Supplemental Material

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Towards the Upper-Ocean Unbalanced Submesoscale Motions in the *Oleander* Observations

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Towards the upper-ocean unbalanced submesoscale motions in the *Oleander* observations

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Contents of this file

Figures S1 to S5
Text of wave-vortex decomposition

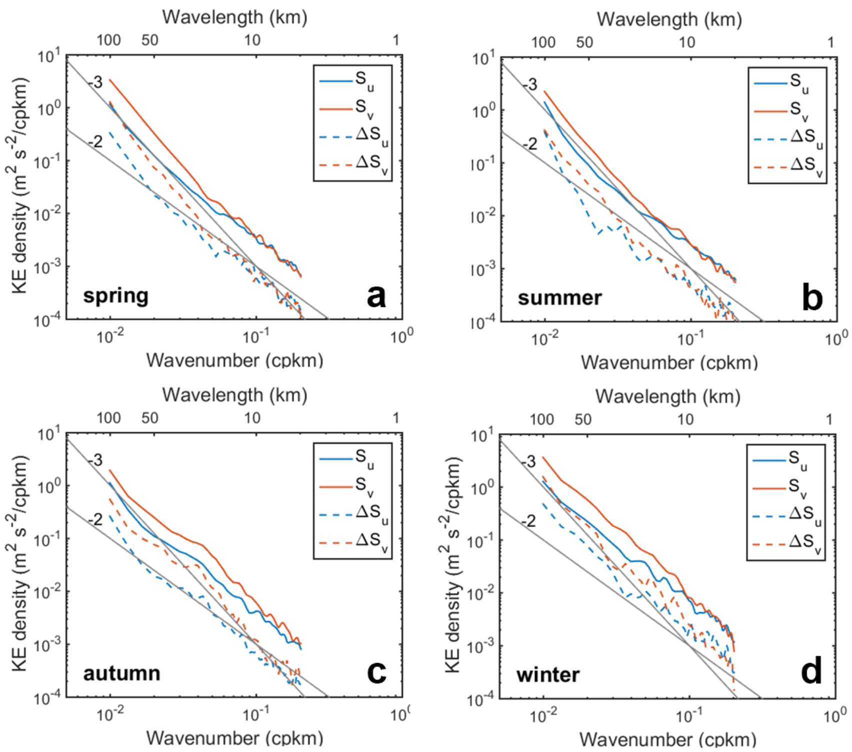

Figure S1. Wavenumber spectra of along-track (blue) and across-track (brown) velocity and the estimated spectral errors at 30-m depth following the method of Cao et al. (2019) in four seasons: (a) spring, (b) summer, (c) autumn, and (d) winter. k^{-2} and k^{-3} power lines are marked in grey for reference. Here we take the range of these estimates (or the 95% confidence interval) as the uncertainty estimate. The errors are smaller than the true values for both velocity components.

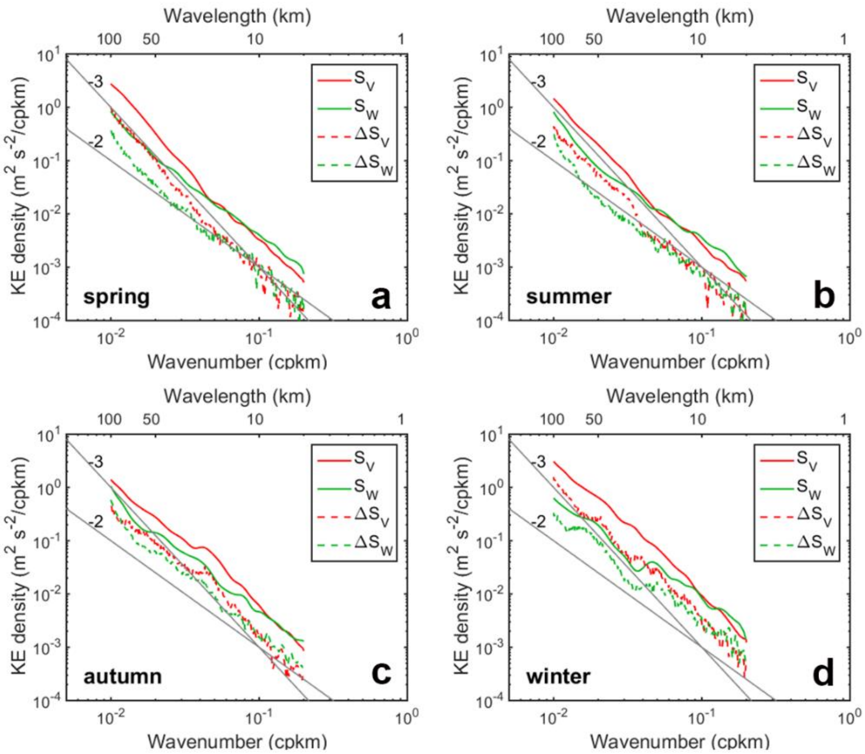

Figure S2. Decomposed wavenumber spectra for vortex (red) and wave (green) parts and the estimated spectral errors at 30-m depth following the method of Cao et al. (2019) in four seasons: (a) spring, (b) summer, (c) autumn, and (d) winter. k^{-2} and k^{-3} power lines are marked in grey for reference. The errors are smaller than the true values for both wave and vortex components.

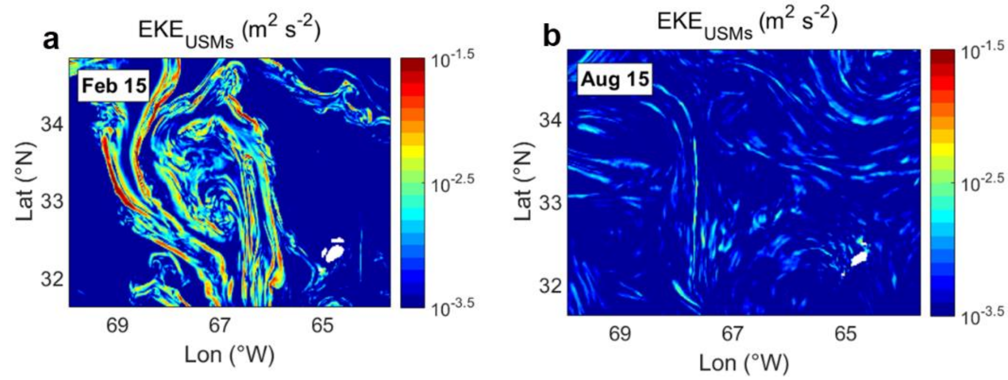

Figure S3. Submesoscale eddy kinetic energy (EKE) by unbalanced submesoscale motions (USMs) on (a) February 15 and (b) August 15, respectively. The USMs occur mostly along the front, especially in winter.

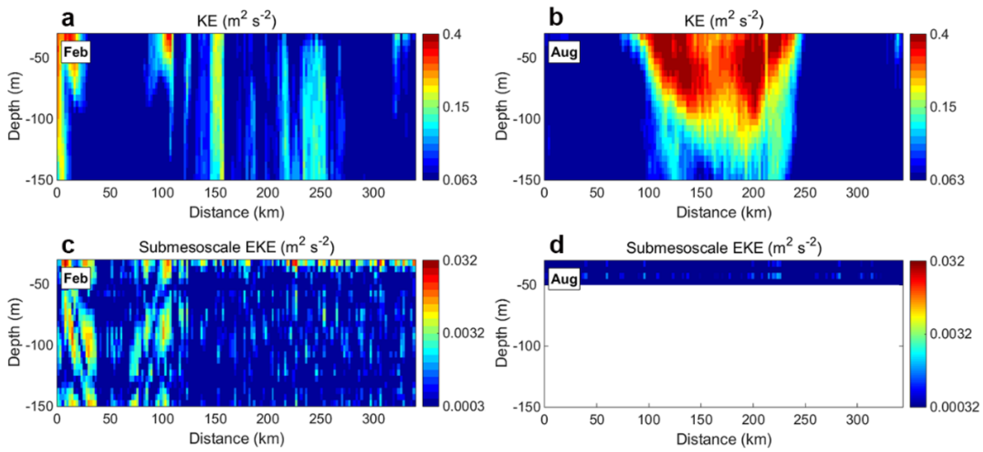

Figure S4. Snapshots of (a and b) kinetic energy (KE) and (c and d) submesoscale eddy kinetic energy (EKE) by USMs in February and August.

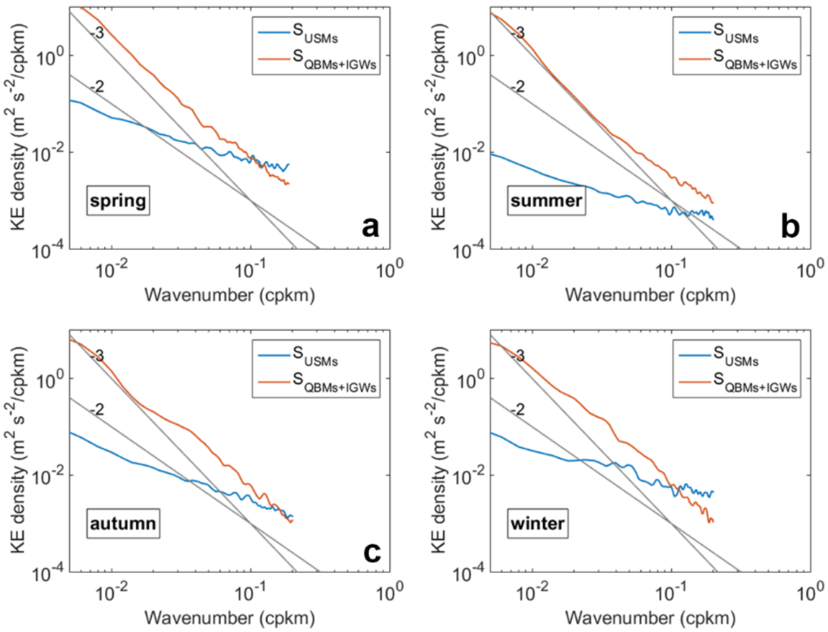

Figure S5. Decomposed wavenumber spectra for QBMs+IGWs part (brown) and USMs (blue) part at 30-m depth in four seasons: (a) spring, (b) summer, (c) autumn, and (d) winter. k^{-2} and k^{-3} power lines are marked in grey for reference.

The 1-D decomposition developed by Bühler et al. (2014) assumes stationarity, homogeneity, and horizontal isotropy for the flow data. Following these assumptions, the velocities are rotated into the along-track (*u*) and across-track (*v*) velocity with the *x* axis aligned with the ship track before spectral decomposition. The Helmholtz decomposition, $S = S^u + S^v = S^\psi + S^\phi$, requires ensemble estimates of the power spectra, S^u, S^v, S^ψ, and S^ϕ, with S^u the along-track velocity spectrum, S^v the across-track velocity spectrum, S^ψ the rotational spectrum, and S^ϕ the divergent spectrum. Based on above assumptions, Bühler et al. (2014) show that:

$$S^\Psi = (1 - k\frac{d}{dk})K^\Psi = K^\Psi - K^\Phi + S^v, \text{ (S1)}$$

$$S^\Phi = (1 - k\frac{d}{dk})K^\Phi = K^\Phi - K^\Psi + S^u, \text{ (S2)}$$

with two intermediary functions constructed to solve (S1-S2) that are found by

$$K^\psi(r) = \int_r^\infty \left[S^u(\bar{r})\cosh(r-\bar{r}) + S^v(\bar{r})\sinh(r-\bar{r})\right]d\bar{r}, \text{ (S3)}$$

$$K^\phi(r) = \int_r^\infty \left[S^u(\bar{r})\sinh(r-\bar{r}) + S^v(\bar{r})\cosh(r-\bar{r})\right]d\bar{r}, \text{ (S4)}$$

where $r = ln(k)$ and *k* is the along-track wavenumber. To find the possible wave effects in S^ψ (as large-scale waves could also have vortical effects by the Coriolis force), the Garrett-Munk spectrum is assumed to hold for the wave field (Polzin & Lvov, 2011), which obtains the scale-dependent ratio between rotational and divergent components for waves, $\frac{f^2}{\omega_*^2}(k) = \frac{S_W^\psi}{S_W^\phi}(k)$. The subscript *W* denotes waves, *f* is the Coriolis parameter, and ω_* is the wave frequency. Finally, by assuming the divergence being dominated by waves, the separated vortex and wave components can be obtained as follows,

$$S_V = S_V^u + S_V^v = \left(K^\Psi - \frac{f^2}{\omega_*^2}K^\Phi\right) - k\frac{\mathrm{d}}{\mathrm{d}k}\left(K^\Psi - \frac{f^2}{\omega_*^2}K^\Phi\right), \text{ (S5)}$$

$$S_W = S_W^u + S_W^v = \left(\frac{f^2}{\omega_*^2}K^\Phi - k\frac{\mathrm{d}}{\mathrm{d}k}K^\Phi\right) + K^\Phi - k\frac{\mathrm{d}}{\mathrm{d}k}\left(\frac{f^2}{\omega_*^2}K^\Phi\right), \text{ (S6)}$$

where the subscript *V* denotes the vortex contribution. Bühler et al. (2014) can be consulted for the more detailed derivation.