A Statistical Model for the Mid-Latitude Tropopause and Jet Stream Layer

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ABSTRACT

Tropopause and level of maximum wind charts are derived from objective analyses of the 500-, 300-, 150-
and 100-mb height and wind fields. Regression equations, based on 24-hr changes, are developed to permit
reconstruction of the layer given the history and current 500-mb data. This new "derived" layer is examined
statistically with respect to its adequacy as a three-dimensional first-guess field for the current objective
analyses for this layer, and also as a method for forecasting the layer from a barotropic 500-mb
prognostic chart.

1. Introduction

Numerical analyses and prognoses for mid-tropospheric levels have become accepted parts of meteorological practice. At higher levels, mainly because of the wind reversal associated with sloping tropopause and maximum wind surfaces, progress has been less notable. In the initial approach to this latter problem a direct method was adopted in order to give some insight into the layer structure as well as an interim solution for operational requirements.

This may most readily be achieved by combining regression techniques with objective analysis procedures. This phase of the experiment may be conveniently phrased as follows:

On the basis of objective analyses for

a) the 500-, 300-, 150-, and 100-mb levels for time, \( t_{24} \) hr,
b) the 500-mb level at time, \( t_0 \),
produce

a) derived tropopauses for times, \( t_{24} \) and \( t_0 \) hr;
b) derived levels of maximum wind, LMW, for times \( t_{24} \) and \( t_0 \) hr;
c) first guess fields for the objective analysis routine at the 300-, 150- and 100-mb levels at time, \( t_0 \) hr.

2. The regression technique

Grid-point data. Since only the 500-mb height field was given at time \( t=0 \) hr, all changes in the structure of the layer were correlated with the 500-mb height change. The data have been used in two ways to produce the necessary regression equations. In the first attempt, raw station data which had been stored on punch cards were used to obtain the relationships. The results were surprisingly poor. The second method took the raw data and transformed both heights and winds into grid-point heights making use of a standard objective analysis procedure. These grid-point values were then used in place of the raw data to produce the necessary regression equations. The results were very much better.

An examination of the two methods of assessing data is of some interest. In the first method one is concerned with reports which can be inconsistent internally because of instrumental or observational error as well as being inconsistent with respect to surrounding reports. In addition, the raw data contain scales of perturbations which are outside (smaller than) the scope of the problem being studied (Reiter, 1958). The random portion of these undesirable features is markedly reduced when the reports from several "stations" are combined to make a "grid-point" value. Thus the "grid-point" data are considered to be much more useful for statistical examination in the scale under consideration.

Derived tropopauses. Synoptic experience indicates that an approximate tropopause may be defined simply and objectively. The point of intersection between an isothermal lapse rate fitting the 100-mb to 150-mb thickness and a pseudo-adiabatic lapse rate fitting the 500-mb to 300-mb thickness was chosen for the purpose. This "derived" tropopause is highly correlated to the observed (WMO definition) tropopause. This scheme for deriving the tropopause has two features vital to computer procedures:

1) The derived tropopause exists at each of the grid-points on the chart and
2) The derived tropopause chart is continuous, because the parameters going into its makeup are themselves horizontally continuous. (A result of the statistical objective analysis procedure.)

The equation for the derived tropopause was determined in the form:

\[ Z_D = A_L T_L + A_W T_W + K, \]  
(1)
where $Z_D =$ height of the derived tropopause, $T_L = Z_{100} - Z_{500}$, $T_U = Z_{100} - Z_{100}$.

While the resultant tropopause ($Z_D$) chart (for values falling below 400 mb and above 125 mb) is not entirely realistic, it is conservative. It is possible to make the coefficients $A_L$ and $A_T$ functions of $Z_D'$, where $Z_D'$ would be found to a first order of approximation by equation (1). Such a modification of $A_L$ and $A_T$ would require considerable data study in polar areas in the winter (for $A_L$) and low latitudes in summer (for $A_T$). Although this could provide a valuable second order approximation to (1), the possibility has not been explored in this report.

It was found that a more conservative $Z_D$ map could be produced if $T_L$ and $T_U$ were smoothed using a regular five-point smoother, before applying them in (1).

After some investigation (1) took the following final form:

$$Z_D = 9.6T_L - 12T_U + 23827.2.$$  

(2)

The constants, $A_T$ and $A_L$, were determined graphically from the foregoing lapse rate intersection with reference to standard atmosphere heights. The height constant was reduced by 2300 ft to the value 23,872 ft on a statistical basis to be discussed later.

A set of charts was obtained using the derived tropopause criterion (Eq 2) and compared to one obtained from the standard WMO criterion. A comparison of typical charts is shown by Figs. 1 and 2. The following features were revealed by the comparison:

1) All the main synoptic features which show on the WMO chart are portrayed.
2) The main steep gradients show a close correspondence to the actual ones.
3) The chart is a smooth one. (Noise has been eliminated.)

**Derived 300, 150 and 100-mb charts.** Since the tropopause and 24-hr height change at 500 mb have been derived, the remainder of the layer may be defined in terms of regression equations. The equations require:

$$\frac{\partial Z_D}{\partial t}, \frac{\partial Z_{100}}{\partial t}, \frac{\partial T_L}{\partial t}, \frac{\partial T_U}{\partial t}$$

as functions of $Z_D$ and $\partial Z_{100}/\partial t$.

The equations for all four dependent variables are similar and take the following form:

$$\frac{\partial Z_D}{\partial t} = a' \frac{\partial Z_{500}}{\partial t} + C_1$$  

(3)

where $a'$ is found to be a function of $Z_D$, hence,

$$\frac{\partial Z_D}{\partial t} = f(Z_D) \frac{\partial Z_{500}}{\partial t} - C_1.$$  

(4)

As a first attempt to find $f(Z_D)$ a multiple regression screening technique (Efroymsen, 1959) was used where $\partial Z_{500}/\partial t$ and $Z_D \partial Z_{500}/\partial t$ were the independent variables and the remaining four the dependent ones. Subsequently the data were sorted with respect to $Z_D$ and plotted on a graph. In this way the curves which best represented a combination of statistical and synoptic experience were produced.

Twenty-four hour changes in $Z_{100}$, $T_L$, $T_U$, $Z_{100}$, and $Z_D$ were then examined with respect to their intercorrelations and standard deviations. These changes were sorted using three ranges of $Z_D$. Two sets of grid-point data were used: one including most of the northern hemisphere and the other confined to the good coverage of data for North America. Very little difference was indicated for the regression coefficients, but the correlation coefficients given by the North American data were generally somewhat higher. The results for the smaller area are given in Tables 1 and 2.

**Table 1. Correlation coefficients for May 1959.**

<table>
<thead>
<tr>
<th>$ft \times 10^4$</th>
<th>$\partial Z_D/\partial t$</th>
<th>$\partial Z_{100}/\partial t$</th>
<th>$\partial T_L/\partial t$</th>
<th>$\partial T_U/\partial t$</th>
<th>$\partial Z_{500}/\partial t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_D &lt; 30$</td>
<td>0.59</td>
<td>0.76</td>
<td>0.34</td>
<td>-0.37</td>
<td>1.0</td>
</tr>
<tr>
<td>$30 &lt; Z_D &lt; 43$</td>
<td>0.60</td>
<td>0.76</td>
<td>0.34</td>
<td>-0.36</td>
<td>0.99</td>
</tr>
<tr>
<td>$43 &lt; Z_D$</td>
<td>0.61</td>
<td>0.78</td>
<td>0.67</td>
<td>-0.35</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Table 2. Standard deviations for May 1959.**

<table>
<thead>
<tr>
<th>$ft \times 10^4$</th>
<th>$\partial Z_{500}/\partial t$</th>
<th>$\partial Z_D/\partial t$</th>
<th>$\partial Z_{100}/\partial t$</th>
<th>$\partial T_L/\partial t$</th>
<th>$\partial T_U/\partial t$</th>
<th>No. points</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_D &lt; 30$</td>
<td>185</td>
<td>514</td>
<td>1827</td>
<td>123</td>
<td>150</td>
<td>88</td>
</tr>
<tr>
<td>$30 &lt; Z_D &lt; 43$</td>
<td>228</td>
<td>779</td>
<td>2278</td>
<td>163</td>
<td>177</td>
<td>77</td>
</tr>
<tr>
<td>$43 &lt; Z_D$</td>
<td>95</td>
<td>393</td>
<td>1001</td>
<td>112</td>
<td>61</td>
<td>49</td>
</tr>
</tbody>
</table>
Fig. 1. Subjective tropopause analysis based on reported values. Note the regions of strong gradient and the complex structure over central North America.

Fig. 2. Objective tropopause analysis derived from the thickness fields in the tropopause layer. Note the correspondence with Fig. 1, particularly in regions of strong gradients.
It is apparent that there is a very high degree of correlation between the two independent variables \(\partial Z_{600}/\partial t\) and \(Z_D'\partial Z_{600}/\partial t\). This result is significant when one considers that the data came from one series of maps (May, 1959). Synoptically, the result can be related to the persistence of one main storm track throughout the period. This storm track was delineated by the major jet stream which closely followed one \(Z_D\) contour line. Thus, where there were areas of significant \(\partial Z_{600}/\partial t\) there was no large, uncorrelated change in \(Z_D\). The variable \(Z_D'\partial Z_{600}/\partial t\) would become important should two main storm tracks exist, or if one should want one set of regression equations to suffice for all seasons. However, the best set of equations for a 24-hr forecast will probably result through the use of weighted historical data. A scheme for achieving this will be presented later in this paper.

The regression equations derived using equal weighting for each of a series of charts from 15 May to 19 May inclusive are:

\[
\partial Z_D/\partial t = (-2Z_D^2 + 14Z_D - 16)\partial Z_{600}/\partial t, \tag{5}
\]

\[
\partial Z_{100}/\partial t = (0.15Z_D + 0.1)\partial Z_{600}/\partial t, \tag{6}
\]

\[
\partial T_L/\partial t = 0.4\partial Z_{600}/\partial t \quad (Z_D > 30T), \tag{7}
\]

\[
= (0.53Z_D - 1.2)\partial Z_{600}/\partial t \quad (Z_D < 30T),
\]

\[
\partial T_U/\partial t = (-0.4)\partial Z_{600}/\partial t \quad (Z_D < 43T), \tag{8}
\]

\[
= (1.33Z_D - 6.1)\partial Z_{600}/\partial t \quad (Z_D > 43T),
\]

where

\[
Z_D = (9.6T_L - 12T_U + 23872) \times 10^{-4} \tag{9}
\]

in the above equations.

**Level of maximum wind chart (LMW).** An attempt was made to produce a derived level of maximum wind chart by fitting the general shape of the usual jet core described by Endlich (1960) to the objective analyses at the 500-, 300-, 150- and 100-mb levels. The results are extremely difficult to verify in an objective manner, and so a typical chart is presented (Fig. 3), as well as some horizontal and vertical wind shear curves (Fig. 4) to aid in a subjective assessment. A brief description of the method follows. The height of the level of maximum wind (LMW) is taken to be that of the derived tropopause \((Z_D)\). The direction of the wind at the LMW is taken normal to the gradient of \(Z_D\). The speed of the wind is obtained by using a weighted mean of a wind derived from that pressure surface immediately below \(Z_D\) and another wind derived from that pressure surface immediately above \(Z_D\). This weighting scheme insures that, should \(Z_D\) be found at, say 150 mb, then \(V_{150} = V_D\) where \(V_D\) is the wind at LMW.

An illustrative example follows:

\[
V_D = \frac{(d-x)V_U \exp[-0.00066x] + (x)V_L \exp[-0.00066(d-x)]}{(d-x) + (x)}, \tag{10}
\]

where

\[
V_U = \text{magnitude of geostrophic wind at the pressure level a distance } d \text{ above the } Z_D \text{ or LMW at point in question;}
\]

\[
V_L = \text{magnitude of geostrophic wind at the pressure level a distance } d-x \text{ below the } Z_D \text{ or LMW at point in question;}
\]

\[
d = \text{local thickness between the two pressure levels providing the } V_U \text{ and } V_L \text{ described above lie one on each side of the LMW for which the calculation of } V_D \text{ is being made.}
\]

In order to verify the hypothesis that the \(Z_D\) map was a good approximation to the LMW data for four seasons over a 5-yr period were examined. In general the results supported those of Avvakumov (1959) and of Austin and Bannon (1952). According to these papers the jet core, on the average, lies 3000 ft below the tropopause. The wind maximum is found well below on the anticyclonic side while it is somewhat: weakly defined and only a little above the tropopause on the cyclonic side. It was found that the \(Z_D\) chart as derived by the intersecting lapse rate curves lay about 700 ft, on the average, below the observed tropopause. Hence, a good first approximation is found by lowering \(Z_D\) 2300 ft so that it will lie, on the average, about 3000 ft below the observed tropopause.

Eq (2) has this 3000 ft constant incorporated in it.

A second order approximation to the orientation of the jet core can be made using the formula:

\[
Z_D = 9.6T_L - 12T_U + 23872 + K |\nabla(\nabla^2 Z_D)|. \tag{11}
\]

Investigations have shown that the addition of the fourth term on the right hand side of (11) will produce a “smooth” map with cross contour flow at the entrances and exits of jet maxima. There seemed to be some doubt that this second order refinement would add enough to the value of the result to warrant its incorporation in an operational program at this time.

Fig. 4 shows cross sections through a derived jet stream maximum portraying the horizontal and vertical wind shears “built in” by the technique described above. One can see that the maximum shear in the vertical is near the jet core and corresponds in magnitude to those found by Reiter (1961). Note that the horizontal shear is greater on the cyclonic than on the anticyclonic side.

**3. Verification of results**

Both the derived height fields and the wind fields have been examined. Table 3 shows the standard deviation of the height data in various aspects for the charts.
Fig. 3. Level of maximum wind chart. Speeds were obtained from an empirical fit of constant pressure geostrophic winds and E Pellici-type vertical profiles. Directions and levels conform to the tropopause isolines of Fig. 2. Note that the maximum wind belt lies along the steep tropopause regions of Figs. 1 and 2.

Fig. 4. Vertical profiles of the wind on a line across the jet stream (C-A in Fig. 3). Note the large horizontal cyclonic shear (curves 2dc to the jet core).
Table 3. Standard deviations of heights (May and February 1959).

<table>
<thead>
<tr>
<th></th>
<th>500 mb</th>
<th>300 mb</th>
<th>150 mb</th>
<th>100 mb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derived field vs. O.A.*</td>
<td>—</td>
<td>195 ft</td>
<td>129 ft</td>
<td>129 ft</td>
</tr>
<tr>
<td>O.A. ((t=0)) vs. O.A. ((t=24)) (May only) O.A. ((t=24)) vs. 24 hr barotropic prog. to (t=24)</td>
<td>240 ft</td>
<td>308 ft</td>
<td>249 ft</td>
<td>150 ft</td>
</tr>
<tr>
<td></td>
<td>148 ft</td>
<td>282 ft</td>
<td>172 ft</td>
<td>132 ft</td>
</tr>
</tbody>
</table>

Table 4. Standard vector deviations of geostrophic wind. (February and May 1959)

<table>
<thead>
<tr>
<th></th>
<th>500 mb</th>
<th>300 mb</th>
<th>150 mb</th>
<th>100 mb</th>
</tr>
</thead>
<tbody>
<tr>
<td>O.A. vs. zero wind</td>
<td>43 kt</td>
<td>66 kt</td>
<td>48 kt</td>
<td>42 kt</td>
</tr>
<tr>
<td>O.A. ((t=0)) vs. O.A. ((t=24))</td>
<td>31 kt</td>
<td>43 kt</td>
<td>28 kt</td>
<td>24 kt</td>
</tr>
<tr>
<td>O.A. vs. derived field</td>
<td>—</td>
<td>27 kt</td>
<td>17 kt</td>
<td>18 kt</td>
</tr>
<tr>
<td>Improvement over persistence</td>
<td>—</td>
<td>32%</td>
<td>42%</td>
<td>23%</td>
</tr>
<tr>
<td>May only O.A. ((t=24)) vs. 24 hr barotropic prog. to (t=24)</td>
<td>19 kt</td>
<td>37 kt</td>
<td>21 kt</td>
<td>19 kt</td>
</tr>
</tbody>
</table>

* O.A. = Objective analysis.

actual winds and consequently his errors are likely to be higher than if he had verified his results using geostrophic winds.

4. Updating of regression coefficients

In an operational system it is desirable to incorporate an automatic updating procedure for the various regression coefficients in order that seasonal and subseasonal changes would be incorporated into the product. One scheme for doing this: follows:

Suppose we wish to predict \(\partial S / \partial t\) where

\[
\partial S / \partial t = a_1 Z_{500} / \partial t + b_1 Z_{500} / \partial t + c_1.
\]  

(12)

Suppose further that each day for \(n\) consecutive days we have produced an Eq (12) using the objective analyses of that day to determine the coefficients. This means that to make a prediction for the day \(n=0\), we have \(n\) sets of coefficients to choose from.

For day 1
\[
\partial S / \partial t(1) = a_1 Z_{500} / \partial t + b_1 Z_{500} / \partial t + c_1.
\]  

(13)

For day 2
\[
\partial S / \partial t(2) = a_2 Z_{500} / \partial t + b_2 Z_{500} / \partial t + c_2.
\]  

(14)

For day \(n\)
\[
\partial S / \partial t(n) = a_n Z_{500} / \partial t + b_n Z_{500} / \partial t + c_n.
\]  

(15)

To obtain the best estimate for \(\partial S / \partial t(0)\), write

\[
\partial S / \partial t(0) = w_1 \partial S / \partial t(1) + w_2 \partial S / \partial t(2) + \cdots + w_n \partial S / \partial t(n),
\]  

(16)

The subscript \((0)\) signifies the current day while other subscripts signify historical data. Since the current \(\partial S / \partial t(0)\) is available for verification, the usual minimization processes to find the \(w_i\) can be applied. This will give the final prediction equation:

\[
\partial S / \partial t = AZ_{500} / \partial t + BZ_{500} / \partial t + C,
\]  

(17)

where

\[
A = \sum_{i=1}^{n} w_i a_i, \quad B = \sum_{i=1}^{n} w_i b_i, \quad C = \sum_{i=1}^{n} w_i c_i.
\]

This is clearly not the only way to use history; however, it is effective, easily done, conservative, and completely objective.

Any computer could update the coefficients automatically each day and thus take account of certain climatological trends in the prediction of the derived fields.

Table 5. Comparison of 24-hr forecast errors and wind variations. (Values shown represent standard vector deviation in knots.)

<table>
<thead>
<tr>
<th></th>
<th>500 mb</th>
<th>300 mb</th>
<th>200 mb</th>
<th>100 mb</th>
</tr>
</thead>
<tbody>
<tr>
<td>24-hr forecast error</td>
<td>23.7</td>
<td>29.0</td>
<td>22.0</td>
<td>17.6</td>
</tr>
<tr>
<td>24-hr wind variation</td>
<td>30.8</td>
<td>42.0</td>
<td>32.5</td>
<td>15.0</td>
</tr>
</tbody>
</table>
5. Conclusions

In this experiment rather simple and direct synoptic results have been applied in an attempt to obtain a statistically useful, objective representation of the tropopause-jet stream layer. By this method it has been found possible to produce tropopause charts and level of maximum wind charts which show all the macroscopic features implied by current forecast models of the jet stream. In addition, standard upper level charts within the layer can be derived from history and a 500-mb analysis or prognostic chart to give either (i) good first-guess fields for further objective analysis or (ii) prognostic charts which are better than 24-hr persistence. The success of this approach suggests that further progress would be likely with more complex tropopause models.

REFERENCES


