On the Nature of the Nonexistence of Ordinal Relationships between Measures of the Accuracy and Value of Probability Forecasts: An Example

ALLAN H. MURPHY

National Center for Atmospheric Research, Boulder, Colo. 80303

JACK C. THOMPSON

Department of Meteorology, San Jose State University, San Jose, Calif. 95192

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ABSTRACT

It has been shown previously that ordinal relationships between measures of the accuracy and value of probability forecasts do not exist, in general, in N-state (N > 2) situations. Some implications of this result are illustrated by comparing the accuracy and value of such forecasts in a realistic decision-making situation—a three-action, three-state situation involving the protection of a fruit orchard against frosts and freezes. Geometrical interpretations of the forecasts and measures are described and then used to investigate the existence of ordinal relationships in this so-called fruit-frost situation. The results indicate, as expected, that an increase in forecast accuracy can lead to a decrease in forecast value. Some generalizations and speculations related to the existence and nonexistence of such ordinal relationships are presented.

1. Introduction

Recently, Murphy (1972, 1974, 1975) has investigated the existence of ordinal relationships between measures of the accuracy and measures of the (ex post) value of probability forecasts for a general class of decision-making situations. In these studies, ordinal relationships between the measures of concern were said to exist if, for two forecast systems or forecasters A and B, the fact that A's forecast(s) is (are) more accurate than B's forecast(s) implies that the value of A's forecast(s) is (are) greater than or equal to the value of B's forecast(s). It was demonstrated that such relationships exist for individual pairs of forecasts in all two-state situations. However, for sets of forecasts in two-state situations and for individual pairs or sets of forecasts in N-state (N > 2) situations, ordinal relationships were found to exist only under very restrictive conditions on the forecasts themselves. This latter result implies that, when these conditions are not met, more accurate forecasts can lead to consequences which are (ex post) less valuable to decision makers.

The purpose of this paper is to illustrate the fact that increases in accuracy can be associated with decreases in value, by comparing the accuracy and value of hypothetical probability forecasts in a realistic decision-making situation. The situation of concern, a three-action, three-state situation involving the protection of a fruit orchard against frosts and freezes, is described in Section 2. In Section 3, specific measures of accuracy and value are defined, a framework which provides geometrical interpretations of these measures is described, and the existence of ordinal relationships between the measures of concern is investigated in this so-called fruit-frost situation using these geometrical interpretations. Section 4 consists of a brief summary and conclusion, including a discussion of some implications of these results.

2. A fruit-frost decision-making situation

It is well known that damage can occur to fruit orchards during periods of below-freezing temperatures. The impact of such temperature events depends upon many factors, including the nature and severity of the events themselves, and, in certain situations, orchardists take different protective actions to reduce or eliminate damage under different temperature conditions (e.g., Rogers and Swift, 1970). In the citrus-growing regions of southern California, the most frequent adverse event of this type is the so-called “frost” event which is associated with clear skies, little or no wind, and nighttime radiational cooling of the air near the surface. Under these conditions, a marked temperature inversion forms in the lower atmosphere, with warm air (generally well above the freezing point of the fruit crop) present only a short distance above the surface. In such situations, a wind machine—a power-driven fan, usually mounted on a tower or pole 30-40 ft above the ground, which can mix the relatively warm air above the
Table 1. The expense matrix $E = (e_{mn}) \ (m,n = 1,2,3)$ and the expected expenses $E_m \ (m = 1,2,3)$ in the fruit-frost situation described in Section 2. The expenses are expressed in relative terms on a scale from zero to one.

<table>
<thead>
<tr>
<th>Expense matrix $E$</th>
<th>States</th>
<th>$s_1$ (no danger)</th>
<th>$s_2$ (frost)</th>
<th>$s_3$ (freeze)</th>
<th>Expected expense ($E$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$ (no protection)</td>
<td>0.00</td>
<td>0.50</td>
<td>0.80</td>
<td>$E_1 = 0.50 r_1 + 0.80 r_2$</td>
<td></td>
</tr>
<tr>
<td>Actions $a_2$ (wind machines)</td>
<td>0.10</td>
<td>0.20</td>
<td>0.90</td>
<td>$E_2 = 0.10 r_1 + 0.20 r_2 + 0.90 r_3$</td>
<td></td>
</tr>
<tr>
<td>$a_3$ (orchard heaters)</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>$E_3 = 0.20 r_1 + 0.30 r_2 + 0.40 r_3$</td>
<td></td>
</tr>
</tbody>
</table>

The inversion with the cold air at the surface—will effectively reduce or eliminate frost damage.

Less frequent, but potentially more damaging, is the so-called “freeze” event, in which cold polar or arctic air invades the citrus-growing region. Under these conditions, temperature inversions generally do not exist and the only recourse available to the orchardist is to attempt to warm the surface air with orchard heaters—generally metal containers in which fossil fuels are burned to produce heat. In some situations, it may be desirable to use both wind machines and orchard heaters. For example, wind machines may be used in conjunction with orchard heaters to mix the heated air more efficiently throughout the orchard. On the other hand, the operation of wind machines alone under freeze conditions can actually increase crop damage, since the air aloft is generally at least as cold as the surface air and the mixing produced by wind machines can cause the fruit to freeze more rapidly than it would have otherwise. Orchard heaters may also be used under frost conditions, but the associated fuel and labor costs are generally greater than the costs of operating wind machines.

In the realistic, but somewhat idealized, fruit-frost situation to be considered in this paper, we shall assume that the orchardist or decision maker has available three different courses of action: $a_1$, to take no protection (no protection); $a_2$, to use wind machines alone (wind machines); and $a_3$, to use orchard heaters alone (orchard heaters). We also assume that only three distinct weather events or states can occur: $s_1$, temperature above critical value (no danger); $s_2$, frost; and $s_3$, freeze. In the terminology of decision analysis, the situation of concern is a three-action ($M = 3$), three-state ($N = 3$) decision-making situation, and this situation is described in Table 1.

Each action-state pair $\{a_m, s_n\} \ (m,n = 1,2,3)$ leads to a different consequence. We shall assume that the values of the consequences to the orchardist are expressed in terms of monetary expenses and that these expenses include both the relevant costs of protection and any losses due to crop damage (hereafter, the term value will be taken to mean negative expense). With the cooperation of a California citrus grower, we have been able to obtain an estimate of the expense $e_{mn}$ associated with each action-state pair $\{a_m, s_n\}$ in this situation, and the grower’s expenses are presented in the form of an expense matrix $E = (e_{mn})$ in Table 1 ($m,n = 1,2,3$). These expenses are expressed in relative terms on a scale from zero to one, where zero represents no protective costs or losses due to crop damage and one represents complete loss of the crop and severe damage to the trees themselves. Note that the minimum expense (i.e., zero) is associated with action $a_1$ (no protection) when state $s_1$ (no danger) occurs, while the maximum expense (i.e., 0.90) is associated with action $a_3$ (wind machines) when state $s_3$ (freeze) occurs. We believe that these expense figures are reasonable estimates of the orchardist’s actual (relative) expenses in the decision-making situation of concern. In this context, the weather forecast is a probability vector

$$r = (r_1, r_2, r_3), \ r_n \geq 0, \ \sum_n r_n = 1,$$

in which $r_n$ represents the probability of occurrence of state $s_n \ (n = 1,2,3)$.

Finally, we shall assume that the orchardist wants to select the action which minimizes his expected expense, where the expected expense of an action is the weighted average of the expenses associated with that action and where the weights are the relevant probabilities. Thus, if we denote the expected expense associated with action $a_m$ by $E_m$, then

$$E_m = \sum_n r_n e_{mn} \ (m, n = 1, 2, 3).$$

According to this decision rule, the orchardist will select action $a_m$ if $E_m < E_{m'}$ for all $m' \neq m$ and he will be indifferent between actions $a_m$ and $a_{m'}$ if $E_m = E_{m'} \ (m,m' = 1,2,3; m \neq m')$. The expected expenses in the fruit-frost situation are presented in Table 1.

3. Ordinal relationships between measures of accuracy and value in the fruit-frost situation

a. Definitions of measures

As a measure of accuracy, we shall use the probability score (Brier, 1950), a measure frequently used to
evaluate probability forecasts in meteorology. If we denote this measure by \( PS(r,d) \), then for an individual forecast

\[
\mathbf{r} = (r_1, \ldots, r_N), \quad r_n \geq 0, \quad \sum_n r_n = 1; \quad n = 1, \ldots, N,
\]

\[
PS(r,d) = \sum_{n=1}^{N} (r_n - d_n)^2,
\]

where \( d = (d_1, \ldots, d_N) \) is the relevant observation, in which \( d_n = 1 \) if state \( s_n \) occurs and \( d_n = 0 \) otherwise. The probability score is defined such that a smaller score is better. Specifically, \( PS(r,d) = 0 \) for a correct, categorical (i.e., perfect) forecast and \( PS(r,d) = 2 \) for an incorrect, categorical forecast. For a set of forecasts, \( PS(r,d) \) is simply the mean square error of the forecasts.

In accordance with the discussion in Section 2, the general measure of value in this paper is an expense measure based upon an \( M \)-action, \( N \)-state expense matrix \( \mathbf{E} = (e_{mn}) \) (\( m = 1, \ldots, M; \quad n = 1, \ldots, N \)). If we denote this measure by \( EM(E,r,d) \), then

\[
EM(E,r,d) = \sum_{m=1}^{M} \delta_m \left( \sum_{n=1}^{N} d_n e_{mn} \right),
\]

in which \( \delta_m = 1 \) if the decision maker selects action \( a_m \) and \( \delta_m = 0 \) otherwise. If the decision maker chooses action \( a_i \) and state \( s_j \) occurs, then the value of \( EM(E,r,d) \) in (2) is simply \( e_{ij} \) (\( i = 1, \ldots, M; \quad j = 1, \ldots, N \)). Thus, for an individual forecast, the values assumed by the measure \( EM(E,r,d) \) are a discrete set of values corresponding to the values of the elements of the expense matrix \( \mathbf{E} \). The expense measure is also defined such that a smaller score is better.

b. Geometrical interpretations of measures

A one-to-one correspondence exists between a probability forecast \( \mathbf{r} = (r_1, \ldots, r_N) \) and a point in an \((N-1)\) dimensional simplex \( R^N \), where

\[
R^N = \{ (r_1, \ldots, r_N) \mid r_n \geq 0, \quad \sum_n r_n = 1; \quad n = 1, \ldots, N \}
\]

(e.g., see Murphy, 1972). This simplex is a unit line segment \( R^2 \) when \( N = 2 \), an equilateral triangle \( R^3 \) when \( N = 3 \), and a regular tetrahedron \( R^4 \) when \( N = 4 \). Since the fruit-frost situation is a three-state \((N=3)\) situation, the appropriate geometrical framework in this situation is the equilateral triangle \( R^3 \); such a triangle is presented in Fig. 1. The vertices of the triangle represent the three observations \( \mathbf{d}_j \) (\( j = 1,2,3 \)), where \( \mathbf{d}_1 = (1,0,0) \), \( \mathbf{d}_2 = (0,1,0) \) and \( \mathbf{d}_3 = (0,0,1) \). A probability forecast \( \mathbf{r} = (0.25, 0.45, 0.30) \) is depicted within \( R^3 \) in Fig. 1. Note that the component \( r_n \) of the forecast \( \mathbf{r} \) is the Euclidean distance between \( \mathbf{r} \) and the side of the triangle opposite the vertex for which \( d_n = 1 \) (\( n = 1,2,3 \)). Thus, the vertices of \( R^3 \) also represent the three categorical forecasts \( r_j (j = 1,2,3) \), where \( r_1 = (1,0,0) \), \( r_2 = (0,1,0) \) and \( r_3 = (0,0,1) \).

If we denote the Euclidean distance between two points \( \mathbf{r} \) and \( \mathbf{r}' \) in the simplex \( R^N \) by \( D(r,r') \), then

\[
D(r,r') = \left[ \sum_{n=1}^{N} (r_n - r'_n)^2 \right]^{1/2}.
\]
A comparison of (1) and (4) indicates that the probability score $PS(r,d)$ can be given a geometrical interpretation within this framework. Specifically,

$$PS(r,d) = D^2(r,d).$$  \hspace{1cm} (5)

That is, the probability score is the square of the Euclidean distance between the forecast $r$ and the observation (or vertex) $d$. Now if state $s_j$ occurs, then $d_j = 1$ and $PS(r,d)$ in (1) becomes $PS_j(r)$, where

$$PS_j(r) = 1 - 2r_j + \sum_{n=1}^{N} r_n^2. \hspace{1cm} (6)$$

In geometrical terms the set of forecasts for which $PS_j(r)$ in (6) is a constant is (a portion of) an $(N-1)$ dimensional hypersphere centered at the point $d_j$ ($j = 1, \ldots, N$). Thus, in three-state situations, the isograms of $PS_j(r)$ are arcs of circles centered at the vertex $d_j$ ($j = 1, 2, 3$). The arc for $PS_2(r) = 0.72$ is depicted in Fig. 1.

The expense measure $EM(E,r,d)$ can also be given a geometrical interpretation. In this regard, the simplex (or set) $R^N_m$ consists of $M$ convex polyhedra (or subsets) $R^N_m$, where

$$R^N_m = \{ (r_1, \ldots, r_N) | r_n \geq 0, \sum_n r_n = 1, \sum_n r_n c_m < \sum_n r_n c_m', \text{ for all } m' \neq m; m, m' = 1, \ldots, M; n = 1, \ldots, N \}. \hspace{1cm} (7)$$

Subset $R^N_m$ represents the set of all forecasts for which the decision maker prefers action $a_m$ to the other $M - 1$ actions. These polyhedra are separated by $(N-2)$ dimensional hyperplanes $R^N_{m,m'}$, where

$$R^N_{m,m'} = \{ (r_1, \ldots, r_N) | r_n \geq 0, \sum_n r_n = 1, \sum_n r_n c_m = \sum_n r_n c_m', m, m' = 1, \ldots, M; m \neq m', n = 1, \ldots, N \}. \hspace{1cm} (8)$$

The hyperplane $R^N_{m,m'}$ consists of the set of all forecasts for which the decision maker is indifferent between actions $a_m$ and $a_{m'}$. Now, the measure $EM(E,r,d)$ is also defined on the set $R^N$ (and its various subsets) and represents, in general, an $(N-1)$ dimensional analogue of a step function in one dimension. Specifically, $EM(E,r,d)$ assumes a constant value for all forecasts in a given subset $R^N_m$ (this constant value depends upon the state that occurs; see below), and this measure takes a "step" up or down to a different value on the hyperplanes $R^N_{m,m'}$ separating that subset from other subsets ($m, m' = 1, \ldots, M; m \neq m'$).

In three-state situations, in which the simplex $R^3$ is the equilateral triangle $R^3$, the subsets $R^3_m$ are convex polygons and the hyperplanes $R^3_{m,m'}$ are lines. The measure $EM(E,r,d)$ is, then, a "plateau function" (i.e., the two-dimensional analogue of a step function). The expense measure $EM(E,r,d)$ which corresponds to
Table 2. Three hypothetical pairs of probability forecasts prepared by two forecast systems (or forecasters) A and B, the relevant observations, and the values of the probability score $PS(r,d)$ and the expense measure $EM(E,r,d)$. The values of $EM(E,r,d)$ relate to the fruit-frost situation described in Table 1.

<table>
<thead>
<tr>
<th>Forecast pair</th>
<th>Forecasts $r = (r_1, r_2, s_2)$</th>
<th>Observation $d = (d_1, d_2, d_3)$</th>
<th>Brier score $PS(r,d)$</th>
<th>Expense measure $EM(E,r,d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(0.10, 0.80, 0.10)$</td>
<td>$(0.45, 0.50, 0.05)$</td>
<td>$(0.10, 0.00, 0.30)$</td>
<td>$(0.060, 0.455, 0.20, 0.20)$</td>
</tr>
<tr>
<td>2</td>
<td>$(0.10, 0.70, 0.20)$</td>
<td>$(0.60, 0.30, 0.10)$</td>
<td>$(0.10, 0.10)$</td>
<td>$(0.140, 0.860, 0.30, 0.30)$</td>
</tr>
<tr>
<td>3</td>
<td>$(0.35, 0.50, 0.15)$</td>
<td>$(0.70, 0.00, 0.30)$</td>
<td>$(0.10, 0.10)$</td>
<td>$(0.395, 1.580, 0.20, 0.50)$</td>
</tr>
</tbody>
</table>

The expense matrix $E$ in the fruit-frost situation described in Table 1 is depicted in Fig. 2. The triangle $R^3$ is divided into three convex regions $R^3_m$ corresponding to the three actions $a_m$ ($m = 1, 2, 3$). The region $R^3_m$ represents the set of all forecasts for which the orchardist of concern prefers action $a_m$ to the other two actions ($m = 1, 2, 3$). The lines $R^3_m$, represent sets of forecasts for which the orchardist is indifferent between actions $a_m$ and $a_{m'}$ ($m, m' = 1, 2, 3; m \neq m'$). The line $R^3_{12}$, for example, consists of the set of all points for which $E_1 = E_2$ and for which both $E_1$ and $E_2$ are greater than $E_3$. The point at which the lines $R^3_{12}$, $R^3_{13}$ and $R^3_{23}$ intersect is the set of probabilities for which $E_1 = E_2 = E_3$ (viz., $r_1 = 7/12$, $r_2 = 3/12$ and $r_3 = 2/12$). The measure $EM(E,r,d)$ "covers" each region with a plateau, the height of which varies from region to region and also depends upon the state that occurs. For example, if state $s_2$ occurs, then $EM_2(E,r)$—the height of the plateau—is 0.50 in $R^3_1$, 0.20 in $R^3_2$ and 0.30 in $R^3_3$. The values of $EM_j(E,r)$ for $j = 1, 2, 3$ are indicated in Fig. 2 in square brackets for each of the three regions.

### c. Ordinal relationships in the fruit-frost situation

Hypothetical probability forecasts prepared by two forecast systems or forecasters, A and B, on each of three different occasions are presented in Table 2. These three pairs of forecasts, $r^A_k$ and $r^B_k$ ($k = 1, 2, 3$), are depicted within the geometrical framework of the fruit-frost situation in Fig. 3. We shall assume, without any loss of generality, that state $s_2$ (i.e., frost) occurs on all three occasions. Thus, $d_k = (0,1,0)$ for $k = 1, 2, 3$ (see Table 2).

Consider the first pair of forecasts, $r^A_1$ and $r^B_1$. Since $r^A_1$ is closer than $r^B_1$ to the vertex (or observation) $d_1$, A's forecast is more accurate than B's forecast. Specifically, $PS^A_1 = 0.060$ and $PS^B_1 = 0.455$ (see Table 2). However, since $r^A_1$ and $r^B_1$ both fall in region $R^3_2$, the expenses associated with these two forecasts are equal. Specifically, $EM^A_1 = EM^B_1 = 0.20$ (see Table 2). In fact, it is true in general that if two forecasts fall in the same region, then the expenses (or values) associated with these forecasts are equal regardless of which event (or

![Fig. 3. The three hypothetical pairs of probability forecasts, $r^A_k$ and $r^B_k$ ($k = 1, 2, 3$), presented in Table 2, depicted within the framework of the fruit-frost situation. The dashed line represents a directed line segment from the vertex $d_k$ (see text).](image-url)
state) occurs. In any case, since the expense associated with A’s forecast is equal to the expense associated with B’s forecast on this occasion, the relationship between the accuracy and expense (or value) of this pair of forecasts does not violate the conditions for the existence of an ordinal relationship between the measures of concern.

Now consider the second pair of forecasts, \( r_1 \) and \( r_2 \). Once again, \( r_1 \) is closer than \( r_2 \) to the observation \( d = (0,1,0) \), which indicates that \( PS_1 \) is less than \( PS_2 \) (see Table 2). In this case, however, note that \( r_1 \) falls in region \( R_1 \), while \( r_2 \) falls in region \( R_2 \). As a result, \( EM_1 \) (= 0.30) is greater than \( EM_2 \) (= 0.20). Thus, even though A’s forecast is more accurate than B’s forecast, the expense associated with A’s forecast is greater than the expense associated with B’s forecast. Obviously, this pair of forecasts violates the conditions for the existence of an ordinal relationship between the measures of accuracy and value of concern in this situation.

The fact that it is possible to find one pair of forecasts which violates the conditions for the existence of an ordinal relationship between these measures is, of course, sufficient to demonstrate the nonexistence of such relationships in the decision-making situation of concern. However, it is clear from an examination of Fig. 3 that many other hypothetical pairs of forecasts would also violate these conditions in the fruit-frost situation. Moreover, we are now in a position to generalize this result to other decision-making situations. In this regard, the fact that points of equal accuracy, according to the probability score, lie on arcs of circles centered at the relevant vertex while points of equal expense, according to the expense measure, constitute the interior of a convex polygon, indicates that pairs of forecasts can be found for all three-state situations which violate the conditions for the existence of such ordinal relationships. Since the circles and polygons in three-state situations become hyperspheres and polyhedra in N-state \( (N > 3) \) situations, it is clear that this result also generalizes to these latter situations.

As Murphy (1974, 1975) has demonstrated, ordinal relationships for individual forecasts do exist in N-state \( (N > 2) \) situations when certain very restrictive conditions are placed upon the forecasts. In particular, the conditions require that we consider only comparable forecasts. In three-state situations, two forecasts are said to be comparable if they lie on the same directed line segment through the vertex of the triangle which corresponds to the event that occurs. If the forecasts of concern are comparable, then the expense (value) associated with the more accurate forecast must be less (greater) than or equal to the expense (value) associated with the less accurate forecast. That is, ordinal relationships do exist for individual, comparable forecasts in N-state \( (N > 2) \) situations. The third pair of forecasts presented in Table 2, \( r_3 \) and \( r_6 \), is comparable; i.e., the points lie on the same directed line segment through the vertex \( d_4 \) (see Fig. 3). In this case, A’s forecast is more accurate and more valuable (i.e., involves less expense) than B’s forecast. This result can be generalized by stating that the probability score \( PS(r,d) \) is an increasing function of distance from the relevant vertex, while the expense measure \( EM(E,r,d) \) is a nondecreasing function of directed distance—i.e., distance along directed line segments—from the vertex of concern. Obviously, however, this result has very limited applicability, since two forecasts will seldom if ever be comparable in the sense of lying on the same directed line segment.

Ordinal relationships do not exist, in general, on the statistics (e.g., the averages) of sets of forecasts in two-state \( (N = 2) \) or N-state \( (N > 2) \) situations unless equally restrictive conditions are placed upon the relationships between the accuracies of the respective forecasts (see Murphy, 1974, 1975). In any case, since we have demonstrated that ordinal relationships do not exist for individual pairs of forecasts in N-state \( (N > 2) \) situations, we need not investigate the existence of these relationships for sets of forecasts in such situations.

4. Summary and conclusion

Here we have investigated some implications of the result, obtained in previous studies, that ordinal relationships between measures of the accuracy and (ex post) value of probability forecasts do not exist, in general, in N-state \( (N > 2) \) decision-making situations. Specifically, this result indicates that the fact that A’s forecast is (are) more accurate than B’s forecast(s) in such situations does not guarantee that A’s forecast is (are) more valuable than B’s forecast(s) to the decision makers of concern. We illustrated this fact by comparing the accuracy and value of pairs of hypothetical probability forecasts within the framework of a realistic decision-making situation—a three-action, three-state situation involving the protection of a particular fruit orchard against frosts and freezes. It was found that, as expected, an increase in forecast accuracy can be associated with a decrease in forecast value in such a situation.

The previous results and the example considered in this paper indicate that, in the absence of specific information concerning the expense (or value) measures of the decision makers of concern, we as meteorologists cannot be certain that an increase in forecast accuracy will be accompanied by an increase in forecast value. This somewhat surprising state of affairs is due primarily to the fact that different evaluation measures are used to determine the accuracy and value of forecasts. Specifically, measures of accuracy are generally scalar measures, while measures of value are almost always vector measures. In view of the fact that these two attributes of the forecasts relate to different types
of evaluation (see Murphy, 1974), it is unlikely that a single, generally acceptable measure of accuracy and value can be formulated. Notwithstanding this apparently insurmountable difficulty, it is not unreasonable to conjecture that, over a sufficiently large set of forecasts, more accurate forecasts will also be more valuable forecasts for most if not all decision makers (see Murphy, 1974). In any case, the problem raised by the nonexistence of ordinal relationships of the type considered in this paper emphasizes the importance of obtaining more information concerning the value (or expense) measures of relevant decision makers. This information will be needed to demonstrate convincingly that increases in forecast accuracy are indeed accompanied by increases in forecast value.

REFERENCES


