^\top, \hat{a})^\top$. Since $\hat{\gamma}_T = (\hat{\beta}_T^\top, \hat{a}_T)^\top$ solves the Equations (6) and (12) in the paper, a Taylor expansion of the equation at the true parameter value γ gives

$$\begin{aligned} 0 &= T^{-1/2} \sum_{t=1}^{T} V_t(\hat{\gamma}_T; \hat{\Psi}_+) \\ &= T^{-1/2} \sum_{t=1}^{T} V_t(\gamma; \hat{\Psi}_+) + \left\{ T^{-1} \sum_{t=1}^{T} \partial \frac{V_t(\gamma; \hat{\Psi}_+)}{\partial \gamma} \right\} T^{1/2}(\hat{\gamma}_T - \gamma) + o_p(1), \end{aligned}$$

where $V_t(\hat{\gamma}_T; \hat{\Psi}_+) = \left\{ G_t^\top(\hat{\gamma}_T; \hat{\Psi}_+), Q_t(\hat{\gamma}_T; \hat{\Psi}_+) \right\}^\top$. Thus,

$$T^{1/2}(\hat{\gamma}_T - \gamma) = - \left\{ T^{-1} \sum_{t=1}^{T} \partial \frac{V_t(\gamma; \hat{\Sigma}_+)}{\partial \gamma} \right\}^{-1} T^{-1/2} \sum_{t=1}^{T} V_t(\gamma) + o_p(1).$$

Since V_t is stationary over time, under mild conditions $T^{-1/2} \sum_{t=1}^{T} V_t(\gamma)$ converges in distribution to a normal distribution $N(0, D)$. Further, $T^{-1} \sum_{t=1}^{T} \partial V_t(\gamma; \hat{\Psi}_+)/\partial \gamma \to C$ by the law of large number. The asymptotic normality of $\hat{\gamma}_T$ then follows. Thus, as $T \to \infty$,

$$\sqrt{T}(\hat{\gamma}_T - \gamma) \to N(0, C^{-1} D (C^{-1})^\top). \tag{2}$$

Note that the variance of $\hat{\beta}_T$ from Equation (2) is the same as before since C is a lower triangular block matrix.

S-2 Additional Simulation Results

Here we provide additional simulation results on various settings: (1) $a \in \{0.5, 1\}$, (2) ϵ follows a multivariate normal or t distribution with degree of freedom 15, and (3) the true Σ is set as a block Toeplitz matrix or the linear shrinkage estimator from control runs. Besides, to evaluate the performance of the three methods with higher dimensional data, we conduct simulations at the global scale with $S = 108$ grid boxes of size $40° \times 15°$. We also performed

simulations based on the CanESM5 climate model, where X_{NAT} and X_{ANT} are correlated. The data preparation is the same as Section 4.1 and the true X_{NAT} and X_{ALL} are set as the means of $\tilde{X}_{\text{NAT}}$ and $\tilde{X}_{\text{ALL}}$. After having $\hat{\beta}_{\text{ALL}}$ and $\hat{\beta}_{\text{NAT}}$, we used the transformation described in Section 4.1 to obtain $\hat{\beta}_{\text{ANT}}$ and $\hat{\beta}_{\text{NAT}}$.

To investigate whether the temporal stationarity assumption helps the TLS estimator, we implement the TLS method with an identical block-diagonal covariance matrix Σ. Though one can evaluate the point estimator through bias and RMSE, how to construct valid confidence intervals for the TLS estimator with this modified covariance matrix is unclear. Thus, to assess the performance of interval estimation, we propose to adapt the pseudo-bootstrap procedure described in Section 2.3 to construct the modified TLS estimator. The details are presented below.

Denote TLS estimated coefficient as $\hat{\beta}_{\text{TLS}}$ and set $a = 1$. The procedure of constructing confidence intervals based on pseudo-bootstrap for the TLS method is described as follows. First, we get the pseudo residuals $r_{\text{pseudo}} = \left\{\delta(\hat{\beta}_{\text{TLS}})\epsilon^{(1)}, \ldots, \delta(\hat{\beta}_{\text{TLS}})\epsilon^{(L)}\right\}$. Then, we can construct pseudo response $Y^{(\ell)}_{t,\text{pseudo}} = r_{t,\text{pseudo}} + \tilde{X}_t\hat{\beta}_{\text{TLS}}$ for $t = 1, ..., T$, $\ell = 1, ..., L$. $\tilde{X}_t$ is the observed data with measurement errors. Then, we re-estimate β by TLS based on $\{Y^{(\ell)}_{t,\text{pseudo}}, \tilde{X}_t\}_{t=1}^{T}$, denoted by $\hat{\beta}^{(\ell)}_{\text{TLS}}$. The variance of $\hat{\beta}_{\text{TLS}}$ can be estimated as $\text{var}(\hat{\beta}^{(\ell)}_{\text{TLS}})$. The confidence interval for $\hat{\beta}_{\text{TLS}}$ can be constructed based on asymptotic normality, or empirical quantiles of $\hat{\beta}^{(\ell)}_{\text{TLS}}$'s.

We denote the new adapted TLS method as TLS-A. As the purpose of constructing the TLS-A estimator is to evaluate the performance improvement caused by the temporal stationarity assumption, we only present TLS-A in the settings with $a = 1$ because TLS-type methods are already heavily biased when $a = 0.5$, making the comparison meaningless.

Tables S-1 to S-8 summarize the bias and the RMSE of the estimated coefficients $\hat{\beta}$. When the true a is 1, TLS-TS, TLS-PBC, EE, and TLS-A all have unbiased estimators. However, EE has the smallest RMSE for most settings, especially for the ANT scaling factor. Even when the true covariance matrix does not have the block Toeplitz structure, the EE

Table S-1: Summaries of the bias and RMSE from four methods under the simulation settings where the true a is 1, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate normal distribution.

				$L = 50$		$L = 100$		$L = 200$	
Scale	Forcing	m	Method	Bias	RMSE	Bias	RMSE	Bias	RMSE
Global	ANT	40	TLS-TS	−0.003	0.148	−0.003	0.145	−0.000	0.137
			TLS-PBC	−0.001	0.145	−0.001	0.137	0.002	0.131
			EE	0.000	0.112	0.003	0.111	0.004	0.109
			TLS-A	−0.004	0.148	−0.005	0.133	0.000	0.127
		20	TLS-TS	0.003	0.151	−0.002	0.148	0.000	0.140
			TLS-PBC	0.003	0.149	−0.003	0.139	0.003	0.134
			EE	−0.005	0.117	−0.003	0.118	−0.003	0.115
			TLS-A	0.000	0.162	−0.003	0.141	−0.002	0.136
	NAT	40	TLS-TS	−0.006	0.299	−0.003	0.285	−0.003	0.262
			TLS-PBC	0.001	0.282	−0.002	0.260	0.001	0.241
			EE	−0.005	0.244	0.006	0.242	0.009	0.241
			TLS-A	0.010	0.357	0.004	0.244	0.004	0.229
		20	TLS-TS	0.001	0.333	0.002	0.310	−0.007	0.280
			TLS-PBC	−0.003	0.309	0.001	0.280	−0.004	0.264
			EE	−0.043	0.288	−0.023	0.289	−0.010	0.291
			TLS-A	0.002	0.407	−0.007	0.267	−0.004	0.252
Regional	ANT	40	TLS-TS	−0.001	0.175	−0.003	0.179	−0.007	0.178
			TLS-PBC	−0.007	0.181	−0.001	0.176	−0.006	0.174
			EE	−0.003	0.159	−0.002	0.157	−0.001	0.157
			TLS-A	0.002	0.159	0.000	0.162	0.005	0.164
		20	TLS-TS	−0.005	0.188	−0.003	0.189	−0.001	0.192
			TLS-PBC	−0.006	0.194	−0.003	0.192	−0.003	0.184
			EE	0.003	0.172	0.005	0.170	0.005	0.171
			TLS-A	−0.003	0.168	0.001	0.172	0.002	0.176
	NAT	40	TLS-TS	0.005	0.334	0.004	0.343	−0.006	0.328
			TLS-PBC	0.010	0.341	0.007	0.336	0.000	0.327
			EE	0.010	0.327	0.012	0.325	0.011	0.324
			TLS-A	0.002	0.308	0.001	0.307	−0.003	0.310
		20	TLS-TS	0.010	0.348	0.011	0.353	0.013	0.355
			TLS-PBC	0.013	0.355	0.006	0.348	0.012	0.350
			EE	−0.001	0.365	−0.000	0.369	0.004	0.370
			TLS-A	0.018	0.321	0.019	0.327	0.016	0.331

Table S-2: Summaries of the bias and RMSE from four methods under the simulation settings where the true a is 1, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate t distribution with degree of freedom 15.

				$L = 50$		$L = 100$		$L = 200$	
Scale	Forcing	m	Method	Bias	RMSE	Bias	RMSE	Bias	RMSE
Global	ANT	40	TLS-TS	−0.010	0.155	−0.010	0.148	−0.005	0.141
			TLS-PBC	−0.009	0.150	−0.009	0.141	−0.003	0.132
			EE	−0.004	0.105	−0.001	0.104	−0.000	0.104
			TLS-A	−0.004	0.151	−0.003	0.138	−0.003	0.127
		20	TLS-TS	−0.011	0.160	−0.011	0.152	−0.006	0.147
			TLS-PBC	−0.010	0.154	−0.011	0.145	−0.005	0.139
			EE	−0.007	0.117	−0.004	0.112	−0.002	0.113
			TLS-A	−0.007	0.156	−0.009	0.143	−0.016	0.157
	NAT	40	TLS-TS	0.032	0.341	0.032	0.322	0.024	0.298
			TLS-PBC	0.027	0.313	0.030	0.289	0.029	0.280
			EE	−0.026	0.241	−0.013	0.240	−0.007	0.238
			TLS-A	0.032	0.389	0.020	0.292	0.020	0.260
		20	TLS-TS	0.060	0.442	0.058	0.417	0.052	0.417
			TLS-PBC	0.051	0.401	0.060	0.395	0.063	0.418
			EE	−0.024	0.297	−0.005	0.296	0.008	0.300
			TLS-A	0.053	0.559	0.052	0.434	0.066	0.484
Regional	ANT	40	TLS-TS	−0.002	0.177	−0.002	0.175	−0.003	0.177
			TLS-PBC	−0.005	0.179	−0.002	0.173	−0.005	0.172
			EE	−0.001	0.159	−0.001	0.158	−0.001	0.156
			TLS-A	0.002	0.168	0.001	0.171	−0.003	0.168
		20	TLS-TS	−0.001	0.183	−0.003	0.182	−0.002	0.186
			TLS-PBC	−0.003	0.187	−0.002	0.181	−0.004	0.184
			EE	−0.006	0.169	−0.005	0.167	−0.005	0.167
			TLS-A	−0.008	0.164	−0.008	0.164	−0.013	0.173
	NAT	40	TLS-TS	−0.000	0.332	0.006	0.324	0.012	0.331
			TLS-PBC	0.004	0.334	0.007	0.336	0.010	0.329
			EE	−0.004	0.322	0.001	0.320	−0.003	0.318
			TLS-A	0.005	0.333	0.008	0.331	0.011	0.337
		20	TLS-TS	0.005	0.347	0.016	0.351	0.018	0.372
			TLS-PBC	0.010	0.361	0.015	0.373	0.020	0.389
			EE	0.024	0.419	0.022	0.410	0.021	0.411
			TLS-A	0.030	0.338	0.030	0.355	0.042	0.399

Table S-3: Summaries of the bias and RMSE from four methods under the simulation settings where the true a is 1, the true Σ is the linear shrinkage estimator from control runs, and ϵ follows a multivariate normal distribution.

				$L = 50$		$L = 100$		$L = 200$	
Scale	Forcing	m	Method	Bias	RMSE	Bias	RMSE	Bias	RMSE
Global	ANT	40	TLS-TS	0.002	0.122	0.003	0.116	0.006	0.102
			TLS-PBC	0.002	0.115	0.005	0.102	0.004	0.092
			EE	−0.003	0.106	−0.001	0.103	−0.001	0.102
			TLS-A	0.003	0.137	0.002	0.126	0.002	0.122
		20	TLS-TS	0.006	0.125	0.004	0.117	0.004	0.105
			TLS-PBC	0.007	0.117	0.005	0.105	0.004	0.089
			EE	0.004	0.114	0.005	0.112	0.006	0.111
			TLS-A	0.004	0.147	0.005	0.140	0.006	0.132
	NAT	40	TLS-TS	0.009	0.308	0.009	0.266	0.002	0.222
			TLS-PBC	0.006	0.269	−0.001	0.226	0.006	0.185
			EE	−0.020	0.251	−0.010	0.251	−0.005	0.248
			TLS-A	−0.001	0.387	0.006	0.264	0.006	0.248
		20	TLS-TS	−0.013	0.335	−0.005	0.291	−0.016	0.245
			TLS-PBC	−0.011	0.293	−0.012	0.249	−0.011	0.204
			EE	−0.032	0.313	−0.012	0.318	−0.000	0.317
			TLS-A	0.011	0.426	0.009	0.290	0.006	0.279
Regional	ANT	40	TLS-TS	−0.001	0.159	0.000	0.159	0.001	0.155
			TLS-PBC	0.006	0.160	0.003	0.148	0.001	0.142
			EE	0.002	0.145	0.003	0.145	0.002	0.144
			TLS-A	−0.001	0.153	0.002	0.158	0.003	0.161
		20	TLS-TS	−0.001	0.172	0.000	0.169	−0.004	0.154
			TLS-PBC	0.002	0.167	−0.002	0.158	−0.001	0.145
			EE	0.002	0.157	0.004	0.156	0.004	0.155
			TLS-A	0.001	0.158	−0.001	0.161	0.001	0.161
	NAT	40	TLS-TS	0.002	0.331	−0.005	0.324	−0.004	0.305
			TLS-PBC	−0.005	0.320	−0.001	0.301	0.002	0.286
			EE	0.005	0.336	0.006	0.335	0.006	0.335
			TLS-A	0.011	0.315	0.012	0.306	0.012	0.304
		20	TLS-TS	0.017	0.350	0.006	0.336	0.008	0.318
			TLS-PBC	0.009	0.340	0.002	0.314	0.000	0.301
			EE	0.017	0.399	0.016	0.395	0.018	0.397
			TLS-A	0.012	0.317	0.014	0.317	0.010	0.325

Table S-4: Summaries of the bias and RMSE from four methods under the simulation settings where the true a is 1, the true Σ is the linear shrinkage estimator from control runs, and ϵ follows a multivariate t distribution with degree of freedom 15.

				$L = 50$		$L = 100$		$L = 200$	
Scale	Forcing	m	Method	Bias	RMSE	Bias	RMSE	Bias	RMSE
Global	ANT	40	TLS-TS	−0.006	0.124	−0.009	0.112	−0.008	0.100
			TLS-PBC	−0.004	0.115	−0.008	0.102	−0.006	0.086
			EE	−0.008	0.101	−0.009	0.100	−0.007	0.097
			TLS-A	−0.003	0.133	−0.003	0.123	−0.003	0.119
		20	TLS-TS	−0.007	0.127	−0.011	0.115	−0.009	0.105
			TLS-PBC	−0.005	0.119	−0.010	0.104	−0.007	0.091
			EE	−0.011	0.110	−0.010	0.109	−0.007	0.106
			TLS-A	−0.007	0.137	−0.014	0.142	−0.018	0.152
	NAT	40	TLS-TS	0.024	0.335	0.026	0.301	0.016	0.240
			TLS-PBC	0.027	0.288	0.021	0.252	0.015	0.203
			EE	−0.007	0.245	0.002	0.243	0.008	0.245
			TLS-A	0.026	0.415	0.011	0.294	0.010	0.276
		20	TLS-TS	0.048	0.432	0.044	0.387	0.028	0.318
			TLS-PBC	0.046	0.372	0.036	0.330	0.024	0.269
			EE	−0.017	0.324	−0.003	0.321	0.010	0.321
			TLS-A	0.069	0.601	0.058	0.460	0.075	0.538
Regional	ANT	40	TLS-TS	−0.002	0.162	−0.001	0.154	0.004	0.152
			TLS-PBC	−0.001	0.161	0.002	0.146	−0.000	0.144
			EE	−0.002	0.142	−0.000	0.140	−0.000	0.140
			TLS-A	0.003	0.158	0.000	0.170	0.000	0.170
		20	TLS-TS	−0.003	0.167	−0.001	0.160	0.004	0.158
			TLS-PBC	−0.002	0.166	0.000	0.153	−0.002	0.152
			EE	−0.003	0.156	−0.002	0.155	−0.003	0.154
			TLS-A	−0.009	0.150	−0.009	0.160	−0.013	0.167
	NAT	40	TLS-TS	0.016	0.326	0.019	0.324	0.008	0.295
			TLS-PBC	0.024	0.314	0.016	0.300	0.018	0.284
			EE	0.011	0.330	0.010	0.321	0.009	0.320
			TLS-A	0.000	0.331	0.006	0.331	0.004	0.341
		20	TLS-TS	0.022	0.355	0.027	0.363	0.021	0.333
			TLS-PBC	0.031	0.345	0.029	0.347	0.034	0.343
			EE	0.027	0.418	0.031	0.416	0.027	0.406
			TLS-A	0.025	0.349	0.029	0.371	0.038	0.417

Table S-5: Summaries of the bias and RMSE from three methods under the simulation settings where the true a is 0.5, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate normal distribution.

				$L = 50$		$L = 100$		$L = 200$	
Scale	Forcing	m	Method	Bias	RMSE	Bias	RMSE	Bias	RMSE
Global	ANT	40	TLS-TS	0.002	0.148	0.003	0.141	0.005	0.132
			TLS-PBC	0.003	0.146	0.008	0.136	0.013	0.127
			EE	−0.003	0.110	−0.001	0.107	−0.000	0.105
		20	TLS-TS	0.011	0.157	0.013	0.154	0.018	0.150
			TLS-PBC	0.012	0.152	0.020	0.148	0.024	0.145
			EE	0.001	0.113	0.002	0.112	0.002	0.110
	NAT	40	TLS-TS	0.127	0.351	0.128	0.333	0.131	0.310
			TLS-PBC	0.134	0.335	0.130	0.316	0.144	0.300
			EE	−0.003	0.222	−0.001	0.214	0.002	0.215
		20	TLS-TS	0.252	0.463	0.248	0.445	0.268	0.426
			TLS-PBC	0.256	0.437	0.260	0.435	0.287	0.432
			EE	−0.025	0.242	−0.016	0.241	−0.007	0.239
Regional	ANT	40	TLS-TS	0.001	0.179	0.003	0.188	0.002	0.182
			TLS-PBC	0.002	0.185	0.007	0.187	0.007	0.180
			EE	−0.001	0.155	0.000	0.154	−0.000	0.153
		20	TLS-TS	−0.004	0.181	0.001	0.191	0.009	0.188
			TLS-PBC	−0.004	0.186	0.007	0.188	0.015	0.187
			EE	−0.001	0.157	−0.000	0.156	0.001	0.156
	NAT	40	TLS-TS	0.029	0.350	0.036	0.359	0.040	0.367
			TLS-PBC	0.034	0.356	0.043	0.358	0.066	0.372
			EE	−0.012	0.290	−0.007	0.293	−0.005	0.291
		20	TLS-TS	0.096	0.380	0.100	0.382	0.133	0.406
			TLS-PBC	0.106	0.387	0.133	0.405	0.186	0.441
			EE	0.021	0.334	0.024	0.333	0.022	0.332

method still has unbiased estimators and competitive RMSE. When the regression errors are normally distributed, TLS-A has a smaller RMSE than the other two TLS methods at the regional scale but not the global scale, possibly because of the higher spatial dimension at the global scale. Meanwhile, the RMSE of the TLS-A estimator is larger than EE's in most settings, which demonstrates the superiority of the proposed EE method is not simply from the temporal stationarity assumption. When the true a is 0.5, all point estimators

Table S-6: Summaries of the bias and RMSE from three methods under the simulation settings where the true a is 0.5, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate t distribution with degree of freedom 15.

				$L = 50$		$L = 100$		$L = 200$	
Scale	Forcing	m	Method	Bias	RMSE	Bias	RMSE	Bias	RMSE
Global	ANT	40	TLS-TS	−0.006	0.151	−0.001	0.149	0.001	0.139
			TLS-PBC	−0.004	0.148	0.002	0.141	0.007	0.130
			EE	−0.004	0.105	−0.002	0.102	−0.002	0.100
		20	TLS-TS	−0.008	0.158	−0.003	0.158	0.001	0.148
			TLS-PBC	−0.006	0.155	0.001	0.152	0.007	0.143
			EE	−0.007	0.109	−0.007	0.110	−0.006	0.107
	NAT	40	TLS-TS	0.153	0.411	0.143	0.382	0.149	0.361
			TLS-PBC	0.155	0.389	0.152	0.375	0.162	0.358
			EE	−0.006	0.216	−0.000	0.212	0.001	0.213
		20	TLS-TS	0.327	0.660	0.329	0.649	0.344	0.663
			TLS-PBC	0.342	0.672	0.353	0.690	0.390	0.737
			EE	−0.013	0.242	−0.002	0.239	0.005	0.238
Regional	ANT	40	TLS-TS	−0.005	0.170	0.008	0.173	0.011	0.172
			TLS-PBC	−0.005	0.176	0.011	0.173	0.011	0.174
			EE	0.003	0.153	0.003	0.152	0.003	0.152
		20	TLS-TS	−0.001	0.176	0.013	0.180	0.016	0.180
			TLS-PBC	0.001	0.184	0.017	0.183	0.015	0.185
			EE	−0.003	0.151	−0.001	0.150	−0.002	0.150
	NAT	40	TLS-TS	0.056	0.347	0.061	0.353	0.069	0.355
			TLS-PBC	0.060	0.350	0.073	0.363	0.095	0.379
			EE	−0.011	0.308	−0.013	0.308	−0.012	0.307
		20	TLS-TS	0.091	0.379	0.106	0.397	0.136	0.426
			TLS-PBC	0.107	0.398	0.137	0.429	0.202	0.513
			EE	0.009	0.334	0.012	0.330	0.013	0.333

of $\hat{\beta}$ from the EE method still have negligible bias. Since the true a is no longer 1, the point estimators from TLS-TS and TLS-PBC have a larger bias than the setting $a = 1$. Specifically, the $\hat{\beta}$'s for ANT scaling factor from the two TLS methods look biased. For the RMSE, the EE estimator has the smallest RMSE for every setting. In summary, the EE method has unbiased estimators with the smallest RMSE, while TLS methods have many biased estimators and larger RMSE than EE.

Table S-7: Summaries of the bias and RMSE from three methods under the simulation settings where the true a is 0.5, the true Σ is the linear shrinkage estimator from control runs, and ϵ follows a multivariate normal distribution.

				$L = 50$		$L = 100$		$L = 200$	
Scale	Forcing	m	Method	Bias	RMSE	Bias	RMSE	Bias	RMSE
Global	ANT	40	TLS-TS	0.001	0.120	0.004	0.115	0.007	0.103
			TLS-PBC	0.001	0.116	0.003	0.101	0.008	0.089
			EE	−0.002	0.099	−0.001	0.097	−0.001	0.097
		20	TLS-TS	0.008	0.126	0.008	0.117	0.016	0.105
			TLS-PBC	0.011	0.119	0.014	0.104	0.019	0.089
			EE	0.002	0.101	0.003	0.097	0.005	0.097
	NAT	40	TLS-TS	0.117	0.347	0.114	0.306	0.106	0.258
			TLS-PBC	0.110	0.305	0.108	0.256	0.098	0.216
			EE	−0.002	0.222	0.003	0.218	0.003	0.214
		20	TLS-TS	0.232	0.438	0.223	0.398	0.205	0.338
			TLS-PBC	0.222	0.393	0.206	0.344	0.191	0.294
			EE	−0.016	0.253	−0.012	0.251	−0.009	0.251
Regional	ANT	40	TLS-TS	0.005	0.159	0.003	0.161	0.013	0.152
			TLS-PBC	0.009	0.157	0.007	0.150	0.014	0.146
			EE	0.009	0.138	0.010	0.138	0.009	0.137
		20	TLS-TS	0.007	0.166	0.006	0.165	0.010	0.154
			TLS-PBC	0.009	0.160	0.011	0.159	0.012	0.147
			EE	0.002	0.144	0.003	0.142	0.003	0.141
	NAT	40	TLS-TS	0.036	0.343	0.045	0.332	0.052	0.308
			TLS-PBC	0.037	0.333	0.054	0.309	0.070	0.301
			EE	0.001	0.294	−0.001	0.294	0.001	0.294
		20	TLS-TS	0.070	0.349	0.079	0.350	0.113	0.360
			TLS-PBC	0.083	0.347	0.108	0.348	0.153	0.367
			EE	−0.007	0.324	−0.007	0.322	−0.008	0.322

Tables S-9 to S-16 summarize the average widths, the empirical coverage rates, and the average interval score of the 90% confidence intervals under the other simulation settings. At both global and regional levels, the EE interval has a close-to-nominal coverage rate in almost all settings. TLS-TS has a low coverage rate across all settings. TLS-PBC has a close-to-nominal coverage rate when L is 100 or 200, except for the NAT scaling factor in the global region. TLS-A interval has a close-to-nominal coverage rate in the setting where

Table S-8: Summaries of the bias and RMSE from three methods under the simulation settings where the true a is 0.5, the true Σ is the linear shrinkage estimator from control runs, and ϵ follows a multivariate t distribution with degree of freedom 15.

				$L = 50$		$L = 100$		$L = 200$	
Scale	Forcing	m	Method	Bias	RMSE	Bias	RMSE	Bias	RMSE
Global	ANT	40	TLS-TS	−0.002	0.128	−0.000	0.121	0.002	0.106
			TLS-PBC	0.001	0.121	0.004	0.106	0.005	0.091
			EE	−0.005	0.102	−0.002	0.100	−0.004	0.098
		20	TLS-TS	−0.004	0.134	−0.001	0.127	0.003	0.111
			TLS-PBC	0.000	0.127	0.004	0.112	0.007	0.096
			EE	−0.006	0.098	−0.004	0.096	−0.005	0.095
	NAT	40	TLS-TS	0.126	0.368	0.120	0.328	0.106	0.286
			TLS-PBC	0.125	0.333	0.107	0.284	0.102	0.245
			EE	−0.012	0.222	−0.010	0.223	−0.007	0.219
		20	TLS-TS	0.290	0.635	0.270	0.563	0.252	0.515
			TLS-PBC	0.278	0.571	0.247	0.501	0.233	0.438
			EE	−0.016	0.248	−0.006	0.249	−0.000	0.251
Regional	ANT	40	TLS-TS	0.004	0.160	0.004	0.153	0.009	0.148
			TLS-PBC	0.006	0.153	0.005	0.143	0.011	0.142
			EE	−0.003	0.139	−0.002	0.138	−0.001	0.137
		20	TLS-TS	0.007	0.164	0.007	0.157	0.012	0.154
			TLS-PBC	0.010	0.159	0.009	0.148	0.012	0.149
			EE	0.000	0.145	−0.000	0.143	−0.001	0.143
	NAT	40	TLS-TS	0.039	0.335	0.044	0.336	0.060	0.317
			TLS-PBC	0.050	0.333	0.053	0.325	0.074	0.314
			EE	−0.002	0.286	−0.001	0.281	−0.003	0.279
		20	TLS-TS	0.077	0.371	0.092	0.391	0.120	0.382
			TLS-PBC	0.098	0.377	0.115	0.392	0.159	0.422
			EE	0.002	0.328	0.000	0.327	−0.001	0.322

$a = 1$. However, when ϵ follows a multivariate t distribution with degree of freedom 15, the interval width and score of TLS-A could be much larger than EE, leading to very conservative confidence intervals. Hence, the temporal stationarity and the pseudo-bootstrap procedure can help improve the TLS method, but it is clear that the EE method provides a unified framework for both the point and interval estimators with more satisfied performances.

Tables S-17 and S-18 summarize the simulation results for a larger number of sites $S = 108$

Table S-9: Summaries of the average length, empirical coverage percentages (CP), and the average interval score of the 90% confidence intervals constructed from four methods under the simulation settings where the true a is 1, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate normal distribution.

				$L = 50$			$L = 100$			$L = 200$		
Scale	Forcing	m	Method	Widths	CP(%)	Score	Widths	CP(%)	Score	Widths	CP(%)	Score
Global	ANT	40	TLS-TS	0.285	61.1	1.235	0.275	65.8	1.087	0.294	71.3	1.004
			TLS-PBC	0.411	84.4	1.042	0.446	89.9	1.029	0.458	91.9	1.008
			EE	0.362	89.3	0.818	0.355	90.0	0.804	0.353	89.4	0.795
			TLS-A	0.504	90.8	1.118	0.434	89.4	0.977	0.409	88.7	0.931
		20	TLS-TS	0.405	65.3	1.403	0.312	68.3	1.110	0.320	73.8	1.012
			TLS-PBC	0.423	83.3	1.064	0.466	90.4	1.039	0.479	91.4	1.042
			EE	0.373	87.0	0.874	0.371	87.9	0.862	0.372	88.6	0.861
			TLS-A	0.519	88.6	1.201	0.448	89.4	1.035	0.429	89.0	1.002
	NAT	40	TLS-TS	0.903	62.4	3.048	0.595	68.0	2.187	0.588	71.6	1.899
			TLS-PBC	0.793	83.0	1.964	0.798	86.7	1.875	0.786	89.9	1.782
			EE	0.768	88.0	1.766	0.761	87.7	1.746	0.760	87.6	1.738
			TLS-A	1.168	89.2	2.638	0.778	89.1	1.800	0.719	88.2	1.670
		20	TLS-TS	2.861	65.8	7.023	0.819	70.4	2.665	0.695	77.3	2.057
			TLS-PBC	0.906	83.9	2.228	0.900	89.2	2.041	0.887	90.5	1.986
			EE	0.879	87.4	2.044	0.893	88.1	2.071	0.908	88.9	2.086
			TLS-A	1.303	89.0	3.033	0.828	87.5	1.945	0.772	87.2	1.835
Regional	ANT	40	TLS-TS	0.316	61.7	1.371	0.356	67.8	1.305	0.411	73.9	1.284
			TLS-PBC	0.489	81.5	1.289	0.589	90.7	1.309	0.638	93.5	1.368
			EE	0.510	87.8	1.178	0.508	89.5	1.159	0.508	88.6	1.158
			TLS-A	0.531	90.1	1.181	0.533	89.1	1.195	0.532	89.0	1.208
		20	TLS-TS	0.373	61.5	1.553	0.385	66.0	1.423	0.447	73.1	1.375
			TLS-PBC	0.509	80.0	1.399	0.614	87.6	1.423	0.666	92.5	1.447
			EE	0.538	89.1	1.259	0.539	88.9	1.243	0.539	88.9	1.246
			TLS-A	0.544	89.6	1.235	0.551	89.7	1.261	0.557	88.7	1.281
	NAT	40	TLS-TS	0.643	65.7	2.607	0.712	68.9	2.555	0.832	80.0	2.317
			TLS-PBC	0.867	79.1	2.467	1.039	86.2	2.455	1.178	92.6	2.548
			EE	1.052	88.5	2.442	1.053	88.6	2.431	1.049	88.4	2.412
			TLS-A	1.001	89.2	2.293	0.977	87.8	2.273	0.972	87.1	2.290
		20	TLS-TS	0.938	65.6	3.087	0.836	73.5	2.677	0.998	82.1	2.604
			TLS-PBC	0.911	78.8	2.540	1.122	87.6	2.609	1.282	92.2	2.761
			EE	1.214	90.6	2.710	1.224	90.5	2.731	1.233	90.4	2.747
			TLS-A	1.035	88.2	2.378	1.015	87.9	2.385	1.027	87.4	2.423

Table S-10: Summaries of the average length, empirical coverage percentages (CP), and the average interval score of the 90% confidence intervals constructed from four methods under the simulation settings where the true a is 1, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate t distribution with degree of freedom 15.

				$L = 50$			$L = 100$			$L = 200$		
Scale	Forcing	m	Method	Widths	CP(%)	Score	Widths	CP(%)	Score	Widths	CP(%)	Score
Global	ANT	40	TLS-TS	0.282	61.1	1.246	0.279	67.1	1.086	0.291	71.3	1.014
			TLS-PBC	0.395	81.2	1.060	0.436	88.9	1.016	0.445	90.7	0.987
			EE	0.362	92.3	0.794	0.352	91.9	0.770	0.349	92.1	0.767
			TLS-A	0.511	89.4	1.167	0.462	89.3	1.063	0.513	90.7	1.127
		20	TLS-TS	0.403	65.3	1.429	0.348	70.2	1.169	0.503	72.6	1.412
			TLS-PBC	0.414	82.7	1.091	0.458	88.4	1.061	0.471	90.7	1.051
			EE	0.383	88.8	0.872	0.376	89.0	0.851	0.372	89.3	0.847
			TLS-A	0.585	89.5	1.335	0.621	91.9	1.371	1.403	94.2	2.959
	NAT	40	TLS-TS	0.933	58.3	3.397	0.657	64.6	2.489	0.625	68.8	2.157
			TLS-PBC	0.780	79.6	2.126	0.794	83.6	2.000	0.797	85.4	1.891
			EE	0.833	92.7	1.809	0.807	91.6	1.781	0.786	91.5	1.740
			TLS-A	1.532	92.5	3.345	1.151	93.5	2.483	1.417	94.7	2.952
		20	TLS-TS	2.048	59.9	5.873	1.249	61.6	3.990	2.703	67.8	6.613
			TLS-PBC	0.926	78.9	2.527	0.936	79.9	2.446	0.939	81.2	2.431
			EE	1.049	93.5	2.265	1.010	91.6	2.213	0.974	90.9	2.176
			TLS-A	3.562	92.8	7.823	2.949	94.3	6.358	5.380	96.1	11.325
Regional	ANT	40	TLS-TS	0.313	60.8	1.402	0.345	68.6	1.291	0.399	72.6	1.272
			TLS-PBC	0.456	79.3	1.261	0.547	88.0	1.272	0.604	91.4	1.325
			EE	0.508	89.8	1.157	0.503	89.3	1.146	0.503	89.2	1.143
			TLS-A	0.533	88.3	1.248	0.537	87.8	1.273	0.551	90.0	1.261
		20	TLS-TS	0.385	63.2	1.514	0.391	68.9	1.356	0.548	75.5	1.521
			TLS-PBC	0.474	79.1	1.330	0.573	87.8	1.329	0.643	90.8	1.417
			EE	0.539	89.4	1.238	0.536	89.9	1.214	0.536	90.0	1.213
			TLS-A	0.553	90.9	1.240	0.598	91.1	1.323	0.936	92.3	1.992
	NAT	40	TLS-TS	0.689	63.8	2.678	0.707	70.5	2.341	0.830	76.2	2.381
			TLS-PBC	0.818	77.5	2.332	0.996	85.7	2.381	1.129	91.3	2.512
			EE	1.072	90.9	2.390	1.061	90.2	2.378	1.050	90.2	2.340
			TLS-A	1.026	87.0	2.462	1.028	88.7	2.476	1.122	90.4	2.606
		20	TLS-TS	1.008	66.8	3.178	0.920	73.0	2.735	1.367	78.7	3.412
			TLS-PBC	0.879	77.3	2.499	1.074	83.8	2.582	1.264	90.2	2.786
			EE	1.307	89.2	3.028	1.285	89.0	2.988	1.270	88.8	2.997
			TLS-A	1.092	89.8	2.545	1.264	90.1	2.910	2.692	93.8	5.695

Table S-11: Summaries of the average length, empirical coverage percentages (CP), and the average interval score of the 90% confidence intervals constructed from four methods under the simulation settings where the true a is 1, the true Σ is the linear shrinkage estimator from control runs, and ϵ follows a multivariate normal distribution.

				$L = 50$			$L = 100$			$L = 200$		
Scale	Forcing	m	Method	Widths	CP(%)	Score	Widths	CP(%)	Score	Widths	CP(%)	Score
Global	ANT	40	TLS-TS	0.178	51.6	1.047	0.181	55.3	0.943	0.186	63.7	0.784
			TLS-PBC	0.331	84.3	0.820	0.333	89.4	0.754	0.315	91.9	0.696
			EE	0.333	87.9	0.770	0.327	89.6	0.746	0.326	90.0	0.740
			TLS-A	0.445	89.0	1.000	0.408	90.4	0.921	0.389	88.5	0.886
		20	TLS-TS	0.194	56.0	1.062	0.191	56.8	0.937	0.194	64.0	0.787
			TLS-PBC	0.338	85.3	0.837	0.342	89.4	0.785	0.326	93.4	0.695
			EE	0.349	87.3	0.833	0.347	87.5	0.827	0.349	88.6	0.825
			TLS-A	0.458	89.1	1.070	0.428	87.8	1.011	0.415	88.2	0.960
	NAT	40	TLS-TS	0.451	51.5	2.719	0.419	57.9	2.190	0.405	61.2	1.697
			TLS-PBC	0.782	85.7	1.914	0.727	87.6	1.665	0.652	92.7	1.422
			EE	0.814	88.0	1.906	0.810	88.2	1.855	0.810	89.4	1.858
			TLS-A	1.229	89.3	2.821	0.818	88.8	1.919	0.787	89.6	1.820
		20	TLS-TS	0.628	55.7	3.081	0.486	61.4	2.390	0.453	63.8	1.875
			TLS-PBC	0.912	87.5	2.130	0.825	89.9	1.916	0.733	91.9	1.570
			EE	0.962	88.5	2.259	0.977	88.4	2.252	0.990	89.9	2.263
			TLS-A	1.399	89.6	3.226	0.898	87.3	2.148	0.861	87.0	2.062
Regional	ANT	40	TLS-TS	0.266	57.1	1.271	0.293	62.7	1.215	0.328	71.4	1.139
			TLS-PBC	0.441	81.9	1.163	0.504	90.8	1.121	0.522	92.5	1.135
			EE	0.469	88.6	1.062	0.468	88.9	1.062	0.466	88.8	1.050
			TLS-A	0.503	88.6	1.136	0.514	89.4	1.156	0.518	88.2	1.174
		20	TLS-TS	0.285	57.9	1.403	0.315	63.1	1.304	0.354	74.1	1.110
			TLS-PBC	0.457	82.2	1.190	0.523	89.2	1.183	0.547	94.1	1.153
			EE	0.500	88.8	1.150	0.499	88.8	1.149	0.501	88.9	1.145
			TLS-A	0.517	90.0	1.169	0.535	90.6	1.184	0.539	90.8	1.185
	NAT	40	TLS-TS	0.563	58.7	2.671	0.620	64.9	2.442	0.693	74.5	2.235
			TLS-PBC	0.857	81.2	2.292	0.985	88.4	2.247	1.033	92.6	2.261
			EE	1.055	88.6	2.414	1.058	87.6	2.430	1.059	88.5	2.422
			TLS-A	1.017	89.9	2.310	0.987	90.1	2.252	0.986	89.8	2.248
		20	TLS-TS	0.665	60.8	2.809	0.732	70.5	2.549	0.810	77.1	2.337
			TLS-PBC	0.928	80.3	2.455	1.070	88.9	2.402	1.131	93.1	2.440
			EE	1.250	88.9	2.868	1.260	90.3	2.878	1.268	89.8	2.871
			TLS-A	1.060	90.0	2.400	1.042	90.3	2.351	1.049	89.2	2.396

Table S-12: Summaries of the average length, empirical coverage percentages (CP), and the average interval score of the 90% confidence intervals constructed from four methods under the simulation settings where the true a is 1, the true Σ is the linear shrinkage estimator from control runs, and ϵ follows a multivariate t distribution with degree of freedom 15.

				$L = 50$			$L = 100$			$L = 200$		
Scale	Forcing	m	Method	Widths	CP(%)	Score	Widths	CP(%)	Score	Widths	CP(%)	Score
Global	ANT	40	TLS-TS	0.176	51.3	1.056	0.179	57.4	0.880	0.180	63.2	0.743
			TLS-PBC	0.316	83.1	0.803	0.321	89.0	0.740	0.304	92.5	0.659
			EE	0.338	89.4	0.754	0.328	89.3	0.736	0.324	88.6	0.728
			TLS-A	0.452	89.9	1.027	0.434	90.5	0.972	0.548	90.7	1.187
		20	TLS-TS	0.197	52.0	1.090	0.191	58.9	0.908	0.190	62.8	0.778
			TLS-PBC	0.329	82.0	0.827	0.333	88.2	0.766	0.316	91.6	0.689
			EE	0.365	89.9	0.827	0.357	88.7	0.811	0.353	89.7	0.802
			TLS-A	0.522	90.2	1.159	0.749	91.9	1.614	3.250	93.9	6.640
	NAT	40	TLS-TS	0.468	49.5	2.888	0.433	54.1	2.427	0.401	60.1	1.815
			TLS-PBC	0.768	83.7	1.952	0.714	85.0	1.751	0.644	89.4	1.461
			EE	0.879	92.9	1.890	0.856	92.9	1.856	0.834	92.3	1.825
			TLS-A	1.607	92.5	3.525	1.218	94.1	2.603	1.642	94.2	3.418
		20	TLS-TS	0.720	52.3	3.730	0.521	51.8	3.073	0.463	56.5	2.379
			TLS-PBC	0.905	81.4	2.361	0.831	83.3	2.135	0.734	85.7	1.764
			EE	1.109	92.9	2.440	1.071	91.3	2.390	1.044	90.9	2.353
			TLS-A	3.800	93.7	8.364	3.575	94.8	7.670	12.345	96.1	25.362
Regional	ANT	40	TLS-TS	0.262	56.4	1.308	0.286	64.5	1.162	0.322	71.3	1.085
			TLS-PBC	0.412	79.5	1.128	0.475	88.7	1.068	0.502	91.6	1.093
			EE	0.465	90.0	1.042	0.462	90.2	1.030	0.461	89.7	1.030
			TLS-A	0.498	87.6	1.175	0.519	87.2	1.246	0.529	88.2	1.255
		20	TLS-TS	0.290	57.7	1.348	0.315	65.9	1.197	0.381	73.6	1.176
			TLS-PBC	0.429	80.2	1.158	0.497	88.4	1.129	0.531	92.3	1.149
			EE	0.509	89.4	1.138	0.507	90.4	1.123	0.505	90.0	1.125
			TLS-A	0.518	89.8	1.146	0.562	91.4	1.244	0.744	91.9	1.595
	NAT	40	TLS-TS	0.569	59.7	2.612	0.615	65.3	2.390	0.695	73.8	2.138
			TLS-PBC	0.817	80.2	2.169	0.941	88.1	2.178	1.007	92.3	2.184
			EE	1.081	90.1	2.487	1.071	89.0	2.478	1.063	89.5	2.466
			TLS-A	1.044	88.2	2.467	1.042	89.7	2.452	1.135	90.1	2.619
		20	TLS-TS	0.718	60.1	2.944	0.762	67.3	2.699	1.022	76.0	2.764
			TLS-PBC	0.879	78.5	2.338	1.019	87.8	2.419	1.113	91.6	2.451
			EE	1.353	90.7	3.120	1.328	90.6	3.060	1.308	90.3	3.035
			TLS-A	1.114	89.3	2.625	1.257	90.6	2.914	2.261	93.1	4.871

Table S-13: Summaries of the average length, empirical coverage percentages (CP), and the average interval score of the 90% confidence intervals constructed from three methods under the simulation settings where the true a is 0.5, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate normal distribution.

				$L = 50$			$L = 100$			$L = 200$		
Scale	Forcing	m	Method	Widths	CP(%)	Score	Widths	CP(%)	Score	Widths	CP(%)	Score
Global	ANT	40	TLS-TS	0.269	62.9	1.187	0.278	65.7	1.076	0.296	73.0	0.944
			TLS-PBC	0.416	83.1	1.058	0.455	90.2	1.021	0.465	93.5	1.008
			EE	0.355	90.6	0.789	0.347	90.1	0.775	0.345	90.7	0.767
		20	TLS-TS	0.389	64.0	1.430	0.380	66.6	1.299	0.339	73.1	1.088
			TLS-PBC	0.432	82.7	1.096	0.482	88.8	1.104	0.502	90.6	1.109
			EE	0.356	86.9	0.859	0.353	87.7	0.846	0.354	88.6	0.840
	NAT	40	TLS-TS	0.713	60.5	3.023	0.639	63.6	2.612	0.649	68.9	2.318
			TLS-PBC	0.824	76.4	2.356	0.840	81.5	2.241	0.848	84.9	2.119
			EE	0.697	88.6	1.616	0.685	88.8	1.571	0.681	88.7	1.571
		20	TLS-TS	2.031	59.3	6.272	1.414	62.7	4.637	0.963	66.8	3.329
			TLS-PBC	1.003	74.1	3.032	1.034	75.8	2.961	1.051	77.7	2.916
			EE	0.752	87.4	1.763	0.756	88.7	1.744	0.758	89.6	1.741
Regional	ANT	40	TLS-TS	0.316	62.2	1.439	0.353	64.4	1.445	0.413	73.0	1.299
			TLS-PBC	0.491	80.5	1.342	0.593	87.3	1.364	0.646	91.3	1.402
			EE	0.494	88.3	1.133	0.492	88.5	1.134	0.492	89.0	1.128
		20	TLS-TS	0.358	64.9	1.439	0.400	68.6	1.446	0.469	77.4	1.348
			TLS-PBC	0.518	82.5	1.352	0.626	89.8	1.431	0.698	92.9	1.507
			EE	0.507	88.9	1.156	0.504	89.0	1.148	0.506	89.2	1.145
	NAT	40	TLS-TS	0.662	64.2	2.788	0.730	66.6	2.657	0.871	76.1	2.649
			TLS-PBC	0.891	77.6	2.548	1.090	87.4	2.590	1.246	90.6	2.789
			EE	0.961	89.9	2.144	0.958	89.9	2.136	0.954	89.7	2.120
		20	TLS-TS	0.860	64.6	3.159	0.945	72.3	2.966	1.138	81.1	2.976
			TLS-PBC	0.964	77.9	2.730	1.203	84.5	2.908	1.451	90.1	3.261
			EE	1.048	89.6	2.414	1.045	88.7	2.406	1.052	88.8	2.431

at the global scale with multivariate normal distributed errors. With a higher resolution, EE is almost unbiased with a smaller RMSE than TLS-TS and TLS-PBC, except for the NAT forcing with a larger measurement error ($m = 20$) and a small number of control runs ($L = 50$). For the confidence interval evaluation, we do not present TLS-PBC for its expensive computational cost. We observe that EE achieves almost nominal coverage rate,

Table S-14: Summaries of the average length, empirical coverage percentages (CP), and the average interval score of the 90% confidence intervals constructed from three methods under the simulation settings where the true a is 0.5, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate t distribution with degree of freedom 15.

				$L = 50$			$L = 100$			$L = 200$		
Scale	Forcing	m	Method	Widths	CP(%)	Score	Widths	CP(%)	Score	Widths	CP(%)	Score
Global	ANT	40	TLS-TS	0.583	62.8	1.798	0.290	65.1	1.130	0.298	72.3	0.990
			TLS-PBC	0.398	83.7	1.015	0.443	87.8	1.043	0.454	91.1	0.981
			EE	0.349	90.3	0.783	0.340	91.3	0.759	0.338	90.8	0.751
		20	TLS-TS	1.116	67.0	2.808	0.355	69.7	1.209	0.438	75.3	1.219
			TLS-PBC	0.426	83.4	1.080	0.483	87.6	1.120	0.509	92.9	1.087
			EE	0.360	89.1	0.845	0.352	88.7	0.828	0.350	88.4	0.822
	NAT	40	TLS-TS	5.091	59.0	11.969	0.806	61.1	3.140	0.711	66.9	2.537
			TLS-PBC	0.819	75.9	2.499	0.853	78.4	2.347	0.859	81.6	2.276
			EE	0.725	91.3	1.609	0.700	91.1	1.564	0.684	89.9	1.544
		20	TLS-TS	10.918	57.6	24.452	1.415	61.1	5.260	2.064	64.5	6.018
			TLS-PBC	1.072	69.5	3.828	1.120	70.4	3.749	1.167	72.8	3.806
			EE	0.852	92.1	1.870	0.819	91.8	1.820	0.791	90.7	1.771
Regional	ANT	40	TLS-TS	0.323	62.2	1.321	0.350	68.0	1.260	0.411	75.6	1.224
			TLS-PBC	0.462	79.5	1.246	0.556	90.3	1.251	0.613	92.5	1.331
			EE	0.490	88.7	1.126	0.487	88.5	1.118	0.487	88.7	1.115
		20	TLS-TS	0.417	66.1	1.464	0.420	70.7	1.365	0.584	78.6	1.528
			TLS-PBC	0.485	80.7	1.307	0.591	89.3	1.334	0.670	92.3	1.448
			EE	0.505	90.8	1.123	0.504	90.6	1.119	0.503	91.2	1.111
	NAT	40	TLS-TS	0.712	64.8	2.784	0.743	70.4	2.499	0.893	78.0	2.593
			TLS-PBC	0.843	77.0	2.425	1.025	85.0	2.491	1.194	88.6	2.697
			EE	0.968	89.4	2.233	0.958	88.5	2.219	0.950	88.2	2.200
		20	TLS-TS	1.179	68.6	3.632	1.091	73.5	3.210	1.855	79.8	4.485
			TLS-PBC	0.930	75.9	2.695	1.155	84.1	2.872	1.415	87.6	3.262
			EE	1.081	90.4	2.434	1.070	90.9	2.390	1.057	90.0	2.391

except for the NAT forcing with $m = 20$, while TLS-TS has a low coverage rate in all cases. When the error ϵ follows a multivariate t distribution with degree of freedom 15 (Tables S-19 and S-20), we observe bias for all three methods for the NAT forcing with $m = 20$ and $L = 50$, whereas the bias is negligible for EE when L gets larger. The RMSE of the two TLS methods is also significantly larger than EE's in many settings. The widths of the confidence

Table S-15: Summaries of the average length, empirical coverage percentages (CP), and the average interval score of the 90% confidence intervals constructed from three methods under the simulation settings where the true a is 0.5, the true Σ is the linear shrinkage estimator from control runs, and ϵ follows a multivariate normal distribution.

				$L = 50$			$L = 100$			$L = 200$		
Scale	Forcing	m	Method	Widths	CP(%)	Score	Widths	CP(%)	Score	Widths	CP(%)	Score
Global	ANT	40	TLS-TS	0.179	52.2	1.018	0.181	54.8	0.939	0.185	62.5	0.800
			TLS-PBC	0.329	83.7	0.826	0.332	90.3	0.754	0.314	91.9	0.688
			EE	0.321	89.2	0.726	0.315	88.9	0.715	0.314	89.2	0.708
		20	TLS-TS	0.215	55.6	1.115	0.191	59.8	0.928	0.196	65.0	0.795
			TLS-PBC	0.343	85.1	0.856	0.348	90.3	0.784	0.330	92.3	0.712
			EE	0.326	87.9	0.786	0.324	88.2	0.765	0.324	88.1	0.763
	NAT	40	TLS-TS	0.472	48.3	3.150	0.444	55.3	2.593	0.421	57.7	2.063
			TLS-PBC	0.795	79.7	2.149	0.743	85.3	1.834	0.669	87.2	1.579
			EE	0.715	87.3	1.676	0.708	88.3	1.675	0.706	88.3	1.645
		20	TLS-TS	0.981	48.6	4.750	0.538	47.7	3.492	0.502	51.5	2.824
			TLS-PBC	0.972	77.5	2.683	0.891	80.4	2.325	0.793	81.8	2.030
			EE	0.796	88.0	1.857	0.801	88.8	1.843	0.805	88.4	1.839
Regional	ANT	40	TLS-TS	0.261	58.4	1.289	0.293	63.9	1.239	0.329	73.6	1.095
			TLS-PBC	0.438	82.0	1.140	0.507	91.0	1.145	0.528	92.7	1.157
			EE	0.452	89.9	1.022	0.449	89.4	1.020	0.450	90.0	1.011
		20	TLS-TS	0.276	58.4	1.336	0.314	67.1	1.246	0.360	74.6	1.133
			TLS-PBC	0.456	84.0	1.154	0.527	89.4	1.216	0.559	93.9	1.186
			EE	0.467	88.7	1.068	0.466	89.4	1.058	0.467	89.6	1.059
	NAT	40	TLS-TS	0.576	60.2	2.817	0.633	67.3	2.484	0.722	74.2	2.206
			TLS-PBC	0.886	81.7	2.426	1.014	90.0	2.317	1.084	93.3	2.345
			EE	0.951	89.3	2.142	0.947	89.5	2.133	0.947	89.2	2.146
		20	TLS-TS	0.672	62.0	2.799	0.770	72.4	2.657	0.894	74.6	2.605
			TLS-PBC	0.950	81.5	2.503	1.123	89.2	2.591	1.224	90.3	2.722
			EE	1.050	89.1	2.374	1.052	89.6	2.372	1.055	90.5	2.350

intervals for TLS-TS are significantly larger than EE's intervals for the NAT forcing with $m = 20$, indicating that this case is challenging due to the high resolution of the grid and non-gaussian errors.

Tables S-21 and S-22 summarize simulation results for correlated forcings as in Section 4.1 with multivariate normal distributed ϵ. Similar to the high-resolution settings, we do not

Table S-16: Summaries of the average length, empirical coverage percentages (CP), and the average interval score of the 90% confidence intervals constructed from three methods under the simulation settings where the true a is 0.5, the true Σ is the linear shrinkage estimator from control runs, and ϵ follows a multivariate t distribution with degree of freedom 15.

				$L = 50$			$L = 100$			$L = 200$		
Scale	Forcing	m	Method	Widths	CP(%)	Score	Widths	CP(%)	Score	Widths	CP(%)	Score
Global	ANT	40	TLS-TS	0.178	50.6	1.109	0.178	55.0	0.987	0.182	62.5	0.805
			TLS-PBC	0.315	79.7	0.846	0.323	86.2	0.764	0.307	90.6	0.676
			EE	0.321	87.8	0.736	0.312	87.6	0.721	0.309	87.8	0.716
		20	TLS-TS	0.204	50.9	1.148	0.203	55.2	1.040	0.195	61.1	0.827
			TLS-PBC	0.334	80.0	0.882	0.344	87.0	0.804	0.327	91.3	0.716
			EE	0.331	90.4	0.728	0.325	91.6	0.704	0.322	92.3	0.698
	NAT	40	TLS-TS	0.498	50.8	3.071	0.444	53.4	2.617	0.424	57.6	2.165
			TLS-PBC	0.791	80.6	2.186	0.734	83.5	1.841	0.669	86.0	1.623
			EE	0.750	89.5	1.683	0.728	89.9	1.648	0.713	90.4	1.617
		20	TLS-TS	0.904	47.8	5.222	0.712	46.7	4.395	0.543	52.3	3.619
			TLS-PBC	1.024	75.3	3.130	0.930	78.6	2.707	0.829	80.3	2.429
			EE	0.885	92.4	1.918	0.854	91.4	1.879	0.832	90.0	1.856
Regional	ANT	40	TLS-TS	0.267	57.0	1.288	0.287	63.2	1.155	0.328	72.3	1.080
			TLS-PBC	0.414	81.9	1.079	0.477	90.3	1.074	0.503	91.9	1.109
			EE	0.444	88.6	1.022	0.442	88.6	1.018	0.442	89.9	1.016
		20	TLS-TS	0.347	59.3	1.406	0.322	64.9	1.191	0.387	74.4	1.165
			TLS-PBC	0.432	82.4	1.117	0.505	90.1	1.121	0.537	91.5	1.169
			EE	0.467	89.2	1.081	0.463	90.5	1.067	0.462	90.1	1.068
	NAT	40	TLS-TS	0.648	59.7	2.824	0.637	65.5	2.497	0.756	74.3	2.284
			TLS-PBC	0.828	78.3	2.274	0.978	87.2	2.342	1.057	91.6	2.316
			EE	0.955	90.7	2.125	0.950	91.3	2.113	0.941	91.2	2.088
		20	TLS-TS	1.137	61.0	3.796	0.836	66.1	2.876	1.131	77.1	3.052
			TLS-PBC	0.908	77.9	2.535	1.102	86.7	2.653	1.229	90.3	2.755
			EE	1.092	90.5	2.474	1.078	91.0	2.428	1.064	90.5	2.406

consider TLS-PBC in evaluating confidence intervals as it is time-consuming. For both global and regional scales, the EE method has a slight bias for the NAT forcing when $L = 50$, and the bias gets negligible with the increase of L in most cases. Similar to previous observations, the coverage rate of EE is close-to-nominal in all cases, and TLS-TS has a low coverage rate. When ϵ follows a multivariate t distribution with degree of freedom 15 (Tables S-23 and

Table S-17: Summaries of the bias and RMSE under the simulation settings where $T = 11$, $S = 108$, the true a is 1, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate normal distribution.

				$L = 50$		$L = 100$		$L = 200$	
Scale	Forcing	m	Method	Bias	RMSE	Bias	RMSE	Bias	RMSE
Global	ANT	40	TLS-TS	0.008	0.139	0.010	0.133	0.009	0.123
			TLS-PBC	0.009	0.134	0.008	0.125	0.006	0.111
			EE	−0.003	0.103	0.000	0.100	0.004	0.098
		20	TLS-TS	−0.000	0.141	0.004	0.138	0.004	0.127
			TLS-PBC	0.003	0.133	0.005	0.130	0.002	0.116
			EE	−0.016	0.103	−0.008	0.101	−0.005	0.100
	NAT	40	TLS-TS	−0.002	0.276	−0.002	0.254	0.003	0.234
			TLS-PBC	0.001	0.257	−0.001	0.235	0.004	0.210
			EE	−0.059	0.216	−0.031	0.208	−0.013	0.206
		20	TLS-TS	0.010	0.311	0.006	0.287	0.005	0.265
			TLS-PBC	0.006	0.288	0.004	0.258	0.005	0.238
			EE	−0.124	0.268	−0.074	0.262	−0.038	0.258

Table S-18: Summaries of the average length, empirical coverage percentages (CP), and the average interval score of the 90% confidence intervals constructed from three methods under the simulation settings where $T = 11$, $S = 108$, the true a is 1, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate normal distribution.

				$L = 50$			$L = 100$			$L = 200$		
Scale	Forcing	m	Method	Widths	CP(%)	Score	Widths	CP(%)	Score	Widths	CP(%)	Score
Global	ANT	40	TLS-TS	0.238	58.640	1.143	0.254	62.800	1.037	0.255	70.160	0.908
			EE	0.299	84.480	0.742	0.297	85.680	0.713	0.299	87.520	0.699
		20	TLS-TS	4.410	66.080	9.382	0.308	66.960	1.096	0.294	74.560	0.921
			EE	0.300	86.320	0.738	0.306	87.600	0.734	0.314	88.000	0.735
	NAT	40	TLS-TS	0.572	60.240	2.393	0.654	68.240	2.180	0.529	73.040	1.718
			EE	0.630	85.600	1.545	0.641	87.120	1.493	0.656	89.040	1.511
		20	TLS-TS	56.169	64.640	113.605	1.098	70.720	3.152	0.734	75.680	2.130
			EE	0.686	78.880	1.894	0.740	83.760	1.878	0.784	86.800	1.877

S-24), the bias of TLS-TS and TLS-PBC is pronounced with the NAT forcing when $m = 20$, and the RMSE of TLS methods is significantly larger than EE.

Table S-19: Summaries of the bias and RMSE under the simulation settings where $T = 11$, $S = 108$, the true a is 1, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate t distribution with degree of freedom 15.

				$L = 50$		$L = 100$		$L = 200$	
Scale	Forcing	m	Method	Bias	RMSE	Bias	RMSE	Bias	RMSE
Global	ANT	40	TLS-TS	0.002	0.137	−0.000	0.133	−0.000	0.124
			TLS-PBC	0.002	0.132	−0.000	0.122	−0.002	0.112
			EE	−0.008	0.096	−0.005	0.092	−0.001	0.090
		20	TLS-TS	−0.027	0.789	−0.017	0.338	−0.038	1.368
			TLS-PBC	−0.006	0.156	−0.015	0.225	−0.322	9.963
			EE	−0.020	0.105	−0.014	0.102	−0.008	0.100
	NAT	40	TLS-TS	0.003	0.317	0.005	0.318	0.011	0.301
			TLS-PBC	0.008	0.315	0.010	0.302	0.018	0.362
			EE	−0.080	0.227	−0.055	0.225	−0.031	0.218
		20	TLS-TS	0.346	8.988	0.217	4.454	0.423	13.652
			TLS-PBC	0.113	1.560	0.171	2.541	2.553	78.347
			EE	−0.121	0.290	−0.068	0.291	−0.024	0.291

Table S-20: Summaries of the average length, empirical coverage percentages (CP), and the average interval score of the 90% confidence intervals constructed from three methods under the simulation settings where $T = 11$, $S = 108$, the true a is 1, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate t distribution with degree of freedom 15.

				$L = 50$			$L = 100$			$L = 200$		
Scale	Forcing	m	Method	Widths	CP(%)	Score	Widths	CP(%)	Score	Widths	CP(%)	Score
Global	ANT	40	TLS-TS	0.265	60.400	1.132	0.254	62.500	1.017	0.264	68.900	0.885
			EE	0.313	89.400	0.725	0.306	89.200	0.691	0.305	90.200	0.684
		20	TLS-TS	0.813	65.800	2.147	0.448	68.500	1.322	1.317	74.000	2.929
			EE	0.338	89.900	0.791	0.332	88.500	0.764	0.330	89.500	0.754
	NAT	40	TLS-TS	0.967	58.000	3.369	0.702	63.900	2.512	0.683	67.000	2.235
			EE	0.773	90.700	1.757	0.757	90.900	1.728	0.738	90.700	1.683
		20	TLS-TS	8.873	57.800	19.945	2.961	59.500	7.814	12.780	62.300	27.092
			EE	0.999	90.500	2.239	0.974	89.800	2.206	0.946	90.000	2.160

Table S-21: Summaries of the bias and RMSE under the simulation settings where the data is the same as Section 4.1, $T = 13$, the true a is 1, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate normal distribution.

				$L = 50$		$L = 100$		$L = 200$	
Scale	Forcing	m	Method	Bias	RMSE	Bias	RMSE	Bias	RMSE
Global	ANT	40	TLS-TS	−0.002	0.065	−0.001	0.055	−0.001	0.046
			TLS-PBC	−0.001	0.055	0.000	0.044	−0.000	0.039
			EE	−0.000	0.047	−0.000	0.047	−0.000	0.046
		20	TLS-TS	0.002	0.067	0.001	0.056	0.000	0.046
			TLS-PBC	−0.000	0.057	0.001	0.046	0.001	0.040
			EE	−0.000	0.048	−0.000	0.048	−0.000	0.048
	NAT	40	TLS-TS	0.020	0.406	0.017	0.345	0.014	0.306
			TLS-PBC	0.017	0.351	0.009	0.305	0.010	0.277
			EE	−0.031	0.311	−0.011	0.312	−0.002	0.316
		20	TLS-TS	−0.007	0.435	−0.002	0.387	−0.003	0.330
			TLS-PBC	−0.005	0.382	−0.004	0.336	−0.002	0.302
			EE	−0.051	0.368	−0.012	0.376	0.004	0.386
Regional	ANT	40	TLS-TS	0.000	0.069	0.002	0.068	−0.001	0.068
			TLS-PBC	−0.001	0.069	−0.001	0.066	−0.001	0.064
			EE	−0.000	0.068	−0.001	0.068	−0.001	0.068
		20	TLS-TS	−0.002	0.071	−0.003	0.070	−0.004	0.068
			TLS-PBC	−0.002	0.070	−0.002	0.069	−0.001	0.067
			EE	−0.002	0.071	−0.002	0.071	−0.002	0.071
	NAT	40	TLS-TS	0.004	0.488	−0.012	0.487	−0.004	0.512
			TLS-PBC	0.005	0.491	−0.008	0.496	−0.014	0.520
			EE	−0.017	0.570	−0.009	0.574	−0.007	0.574
		20	TLS-TS	0.032	0.534	0.040	0.558	0.017	0.557
			TLS-PBC	0.035	0.539	0.047	0.555	0.026	0.574
			EE	0.064	0.805	0.086	0.848	0.093	0.826

Table S-22: Summaries of the average length, empirical coverage percentages (CP), and the average interval score of the 90% confidence intervals under the simulation settings where the data is the same as Section 4.1, $T = 13$, the true a is 1, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate normal distribution.

				$L = 50$			$L = 100$			$L = 200$	
Scale	Forcing	m	Method	Widths	CP(%)	Score	Widths	CP(%)	Score	Widths	CP(%)	Score
Global	ANT	40	TLS-TS	0.105	55.680	0.573	0.089	58.480	0.446	0.084	64.160	0.356
			EE	0.147	88.720	0.346	0.146	89.040	0.341	0.146	89.920	0.337
		20	TLS-TS	0.157	57.120	0.668	0.102	60.960	0.454	0.089	67.120	0.353
			EE	0.149	87.600	0.351	0.150	88.080	0.349	0.151	88.160	0.350
	NAT	40	TLS-TS	1.245	59.520	4.368	0.675	64.640	2.736	0.632	68.720	2.290
			EE	0.956	87.520	2.254	0.969	87.120	2.257	0.980	87.120	2.283
		20	TLS-TS	2.723	64.160	7.244	1.157	66.640	3.778	0.785	74.640	2.449
			EE	1.125	87.360	2.672	1.182	88.400	2.743	1.214	88.720	2.803
Regional	ANT	40	TLS-TS	0.116	60.000	0.562	0.133	65.600	0.519	0.152	72.900	0.502
			EE	0.219	88.300	0.505	0.219	88.100	0.502	0.219	88.300	0.504
		20	TLS-TS	0.209	62.500	0.723	0.167	68.100	0.578	0.184	76.000	0.521
			EE	0.230	89.500	0.525	0.231	89.300	0.525	0.231	89.600	0.524
	NAT	40	TLS-TS	1.130	65.300	4.089	1.295	73.800	3.919	1.538	81.500	3.981
			EE	1.859	90.600	4.159	1.877	91.400	4.160	1.888	90.400	4.197
		20	TLS-TS	3.833	72.200	9.289	2.637	79.700	6.389	2.872	85.000	6.453
			EE	2.492	89.100	5.619	2.562	89.400	5.736	2.573	89.600	5.700

Table S-23: Summaries of the bias and RMSE under the simulation settings where the data is the same as Section 4.1, $T = 13$, the true a is 1, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate t distribution with degree of freedom 15.

				$L = 50$		$L = 100$		$L = 200$	
Scale	Forcing	m	Method	Bias	RMSE	Bias	RMSE	Bias	RMSE
Global	ANT	40	TLS-TS	−0.001	0.063	−0.002	0.052	−0.002	0.045
			TLS-PBC	−0.002	0.053	−0.002	0.044	−0.001	0.040
			EE	−0.001	0.045	−0.001	0.044	−0.001	0.044
		20	TLS-TS	−0.005	0.077	−0.005	0.065	−0.009	0.067
			TLS-PBC	−0.006	0.074	−0.003	0.122	−0.009	0.074
			EE	−0.002	0.047	−0.002	0.046	−0.001	0.046
	NAT	40	TLS-TS	0.040	0.531	0.039	0.451	0.040	0.466
			TLS-PBC	0.038	0.510	0.042	0.456	0.036	0.445
			EE	−0.026	0.297	−0.004	0.295	0.005	0.298
		20	TLS-TS	0.072	2.070	0.105	1.222	0.174	1.379
			TLS-PBC	0.050	2.334	0.018	3.068	0.179	1.553
			EE	−0.031	0.372	0.020	0.385	0.030	0.394
Regional	ANT	40	TLS-TS	−0.003	0.063	−0.005	0.068	−0.004	0.064
			TLS-PBC	−0.004	0.063	−0.006	0.069	−0.005	0.075
			EE	−0.003	0.063	−0.004	0.064	−0.003	0.063
		20	TLS-TS	−0.002	0.065	−0.005	0.080	−0.007	0.078
			TLS-PBC	−0.004	0.067	−0.006	0.080	−0.010	0.109
			EE	−0.006	0.069	−0.005	0.068	−0.005	0.067
	NAT	40	TLS-TS	0.008	0.551	−0.011	0.520	0.017	0.698
			TLS-PBC	0.016	0.685	0.024	0.964	0.064	1.354
			EE	0.009	0.589	0.012	0.587	0.012	0.588
		20	TLS-TS	0.035	0.845	−0.005	0.898	0.095	1.307
			TLS-PBC	0.052	1.059	0.083	1.666	0.255	2.724
			EE	0.119	1.095	0.096	0.934	0.107	1.004

Table S-24: Summaries of the average length, empirical coverage percentages (CP), and the average interval score of the 90% confidence intervals under the simulation settings where the data is the same as Section 4.1, $T = 13$, the true a is 1, the true Σ has a block Toeplitz structure, and ϵ follows a multivariate t distribution with degree of freedom 15.

				$L = 50$			$L = 100$			$L = 200$		
Scale	Forcing	m	Method	Widths	CP(%)	Score	Widths	CP(%)	Score	Widths	CP(%)	Score
Global	ANT	40	TLS-TS	0.106	54.800	0.544	0.100	60.200	0.428	0.165	66.500	0.483
			EE	0.148	89.300	0.342	0.147	89.300	0.336	0.146	89.800	0.332
		20	TLS-TS	0.123	59.091	0.554	0.138	62.175	0.504	0.115	68.182	0.392
			EE	0.151	88.799	0.351	0.152	88.799	0.352	0.152	90.584	0.345
	NAT	40	TLS-TS	1.365	61.900	4.748	1.024	66.900	3.370	2.566	68.900	6.369
			EE	1.109	92.700	2.385	1.089	93.000	2.356	1.064	91.900	2.341
		20	TLS-TS	1.984	59.578	7.430	2.132	63.961	6.949	1.482	65.097	5.215
			EE	1.459	95.455	3.090	1.436	93.994	3.080	1.385	92.857	3.029
Regional	ANT	40	TLS-TS	0.147	62.800	0.550	0.141	67.000	0.515	0.156	74.100	0.469
			EE	0.219	92.900	0.477	0.218	92.000	0.479	0.218	92.300	0.475
		20	TLS-TS	0.523	63.896	1.286	0.163	68.182	0.561	0.300	77.922	0.746
			EE	0.235	92.208	0.508	0.231	92.987	0.504	0.232	93.117	0.503
	NAT	40	TLS-TS	2.350	68.700	6.239	1.622	73.900	4.465	1.941	81.800	4.924
			EE	2.041	90.900	4.481	2.014	90.900	4.490	1.979	90.100	4.458
		20	TLS-TS	8.536	71.039	18.928	2.715	75.325	7.137	6.635	84.286	14.545
			EE	3.090	93.247	6.610	2.887	93.506	6.256	2.869	92.208	6.245