Measurement of Rainfall Rates by Lidar

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ABSTRACT

Monostatic lidar is explored as a means for determining the rainfall rate over an extended atmospheric path with a spatial resolution comparable to that of rain gages. An empirical relationship is established between the optical extinction coefficient of rain $\beta_r$ (km$^{-1}$) and the rainfall rate $R$ (mm hr$^{-1}$). Correlation of lidar-derived rainfall extinction and gage rainfall rates at Madison gives

$$\beta_r = 0.16 R^{0.74}.$$ 

The $\beta_r-R$ relations obtained from the work of other authors compare well with this relationship.

A lidar equation which accounts for the multiple scattering of light in rain is presented. A numerical procedure which derives estimates of $\beta_r$ as a function of range from lidar returns is developed. Examples of lidar-derived rainfall rate range profiles in spatially inhomogeneous thunderstorms are given.

1. Introduction

Remote sensing of precipitation is performed by radar through the measurement of radar reflectivity. The radar reflectivity factor $Z$ (mm$^4$ m$^{-3}$) is related to the rainfall rate $R$ (mm hr$^{-1}$) by an equation of the form

$$Z = DR^\delta,$$  \hspace{1cm} (1)

where $D$ and $\delta$ are empirically determined constants (see Battan, 1973). It is customary to deploy rain gages under the radar-sampled atmospheric volume to calibrate weather radars for the constants $D$ and $\delta$. There is a large difference in the volumes sampled by gages and radar, however, and consistent correlations between $Z$ and gage rainfall measurements are difficult to obtain. Smith (1973) has noted that $Z-R$ relations could be adjusted through comparison of radar measurements with real-time gage measurements of the same rainfall. If the number of gage measures utilized in such a calibration is small then the accuracy of the rainfall measurements may still be unsatisfactory.

Lidar may provide a solution to the problem of calibrating weather radars. A monostatic lidar yields information from the atmosphere over an extended path length (several kilometers) with a spatial resolution comparable to that of a rain gage (several meters). The rainfall optical extinction coefficient is related to the rainfall rate by

$$\beta_r = GR^\gamma,$$  \hspace{1cm} (2)

where $G$ and $\gamma$ are constants. This study explores the capability of monostatic lidar to determine the rainfall rate from optical extinction by means of (2).

Several $\beta_r-R$ relationships determined from the data of other investigators are presented in Table 1. Most of these $\beta_r-R$ relations were computed from measured rainfall drop size distributions. Chu and Hogg (1968) directly measured the $\beta_r-R$ relation by means of laser transmission. The liquid water content of rainfall $W$ (mg m$^{-3}$) is also related to $\beta_r$ by a power law, and this quantity finds application in cloud modeling. The $\beta_r-W$ relationships determined from the drop size distributions of these investigators are also presented in Table 1.

The time-dependent lidar return signal from rainfall can be specified by a lidar equation which is similar in form to the conventional single-scatter lidar equation. The lidar equation is simply modified to include the effect of multiple small-angle forward scattering. A numerical procedure which derives $\beta_r$ as a function of range is developed, and it is used to obtain rainfall extinction profiles from lidar returns from the leading edge of a thunderstorm. An analytic solution to the approximate small-angle multiple scatter lidar equation for average $\beta_r$ in spatially homogeneous rainfall is also given. Both the numerical procedure and the analytic solution can be used to derive an average rainfall rate over rain gages. It is shown that the lidar method can reliably determine the rainfall rate over an extended path in markedly inhomogeneous rainfall.

2. The lidar equation

The conventional monostatic lidar system consists of a Q-switched laser transmitter and a collimated receiver telescope with suitable photon detection elec-
Table 1. Summary of β_r-R and β_r-W relationships.

<table>
<thead>
<tr>
<th>Author</th>
<th>Distribution</th>
<th>Rainfall type</th>
<th>β_r-R</th>
<th>β_r-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlas (1958)</td>
<td>MP</td>
<td>Mean</td>
<td>β_r = 0.31R^{0.67}</td>
<td>β_r = 0.013R^{0.75}</td>
</tr>
<tr>
<td>Best (1958)</td>
<td>B</td>
<td>Mean</td>
<td>β_r = 0.25R^{0.44}</td>
<td>β_r = 0.017R^{0.75}</td>
</tr>
<tr>
<td>Chu and Hogg (1968)</td>
<td>NUM</td>
<td>Thunderstorm</td>
<td>β_r = 0.18R^{0.97}</td>
<td>Not available</td>
</tr>
<tr>
<td>Miller (1973)</td>
<td>NUM</td>
<td>Thunderstorm</td>
<td>β_r = 0.12R^{0.78}</td>
<td>β_r = 0.004R^{0.80}</td>
</tr>
<tr>
<td>Polakova (1960)</td>
<td>NUM</td>
<td>Mean</td>
<td>β_r = 0.21R^{0.74}</td>
<td>β_r = 0.002R^{0.80}</td>
</tr>
<tr>
<td>Simms and Mueller (1972)</td>
<td>NUM</td>
<td>Thunderstorm</td>
<td>β_r = 0.138R^{0.82}</td>
<td>β_r = 0.001R^{0.80}</td>
</tr>
<tr>
<td>Wessels (1972)</td>
<td>B</td>
<td>Mean</td>
<td>β_r = 0.25R^{0.49}</td>
<td>β_r = 0.011R^{0.90}</td>
</tr>
</tbody>
</table>

* A discussion of the data of each author and the procedure followed to determine the β_r-R relations is presented in Appendix A of Shipley (1973). A similar procedure was followed to obtain the β_r-W relations. These relations are given for β_r in km^{-1}, R in mm hr^{-1} and W in mg m^{-2}.  
* Abbreviations refer to Marshall-Palmer (MP), Best (B) and numerical (NUM) drop size distributions. The result of Chu and Hogg was obtained by direct measurement.  
* Private communication.

In general, the transmitter and receiver axes are aligned so that the receiver field of view overlaps a large part of the transmitted beam. Those photons scattered through θ radians by atmospheric gases and particles are gathered by the receiver telescope. The monostatic lidar equation for multiply-scattered return power from a non-absorbing atmosphere is given by

$$P(\theta) = \frac{c A_r}{2\pi} \left( \sum_j \frac{\phi_j(\theta)}{4\pi} \beta_j \right) \exp \left( -2 \int_0^\infty \sum_j \epsilon_j \gamma_j d\gamma \right),$$

where:
- $c$ is the speed of light
- $A_r$ is the effective receiver aperture
- $\phi_j(\theta)$ is the normalized backscattering phase function of the $j$th atmospheric scattering component
- $\beta_j$ is the extinction coefficient
- $\epsilon_j$ is the fraction of scattered light which is removed from the field of view by the $j$th atmospheric scattering component.

The return power $P$, the volume backscattering cross section $\sum_j \phi_j(\theta)\beta_j$, and the effective extinction coefficient $\sum_j \epsilon_j \beta_j$ are general functions of range.

Let $N_j(\alpha)$ represent the size distribution of a non-absorbing spherical polydispersion with a real refractive index $m_j$ as a function of the size parameter $\alpha = \pi x / \lambda$, where $x$ is the particle diameter. $\phi_j(\theta)$ and $\beta_j$ are defined by

$$\phi_j(\theta) = \frac{4\pi}{(2\pi)^3} \int_0^\infty N_j(\alpha) i_x(m_j,\alpha) d\alpha,$$

$$\beta_j = \frac{\pi}{(2\pi)^3} \int_0^\infty \alpha^2 N_j(\alpha) Q_{sca}(m_j,\alpha) d\alpha,$$

where $Q_{sca}(m_j,\alpha)$ is the scattering efficiency and $i_x(m_j,\alpha)$ the dimensionless backscatter intensity (see Deirmendjian, 1969). 1 The scattering efficiency of a raindrop at visible wavelengths is nearly a constant of 2.

3. Extinction component parameters

Atmospheric extinction in an atmosphere containing rainfall is due to gas molecules, aerosol and rain. The properties of these constituents are now considered.

a. Rayleigh scattering

The extinction coefficient of atmospheric gases is small in comparison to the aerosol extinction coefficient during rainfall, and is therefore neglected in the present analysis. Tropospheric absorption near the characteristic ruby laser wavelength is due primarily to water vapor; however, Bradley and Schotland (1969) showed that water vapor absorption is negligible when the ruby laser wavelength lies between the $H_2O$ absorption lines at 694.237 and 694.380 nm.

b. Aerosol scattering

Waggoner et al. (1972) utilized a ruby lidar and an integrating nephelometer to measure $\phi_a(\theta)$ in an urban environment. Analysis of their data taken with relative humidity $> 75\%$ gives the aerosol backscattering phase function $\phi_a(\theta) = 0.40 \pm 0.25$. Absorption is assumed negligible.

c. Scattering by large hydrometeors

Querfeld (1973, personal communication) has computed the Mie backscattering phase functions for large water spheres with a real refractive index $m_s = 1.33$ at a wavelength of 694.3 nm. An average phase function in the angular interval 179.7° to 180.0° was computed for several sphere diameters near 1 mm.

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1 In Deirmendjian's notation $\phi(\theta) = [\phi_1(\theta) + \phi_2(\theta)]$, where $\phi_1(\theta)$ and $\phi_2(\theta)$ are the first two elements of the Stokes scattering matrix. In this paper, phase function subscripts refer only to the composition of the scatters ($a$ for aerosol and $r$ for rain).
The mean average backscattering phase function is \( \theta_{\nu}(\pi) = 0.29 \pm 0.15 \).

d. Multiple-scattering correction factors

Atmospheric scattering during rainfall is primarily due to aerosol and rain, which can both be represented as ensembles of spherical particles. The Fraunhofer diffraction pattern for a circular aperture consists of a bright central maximum surrounded by concentric dark and bright rings (see, e.g., Born and Wolf, 1970). One-half of the energy scattered by a sphere is diffracted in the forward direction, and approximately 84% of the energy diffracted by a sphere is directed into the characteristic central maximum. The first dark ring has a full angular width \( \theta_\ell \) such that

\[
\frac{\theta_\ell}{\alpha \sin \frac{\pi}{2}} = 3.832,
\]

where \( \theta_\ell = 3.4 \) mrad for a 0.5-mm diameter sphere at a wavelength of 694.3 nm.

The Wisconsin lidar has a transmitted beam divergence of 3 mrad and a full angle receiver field of view \( \eta \) which can be varied from 1 to 8 mrad. Most of the light diffracted by rain continues to propagate in nearly the same direction as unscattered light and does not leave the receiver field of view when \( \eta = 8 \) mrad. A lidar with a sufficiently large \( \eta \) will thus measure an effective rainfall extinction coefficient which is approximately half that indicated by (5). It follows that \( \epsilon_\tau \approx 0.5 \) for the Wisconsin lidar.

By a similar argument, light diffracted by aerosols is distributed over correspondingly greater angles and is removed from the incident beam for \( \eta = 8 \) mrad. Thus, the value of \( \epsilon_\tau \) is assumed to be 1.0 for the Wisconsin lidar.

4. Solutions to the lidar equation for \( \beta_r \)

The lidar equation is solved for the extinction coefficient in an atmosphere containing two non-absorbing extinction components. The extinction coefficient \( \beta_r \) is due to aerosol and is assumed to be spatially homogeneous, while \( \beta_r \) is the spatially inhomogeneous extinction coefficient of rain. The effective atmospheric extinction coefficient during rainfall is

\[
\beta_{\text{atmos}}(s) = \epsilon_r \beta_r(s) + \epsilon_\alpha \beta_\alpha.
\]

The backscattering phase function \( \Theta_\nu(\pi) \) is assumed constant for each component. Taking the natural logarithm of (3), the two-component rainfall lidar equation is

\[
\ln(Ps^2) = \ln C + \ln[\Theta_\nu(\pi)\beta_\alpha + \Theta_\nu(\pi)\beta_r]
- \frac{2}{\epsilon_r \beta_r + \epsilon_\alpha \beta_\alpha} \int_0^\pi d\theta' \epsilon_r \beta_r + \epsilon_\alpha \beta_\alpha, \tag{8}
\]

where \( C \) is a lidar system constant.

a. Average slope solution

The terms \( \Theta_\nu(\pi), \epsilon_\alpha \) and \( \Theta_\nu(\pi) \) are assumed to be constants in this analysis. If \( ds'/ds = 0 \), then the derivative with respect to range of (9) gives

\[
\frac{d}{ds} \ln(Ps^2) = \frac{1}{2} \ln(Ps^2), \tag{9}
\]

and the extinction coefficient is readily found from an average slope of the lidar return; \( \beta_r \) is then found by means of (7). An examination of lidar returns from rainfall indicates that \( \beta_r \) is often a constant over a range on the order of kilometers.

b. Solution by successive approximations

An average slope cannot be determined for lidar returns from spatially inhomogeneous rainfall. A numerical procedure which derives range profiles of \( \beta_r \) by means of (8) was developed for such cases. This procedure is outlined as follows:

Let the term \( \beta^{(j)}_r \) represent the \( j \)th guess to the nonhomogeneous extinction coefficient at range \( s_j, j = 1, \ldots, J \). A lidar return can then be constructed by means of (8), such that the departure of the \( j \)th guess lidar return from the measured lidar return at range \( s_j \) is

\[
g^{(j)}_r = \ln(Ps^2) - \ln\left[\Theta_\nu(\pi)\beta^{(j)}_r + \Theta_\nu(\pi)\beta_\alpha\right] + 2\epsilon_r \beta^{(j)}_r \sum_{k=1}^J \left( \beta^{(j-1)}_r + \epsilon_\alpha \beta_\alpha \right). \tag{10}
\]

The departure \( g^{(j)}_r \) can also be expressed in terms of the difference \( \beta^{(j+1)}_r - \beta^{(j)}_r \) and a new guess \( \beta^{(j+1)}_r \), such that

\[
g^{(j)}_r = \ln\left[\frac{P^{(j+1)}}{P^{(j)}}\right] = \ln\left[\frac{\Theta_\nu(\pi)\beta^{(j+1)}_r + \Theta_\nu(\pi)\beta_\alpha}{\Theta_\nu(\pi)\beta^{(j)}_r + \Theta_\nu(\pi)\beta_\alpha}\right] - 2\epsilon_r \beta^{(j)}_r \sum_{k=1}^J \left( \beta^{(j+1)}_r + \beta^{(j)}_r \right). \tag{11}
\]

Noting that \( \ln(1+u) \approx u \), the correction to the rainfall extinction coefficient is

\[
\Delta \beta^{(j)}_r = \frac{g^{(j+1)}_r - 2\epsilon_r \Delta s \sum_{k=1}^J \Delta \beta^{(j)}_r}{g^{(j+1)}_r + g^{(j)}_r}, \tag{12}
\]

where \( \Delta \beta^{(j)}_r = \beta^{(j+1)}_r - \beta^{(j)}_r, j = 1, \ldots, J \), and where \( g^2 = \Theta_\nu(\pi)/\Theta_\nu(\pi) \).

A common problem for a monostatic lidar having a lateral separation between the laser and the receiving telescope is the absence of a signal at close range \( s_j, j = 1, \ldots, m-1 \), before the transmitter and receiver
fields of view fully overlap. The close-range extinction

\[-2\Delta s \sum_{k=1}^{m-1} (\epsilon_b \beta_{r,k} + \epsilon_a \beta_a),\]

is then an additional unknown which may vary from shot to shot. The close-range extinction can be combined with the lidar equation constant in (10), such that

\[g^{[2]} = \ln(P_s S^2) - \left[\ln(C' + \ln[\Phi_r(\pi) \beta_r^{[2]} + \Phi_a(\pi) \beta_a] - 2\Delta s \sum_{k=m}^{j} (\epsilon_b \beta_{r,k}^{[2]} + \epsilon_a \beta_a)),\]

for \(m \leq j \leq J\). The unknown \(C'\) can be found by choosing a \(C'\) for each iteration such that

\[\sum_{j=m}^{J} (g^{[2]})^2\]

is a minimum. The solution then converges to a representative constant \(C'\) and extinction coefficient profile after several iterations.

\[c. \text{Comparison of the solutions}\]

A lidar return from the leading edge of a thunderstorm on 20 April 1973 is given in Fig. 1. Three range segments having distinct average slope values are apparent. Using the value \(\beta_a = 0.14 \text{ km}^{-1}\) cited by Elterman (1968), and letting \(\epsilon_a = 1.0\) and \(\epsilon_r = 0.5\), the three average slopes give \(\beta_r\) values of 0.0, 0.4 and 1.0 \text{ km}^{-1}. The average return slopes are also shown in Fig. 1. Eq. (9) cannot be applied to lidar returns where

\[\epsilon_0 \beta_a > -d \ln(P_s S^2)/ds,
\]

or where an average slope cannot be reliably estimated.

The lidar return of Fig. 1 was analyzed for \(\beta_r\) by means of the successive approximations technique. This solution was optimized through variation of the rainfall backscattering phase function \(\Phi_r(\pi)\), and the resulting \(\beta_r\) profiles were compared to that \(\beta_r\) profile found by means of (9). The value \(\Phi_r(\pi) = 0.18 \pm 0.05\) gave best agreement, and it is within the theoretical limits found by Querfeld.

Several lidar returns from the same rainfall were then analyzed by successive approximations using \(\Phi_a(\pi) = 0.40\) and \(\Phi_r(\pi) = 0.18\). All of these solutions were in agreement with the results of the average slope method where that method was applicable. The successive approximations solution for the lidar return of Fig. 1 is shown in the lower half of that figure.

The close-range problem was handled by calculating

\[E = \sum_{j=m}^{J} (g^{[2]})^2\]

for the tenth iteration of (12) for various values of \(C'\),

\[\text{Fig. 1. Upper: average slope solution for } \beta_r \text{ on a lidar return from the leading edge of a thunderstorm on 20 April 1973. Lower: successive approximations solution for } \beta_r \text{ on the same return.}\]

and selecting that value of \(C'\) corresponding to a minimum in \(E\). The minima of \(E(C')\) were found to be unique.

\[5. \text{Experimental method}\]

The Wisconsin monostatic lidar was operated at an inclination of 5° above the horizontal from the penthouse of the building which houses the Meteorology Department at Madison. Tipping-bucket rain gages deployed under the beam path were used to measure the rainfall rate. The characteristics of the Wisconsin lidar are summarized in Table 2. The laser transmitter and telescope receiver axes were aligned parallel with a lateral separation of 1.5 m. The beam divergence and receiver field of view gave full beam overlap at a range of 1.0 km.

Rain gage types, ranges and headings, and heights of the laser beam above each gage are given in Table 3. The rain gages are of the tipping-bucket type. The Weather Measure P-501 tipping-bucket rain gages measure 0.25 mm (±10%) rainfall per tip with a 20.3 cm diameter catch. The Wisconsin gages have a 45.7 cm diameter catch. Gages UWI and UWII measure 0.20 mm and 0.23 mm rainfall per tip (±15%), respectively. The times of bucket tip events were recorded on strip chart recorders.

The constants \(G\) and \(\gamma\) of (2) were found by linear regression of lidar measurements of \(\beta_r\) by means of (9).
TABLE 2. Parameters of the University of Wisconsin lidar system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitter</td>
<td>Operating wavelength: 694.30±0.05 nm (Ruby)</td>
</tr>
<tr>
<td></td>
<td>Output energy per pulse: 1 J (real-time monitored)</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>&lt;3 min^-1</td>
</tr>
<tr>
<td>Pulse duration</td>
<td>20 nsec (Pockels Cell Q Switch)</td>
</tr>
<tr>
<td>Beam divergence</td>
<td>3 mrad, 5.1 cm diameter telescope</td>
</tr>
<tr>
<td>Temperature regulation</td>
<td>water cooled, 17±5°C</td>
</tr>
<tr>
<td>Receiver</td>
<td>Telescope: 20.3 cm diameter Newtonian (astrometrical quality)</td>
</tr>
<tr>
<td></td>
<td>Spectral resolution: 20 Å (interference filter)</td>
</tr>
<tr>
<td></td>
<td>Detector: RCA C7042K photomultiplier with extended response multi-alkali cathode</td>
</tr>
<tr>
<td></td>
<td>Field of view: 8 mrad, full angle</td>
</tr>
<tr>
<td>Data logging system</td>
<td>Amplifiers: a) linear, gain stepping, 3 stages</td>
</tr>
<tr>
<td></td>
<td>b) log: 80-db range</td>
</tr>
<tr>
<td>Storage</td>
<td>1024 8-bit words</td>
</tr>
<tr>
<td>Sampling rate</td>
<td>5-10 words sec^-1</td>
</tr>
<tr>
<td>Multiplexing</td>
<td>4 input, 70 nsec settling time manually programmable automatic sequence</td>
</tr>
<tr>
<td>Output</td>
<td>Real-time scope, and paper tape</td>
</tr>
</tbody>
</table>

and gage rainfall rates on a log-log scale. Tipping bucket rain gages provide an average rainfall rate in the time interval between bucket tips, whereas lidar provides instantaneous estimates of $\beta_r$ as a function of range. Thus, time averages of $\beta_r$ over a bucket tip interval are needed in the comparison of $\beta_r$ and $R$.

Lidar returns were excluded from this analysis when (i) the fractional variance of $\tilde{\alpha} \ln(P^2)/ds$ was greater than 0.09, and when (ii) spatial inhomogeneity of rainfall or aerosol extinction coefficients was grossly apparent in the sampled range interval. Further, $\beta_r$ and $R$ data were not used in the linear regression when the fractional variance of $\beta_r$ in the tip time interval was greater than 0.09. This error analysis is presented in greater detail by Shipley (1973).

6. Experimental results

The University of Wisconsin monostatic lidar was operated during precipitation in the period from August 1972 to May 1973. Empirical $\beta_r-R$ relations were found by correlating lidar measurements of $\beta_r$ by means of (9) and gage rainfall rates. The most reliable data were taken in stratus rainfall on 20 October 1972 and in thunderstorm rainfall on 20 April 1973. The $\beta_r-R$ relations found for each of these events were not significantly different. Linear regression over the combined 46 values of ln$\beta_r$ and ln$R$ gives

$$\beta_r = 0.16 \pm 0.04 R^{0.74 \pm 0.12},$$  (14)

for $\beta_r$ in km^-1 and $R$ in mm hr^-1. Log $\beta_r$ and log$R$ have a linear correlation coefficient of 0.97. The data and (14) are plotted in Fig. 2.

Lidar measurements can be used to obtain estimates of the rainfall rate over rain gages by means of (14). Lidar and gage rainfall rates are compared for two thunderstorm cell passes of 20 April in Fig. 3. The integration of these rainfall rates over time gives the following rainfall totals:

Shower I  | Shower II
--- | ---
Lidar | 3.84 mm | 4.26 mm
Gage B1 | 3.68 mm | 2.92 mm

The large difference in rainfall totals of Shower II points out the importance of temporal aerosol extinction variability in the lidar determination of the rainfall rate. Fractocumulus was frequently observed during lidar operations in rainfall. The lidar method cannot differentiate between aerosol and rainfall extinction, and it may overestimate the total rainfall when an unexpected aerosol extinction component is present.

A sequence of rainfall-rate range profiles obtained from the leading edge of a thunderstorm cell on 20 April is given in Fig. 4, where $\beta_r$ has been transformed into $R$ by means of (14). The successive approximations technique was used to obtain $\beta_r$ profiles for $\beta_r=0.14$ km^-1, $\epsilon_0=1.0$, $\epsilon_1=0.5$, $\beta_r=0.40$ and $\beta_r=0.18$. The rainfall rates as measured by gages B1 and B2 at the nearest bucket tip intervals are included in Fig. 4. The differences in lidar and gage rainfall rates are not significant with respect to experimental error. However, both measurements do show the same qualitative temporal variation. These rainfall-rate range profiles were obtained using an 8-in diameter receiver mirror. The Wisconsin lidar has an operating range of about 5 km in typical showers ($R=10$ mm hr^-1), and about 1 km in heavy showers ($R=100$ mm hr^-1). A larger receiver area would provide greater operating ranges.

The profiles of Fig. 4 show the capability of lidar to determine the spatial distribution of rainfall rate with high resolution. The lidar method provides rainfall information on a spatial scale which could be used in the determination of $Z-R$ relations for weather radar calibration.
7. Conclusions

The lidar-derived extinction coefficient of rainfall was correlated with simultaneous gage measurements of the rainfall rate. Data taken at Madison in stratus and thunderstorm rainfall yield the relation

$$\beta_r = 0.16 \pm 0.04 R^{0.74 \pm 0.12},$$  \hspace{1cm} (14)

for $\beta_r$ in km$^{-1}$ and $R$ in mm hr$^{-1}$. As shown in Table 1, this $\beta_r$-$R$ relations is consistent with the results obtained from the data of other authors.

The extinction coefficient of an atmosphere is readily found from a lidar return when that extinction coefficient is spatially homogeneous. The total atmospheric extinction coefficient is then found from an average slope of the lidar return power $P$, such that

$$\beta_{\text{atmos}} = \frac{1}{2 \, ds} \ln(Ps^2),$$  \hspace{1cm} (9)

where $s$ is range; $\beta_{\text{atmos}}$ is often constant over a range on the order of kilometers in rainfall.

A numerical solution was developed to derive $\beta_r$ range profiles from digitized lidar returns from spatially inhomogeneous rainfall. A comparison of the $\beta_r$ profiles found by means of this solution and the results of the average slope method yields the optimal value $\sigma_r(\pi) = 0.18 \pm 0.05$ for the backscattering phase function of raindrops. This measurement is consistent with the computed values of $\sigma_r(\pi)$ derived from Mie theory.

A sequence of rainfall-rate range profls obtained from the leading edge of a thunderstorm cell is given in Fig. 4. These profiles illustrate the capability of lidar to determine the spatial distribution of rainfall rate with

Fig. 2. Empirical $\beta_r$-$R$ relation determined by lidar and rain gages. Data from stratus (triangles, 20 October 1972) and thunderstorm (circles, 20 April 1973) rainfall. Error bars show typical experimental errors.

Fig. 3. Rainfall rates determined by lidar and rain gage B1 for two thunderstorm cell passes on 20 April 1973. The lidar-derived $R$ (circles) was found by means of (14). Rain gage measurements are given by solid lines.
high resolution. Lidar can potentially determine the distribution of rainfall rate throughout an atmospheric volume under cloud bases. This is a capability which cannot be provided by rain gages. Lidar can therefore provide additional information which would aid in the determination of \( Z-R \) relations for weather radar calibration.

The \( \beta_r \) solutions presented in this paper assume that the extinction component not due to rainfall is spatially homogeneous. It was visually noted, however, that fractocumulus often drifts through the sampled atmospheric volume. The present lidar rainfall technique cannot determine whether variations in \( \beta_{atmos} \) are due to variations in rainfall or aerosol extinction. The \( \beta_r \) solutions also assume that the backscattering phase functions for rainfall and aerosol are constants. The validity of these assumptions is left for further study.

The Wisconsin lidar has a useful operating range of about 5 km in a typical shower (\( R \approx 10 \text{ mm hr}^{-1} \)), and about 1 km in heavy showers (\( R \approx 100 \text{ mm hr}^{-1} \)). The rainfall rate profiles of Fig. 4 were obtained using an 8-inch diameter receiver mirror. These operating ranges can be readily extended by employing a larger receiver.

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