



**AMS**  
American Meteorological Society

# Supplemental Material

*Journal of Physical Oceanography*

Towards the Upper-Ocean Unbalanced Submesoscale Motions in the *Oleander*  
Observations

<https://doi.org/10.1175/JPO-D-22-0134.1>

© Copyright 2023 American Meteorological Society (AMS)

For permission to reuse any portion of this work, please contact [permissions@ametsoc.org](mailto:permissions@ametsoc.org). Any use of material in this work that is determined to be “fair use” under Section 107 of the U.S. Copyright Act (17 USC §107) or that satisfies the conditions specified in Section 108 of the U.S. Copyright Act (17 USC §108) does not require AMS’s permission. Republication, systematic reproduction, posting in electronic form, such as on a website or in a searchable database, or other uses of this material, except as exempted by the above statement, requires written permission or a license from AMS. All AMS journals and monograph publications are registered with the Copyright Clearance Center (<https://www.copyright.com>). Additional details are provided in the AMS Copyright Policy statement, available on the AMS website (<https://www.ametsoc.org/PUBSCopyrightPolicy>).

# **Towards the upper-ocean unbalanced submesoscale motions in the *Oleander* observations**

Haijin Cao<sup>a,b,\*</sup>, Baylor Fox-Kemper<sup>c</sup>, Zhiyou Jing<sup>d</sup>, Xiangzhou Song<sup>a,b</sup>, and Yuyi Liu<sup>d</sup>

<sup>a</sup>*Key Laboratory of Marine Hazards Forecasting, Ministry of Natural Resources, Hohai University,  
Nanjing, China*

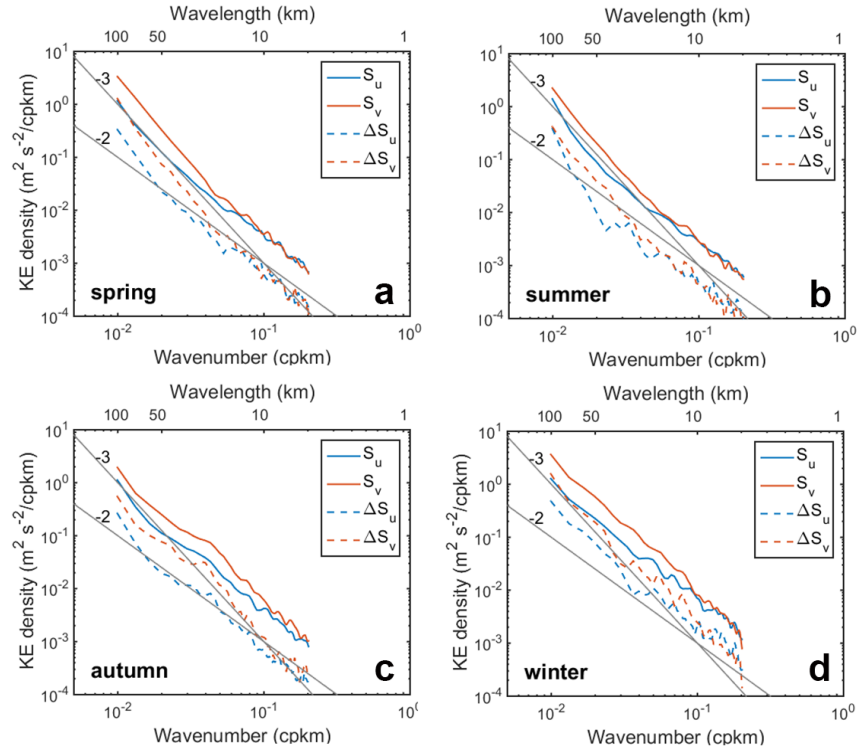
<sup>b</sup>*College of Oceanography, Hohai University, Nanjing, China*

<sup>c</sup>*Department of Earth, Environmental, and Planetary Sciences, Brown University, Providence, RI, USA*

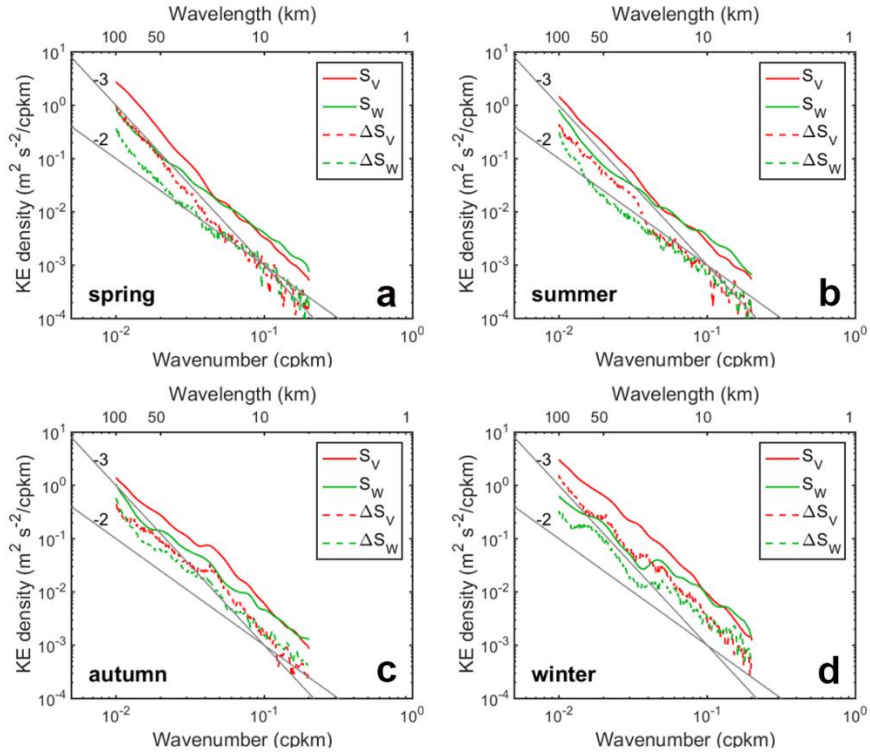
<sup>d</sup>*State Key Laboratory of Tropical Oceanography, South China Sea Institute of Oceanology, Chinese  
Academy of Sciences, Guangzhou, China*

## **Contents of this file**

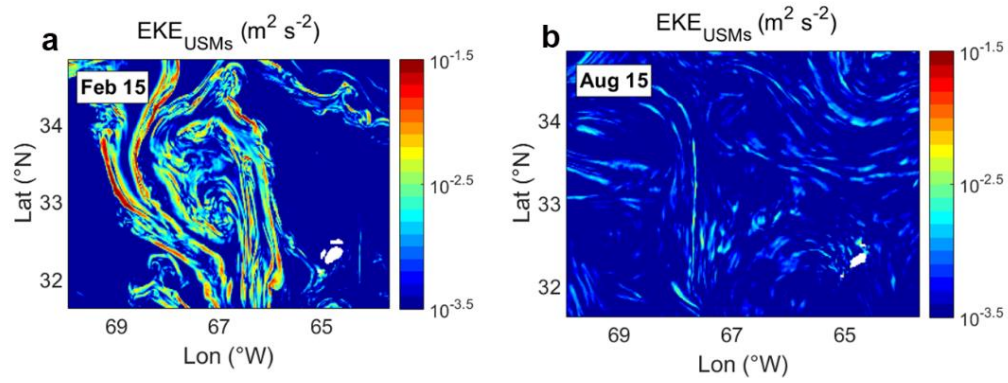
Figures S1 to S5  
Text of wave-vortex decomposition



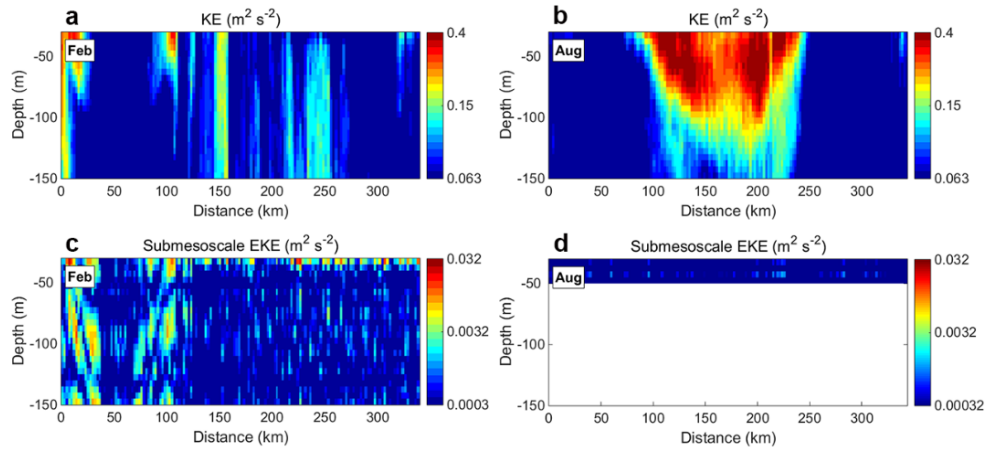
**Figure S1.** Wavenumber spectra of along-track (blue) and across-track (brown) velocity and the estimated spectral errors at 30-m depth following the method of Cao et al. (2019) in four seasons: (a) spring, (b) summer, (c) autumn, and (d) winter.  $k^{-2}$  and  $k^{-3}$  power lines are marked in grey for reference. Here we take the range of these estimates (or the 95% confidence interval) as the uncertainty estimate. The errors are smaller than the true values for both velocity components.



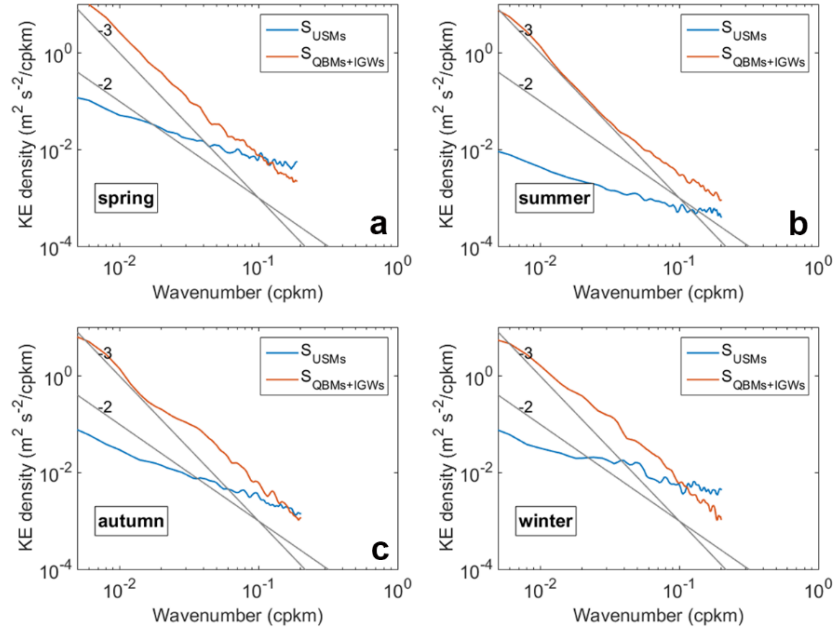
**Figure S2.** Decomposed wavenumber spectra for vortex (red) and wave (green) parts and the estimated spectral errors at 30-m depth following the method of Cao et al. (2019) in four seasons: (a) spring, (b) summer, (c) autumn, and (d) winter.  $k^{-2}$  and  $k^{-3}$  power lines are marked in grey for reference. The errors are smaller than the true values for both wave and vortex components.



**Figure S3.** Submesoscale eddy kinetic energy (EKE) by unbalanced submesoscale motions (USMs) on (a) February 15 and (b) August 15, respectively. The USMs occur mostly along the front, especially in winter.



**Figure S4.** Snapshots of (a and b) kinetic energy (KE) and (c and d) submesoscale eddy kinetic energy (EKE) by USMs in February and August.



**Figure S5.** Decomposed wavenumber spectra for QBMs+IGWs part (brown) and USMs (blue) part at 30-m depth in four seasons: (a) spring, (b) summer, (c) autumn, and (d) winter.  $k^{-2}$  and  $k^{-3}$  power lines are marked in grey for reference.

The 1-D decomposition developed by Bühler et al. (2014) assumes stationarity, homogeneity, and horizontal isotropy for the flow data. Following these assumptions, the velocities are rotated into the along-track ( $u$ ) and across-track ( $v$ ) velocity with the  $x$  axis aligned with the ship track before spectral decomposition. The Helmholtz decomposition,  $S = S^u + S^v = S^\Psi + S^\Phi$ , requires ensemble estimates of the power spectra,  $S^u$ ,  $S^v$ ,  $S^\Psi$ , and  $S^\Phi$ , with  $S^u$  the along-track velocity spectrum,  $S^v$  the across-track velocity spectrum,  $S^\Psi$  the rotational spectrum, and  $S^\Phi$  the divergent spectrum. Based on above assumptions, Bühler et al. (2014) show that:

$$S^\Psi = (1 - k \frac{d}{dk}) K^\Psi = K^\Psi - K^\Phi + S^v, \quad (\text{S1})$$

$$S^\Phi = (1 - k \frac{d}{dk}) K^\Phi = K^\Phi - K^\Psi + S^u, \quad (\text{S2})$$

with two intermediary functions constructed to solve (S1-S2) that are found by

$$K^\Psi(r) = \int_r^\infty [S^u(\bar{r}) \cosh(r - \bar{r}) + S^v(\bar{r}) \sinh(r - \bar{r})] d\bar{r}, \quad (\text{S3})$$

$$K^\Phi(r) = \int_r^\infty [S^u(\bar{r}) \sinh(r - \bar{r}) + S^v(\bar{r}) \cosh(r - \bar{r})] d\bar{r}, \quad (\text{S4})$$

where  $r = \ln(k)$  and  $k$  is the along-track wavenumber. To find the possible wave effects in  $S^\Psi$  (as large-scale waves could also have vortical effects by the Coriolis force), the Garrett-Munk spectrum is assumed to hold for the wave field (Polzin & Lvov, 2011), which obtains the scale-dependent ratio between rotational and divergent components for waves,  $\frac{f^2}{\omega_*^2}(k) = \frac{S_W^\Psi}{S_W^\Phi}(k)$ . The subscript  $W$  denotes waves,  $f$  is the Coriolis parameter, and

$\omega_*$  is the wave frequency. Finally, by assuming the divergence being dominated by waves, the separated vortex and wave components can be obtained as follows,

$$S_V = S_V^u + S_V^v = \left( K^\Psi - \frac{f^2}{\omega_*^2} K^\Phi \right) - k \frac{d}{dk} \left( K^\Psi - \frac{f^2}{\omega_*^2} K^\Phi \right), \quad (\text{S5})$$

$$S_W = S_W^u + S_W^v = \left( \frac{f^2}{\omega_*^2} K^\Phi - k \frac{d}{dk} K^\Phi \right) + K^\Phi - k \frac{d}{dk} \left( \frac{f^2}{\omega_*^2} K^\Phi \right), \quad (\text{S6})$$

where the subscript  $V$  denotes the vortex contribution. Bühler et al. (2014) can be consulted for the more detailed derivation.