Supplementary material for: Spatial and Temporal Scales of Sverdrup Balance

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1. Calculation of Vorticity in ECCO-GODAE

Here we describe the method by which we calculate terms in the vorticity equation in ECCO-GODAE. The accuracy of this calculation can be sensitive to the treatment of variables located at different points on the model grid. ECCO-GODAE solves the discretised momentum equation on an Arakawa C grid (Arakawa and Lamb 1977). It also employs the so-called CD scheme (Adcroft et al. 1999) to calculate the Coriolis term, which uses additional velocities located in the D grid configuration (Fig. 1).

The C grid zonal velocities, $u_c$, and D grid meridional velocities, $v_d$, lie on the east and west faces of the grid boxes (Fig. 1a). C grid vertical velocities, $w_c$, lie in the centres of the top and bottom faces and D grid vertical velocities, $w_d$, lie on the corners of the top and bottom faces (Fig. 1b), here calculated respectively as

$$w_c = -\int_{-H}^{H} \nabla_h \cdot u_c \, dz,$$

$$w_d = -\int_{-H}^{H} \nabla_h \cdot u_d \, dz,$$

where $\nabla_h$ is the horizontal gradient operator. All vertical velocities are located at the top and bottom of the grid cell on the vertical axis and all horizontal velocities are located in the middle of the grid cell in the vertical axis. All terms in the zonal (meridional) momentum equation are located at the point where $u_c$ ($v_c$) lies. The use of the D-grid velocities in calculating the Coriolis term thereby means that the term naturally lies at the correct point without requiring spatial averaging. Terms in the vorticity equation lie at the corner of the grid in the middle of the vertical axis of the cell (the vorticity point).

In ECCO-GODAE the final three terms of the vorticity equation, (Eq. 1 in the main text) are calculated as the finite difference curl of the equivalent terms in the time-mean momentum equation as described in Adcroft et al. (2013). However, since the curl of the Coriolis term in the momentum equation gives only the difference between the linear vorticity terms, i.e.

$$\hat{k} \cdot \nabla \times (f\hat{k} \times u) = \beta v - f \partial_z w,$$
the two linear vorticity terms are calculated separately as $\beta v_d^j$ (where $v_d^j$ signifies a meridional average over two neighbouring grid cells following the notation of Adcroft et al. 2013) and $-f \nabla_h \cdot u_d$ respectively. If the latter is instead calculated according to $-f \nabla_h \cdot \bar{u}_d^{ij}$ (where $\bar{u}_d^{ij}$ is a four-point horizontal average of each velocity component, as is done when the CD scheme is not employed) then the vorticity equation does not balance. It should be noted that $w_d$ is not calculated in the model code of ECCO-GODAE and so our method assumes that the continuity equation holds with the D grid velocities. However, the vorticity equation closes to a high order of accuracy when calculated in this manner.

If the CD scheme is not properly accounted for then the error incurred in the 15 year mean of the momentum equation is only small. However, the error becomes relatively much larger in the vorticity equation (Eq. 1 in the main text) due to large errors that arise in the first term on the RHS. Therefore, any vorticity calculations that involve the vertical velocity will be erroneous if not calculated using the D grid velocities (Fig. 2).

Differences in the vertical velocity at the different grid locations may explain the inconsistencies between our study and that of Lu and Stammer (2004) who use an older version of ECCO-GODAE. In particular, they attribute a much larger portion of the Sverdrup error to $\hat{k} \cdot \nabla \times \text{ADV}$ (and therefore to $\Delta_{LV}$) which they calculate as a residual of the other terms. Similarly, in the recent study by Wunsch (2011) vertical velocities at 117.5 m were used to approximate the Sverdrup transport from Ekman velocities. The C grid vertical velocities at this depth are smoother than the D grid velocities and so their use may result in an error in the estimate of the Sverdrup transport.

2. Methods of Determining a Geovarying Level Of No Motion in ECCO-GODAE

We have used a variety of methods to determine a LONM which use either the density field or the velocity field. We will first describe the two methods that give a physically
meaningful LONM. The first of these considers potential density surfaces (referenced to 2000 m) and the other considers the magnitude of the horizontal velocity vector, $|v_h|$. These methods are then contrasted with simpler methods, such as the $V_{\text{plane}}$ method described in the main text, and estimates that satisfy the theoretical considerations of Sverdrup balance but that we do not consider to provide a physically meaningful depth surface. Each of the methods are optimised according to the pointwise Sverdrup metric, $M_{pw}$.

Sverdrup balance requires, in the strict sense of the mid-depth integrated vorticity equation, a region where the vertical velocities go to zero since $\Delta_{LONM} = -fw_h/\beta$. This can theoretically therefore be a ‘zero-crossing’ depth where $w$ equals zero but $\partial_z w$ is not zero, and can exist at a depth where wind-induced vertical velocities cancel out with BPT-induced vertical velocities. The depth at which this would occur would be influenced by both the wind and the BPT. Therefore, if Sverdrup balance is considered to represent the transport that is driven only by the wind stress curl then such a zero-crossing depth is not considered here to be a very satisfactory LONM since it does not necessarily mark the bottom of the wind-driven flow. It is instead more meaningful to search for a depth where the horizontal velocities go to zero because they (along with their vertical derivatives) would go to zero at the base of the wind-driven layer when other forcing is absent. This then relates to $\partial_z w$ being small through consideration of the continuity equation as well as the linear vorticity balance. Nevertheless, it is interesting to compare the ‘theoretical Sverdrup balance’, at the depth where $w = 0$, to a more practically defined Sverdup balance. The LONM associated with this theoretical Sverdrup balance is simply determined as the depth, at each geographical location, where the magnitude of the vertical velocity is at a minimum. Unsurprisingly, this LONM varies greatly with location and is therefore not shown since we do not consider it to have any physical meaning. Quantitative estimates from this method, referred to as $w_{\text{min}}$, will however be referred to further on.

The first method to find a meaningful geovarying LONM assumes that mid-depth density surfaces will follow a horizontal depth structure resembling the variation of the wind stress
curl. The depth of the 1037.3 kg m\(^{-3}\) potential density (referenced to 2000 m) surface has been mapped (Fig. 3a) and used as an integration depth in Eq. (2) in the main text. The associated Sverdrup errors are evaluated according to \(M_{pw}\) applied over the unmasked subtropical domain, which for this density surface is 53%. The Sverdrup errors obtained from this surface are the smallest from the range of all possible density surfaces in ECCO-GODAE. This method is called \(\rho_{\text{thresh}}\). (Associated maps of the Sverdrup errors produced using both methods of finding a geovarying LONM are not shown since they are only subtly different to those produced from \(V_{\text{plane}}\;\); Fig. 2c in the main article).

The second method to determine a meaningful geovarying LONM finds depths at which the horizontal currents are quiescent. This occurs at a depth where both \(|v_h|\) and its vertical derivative, \(\partial_z |v_h|\), are small. The shallowest depth for which values of \(|v_h|\) and \(\partial_z |v_h|\) are respectively smaller than \(4.8 \times 10^{-3}\) m s\(^{-1}\) and \(1.1 \times 10^{-5}\) s\(^{-1}\) is mapped (Fig. 3b) and subsequently used as the integration depth in Eq. (2) in the main article. The associated \(M_{pw}\) is 54% and is the smallest that can be achieved for all possible variations of both \(|v_h|\) and \(\partial_z |v_h|\). This method is called \(v_{\text{thresh}}\). It should be noted that at the high threshold limit of \(\partial_z |v_h|\) the only criterion is \(|v_h|\) (and vice versa). However, the combination that provides the optimum \(M_{pw}\) is found when limited by both. Although there are some spatial discontinuities, the LONM found using \(v_{\text{thresh}}\) mostly forms a continuous surface that bears some similarity to the spatial pattern of the wind driven gyres. It also exhibits some features of the Luyten et al. (1983) theory of the thermocline depth, namely that a bowl shaped thermocline is deep in the west of the basin and shallows eastwards.

The velocity based method has also been applied to \(|w|\) instead of \(|v_h|\). This is to try and satisfy the true Sverdrup balance assumption that vertical velocities are small. However, we found that this approach produces a LONM that is extremely variable horizontally and does not represent a physically meaningful surface. The surface produced by using \(|w|\) instead of \(|v_h|\) in the second method is therefore not shown, but for quantitative comparison to the other methods the method is referred to as \(w_{\text{thresh}}\) below.
The results are summarised in Table 1, which shows \( M_{pw} \) and \( M_{zi} \) for each method. Also shown in Table 1 are the Sverdrup metrics when calculated from 5° horizontally-smoothed transport fields (symbolised as \( \overline{M}_{pw} \) and \( \overline{M}_{zi} \)). Differences between methods \( \rho_{\text{thresh}}, v_{\text{thresh}}, w_{\text{thresh}} \) and \( V_{\text{plane}} \) are minimal, particularly in larger scale considerations when the transports are zonally integrated or smoothed. This is especially important given the large differences between the depths that are output by each method. The Sverdrup error is therefore, for practical purposes, independent of depth as long as the ocean is everywhere integrated to below the main thermocline depth, but not so deep that non-wind driven deep transports are included. Although method \( w_{\text{min}} \) gives an improvement of approximately 20 percentage points in \( M_{pw} \) compared to \( V_{\text{plane}} \), this is the best possible LONM that can be achieved in ECCO-GODAE and is unlikely to be a level that represents the mechanisms associated with Sverdrup balance. This difference is reduced to only 4 percentage points in \( M_{zi} \), and on scales greater than 5° the difference between the methods for the metrics \( \overline{M}_{pw} \) and \( \overline{M}_{zi} \) is only 10 and 1 percentage points respectively.

3. HiGEM Sverdrup errors

Figure 4 shows the LONM error, \( \Delta_{LONM} \), and linear vorticity error, \( \Delta_{LV} \), in HiGEM. Due to the availability of only annual mean output there remain non-linear correlation terms that cannot be calculated. Therefore only the ocean transport, Sverdrup transport and \( \Delta_{LONM} \) can be directly calculated, while \( \Delta_{LV} \) (which encorporates \( \hat{k} \cdot \nabla \times H V, \hat{k} \cdot \nabla \times A D V \), and the correlation terms), is calculated as a residual. Unlike in ECCO-GODAE, however, the absence of a CD-scheme in HiGEM makes the calculation relatively straightforward. We therefore consider the calculation of \( \Delta_{LONM} \) to be robust.
REFERENCES


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<table>
<thead>
<tr>
<th>Method</th>
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<th>$M_{zi}$ (%)</th>
<th>$\overline{M}_{pw}$ (%)</th>
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<td>$w_{min}$</td>
<td>36</td>
<td>16</td>
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