Supplementary Information:

The relation of wind-driven coastal and offshore upwelling in the Benguela Upwelling System

Journal of Physical Oceanography https://doi.org/10.1175/JPO-D-20-0297.1

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S1. The computation of the coastal Ekman upwelling

There are several reasons to estimate upwelling associated with the alongshore wind stress and wind-stress-curl. First, near-real-time wind data from remote sensing scatterometers is available. With such an approach available, the fast evaluation of wind-driven upwelling state as the basic driver of the nutrient cycle in the BUS becomes possible. Such a fast and reliable evaluation of the ecosystem state meets the needs of researchers in different disciplines of ocean science who might need a real-time estimate of upwelling when they are on the cruise. Second, having a perfect numerical model available does not ensure a separate evaluation of coastal and wind-stress-curl-driven (WSCD) upwelling. Here we present two conceptual approaches to evaluate upwelling solely in terms of wind speed and stratification represented by a climatological Rossby radius. We shall show that they basically yield very similar results. We use the analytical approach to investigate the upwelling variability for several distinct upwelling cells in the BUS and disentangle the role of coastal and WSCD upwelling.

S1.1 Revisiting analytical solution for the BUS

Upwelling intensity, its temporal and spatial variability are of great importance for the assessment of the ecosystem dynamics. In the tropical and subtropical eastern ocean boundaries upwelling events are primarily wind-driven. Hence, the Ekman theory is applicable for the estimate of upwelling intensity. Our approach has three major ingredients:

- The Ekman balance between wind-stress and Coriolis acceleration to determine the Ekman transport in the surface layer
- The coastal divergence from the disturbance of the Ekman balance and the rectification of the Ekman transport due to presence of the coast. For details of the horizontal flow pattern near the coast we use the theory of Fennel et al. (1988, 1999).
- Conservation of volume that enables the diagnosis of the vertical velocity at the base of the mixed layer from the divergence of the transport within the surface mixed layer.

The vertical water velocity at the bottom of the mixed surface layer, \( w \), is the key quantity for upward mass transport. For a slowly varying sea surface height, i.e., in the rigid lid approximation, it is related to the divergence of the horizontal transport in the surface layer and reads

\[
w(z = -H_{\text{mix}}) = \frac{\partial}{\partial x} U + \frac{\partial}{\partial y} V
\]  

(S1)

Far away from the coast the Ekman transport,

\[
(U_E, V_E) = \left( \frac{\tau(y), -\tau(x)}{f\rho} \right)
\]

(S2)

is a leading contributor to the total surface layer horizontal transport. It determines the WSCD upwelling that reads

\[
w_{\text{curl}} = w_E(z = -H_{\text{mix}}) = \frac{\partial}{\partial x} U_E + \frac{\partial}{\partial y} V_E = \frac{\text{curl} \tau}{f\rho}
\]

(S3)

Even if other flow components are of the same order of magnitude, \( w_{\text{curl}} \) is a robust measure for the wind-driven upward-directed water flow in the open ocean.

Near the coast so-called coastal divergence needs to be considered. We choose \( U \) and \( V \) as the cross-shore and the alongshore directed flow, respectively. Since the Ekman balance is disturbed, an additional surface transport emerges

\[
U = U_E + U_C
\]

(S4)

that is in opposite direction to the Ekman transport and guaranties the coastal boundary condition \( U(x = 0) = 0 \) or \( U_E = -U_C \) at \( x = 0 \). The detailed shape of \( U_C \) will be discussed below. We do not consider a similar contribution for the alongshore flow. Since wind parallel to the coast is not balanced by the Coriolis force the coastal jet develops. Alongshore variability of the meridional wind stress excites Kevin waves, which tends to reduce the upwelling velocity and establishes a balance between the wind stress and alongshore pressure gradients. For the sake of simplicity, we do not include this current in our analysis. However, this contribution should be
kept in mind, when results from the numerical model and from the analytical approach are compared.

Now we use volume conservation and define another quantity representing the upwelling, i.e., integrated or total coastal upwelling as

\[ W = \int_{-L}^{0} dx \left( \frac{\partial}{\partial x} U + \frac{\partial}{\partial y} V_E \right) = \int_{-L}^{0} dx \frac{\text{curl} \tau}{f \rho} + U_C(x = 0) - U_C(x = -L) \]  

(S5)

If we are sufficiently far distant from the coast, \( U_C \) vanishes, i.e., \( U_C(x = -L) = 0 \). With the coastal boundary condition, we find for the total coastal upwelling

\[ W = \int_{-L}^{0} dx \left( \frac{\partial}{\partial x} U + \frac{\partial}{\partial y} V_E \right) = -U_E(x = -L) - \int_{-L}^{0} dx \frac{1}{f \rho} \frac{\partial}{\partial y} \tau^{(x)} \]  

(S6)

Note, this is a line integral with unit \( \text{m}^2/\text{s} \) and depends on latitude. Hence, for a uniform meridional wind, the integrated coastal upwelling depends solely on the off-shore directed Ekman transport and therefore on the wind at \( x = -L \). Remarkably, it is independent of the coastal wind and a robust quantity to characterize total coastal upwelling. It has two contributions,

\[ W = W_E + W_C \]  

(S7)

the integrated or total WSCD upwelling \( W_E \)

\[ W_E = \int_{-L}^{0} dx \frac{\text{curl} \tau}{f \rho} \]  

(S8)

and the total upwelling from the coastal divergence \( W_C \)

\[ W_C = -U_E(x = 0) = -\frac{\tau^{(y)}(x = 0)}{f \rho} \]  

(S9)

For the spatial pattern of the coastal divergence, a theory beyond the Ekman theory is needed. The analytical result of Fennel 1988, 1999 would be suitable but introduces additional unknowns like stratification (Rossby radius) and mixed layer depth.

We consider Eq. 29 from the theory of Benguela upwelling Fennel (1999) and follow the notation used there. We neglect the alongshore variability in the coastal wind, hence, the Fourier transformation with respect to \( \kappa \) can be carried out and leads to \( Q(y) = 1 \). The time variability of
the wind stress, represented by $T(\omega)$ is fully coupled to the flow field, i.e., the wind variation happens that slowly that the Ekman balance can be reestablished permanently. The cross-shore velocity reads approximately

$$u(x) = -T(\omega) \sum_n \frac{F_n u^2}{f \rho h_n} \left( \Pi(x) - \Pi(0) e^{\frac{x}{R_n}} \right)$$  \hspace{1cm} (S10)$$

$u^*$ is the magnitude of the averaged friction velocity, $\Pi(x)$ is a dimensionless factor standing for the cross-shore variability of the alongshore wind stress. There are more contributions proportional to the wind-stress-curl. Those need not to be small but are only local modifications of the flow. Hence, we leave these terms out. The theory assumes a volume force to describe the momentum flux into the ocean. It is assumed as uniformly distributed over the surface layer and to be zero below. Accordingly, the sum over the vertical eigenfunctions

$$\sum_n \frac{F_n}{h_n} = \frac{\theta(z + H_{mix})}{H_{mix}}$$  \hspace{1cm} (S11)$$

renders a step-like function describing this volume force. The vertical integral over the volume force is the momentum flux (i.e., wind-stress) entering the ocean at the surface

$$\int_{-H_{mix}}^0 dz \sum_n \frac{F_n}{h_n} u^2 \Pi(x) = \tau(y)(x)$$  \hspace{1cm} (S12)$$

From the equation of continuity, $w_z = -(u_x + v_y)$, and by vertical integration starting from a rigid lid sea surface, we get for the WSCD upwelling component at the base of the mixed layer the well-known Ekman theory result:

$$w_E(z = -H_{mix}) = w_{curl} = \frac{\text{curl } \tau}{f \rho}$$  \hspace{1cm} (S13)$$

$$w_c(z) = -\int_0^z dz' F_n \frac{u^2}{\rho f h_n R_n} \Pi(x = 0) e^{\frac{x}{R_n}}$$  \hspace{1cm} (S14)$$

It does not depend solely on the coastal wind, but depend on the mixed layer depth determining the coefficients $h_n$, and stratification via the Rossby radius $R$. More complicated, for a uniform stratification the vertical velocity at the base of the mixed layer ($z=-H_{mix}$) and near the coast becomes logarithmically divergent.

As a check, the horizontally integrated, i.e., the total coastal upwelling reads
\[ W_C = \int_0^x dx w_c = -\int_0^{H_{mix}} dz \sum_n \frac{F_n u^2}{f \rho h_n} \Pi(x = 0) = -\frac{\tau_y(x = 0)}{f \rho} \]  

(S15)

Hence, the approximation (S14) preserves the volume conservation.

The sum in formula (S14) does not converge at the coast for \( x=0 \). Hence an approximation in terms of the coastal upwelling velocity is not possible.

As a final step, we present an approximation suitable to show coastal and the WSCD vertical velocity in one plot. To this end, we start from the analytical result Eq. 25b in Fennel 1988. Away from the coast, we decompose the logarithm in a power series with respect to \( e^{\frac{x}{R_1}} \). The first contribution:

\[ w_c \sim -\frac{\tau_y(x = 0)}{\rho f} \frac{2H_{mix} e^{\frac{x}{R_1}}}{H R_1} \]  

(S16)

gives the wrong horizontal integral, especially in the limit that the mixed layer depth is small compared to the total depth. Including the second order, we get

\[ w_c \sim -\frac{\tau_y(x = 0)}{\rho f} \left( \frac{2H_{mix} e^{\frac{x}{R_1}}}{H R_1} + \left( 1 - \frac{2H_{mix}}{H} \right) \frac{e^{\frac{x}{R_2}}}{R_2} \right) \]  

(S17)

For \( \frac{2H_{mix}}{H} \ll 1 \) this gives the correct horizontal integral. Remember \( R_2 = \frac{R_1}{2} \). Hence we use

\[ w_c = -\frac{\tau_y(x = 0)}{\rho f} \frac{e^{\frac{x}{R_2}}}{R_2} \]  

(S18)

Finally, this provides a choice for the scale \((-L)\) to define integrated coastal upwelling, \( W \).

From the model, only the total vertical velocity, \( w \), can be defined. A separation of the WSCD component and that from the coastal divergence is difficult. Hence, to distinguish this representation from the quantities derived from the Ekman theory, we use the symbol \( w_m \) (see Table S1).

Table S1: Definition of different upwelling related variables used in this study
<table>
<thead>
<tr>
<th>Acronyms</th>
<th>Full Name</th>
<th>Definition</th>
<th>formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{\text{curl}}$</td>
<td>Wind-stress-curl-driven upwelling</td>
<td>Upwelling related to the divergence of Ekman balanced flow at the mixed layer base</td>
<td>$w_{\text{curl}} = \frac{1}{\rho_w f} \left( \frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} \right)$</td>
</tr>
<tr>
<td>$w_c$</td>
<td>Upwelling related to the coastal divergence</td>
<td>Upwelling related to the coastal divergence at the mixed layer base</td>
<td>$w_c \sim - \frac{2 \tau_y (x = 0)}{\rho f} e^{\frac{2x}{R_1}}$</td>
</tr>
<tr>
<td>$W_E$</td>
<td>Integrated coastal divergence of Ekman transport</td>
<td>Upwelling due to the coastal divergence. Equal to the offshore transport right at the coast</td>
<td>$W_E = - \frac{\tau_y}{\rho f} + \frac{\tau_y}{\rho f}$</td>
</tr>
<tr>
<td>$W_C$</td>
<td>Integrated coastal upwelling</td>
<td>Upwelling integrated from the coast to a distance where the coastal influence becomes small ($x=R1$).</td>
<td>$W = W_E + W_C = - \frac{\tau_y}{\rho f}$</td>
</tr>
<tr>
<td>$w_m$</td>
<td>upwelling simulated by the ocean model</td>
<td>Modelled upwelling at the mixed layer base</td>
<td></td>
</tr>
<tr>
<td>$V_C$</td>
<td>Volume of nearshore upwelled water in the analytical theory ($x&gt;R$)</td>
<td>Integrated volume of upwelled water from the coast to the coastal distance of R in the analytical theory</td>
<td></td>
</tr>
<tr>
<td>$V_{\text{curl}}$</td>
<td>Volume of WSCD upwelled water</td>
<td>Integrated volume of upwelled water from the coastal distance of R1 and 250km offshore in the analytical theory</td>
<td></td>
</tr>
<tr>
<td>$V_{\text{m-coast}}$</td>
<td>Volume of nearshore upwelled water in the model simulation ($x&gt;R$)</td>
<td>Integrated volume of upwelled water from the coast to the coastal distance of R in the model simulation</td>
<td></td>
</tr>
<tr>
<td>$V_{\text{m-offshore}}$</td>
<td>Volume of offshore upwelled water in the model simulation ($x&lt;R$)</td>
<td>Integrated volume of upwelled water between coastal distance of R and 250km offshore in the model simulation</td>
<td></td>
</tr>
</tbody>
</table>

S1.2 An alternative estimation of coastal upwelling
We assumed that the cross-shore Ekman transport ($U_{\text{cross-shore}}$) is replaced by vertical water flux within a coastal band equal to $R_1$ which means:

$$U_{\text{cross-shore}} = \int_0^R dx \, w_{\text{coast}} e^{\alpha x}$$

(S19)

or

$$U_{\text{cross-shore}} = \frac{w_{\text{coast}}}{\alpha} (1 - e^{-\alpha R_1})$$

(S20)

$w_{\text{coast}}$ denotes the vertical water velocity near the coast.

In addition, we approximate the coefficient $\alpha$ with the assumption that the vertical water velocity in the coastal distance equal to $R_1$ is reduced to 10\% of its value at the coast (i.e., $w_{\text{coast}}$) which means

$$w_{\text{coast}} e^{-\alpha R_1} = 0.1 \times w_{\text{coast}} \quad \text{or} \quad \alpha = \frac{2.3026}{R_1}$$

(S21)

With the help of Equation (S3), the $w_{\text{coast}}$ is obtained as

$$w_{\text{coast}} = \frac{2.0723 \times U_{\text{cross-shore}}}{R_1}$$

(S22)

Hence, the vertical water velocity within the offshore distance of $R_1$ is computed as

$$w = \frac{2.07 \times U_{\text{cross-shore}}}{R_1} e^{\frac{2.3026}{R_1} x}$$

(S23)

which yields similar result of previous approach in S1.1.
Figure S1: (a) the map of bathymetry across the BUS (m) and (b) coastal distance of 200m isobaths (km). Contours in (a) represent 200m and 500m isobaths.
Figure S2: (a) the coastal orientation which use to decompose the Ekman transport to cross-shore and alongshore transport. The shoreline orientation angle (°) with respect to the north is represented in (b). Note that positive (negative) values indicate clockwise (anticlockwise) deviation from the north.
Figure S3: (a) the spatial pattern of the first baroclinic Rossby Radius (R, km) of deformation across the BUS computed by Chelton et al., (1998). (b) show the size of R (km) near the southwest coast of Africa which is used in this study.
Figure S4: the spatial pattern of the mean-state (color shading; °C) and variability (dark-grey contours; °C) of the simulated and observed SSTs across the BUS. Panels a, b, c represent the results of the simulated, Reynolds and Aqua-Modis data sets over the 2003-2018 period, respectively. The variability is estimated from the standard deviation of 5-day mean values. Note that solid white contours indicate the 17°C isotherm in the SST mean-state.
Figure S5: time series of the DSL (m) in the model (red), satellite measurement (blue), and sea surface height from tide gauge records (green) in Walvis Bay (a), Simon’s Town (b), Port Sonara (c), Takoradi (d), Ascension (e), and Macae (f). The long-term mean and linear trend were subtracted from each data time series. The correlation between each pair of time series was computed and displayed on the bottom side of each panel.
Figure S6: time series of the simulated (red) and observed (blue) ocean dynamic topography averaged over (a) west equatorial Atlantic (45°W-35°W,3°S-3°N), (b) Atlantic Niño (10°W-0,3°S-3°N), (c) northern BUS (10°E-15°E,26°S-17°S), and (d) southern BUS (13.5°E-19°E,34°S-26°S). The long-term mean and linear trend were subtracted from each time series. The correlation between simulated and observed time series is displayed in the bottom-left corner of each panel.
Figure S7: a comparison of the observed and simulated SST-based upwelling index. Top panels represent the spatial pattern of the mean-state (color shading; °C) and variability (contours; °C) in the model (a), Reynolds SST (b), and Aqua-Modis SST (c) over the 2003-2018 period. The variability is estimated from the standard deviation of 5-day mean values. Panel (d) and (e) represent the time series of the areal averaged over northern (10°E-15°E,26°S-17°S) and southern (13.5°E-19°E,34°S-26°S) BUS, respectively. Red, blue, and green time series are the results of the simulated, Aqua-Modis and Reynolds values, respectively. The linear correlation coefficient between each pair of time series is shown at the bottom of panel (d) and (e). The observational indices (i.e., Aqua-Modis and Reynolds) were computed from the SST whereas the modeled index is based on water temperature at 1.5m depth which is the shallowest vertical level of the model.
Figure S8: contribution of the wind-stress-curl-driven upwelling (%) in the total upwelling obtained from the analytical theory. The contribution is computed over the coastal distance between 0.25° (grey contour in a) and R (black contour in a). The contribution is the long-term average of the ratio between wind-stress-curl-driven upwelling and the total upwelling. Over a coastal distance smaller than 0.25° the wind-stress-curl is considered as missing value and beyond the coastal distance of R, the coastal upwelling becomes zero in the analytical theory (see Methods). Note that R is typically close to 0.25° south of Walvis Bay and the contribution is set to missing values.
Figure S9: climatology annual cycle of the meridional wind stress (Nm$^{-2}$) obtained from ASCAT data. Contours represents 0.04 Nm$^{-2}$ and 0.08 Nm$^{-2}$ wind stress.
Figure S10: climatology annual cycle of the wind-stress-curl ($10^{-7}$ Nm$^{-3}$) obtained from ASCAT data.
Figure S11: same as Fig. 7 but the climatological cycle was subtracted prior to computing the correlation.
Figure S12: same as figure 7 but for the simulated vertical velocity taken at the base of mixed layer depth.