Measurement of Atmospheric Temperature Profiles by Raman Backscatter

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ABSTRACT

A technique for measuring instantaneous atmospheric temperature profiles is given. Utilizing the portion of the laser backscatter arising from the Raman rotational spectrum of \(N_2\) profiles up to 2 km with 100 m depth resolution provide temperature to within a degree with signal-to-noise ratios of order unity. Examples are given.

1. Introduction

In this article a method of measuring temperature profiles in the atmosphere is described. The measurement relies on the temperature dependence of the Raman scattering from the atmospheric constituents. In particular, it is possible to obtain vertical temperature profiles by monitoring the frequency-shifted radiation backscattered from the atmosphere, arising from the Raman rotational scattering when illuminating the atmosphere with a high-powered laser. As such it gives rise to an instantaneous profile. A preliminary feasibility estimate was reported earlier (Cooney, 1971).

Raman backscatter has been successfully observed for both major and minor atmospheric constituents (Cooney, 1968, 1970; Mel6, 1969). These measurements have provided line-of-sight (mostly vertical) density profiles of nitrogen and water vapor which have been measured to 9.0 and 2.5 km, respectively.

The fact that the Raman spectrum also contains temperature information is very well known. To cite an appropriate example, the rotational energy levels of atmospheric \(N_2\), because they relax very rapidly to an equilibrium distribution, contain in their relative transitional amplitudes a measure of temperature to a high order of accuracy.

It is shown below that a measure of the signal from two different portions of the \(N_2\) Raman rotational backscatter spectrum from a given volume of the atmosphere, if compared with each other, is a measure of the temperature of that volume. Thus, the differences of the amplitudes of appropriately chosen portions of the Raman spectrum as a function of altitude becomes, in effect, a means of obtaining a temperature profile. The calculations below are concerned with the \(N_2\) rotational Raman spectrum only. In reality, the \(O_2\) rotational spectrum is intermingled with the \(N_2\) spectrum as well as that of other less numerous species. This fact is ignored because, as shown by prior calculations, its inclusion would complicate things without changing anything of significance.

2. Theoretical background

The Raman rotational spectrum contains two branches, an \(O\) (anti-Stokes) and an \(S\) (Stokes) branch of approximately equal amplitude. These branches are positioned symmetrically on the frequency axis on either side of the exciting line. The expression for the intensity of the spectral envelope of either the \(O\) or \(S\) branch is given as

\[
I_{J',J''} = A b_J g_J (2J+1) \exp \left[ \frac{-\beta h c}{kT} J(J+1) \right],
\]

(1)

where:

- \(A\) a normalizing parameter to establish absolute value \((A \approx 1/T)\)
- \(b_J\) relative line strength
- \(g_J\) the nuclear spin weight
- \(\beta\) the molecular rotational constant
- \(k\) Boltzmann's constant
- \(c\) speed of light
- \(J\) rotational quantum number \([\text{mean value of upper} (J') \text{ and lower} (J'') \text{ states}]\)
- \(T\) temperature
- \(h\) Planck's constant

The frequency separation of the exciting and scattering line, \(\Delta \nu\), is given in terms of the rotational quantum number

\[
\Delta \nu = 4 \beta \left( \frac{3}{2} \right)^{-3}\]

(2)

where \(\beta = 1.83 \ \text{cm}^{-1}\) for \(N_2\). Eq. (1) represents the envelope of the spectrum, which, in reality, is a series
of discrete lines a few angstroms apart. However, since only bandwidths 5–10 greater than this are of interest, a continuous approximation of equivalent intensity of the spectrum can be assumed with little error. Thus, in the initial calculations reported earlier (Cooney, 1971), the true spectrum was replaced by its envelope. However, the more extensive calculations reported here were done by computer so that the results reflect use of the actual spectrum.

The capability, in principle, of making temperature measurements was recognized at the outset of the Raman studies involving remote atmospheric probing. Thus, Fig. 1 shows the manner (qualitatively) by which the spectral envelope changes shape as a function of temperature. But while concentration measurements in the field using the Raman backscatter return were begun within a year or so of the advent of the giant pulse laser, temperature measurements have yet to be tried. What has stymied the exploitation of the temperature measurement possibilities thus far is the relative lack of sensitivity of the intensity to temperature changes. For example, the intensity of the $N_2$ rotational backscatter at 300K has a maximum sensitivity as a function of wavelength of 0.2% ($^oK^{-1}$). This small change is extremely difficult to measure. On the other hand, the measurement technique presented below can increase the signal sensitivity by close to two orders of magnitude. Basically the suggested method is a differencing scheme in which two separate input signals are inserted into a differential amplifier. The signals are simultaneous and come (as indicated above) from two discrete portions of the Raman rotational backscattered spectrum. Fig. 2 depicts the disposition of the two filters (and hence the two signals) on the wavelength axis. The changing spectral shape, arising directly from the temperature changes, provides the difference signal whose amplitude is a direct measurement of the temperature. This difference signal is much more sensitive to changes in the shape of the spectral envelope and hence more temperature sensitive.

A feature which makes the difference signal so useful is a property of the operational amplifier called the common mode rejection ratio. It allows for the rejection of the commonality of the two input signals to a very high order (to within the limit of the amplifier noise level, of course). An operational amplifier with a common mode rejection ratio of 70 dB will measure a difference of two signals, say, 1.000 V and 1.001 V to one part in $10^4$. The equivalent input noise level for typical differential amplifiers is about 10 $\mu$V.

3. Description of system

The drawing shown in Fig. 3 depicts, schematically, the essentials of the measuring apparatus. As shown, an incoming signal is divided in half by a pellicle. In one of the resulting optical channels an optical filter with a specified transmission characteristic labelled $F_1$ is inserted. In the other channel a filter labelled $F_2$ is inserted. Both signals, after passing through the optical filtering and after the necessary optics, fall upon their respective photomultipliers. The output electrical signals from the photomultipliers are fed to the input terminals of the operational amplifier (wired as a differential amplifier). Here the difference is amplified and the output signal can then be displayed on an oscilloscope. This is the essence of the system.

One of the most interesting aspects of this particular laser radar configuration is that some of the normal trouble spots producing inaccuracies in the data are mitigated to a large extent. This is due primarily to the fact that power in the two spectral intervals under consideration are part of a larger single signal. Hence, their transmission losses are identical as are the fluctuations in scattering power (e.g., in number density of scatterers). Also the system-dependent part of the signal is cancelled out as is shown, for example, in Eq. (9) where the $1/r^2$ dependence of the individual signals cancels one another. Uncertainty in the optical system parameters

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Fig. 1. Graph of Eq. (1) depicting the shape changes of Raman rotational spectral envelope arising from temperature changes.

Fig. 2. Disposition of optical filters relative to Raman rotational spectrum. (Note: For actual disposition of filters, the abscissa has been changed to wavelength $\lambda$).

Fig. 3. Schematic diagram of electro-optics of receiver system of remote temperature measuring apparatus.
will cause uncertainties primarily in the measured value of the ground temperature, and will have relatively little effect on the relative measurement as a function of altitude. Thus, the measured profile will best be rectified with a ground based measurement. The one important source of error in this relative profile is the shot noise on the signal. Even the ambient sky noise will cancel out to a very marked degree. A detailed error analysis must, however, await the acquisition of experimental data.

4. System computations

Computations are helpful in establishing reasonable values of key system parameters such as temperature sensitivity and minimum temperature resolvability. Also, since certain optical filter characteristics have to be optimized, calculations to determine hardware parameters were performed. Thus, let

\[ I = \int_0^\infty I_2 J' dJ = 1.0 \]  
(normalized to unity for convenience), \( (3) \)

\[ I_1 = \int_0^\infty I_1 J' F_1 dJ, \]  
\( (4) \)

\[ I_2 = \int_0^\infty I_2 J' F_2 dJ, \]  
\( (5) \)

where \( I_2 \)' is given in (1), \( I \) represents the backscattered power in one branch (say the S branch) of the spectrum, \( I_1 \) the power of that portion of the spectrum coming through filter one (F₁), and \( I_2 \) that through filter two (F₂).

The signals \( I_1 \) and \( I_2 \) are fed to the input terminals of the differential amplifier. Thus, let

\[ I_\Delta = a_1 I_1 - a_2 I_2, \]  
\( (6) \)

where \( I_\Delta \) is the input current difference. The \( a \)'s are the photomultipliers' conversion efficiencies (luminous sensitivities times gain). The two electro-optical channels of the system can be balanced for equal incoming intensities, so that it is possible to let \( a_1 = a_2 = \alpha \).

Now one also has an output current from the differential amplifier given by

\[ I_{\Delta 0} = B \alpha (I_1 - I_2), \]  
\( (7) \)

where \( B \) is the gain of amplifier, and \( I_{\Delta 0} \) the basic output signal which is the direct measure of the temperature as a function of range. Displaying \( I_{\Delta 0} \) on an oscilloscope and photographing the screen will provide an altitude temperature profile recording.

It is of utmost importance to determine the practical change of signal for a given change in temperature so that the resulting sensitivity can be compared to typical system noise levels. It is the comparison of the temperature sensitivity to the noise level which will determine to what degree of fineness the temperature changes can be measured. Thus, for these purposes let the fractional change in intensity \( I_{\Delta T} \) for a temperature change \( \Delta T \) be given as

\[ I_{\Delta T} = \frac{I_{\Delta 0}(T + \Delta T) - I_{\Delta 0}(T)}{I_{\Delta 0}(T + \Delta T) + I_{\Delta 0}(T)} \]  
\( (8) \)

It is useful to determine the magnitudes of \( I_{\Delta T} \) for reasonable value of system response and Raman cross sections. Detailed computer calculations have been made along with the optimization calculations mentioned above. However, before proceeding with the calculations it is useful to mention a word of caution concerning the Rayleigh signal.

The Rayleigh component of the backscatter is a source of spurious signal and it has to be made very small [i.e., a magnitude of \( \sim 1\% \) of \( I_{\Delta T} \) in Eq. (8)]. It might be thought that this could be rendered irrelevant due to the cancelling effects inherent in \( I_{\Delta T} \) and then would simply be a finite but constant offset in the signal due to the inability to precisely match transmission characteristics of the both filters of the exiting or Rayleigh wavelength. However, the Rayleigh backscatter will diminish as a function of altitude due to the decrease in the number of scatterers with altitude, and this signal change with altitude would be about what would be expected for the changes in signal with altitude due to temperature changes. Thus, any residual difference in the Rayleigh components in the two channels would exhibit an altitude change similar to the temperature. The most effective way to deal with the Rayleigh signal as noted above is to make its contribution to the total signal less than a few percent of the difference signal given by \( I_{\Delta T} \). In addition, if Mie (particulate) scatter were to exceed Rayleigh by a factor of 5.0–10.0, then this would start to add to the signal error and a new set of filter specifications would be required.

Computations on temperature sensitivity and the signal-to-noise ratio were made based upon hardware parameters which themselves were derived from signal optimizing computer calculations. As a typical example, let filter one (i.e., F₁) have half-width at half-maximum (H.W.H.M.) of 7.0 Å and a maximum transmission of 0.30 centered at 36.0 Å away from the exciting line, and let filter two (F₂) have H.W.H.M. = 10.0 Å and a maximum transmission of 0.3 centered at 65 Å from the exciting line. Such filter specifications are within various manufacturers capabilities. Assuming Eq. (3) to hold, the results of such computations are given in Table 1.

A perusal of the table shows that if it is assumed that the instantaneous power \( I \) is the equivalent of a 100.0 V signal, then the instantaneous difference signal ranges from 280–1000 mV and the temperature sensitivity of the difference in signal ranges from 19–29 mV (°K)⁻¹.

In order to compute the signal-to-noise ratio for such a system, typical return signal levels must be evaluated in order to estimate the shot noise. The basic system assumptions are given in Table 2.
Table 1. Signal computations.

<table>
<thead>
<tr>
<th>Temperature (°K)</th>
<th>$I_{P1}$†</th>
<th>$I_{P2}$‡</th>
<th>$I_{0}(\times 10^{-3})$</th>
<th>$I_{AT}(\times 10^{-6})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>262</td>
<td>0.091347</td>
<td>0.088556</td>
<td>0.27912</td>
<td>0.28775</td>
</tr>
<tr>
<td>263</td>
<td>0.091185</td>
<td>0.088681</td>
<td>0.25035</td>
<td>0.28572</td>
</tr>
<tr>
<td>264</td>
<td>0.091023</td>
<td>0.088805</td>
<td>0.22178</td>
<td>0.28376</td>
</tr>
<tr>
<td>299</td>
<td>0.085537</td>
<td>0.092147</td>
<td>0.66105</td>
<td>0.22123</td>
</tr>
<tr>
<td>300</td>
<td>0.085386</td>
<td>0.092218</td>
<td>0.68317</td>
<td>0.21965</td>
</tr>
<tr>
<td>301</td>
<td>0.085325</td>
<td>0.092287</td>
<td>0.70514</td>
<td>0.21802</td>
</tr>
<tr>
<td>320</td>
<td>0.082446</td>
<td>0.093382</td>
<td>1.0936</td>
<td>0.18999</td>
</tr>
<tr>
<td>321</td>
<td>0.082303</td>
<td>0.093428</td>
<td>1.1126</td>
<td>0.18862</td>
</tr>
<tr>
<td>322</td>
<td>0.082160</td>
<td>0.093474</td>
<td>1.1315</td>
<td>0.18723</td>
</tr>
</tbody>
</table>

† Rayleigh contribution is $6.2 \times 10^{-6}$.
‡ Rayleigh contribution is $5.27 \times 10^{-4}$.

The radar equation is now used to determine the number of photons scattered back into the receiver mirror from 2 km:

$$P_r = \frac{P_T(\sigma T)D\xi A}{4\pi r^2},$$  \hspace{1cm} (9)

where:

- $P_r$: received power (W)
- $P_T$: transmitted power
- $c$: speed of light
- $\tau$: pulse duration (sec)
- $D$: optical system efficiency of 0.07 (does not include Q.E.)
- $\xi$: volume extinction coefficient due to Raman rotational scattering, equal to $1.33 \times 10^{-7}$ m$^{-1}$
- $A$: 1.0 m$^2$, area of mirror
- $r$: altitude (m)

Eq. (9) implies the typical laser radar configuration in which a high-powered laser is mounted alongside a large receiver mirror in a so-called monostatic configuration. Note that in (9) transmission losses can be ignored when estimating signal power. The laser and mirror optic axis are generally aligned. Details for such a system are given in Cooney (1968).

On the basis of the above system parameters the instantaneous power backscattered into the receiver from the 2.0-km level is, for each channel, $8.0 \times 10^{-4}$ W. Now if it is further assumed that we have a receiver electronic bandwidth of $1.5 \times 10^8$ Hz, this places a limit on the minimum resolvable altitude interval of the return signal of 100.0 m. It also integrates the instantaneous return power over a time interval of $\sim 0.667 \mu$sec, corresponding to the above assumed bandwidth. In addition, as can be seen from Table 1, 8–9% of this power gets through the optical filter. Thus, coupled with a 0.85 mirror reflectivity, approximately $5.8 \times 10^{-7}$ W of radiation falls upon the photocathode in each channel.

The signal-to-noise ratio depends upon the quantum efficiency of the photodetector. For $\lambda=0.7 \mu$, an effective quantum efficiency of 10–15% can be realized for a photoemitter while 60–70% is possible for a photoductor. Because of the relatively large light signal into the receiver, the extreme sensitivity of the photoemitter is not required; thus, the high quantum yield of the photoductor becomes attractive because of the role quantum efficiency plays in determining the signal-to-noise ratio. During the so-called receiver integration time of 0.667 μsec, there are approximately $1.35 \times 10^4$ photons falling on the photocathode. If one assumes that $\sigma$ is the number of photoelectrons thereby available (released in the case of the photoemitter), this gives a $\sigma=8.76 \times 10^2$ for a 65.0% quantum efficiency or $\sigma=1.7 \times 10^3$ for a 12.5% Q.E. Since $S(N \approx (\sigma)^0$, then $(S/N)_{p.e.}=933.0$ or $(S/N)_{p.e.}=410.0$. Again, assuming that power $I$ is the equivalent of a 100.0 V signal, then the signal in either channel on this basis (from Table 1) would be 8.0–9.0 V. Thus, the noise is about 9.0 mV for the photoductor or 20.5 mV for the photoemitter. In conjunction with the temperature sensitivity computed above this implies a mean minimum temperature resolvability of 0.31–0.47K for the photoductor or 0.71–1.0K for the photoemitter. Clearly, in a given instance, the actual temperature resolution depends upon the actual shot noise on a particular signal and for a particular altitude interval.

The precision with which the temperature change for a layer of air can be measured, can be computed on the basis of the temperature resolvability. Thus, if a lapse rate of 6.5K km$^{-1}$ existed over a 1-km layer, the error in the laser measured lapse rate would range from about 5–16%, depending upon whether a temperature resolvability of 0.31 or 1.0K were assumed.

It should also be noted that the various sources of noise external to the receiver such as laser fluorescence, laser pump light, airglow, etc., will essentially cancel in the two channels.

The above calculation concerns a single pulse. The signal-to-noise ratio would increase with the square root of the number of pulses (or individual profiles). Thus for a four pulse per minute output pulse rate, the 1-min time average temperature profile would have its mean minimum temperature resolvability cut in half. On the other hand the signal-to-noise ratio diminishes as the reciprocal of the range. Hence, a 0.5K temperature resolvability at 2.0 km would imply a 1.0K temperature resolvability at 4.0 km.

The system described above does not attempt to take advantage of some of the more reasonable prospects in

Table 2. Lidar specifications.

1. Output energy, 20.0 J pulse$^{-1}$
2. Pulse duration <500 nsec
3. Output wavelength, 6943 Å
4. Receiver mirror area, 1 m$^2$
5. Receiver conversion efficiency, 0.07 (not including Q.E.)
6. Raman rotational cross section for $N_2$ for entire S-branch, $6.32 \times 10^{-24}$ m$^2$ or $1.33 \times 10^{-7}$ m$^{-1}$ volume attenuation for N.T.P. (H. Ory, 1965).
laser development. Hence, a narrow band optical amplifier in each of the optical channels could very easily eliminate the shot noise problem and reduce the minimum temperature resolution interval to well below any practical anticipated needs in atmospheric probing. In addition this would extend the altitude range to limits which would then depend upon some new noise criterion.

Finally, and in a sense, perhaps the most notable feature, it appears as though the dual channel approach can remove the restriction of nighttime use for many applications of lidar.

REFERENCES


