CHAPTER 9

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ABSTRACT
In this chapter, a model parameterization for organized tropical convection and convectively coupled tropical waves is presented. The model is based on the main three cloud types, congestus, deep, and stratiform, that are observed to play an important role in the dynamics and morphology of tropical convective systems. The model is based on the self-similarity across scales of tropical convective systems and uses physically sound theory about the mutual interactions between the three cloud types and the environment. Both linear analysis and numerical simulations of convectively coupled waves and the Madden–Julian oscillation are discussed.

1. Introduction
Convection in the tropics is organized on a hierarchy of scales ranging from the convective cell of a few kilometers to planetary-scale disturbances such as the Madden–Julian oscillation (MJO) (Nakazawa 1974). Cloud clusters and superclusters occur on the meso- and synoptic scales and often appear embedded in each other and within the MJO envelope. Analysis of outgoing longwave radiation cross correlated with the reanalysis products helped identify the synoptic-scale superclusters as the moist analogs of the equatorial shallow-water waves of Matsuno (1966) but with a severely reduced phase speed (Takayabu 1994; Wheeler and Kiladis 1999) and a front-to-rear vertical tilt in zonal wind, temperature, heating, and humidity profiles (Wheeler et al. 2000; Straub and Kiladis 2002). These moisture-coupled waves are often referred to as convectively coupled waves (CCWs; Takayabu 1994; Wheeler and Kiladis 1999; Wheeler et al. 2000; Kiladis et al. 2009). Convectively coupled Kelvin waves associated with the deepest baroclinic mode dominate the spectral variability on the synoptic scales and propagate, along the equator, at speeds ranging from 12 to 20 m s⁻¹, unlike their dry counterparts that travel at 50 m s⁻¹ (Kiladis et al. 2009).

For almost a decade, simple primitive equation models involving a single baroclinic vertical mode, forced by deep convection, have been used with some relative success for theoretical and numerical studies of convectively coupled waves (Emanuel 1987; Mapes 1993; Neelin and Yu 1994; Yano et al. 1995, 1998; Majda and Shefter 2001b; Fuchs and Raymond 2002; Majda and Khouider 2002). They were somewhat able to reproduce scale-selective instability, at the synoptic scale, of Kelvin waves with a propagation speed in the observed range. However, these one-baroclinic-mode models failed badly to reproduce the important¹ front-to-rear tilt in

¹ As we will see in the next chapter (Majda and Stechmann 2016), the vertical tilt is important for momentum transport from convective systems toward larger scales.

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the vertical and many other important features of CCWs. Moreover, these early models are based on two major theories for the destabilization of large-scale waves by convection, namely, they are divided into convergence-driven models and quasi-equilibrium models.

Convergence models date back to the work of Charney and Eliassen (1964) and Ooyama (1964) followed by Yamasaki (1969), Hayashi (1971), and Lindzen (1974). The convergence models, also called convective instability of the second kind (CISK) models, sustain convection through large reservoirs of convectively available potential energy (CAPE) driven by low-level moisture convergence. Such models exhibit extreme sensitivity to grid-scale variability, and linearized stability analysis reveals the undesirable feature of catastrophic instability with growth rates increasing with the wavenumber (Yano et al. 1998; Majda and Shefter 2001b). In the quasi-equilibrium thinking, first introduced by Arakawa and Shubert (1974), one assumes a large-scale quasi-equilibrium state where CAPE is nearly constant and deep convection acts as an energy regulator in restoring quickly the equilibrium by consuming any excess of CAPE. The triggering and the amplification of convection in quasi-equilibrium models rely on surface fluxes. Indeed, such quasi-equilibrium models are linearly (Neelin and Yu 1994) and even non-linearly stable (Frierson et al. 2004). The most popular mechanism used in concert with the quasi-equilibrium models to create instability is wind-induced surface heat exchange (WISHE) (Emanuel 1987; Emanuel et al. 1994). Both WISHE and CISK theories were initially proposed for hurricanes (Zehr 2001; Craig and Gray 1996). There is no basic observational evidence for their validity for CCWs (Straub and Kiladis 2003). The phenomenon of phase speed reduction is associated, in these one-baroclinic-mode models, solely with a reduction in the background stratification due to moisture coupling of the waves; but the observed backward vertical tilt is suggestive of a strong projection of the CCWs onto shallower vertical modes, with much slower gravity wave speeds, in addition to the prominence of the fastest first baroclinic mode.

Recent analysis of observations over the Indo-Pacific warm pool in the tropics reveals the ubiquity of three cloud types above the boundary layer: shallow congestus clouds, stratiform clouds, and deep penetrative cumulus clouds (Lin and Johnson 1996; Johnson et al. 1999). Furthermore, recent analysis of convectively coupled waves on large scales reveals a similar multicloud convective structure with leading shallow congestus cloud decks that moisten and precondition the lower troposphere followed by deep convection and finally trailing decks of stratiform precipitation; this structure applies to the eastward-propagating convectively coupled Kelvin waves (Wheeler and Kiladis 1999; Straub and Kiladis 2002) and westward-propagating 2-day waves (Haertel and Kiladis 2004) that reside on equatorial synoptic scales of the order of 1000 to 3000 km in the lower troposphere as well as the planetary-scale Madden-Julian oscillation (Kiladis et al. 2005; Dunkerton and Crum 1995). An inherently multiscale theory for the Madden-Julian oscillation with qualitative agreement with observations that is based on these three cloud types is proposed by Majda and Biello (2004) and Biello and Majda (2005).

Furthermore, despite the observational evidence, none of the models with a single vertical mode mentioned earlier account for the multimode nature of tropical convection and the importance of the different cloud types; they are forced by a heating profile based solely on the deep, penetrative clouds. Parameterizations with two convective heating modes (systematically representing a deep, convective mode and a stratiform mode) have first appeared in the work of Mapes (2000). Majda and Shefter (2001a) proposed and analyzed a simpler version of Mapes' model based on a systematic Galerkin projection of the primitive equations onto the first two linear, baroclinic modes yielding a set of two shallow-water systems. The first baroclinic system is heated by deep convective clouds, while the second baroclinic system is heated aloft and cooled below by stratiform anvils. Linear stability analysis of this model convective parameterization revealed a mechanism of stratiform instability independent of WISHE (Majda and Shefter 2001a; Majda et al. 2004). Numerical simulations carried out in Majda et al. (2004) revealed the resemblance of many features of the moist gravity waves for the Majda and Shefter (2001a) model and the real-world convective superclusters (Straub and Kiladis 2002; Majda et al. 2004). One visible shortcoming of the Majda and Shefter (MS) model, however, is its short-cutting of the role of the congestus heating and the systematic interaction of the cloud types with moisture. In fact, the Majda and Shefter model requires the use of WISHE in order to produce realistic amplification of convectively coupled gravity waves during nonlinear simulations (Majda et al. 2004).

2. The multicloud model

In a seminal paper, the authors (Khouider and Majda 2006b) proposed a new model convective parameterization within the framework of the MS model.
In addition to the deep convective and stratiform clouds, the new model carries cumulus congestus clouds that serve to heat the second baroclinic mode from below and cool it from above as in actual congestus cloud decks.

To help the visualization, a cartoon of the cloud types is shown in Fig. 9-1 together with the associated heating profiles. The first baroclinic mode is heated by deep convection, as in the simple one-baroclinic-mode models discussed above, and the second baroclinic mode is heated and cooled by both congestus and stratiform clouds. Congestus clouds lead tropical convective systems and precondition the environment prior to deep convection. They heat the lower troposphere through condensation and induce a cooling of the upper troposphere through detrainment at the cloud top and by blocking longwave radiation emanating from the surface. Stratiform clouds, on the other hand, are observed to trail behind deep convection, and they heat the upper troposphere as deep clouds enter their freezing phase and cool the lower troposphere because of the evaporation of stratiform rain that falls into the already dry environment.

The minimal dynamical core for the multicloud model consists of two coupled (linear) shallow water systems, forced and coupled through the heating rates associated with the three cloud types:

\[
\begin{align*}
\frac{\partial \mathbf{v}_j}{\partial t} + \beta y \mathbf{v}_j - \nabla \theta_j &= -C_d(u_0)\mathbf{v}_j - \frac{1}{\tau_w} \mathbf{v}_j, \quad j = 1, 2, \\
\frac{\partial \theta_1}{\partial t} - \text{div} \mathbf{v}_1 &= \frac{\pi}{2\sqrt{2}} P + S_1, \\
\frac{\partial \theta_2}{\partial t} - \frac{1}{4} \text{div} \mathbf{v}_2 &= \frac{\pi}{2\sqrt{2}} (-H_s + H_c) + S_2,
\end{align*}
\]

where the two modes \( j = 1 \) and \( 2 \) are coupled through the nonlinear source terms. These equations are derived through a systematic Galerkin projection of the equatorial beta-plane primitive equations onto the first and second baroclinic modes that are directly forced by the heating rates \( H_c, H_d, H_s \) associated with the three cloud types. Following Khouider and Majda (2008b), the total precipitation is set to \( P = H_d + \xi_s H_s + \xi_c H_c \), and it includes contributions from deep convection, stratiform, and congestus clouds. Here, \( \xi_s \) and \( \xi_c \) are parameters, with values between 0 and 1, representing the relative contributions of stratiform and congestus clouds to surface precipitation. In this framework, the horizontal velocity and potential temperature are given in terms of their first and second baroclinic components \( \mathbf{v}_j \) and \( \theta_j \), \( j = 1, 2 \), respectively, via the Galerkin expansions:

\[
\mathbf{v} = \sqrt{2}\mathbf{v}_1 \cos(z) + \sqrt{2}\mathbf{v}_2 \cos(2z),
\]

\[
\theta = \sqrt{2}\theta_1 \sin(z) + 2\sqrt{2}\theta_2 \sin(2z).
\]

Fig. 9-1. (left) Schematic of the three tropical cloud types interacting with the well-mixed planetary boundary layer above the sea surface through convective updrafts and downdrafts: the trade wind inversion, the freezing level with temperature 0°C, and tropopause layers are shown. (right) Vertical profiles of heating and cooling fields associated with the three cloud types. [Figure 1 from Khouider and Majda (2008b). ©2008 American Meteorological Society. Reprinted with permission.]
In (9-1), \( v^+ = k \times v \), where \( k \) is the upward vertical unit vector, and \( V \) and \( \text{div} \) are the gradient and divergence operators, respectively. The nonlinear advection terms are neglected for simplicity; they are believed to play a secondary role in the presence of convective heating. The other source terms represent radiative cooling rates: \( S_j = -Q^0_{R,j} - (\tau_R)^{-1} \theta_j, j = 1, 2. \) Here and elsewhere in the text, the parameters and variables that are not properly defined in the text are described in Table 9-1.

### Table 9-1. Parameters and symbols used in the multicloud model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_d )</td>
<td>Turbulent boundary layer drag coefficient</td>
<td>0.001</td>
</tr>
<tr>
<td>( \tau_W )</td>
<td>Rayleigh damping time scale</td>
<td>75 days</td>
</tr>
<tr>
<td>( \tau_R )</td>
<td>Newtonian cooling time scale</td>
<td>50 days</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Gradient of Coriolis force at the equator</td>
<td>( 2.28 \times 10^{-11} \text{ m}^{-1} \text{s}^{-1} )</td>
</tr>
<tr>
<td>( u )</td>
<td>Turbulence velocity scale</td>
<td>( 2 \text{ m s}^{-1} )</td>
</tr>
<tr>
<td>( \xi_c, \xi_e )</td>
<td>Contribution of respectively stratiform and congestus heating to total precipitation</td>
<td>Typically 05 and 1.25, respectively</td>
</tr>
<tr>
<td>( \tilde{Q} )</td>
<td>Background moisture stratification</td>
<td>0.9</td>
</tr>
<tr>
<td>( \tilde{\alpha} )</td>
<td>Coefficient of second baroclinic nonlinear convergence</td>
<td>0.1</td>
</tr>
<tr>
<td>( \tilde{\lambda} )</td>
<td>Relative contribution of second baroclinic moisture convergence, linear term</td>
<td>0.8</td>
</tr>
<tr>
<td>( a_s )</td>
<td>Stratiform heating adjustment coefficient</td>
<td>0.25 (varies)</td>
</tr>
<tr>
<td>( a_c )</td>
<td>Congestus heating adjustment coefficient</td>
<td>0.1 (varies)</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>Coefficient of ( \theta_{eb} ) in ( Q_d )</td>
<td>0.5 (varies)</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>Coefficient of ( q ) in ( Q_c )</td>
<td>( 1 - a_1 ) (varies)</td>
</tr>
<tr>
<td>( a_0 )</td>
<td>Relative strength of buoyancy fluctuation in ( Q_d )</td>
<td>2 (varies)</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>Relative contribution of ( \theta_2 ) in ( Q_d )</td>
<td>0.1 (varies)</td>
</tr>
<tr>
<td>( a'_0 )</td>
<td>Relative strength of buoyancy fluctuation in ( Q_d )</td>
<td>2 (varies)</td>
</tr>
<tr>
<td>( \gamma'_2 )</td>
<td>Relative contribution of ( \theta_2 ) in ( Q_c )</td>
<td>0.1 (varies)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Contribution of stratiform rain evaporation to downdraft</td>
<td>0.25 (varies)</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>Moisture switch function</td>
<td>1 if ( \theta_{eb} - \theta_{em} \geq 20 \text{ K} ) 0 if ( \theta_{eb} - \theta_{em} \leq 10 \text{ K} ) 0.1(( \theta_{eb} - \theta_{em} )) - 1 if ( 10 \text{ K} \leq \theta_{eb} - \theta_{em} \leq 20 \text{ K} )</td>
</tr>
<tr>
<td>( \overline{X} )</td>
<td>Moisture switch function at RCE</td>
<td>0.1(( \overline{\theta}<em>{eb} - \overline{\theta}</em>{em} )) - 1 (varies)</td>
</tr>
<tr>
<td>( Q_d = \max \left{ \left( \overline{U} + \frac{1}{\tau_{\text{conv}}} \left[ a_1 \theta'_{eb} + a_2 q' - a_0 (\theta'_1 + \gamma_2 \theta'_2) \right] \right), 0 \right} )</td>
<td>Deep convective heating potential</td>
<td>Variable</td>
</tr>
<tr>
<td>( Q_c = \max \left{ \left( \overline{U} + \frac{\theta_{eb}}{\tau_{\text{conv}}} \left[ \theta'_{eb} - a_0 (\theta'_1 + \gamma'_2 \theta'_2) \right] \right), 0 \right} )</td>
<td>Congestus heating potential</td>
<td>Variable</td>
</tr>
<tr>
<td>( D_0 = m_0 \max \left( \frac{\mu H_r - H_c}{\overline{U}}, 0 \right) \left( \theta_{eb} - \theta_{em} \right) )</td>
<td>Downdraft fluxes</td>
<td>Variable</td>
</tr>
<tr>
<td>( m_0 = \frac{H_r q^0_{R,1}}{\left[ 1 + \mu (\alpha_s (1 - \overline{X}) - \overline{\alpha_{eb}}) (\overline{\theta}<em>{eb} - \overline{\theta}</em>{em}) \right] } )</td>
<td>Downdraft mass flux scale</td>
<td>Set by RCE solution</td>
</tr>
<tr>
<td>( \overline{U} = (1 - \overline{X}) q^0_{R,1} )</td>
<td>Convective heating potential at RCE</td>
<td>(Varies)</td>
</tr>
</tbody>
</table>

In addition to this dynamical core [(9-1)], there are equations for the boundary layer equivalent potential temperature \( \theta_{eb} \) and the vertically integrated moisture content \( q \):

\[
\frac{\partial \theta_{eb}}{\partial t} = \frac{1}{H_b} (E - D)
\]

\[
\frac{\partial q}{\partial t} + \text{div}(v_1 + \tilde{\alpha} v_2) q + \tilde{Q} \text{div}(v_1 + \tilde{\lambda} v_2) = -P + \frac{1}{H_r} D
\]

(9-3)
Here, \( h_b \approx 500 \text{ m} \) is the height of the moist boundary layer, and \( H_f = 16 \text{ km} \) is the tropospheric height, while \( \dot{Q}, \dot{\lambda}, \) and \( \dot{\alpha} \) are parameters associated with a prescribed moisture background and perturbation vertical profiles. According to the first equation in (9-3), \( \theta_{eb} \) changes in response to the downdrafts \( D \) and the sea surface evaporation \( E \). When setting the closure for the forcing terms in (9-3), conservation of vertically integrated moist static energy is used as a design principle. The moisture equation in (9-3) is derived through a systematic vertical averaging of the water vapor conservation equation (Khouider and Majda 2006b,a). The parameter \( \lambda \) measures the strength of moisture convergence due to the second baroclinic mode, and it plays an important role in the dynamics of convectively coupled tropical waves (Khouider and Majda 2006b).

The main nonlinearities of the model are in the source terms \( H_c, H_d, H_s, \) and \( D \). The stratiform and congestus heating rates \( H_c \) and \( H_s \) satisfy relaxation-type equations:

\[
\frac{\partial H_c}{\partial t} = \frac{1}{\tau_c} (\alpha_s H_d - H_c),
\]

(9-4)

and

\[
\frac{\partial H_s}{\partial t} = \frac{1}{\tau_c} \left( \alpha_c \Lambda - \Lambda^* Q_c - H_s \right).
\]

(9-5)

The deep convective heating \( H_d \) and the downdrafts \( D \) are given diagnostically by

\[
H_d = \frac{1 - \Lambda}{1 - \Lambda^*} Q_d \quad \text{and} \quad D = \Lambda D_0.
\]

(9-6)

The “moisture switch” function \( \Lambda \) controls the transition between congestus to deep convection, and it depends on the difference between boundary layer and midtropospheric equivalent potential temperatures \( \theta_{eb} - \theta_{em} \). The diagnostic functions \( Q_d, Q_c, D_0, \) and \( \Lambda \) all involve nonlinear switches, and they are described in detail in Khouider and Majda (2008b) (see Table 9-1). These source terms take slightly different forms in different versions of the multicloud model such as Khouider and Majda (2006b, 2008b).

3. Convectively coupled equatorial waves in the multicloud model

In this section, we exhibit some typical solutions of the multicloud model equations (9-1)–(9-5) in the form of linear waves and long-time nonlinear simulations.

For the linear waves, the base state is a state of radiative–convective equilibrium (RCE). RCE is a homogeneous (in space and time) steady-state solution for the governing equations (9-1)–(9-5) around which waves can grow and oscillate. In the multicloud model, an RCE solution is determined by fixing three important climatological parameters: the longwave radiative cooling rate \( Q_{R1}^0 \), the discrepancy between the boundary layer equivalent potential temperature and its saturation value \( \theta_{eb}^* - \theta_{eb} \), and the discrepancy between the boundary layer and midtropospheric equivalent potential temperatures \( \theta_{eb} - \theta_{em} \). They are fixed according to climatological values that are recorded in the tropics (Khouider and Majda 2006b, 2008b): \( Q_{R1}^0 = 1 \text{ K day}^{-1} \) and \( \theta_{eb}^* - \theta_{eb} = 10 \text{ K} \), while \( 10 \leq \theta_{eb} - \theta_{em} \leq 20 \text{ K} \) or higher/lower according to whether we want to study a case of a dry or a moist middle troposphere.

The PDE system [(9-1)–(9-5)] is then linearized around the relevant RCE solution and linear solutions are sought on the form \( \hat{U}(x, t) = \hat{U} \exp[i(kx - \omega t)] \), where \( \hat{U} \) is the vector of diagnostic variables. Here, \( k \) is the zonal wavenumber, and \( \omega = \omega(k) \) is the generalized frequency determined as an eigenvalue of the corresponding linear system for a fixed value of \( k \), while \( U(k) \) is the associated eigenvector. The real part of \( \omega(k) \) defines the phase speed of the wave solution: \( c(k) = \Re[\omega(k)]/k \), while the imaginary part represents its growth rate.

More details on the construction of climatologically sound RCE solutions for the multicloud model are found in Khouider and Majda (2006b,a, 2008b). Before proceeding to the study of linear wave solution to the PDE system, it is important to select RCE’s that are stable to small perturbation. This is easily achieved by looking at solutions of the linear system when \( k = 0 \), which essentially represents solutions to the zonally averaged linear solutions. As shown in Khouider and Majda (2006b,a), this system exhibits interesting bifurcation behavior with respect to the model parameters. A typical stability diagram of the background RCE solution with respect to the parameters \( \alpha_c, \alpha_e, \) and \( \theta_{eb} - \theta_{em} \) is shown in Fig. 9-2. Such diagrams are used as guidelines to select the appropriate parameters for the multicloud model (Khouider and Majda 2006b,a, 2008b). Parameter values for which the homogeneous RCE is unstable cannot possibly support waves since the associated background itself will grow and change considerably during the integration of the model. Further linear wave analysis associated with an unstable background is meaningless. The associated parameter values or range of values are thus automatically discarded.

However, the important characteristic of the multicloud model is that, in the appropriate parameter regime, it exhibits a scale-selective instability of wave solutions that have several key physical and dynamical features resembling observed convectively coupled equatorial waves. In Figs. 9-3 and 9-4, we show linear
and nonlinear solutions for the multicloud equations obtained in the case of a simple flow above the equator, where the beta effect is ignored. When the full (nonlinear) multicloud equations are integrated numerically, for a long enough time, with an initial condition consisting of the RCE solution plus a small random perturbation, the solution goes to a statistical steady state that exhibits the wavelike disturbances that have the same features as their linear equivalents, including a reduced phase speed of roughly 17 m s$^{-1}$ and a front-to-rear vertical tilt in wind, temperature, and heating field. In particular, note that the nonlinear simulation is characterized by packets of synoptic-scale waves moving at about 17 m s$^{-1}$ and have a planetary-scale wave envelope moving in the opposite direction at a slower speed of 5 to 6 m s$^{-1}$, mimicking observed CCWs evolving within the MJO envelope. Consistent with the “self-similarity” of tropical convective systems across scales (Mapes et al. 2006; Majda 2007; Kiladis et al. 2009), the multicloud model exhibits a planetary scale envelope that has the same front-to-rear tilted structure as the synoptic-scale waves, though only the synoptic-scale waves are linearly unstable.

Moreover, when the beta effect is included the multicloud model exhibits instabilities corresponding to the full spectrum of convectively coupled waves seen in the observational records (Takayabu 1994; Wheeler and Kiladis 1999), with comparable length scales and phase speeds, namely, Kelvin, westward inertia-gravity waves, and $n = 0$ eastward mixed Rossby-gravity [MRG, also known as Yanai waves (Yanai and Maruyama 1966)] waves (Khouider and Majda 2008a; Han and Khouider 2010). Similarly, to the rotation-free case, all the simulated CCW solutions exhibit a front-to-rear tilted vertical structure as in nature. While westward MRG waves and Rossby waves are missing in the multicloud model linearized about a homogeneous RCE, they are recovered in the case of a meridional barotropic shear background, mimicking the climatological jet stream (Han and Khouider 2010). The combination of the results shown in Khouider and Majda (2008a) and Han and Khouider (2010), which are summarized in Fig. 9-5, demonstrate clearly that the multicloud model with rotation reproduces the full spectrum of convectively coupled waves that are reported in the observational literature (Takayabu 1994; Wheeler and Kiladis 1999).

To conclude this section, we note that the results in Figs. 9.3–9.5 are obtained with the typical synoptic-scale convective time scales, $t_{\text{conv}} = 2$ h, $t_{\tau} = 3$ h, and $t_{e} = 1$ h, which are the estimated time scales of the deep convective, stratiform, and congestus clouds. Comparable values are also used in the stochastic version of the multicloud model (SMCM) (Khouider et al. 2010; Frenkel et al. 2012). In addition to the self-similar cloud morphology of multiscale tropical convective systems, the SMCM is introduced to capture the missing variability in GCMs due to unresolved convective processes. In a nutshell, it is based on the lattice system overlaid over each GCM grid box with an order parameter taking the values 1, 2, 3, or 0, on each lattice site, according to whether it is occupied by a cloud of a certain type (congestus, deep, or stratiform) or it is a clear-sky site. As time increases, the order parameter makes random transitions, with prescribed time scales, from one state to another according to whether the large-scale state and the microscopic configuration is favorable to that state or not.

### 4. The MJO analog wave

By exploiting the observed self-similarity of tropical convective systems (Majda 2007; Kiladis et al. 2009), the multicloud model can be configured toward a planetary and intraseasonal-scale instability of a wave that resembles the tropical intraseasonal oscillation, that is, the MJO, as regards flows along the equator. This was
accomplished in Majda et al. (2007) by setting the convective time scales to

\[ \tau_{\text{conv}} = 12 \text{ hours, and } \tau_c = \tau_s = 7 \text{ days.} \]

Recall that for the case of the synoptic-scale waves above, we have \( \tau_{\text{conv}} = 2 \text{ h, } \tau_c = 1 \text{ h, and } \tau_s = 3 \text{ h.} \) The main effect of this change in parameter value is to shift the instability band to larger scales in both time and space (see top panels of Fig. 9-3). Consistently, numerical simulations with these parameter values yield a solution with the following attractive features that essentially characterize the east–west flow of the MJO above the equator, as shown in Fig. 9-6 (from Majda et al. 2007):

(i) an actual propagation speed of roughly 5 m s\(^{-1}\) as predicted by linear theory;
(ii) a wavenumber 2 structure for the low-frequency planetary-scale envelope with distinct active and inactive phases of deep convection;
(iii) an intermittent turbulent chaotic multiscale structure within the wave envelope involving embedded westward- and eastward-propagating deep convection events; and
(iv) qualitative features of the low-frequency averaged planetary-scale envelope from the observational record in terms of, for example, vertical structure of heating and westerly wind burst.

5. GCM simulation of the MJO and convectively coupled waves

Here, we show an MJO solution produced by the multicloud model when implemented in the next generation climate model of the National Center for Atmospheric Research (NCAR), namely, the High-Order Method Modeling Environment (HOMME) dynamical core. HOMME is a highly scalable parallel code for the atmospheric general circulation based on the discretization of the primitive equations on the sphere using high-order spectral elements in the horizontal and finite differences in the vertical. The interested reader is invited to look into the code development papers (Dennis et al. 2005; Taylor et al. 2008) and the online documentation found on the NCAR website. Here, the discussion will be limited to a brief description of the strategy adopted to implement the multicloud
parameterization in HOMME, which provides the added condensational heating to the otherwise dry model to produce realistic MJO and convectively coupled waves’ solutions as reported in Khouider et al. (2011).

The first step to incorporate the multicloud model in HOMME consists of designing the proper vertical profiles for the heating field associated with the three cloud types: congestus, deep, and stratiform. We use the vertical normal modes of Kasahara and Puri (1981) in place of the sine and cosine functions in (9-2). More precisely, the vertical normal modes result from a vertical mode expansion of the primitive equations by setting, for example, the horizontal velocity $v = \sum j f_j(z)$, where the $f_j$’s are recovered as the eigensolutions of a Sturm–Liouville problem. They are thus a generalization of the sine and cosine basis functions used above for the general case of a nonuniform stratification and without the rigid-lid assumption. The two eigenfunctions $f_1$, $f_2$, corresponding to the first and second baroclinic modes, are plotted in Fig. 9-7a together with the associated heating profiles given by the temperature basis functions: $\psi_j = (p_B - p_P)^{-1} \int_{\rho_j}^{\rho_0} \phi_j(p') dp'$ and $j = 1, 2$, according to the hydrostatic balance equation. The function $\psi_1$ is used to fix the vertical profile for deep convective heating and $\psi_2$ is accordingly used for both stratiform and congestus clouds. The heating is set to zero above roughly 200 hPa to avoid spurious heating in the upper atmosphere.

With the heating rates $H_d$, $H_s$, and $H_c$ parameterized as in the idealized case presented above and the vertical average moisture equation rederived according to the new basis functions $\phi_j$, $\psi_j$ (instead of the cosines and sines), the HOMME full primitive equations are forced by the total heating field $H(x, y, p) = H_d(x, y)\psi_1(p) + [H_s(x, y) - H_d(x, y)]\psi_2(p)$. The resulting coupled model is run on an aquaplanet with a uniform surface evaporation.

FIG. 9-4. (bottom) Filtered structure of the synoptic-scale moist gravity wave and (top) its low-frequency planetary-scale envelope. (left) Potential temperature contours, while (right) the heating anomalies are contoured with the corresponding $u-w$ velocity arrows that are overlaid on top. The + signs on the temperature panels refer to positive anomalies. [Figure 7 from Khouider and Majda (2008b). ©2008 American Meteorological Society. Reprinted with permission.]
and a standard value of radiative cooling: 1 K day\(^{-1}\). The moisture and temperature anomalies are initialized to a tropical mean profile that is also used to fix the Brunt–Väisälä frequency profile in the Kasahara and Puri code for the normal modes that is solved once for all at the beginning of the simulation.

In the appropriate parameter regime (Khouider et al. 2011), the coupled HOMME–multicloud model

![Fig. 9-5. Dispersion diagrams for the multicloud model with rotation. (top) With a homogeneous background. (bottom) With a barotropic meridional shear zonal wind mimicking the jet stream. (left) Symmetric waves and (right) antisymmetric waves are shown separately. [Figure 2 from Khouider and Majda (2008a) and Fig. 8 of Han and Khouider (2010). ©2008 and ©2010 American Meteorological Society. Reprinted with permission.]

![Fig. 9-6. MJO analog wave obtained by convective time scaling for the multicloud model. (left) \(x-t\) contours of precipitation showing slowing moving wave envelopes of mesoscale chaotic convective events that evolve within the active phase and propagate in the opposite direction. (right) Vertical structure of (a) the total heating, with the \(u-w\) velocity overlaid, and (b) the zonal velocity for the moving average of the planetary-scale envelope. [Figures 2 and 4 from Majda et al. (2007). ©2007 National Academy of Sciences, USA.]

yields a solution consisting of two MJO-like waves that move eastward at roughly $5 \text{ m s}^{-1}$ somewhat similar to the two TOGA COARE MJO events reported in Yanai et al. (2000). This is illustrated by (Fig. 9-7b) the $x$-$t$ contours of the zonal velocity and deep convection, (Fig. 9-7c) the zonal structure of the MJO filtered zonal velocity, (Fig. 9-7d) the filtered vorticity, and (Fig. 9-7e) the vertical structure of the filtered zonal velocity shown in Fig. 9-7. Note in particular that the MJO solution has the same tilted structure, intraseasonal period, and embedded mesoscale turbulent fluctuations in the deep convective heating as in the case of the MJO analog of Fig. 9-6, and, in addition, the vorticity field is characterized by a quadruple vortex surrounding the westerly wind burst. The interested reader is referred to the original paper (Khouider et al. 2011) for further discussion of this solution and others. In particular, the whole spectrum of synoptic-scale convectively coupled waves is reproduced in the appropriate parameter regime.

6. Conclusions

A simple model parameterization based on the systematic multiscale interactions of organized convective system is reviewed in this chapter. The main idea is to use the dynamics of mesoscale cloud systems as a building block to capture the multiscale interactions of tropical convection (Mapes et al. 2006). The model is based on the three cloud types that characterize organized tropical systems at the mesoscale, synoptic, and planetary scales (Mapes 1993).

As it is briefly illustrated here, despite its simplicity, the multicloud model captures the whole spectrum of convectively coupled equatorial waves from both the linear and nonlinear simulations stand points (Khouider and Majda 2006a,b, 2008b,a; Han and Khouider 2010; Khouider et al. 2012b). It is flexible enough to be tested as a cumulus parameterization in a GCM and as such it successfully captures the Madden–Julian oscillation and
In terms of its intermittent variability (Qiang et al. 2014). Moreover, as already mentioned, a stochastic version (SMCM) has been invented and implemented in a GCM (HOMME) (Khouider et al. 2010; Qiang et al. 2014). In addition to recreating the physical features and morphology of organized convective systems, the SMCM captures well the intermittent variability of climate, CCWs, and the MJO when tested in both simplified models and in an aquaplanet GCM (Frenkel et al. 2012, 2013; Qiang et al. 2014). In particular, the coupled SMCM–HOMME model successfully simulates the MJO, in an aquaplanet setting, both in terms of the physical features and morphology listed in section 4 and in terms of its intermittent variability (Qiang et al. 2014).

Also the deterministic multicloud model is successfully used to study the parameterization of convective momentum transport (Majda and Stechmann 2008; Khouider et al. 2012), the evolution of meso- and synoptic-scale convectively coupled waves in the MJO background (Han and Khouider 2010; Khouider et al. 2012b), and the diurnal cycle over land and over the ocean (Frenkel et al. 2011a,b). To make it adaptable to the land situation, the multicloud model is coupled to a dynamical boundary layer (Waite and Khouider 2009).

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