The Effect of Building Shadows on the Vertical Temperature Structure of the Lower Atmosphere in Downtown Denver

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ABSTRACT

Denver’s Continuous Air Monitoring Program (CAMP) site, typically recording the highest carbon monoxide levels in the metropolitan area, lies within a large region of downtown Denver shadowed by tall buildings. Two studies conducted during the winters of 1987/88 and 1988/89 indicated several possible scenarios leading to the high-pollution episodes often reported at CAMP. Sonar records and stability calculations at CAMP indicated that building shadows may be a contributing factor. The building shadowing was simulated by a computer model and its effects were examined from 2 days of detailed vertical temperature profiles taken in the vicinity of CAMP. The vertical temperature structure was mapped both spatially and temporally as it pertains to the shadowed and unshadowed regions. Results show that shadowing at CAMP is quickly followed by the formation of a shadow surface-based inversion and a local rise in carbon monoxide concentrations. Strength of the inversion depends on the meteorology and surface albedo and relates to a difference in solar radiation intensity of >100 W m⁻² between shadowed and unshadowed regions.

1. Introduction

Studies carried out in the Denver metropolitan area during the 1987/88 and 1988/89 winter pollution seasons (Neff 1989a) collected data to examine the meteorology associated with high pollution episodes. Preliminary analysis showed that changes from downtown buildings can affect the microclimate surrounding CAMP, an area typically recording the highest carbon monoxide (CO) readings in the region and close to many parking lots and garages. Shadowing by buildings is analogous to the setting of the sun. How this artificial sunset affects the surface energy budget and contributes to the formation of a local inversion and the trapping of pollutants has not been fully studied.

Research in urban climatology focuses upon surface and atmospheric processes and their interrelationships through numerical models (Todhunter and Terjung 1988). The effect of the surface energy balance as it pertains to urban climatology and the temperature distribution is described by Ross and Oke (1988). They define the energy balance as the partitioning of the radiant energy absorbed near the earth’s surface into sensible and latent fluxes into the atmosphere and a conductive flux of sensible heat into the substrata. Studies of radiative transfer, wind flow patterns and pollution dispersion in the complex urban environment can be found in the literature (Oke 1974; Nicholson 1975; Sievers and Zdunkowski 1985; Eichorn et al. 1988). Those studies incorporating cooling caused by shadowing, which we found to be significant in an urban area, are generally confined to mountain valleys (Whiteman 1987), while building shadows in an urban environment are mentioned as an area in need of further study (Oke 1982).

This paper focuses on the importance of the spatial and temporal distribution of short wave radiation inside an urban area and is divided into two parts. The first part describes a radiation model which quantitatively simulates the direct and diffuse anisotropic radiation distribution on downtown Denver. In part two, measurements of vertical temperature structure in the lowest 200 m at CAMP on two days which were forecast to have high pollution are described. These were clear days with light winds, one with snow cover and one without. Results from the observations and the model are compared to describe the temporal and spatial (vertical) temperature structure present at CAMP during shadowed and unshadowed conditions. Effects of the shadowing on CO concentrations at CAMP are discussed.

The computer model, designed to simulate the shadows (Ruffieux 1989), aided in determining the positioning and timing of tethered-balloon profiles. Vertical profiles were obtained in rapid succession at a single location as well as across the shadow boundaries.
to provide a good temporal and spatial resolution. These profiles were obtained starting early in the afternoon, when flat surface radiation was near its peak, and continued until after sunset. We also examined synoptic and mesoscale influences on local meteorological conditions measured at CAMP.

2. Model description

a. Model domain

The domain used in the model covers only a small fraction of the metropolitan region but encompasses downtown Denver and the CAMP monitoring site. The input matrix comprises two superimposed grids, 2.0 km by 1.7 km, defining the surface topography and building heights. The topographical grid is computed by interpolating between 250 digitized elevations for downtown Denver, which is built on a slope of less than 1 degree running northwest to southeast. The second grid consists of the heights of the 90 tallest buildings in Denver. The resultant matrix is a 200 column by 170 row spatial grid of downtown Denver, specifying the elevation (topography plus building height) at each of the 34 000 matrix cells. The horizontal resolution is 10 m and the vertical resolution is 1 m. Figure 1 shows a three-dimensional representation of the Front Range and the model matrix looking from the northeast across Denver.

b. Model calculations

The effect of shadowing by tall buildings on the city’s energy budget has been simulated using a model that computes direct and diffuse anisotropic radiation reaching the ground for each cell of the topographical matrix. The equations used in the model and described below are presented in the Appendix. The model is divided into three sections: initialization, direct radiation calculation, and diffuse radiation calculation.

The model can perform two different simulations. The first calculates instantaneous values of solar radiation (W m⁻²) at specific times for each grid point, and is useful for resolving the times and positions of shadows. The other simulation calculates cumulative solar radiation values (MJ) for a day or any part of a day, providing a picture of the heating and/or cooling sources and sinks within the city.

1) Initialization

Within the initialization process, the sun’s position and the total energy received at the top of the atmosphere are calculated. Local time is converted to mean solar time (ST) [Appendix, Eq. (A1)] for the month, day, hour, and minute of the desired simulation period using a longitude correction (±4 min/degree of longitude) and the equation of time (EQ) (Paltridge and Platt 1976). The actual elevation and azimuth angles of the sun can then be calculated at any time for a

![Image of topography along the Front Range of Rocky Mountains including model topographical matrix of downtown Denver.](image-url)
specific location [Eqs. (A2) and (A3)] (Hufty and Theriault 1983). The total energy \(E_s\) [Eq. A4] received at the top of the atmosphere is calculated as a function of the sun’s declination angle \(\delta\) and the ratio of the distance between the earth and the sun and their mean distance \(R\). Variations in \(EQ, \delta,\) and \(R\) for an entire year are shown in Fig. 2.

2) DIRECT RADIATION CALCULATION

The solar energy reaching the earth’s surface is primarily a function of the relative air mass, the amount of precipitable water and the concentration of aerosols (turbidity). The relative air mass is computed from the optical path through the atmosphere. Vertically integrated precipitable water and turbidity are estimated or measured in the field. The total atmospheric transmission \(t\), which determines the percentage of \(E_s\) reaching the ground, is then computed as an exponential function of atmospheric extinction, aerosol and water vapor scattering, and relative air mass [Eq. (A5)].

The magnitude of the direct radiation also depends on the angle of incidence. The intensity of radiation per unit area is calculated using the cosine of the angle \(js\) between the “sun vector” and the perpendicular to the slope of the cell [Eq. (A5)]. If the cosine is <0, the cell is in shadow and we have no direct radiation (Fig. 3). Each cell is tested in conjunction with the sun’s angle, starting with the cells closest to the sun, to determine the possibility of shadowing on other cells. Only diffuse radiation is calculated for cells in shadow.

Fig. 2. Annual variation of the equation of time \(EQ\), declination of sun \(\delta\), and distance between earth and sun \(R\).

Fig. 3. The effects of slopes and buildings in determining shadows within the topographical matrix.
3) **Diffuse Anisotropic Radiation Calculation**

The normal equation for computing anisotropic diffuse radiation \(D\) is a double integration of elevation from 0 to 90 degrees and of azimuth 0 to 360 degrees [Eq. (A6)]. In our model a simplified regression equation is used [Eq. (A7)] (Hufty and Theriault 1983) in which diffuse radiation is now the double sum of partial luminances for every 5 degrees of elevation and azimuth. This solution is a good approximation and requires considerably less calculation time than using Eq. (A6). It should be noted that solar radiation reflected from buildings has not been incorporated in the model.

4) **Model Calibration**

Precipitable water in Denver was calculated using Hufty and Theriault’s formula (1983) from an average winter time surface dewpoint temperature and produced values on the order of 0.2–0.3 cm m\(^{-3}\). Dogniaux (1970) gives typical values for turbidity in urban areas ranging from 0.2 to 0.3. Figure 4 depicts the relative weight of precipitable water \((w)\) and turbidity \((\beta)\) in the estimation of the solar radiation reaching a flat surface in Denver.

Because calculations within the model depend on atmospheric parameters that vary from day to day, it is necessary to fine-tune the model, even after initialization, to an individual day of interest. Fine-tuning is accomplished by adjusting the precipitable water and turbidity so the model output matches solar radiation measurements. Figure 5 shows the measured solar radiation from CAMP on 20 January 1989, a clear day, and the model simulation for the grid cell corresponding to CAMP after fine-tuning. The two curves (Fig. 5) show three distinct features: 1) building shadowing, 2) unobstructed sun, and 3) intermittent sun. During the periods of shadowing by buildings (0700–0835, 1345–1415, 1430–1520), direct radiation is equal to zero. Unobstructed sun (0835–1100, 1230–1345, 1415–1430, 1520–1550) has both maximum direct and diffuse radiation and is the period used to fine tune the
A low-level obstruction, specifically a tree south of the pyranometer, caused the period of intermittent sun (1100–1230) at CAMP, and is not part of the model topographic matrix. The differences between the model and measured values are due to the resolution of the model (10 m).

### 3. Experiment description

CAMP served as the focal point for the experiment not only because CO levels are relatively higher than in other regions in Denver, but also because model simulations place it near the boundary of maximum shadowing. Also of interest and importance is CAMP's proximity to many parking lots, garages, and two major rush hour traffic corridors. Logistics within the downtown area limited the methods and equipment used for profiling. To study the temporal evolution, profiles were obtained at CAMP every 30 minutes up to 200 m with a tethered-balloon system. In order to study the spatial effects of shadowing and still obtain a nearly simultaneous vertical cross section, a more portable

### Table 1. Overview of the tethered-balloon profiles obtained on 13 and 20 January 1989.

<table>
<thead>
<tr>
<th>Launch time (MST)</th>
<th>Location*</th>
<th>Max height (m)</th>
<th>Launch time (MST)</th>
<th>Location*</th>
<th>Max height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1400</td>
<td>CAMP</td>
<td>200</td>
<td>1453</td>
<td>CAMP</td>
<td>240</td>
</tr>
<tr>
<td>1423</td>
<td>(B)</td>
<td>110</td>
<td>1525</td>
<td>CAMP</td>
<td>310</td>
</tr>
<tr>
<td>1430</td>
<td>(A)</td>
<td>120</td>
<td>1558</td>
<td>CAMP</td>
<td>209</td>
</tr>
<tr>
<td>1516</td>
<td>(B)</td>
<td>220</td>
<td>1632</td>
<td>CAMP</td>
<td>320</td>
</tr>
<tr>
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<td>CAMP</td>
<td>110</td>
<td>1702</td>
<td>CAMP</td>
<td>215</td>
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<tr>
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<td>1733</td>
<td>CAMP</td>
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<td>CAMP</td>
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<td>1829</td>
<td>CAMP</td>
<td>230</td>
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<tr>
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<td>(B)</td>
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<td>1901</td>
<td>CAMP</td>
<td>215</td>
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<tr>
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<tr>
<td>1640</td>
<td>(A)</td>
<td>130</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* For the locations of (A), (B), and CAMP, see Fig. 1.
system was devised that allowed us to move rapidly between three sites taking profiles within a time span of approximately 20 min. The depth of the shadow determined the maximum height of these profiles. Table 1 lists dates, times, locations, and maximum heights for all the profiles.

4. Experiment days

The two days, 13 and 20 January 1989, were chosen because of their similar synoptic conditions with regionally clear skies and light winds. Synoptic patterns on 13 and 20 January are indicative of synoptic conditions found conducive to high pollution in the Denver area (Summers et al. 1989). The 1700 MST 500-mb upper-air pattern on both days consists of split flow and a weak ridge that is centered over eastern Kansas on 13 January and over eastern Colorado on 20 January. Surface pressure patterns show a lee trough along the Front Range of the Rocky Mountains moving slowly from the United States–Canadian border into eastern Colorado.

Because of the similar synoptic features, the mesoscale meteorology is also similar on both days as is the micrometeorology. At CAMP, winds are light (peak gusts lower than 3 m s\(^{-1}\)) and daily means of 1.5 and 1.2 m s\(^{-1}\) for 13 and 20 January, respectively) and northerly becoming northeasterly in the late afternoon. Weak northeasterly flow, induced by the lee trough, has been shown (Neff 1989b; Wolfe and Gaynor 1989).
Fig. 8. (a) Top: Model simulation of the total solar radiation reaching the ground between 1200 and 1500 MST on 20 January 1989. (b) Bottom: Horizontal cross section of solar radiation from model simulation running SW-NE and bisecting CAMP. Sharp upward peaks are due to edges of buildings which are oriented perpendicular to the solar beams. The 6-MJ level marks the upper limit of incident solar radiation in Fig. 9. (c) Top right: Model simulation of incoming solar radiation at 1445 MST 13 January 1989. This simulation corresponds to the profiles shown in Fig. 8. (d) Bottom right: Model simulation of incident solar radiation at 1520 MST 13 January 1989. This simulation corresponds to the profiles shown in Fig. 9. Note the shadowing structure parallel to the SW-NE running streets.
to produce stagnation on the windward side of downtown Denver, helping to trap the CO.

Both days are clear, but have significantly different surface albedos. Thirteen January had snow on the ground and wet streets; 20 January had no snow and dry streets. Temperature traces for both days have a strong diurnal trend as expected, but also show differences due to the surface albedo and the effects of shadowing. Figure 6 depicts the temperature at 2 m measured at CAMP between 1100 and 1900 MST. The daily means are removed to facilitate the comparison, and periods of shadowing on 13 January are highlighted. Cooling in response to the first afternoon shadow and snow cover is more dramatic on 13 January. The second period of shadowing is barely discernible on 13 January compared with 20 January, because snow cover has limited the heating between shadows.

5. Analysis

a. Temporal evolution of the temperature structure

Figure 7 shows selected temperature profiles obtained at CAMP on 13 and 20 January. Profiles 1a–1b, 2a–2b, and 3a–3b are a time evolution and correspond to surface shadowing, return to direct sun, and the sunset transition periods for similar times on each day. Profile 4b, obtained well after sunset on 20 January, shows the nocturnal inversion. Shadowing by the buildings is reflected in profiles 1a–1b as a shallow surface-based inversion. In the presence of snow cover, this inversion is stronger, 2° versus 1°C, but the depth is the same. Profiles 2a and 2b are back in direct sunlight and the surface inversion has all but disappeared on both days. The sunset transition period profiles (3a and 3b) are very similar to the earlier profiles, but have cooled through nearly the entire depth of the profiles. The nocturnal inversion profile, 4b, on 20 January is very strong and shallow (<75 m). Differences in the profiles between the two days are due primarily to the surface conditions. Because the surface winds were very light, it is assumed that the effects of advection were small compared with those of radiation.

Model simulations of the shadow provide a useful overall view of the contrast between shadowed and unshadowed regions. Preliminary analysis suggests there is a minimum difference in intensity of radiation, from unshadowed to shadowed conditions, of at least 100 W m⁻², which is related to the formation of the surface inversion. In the early afternoon this difference can be as much as 400 W m⁻², compared with the period before sunset transition when the difference is less than 100 W m⁻² (Fig. 5). Figure 8 is a simulation of the cumulative energy received at the ground from 1200 until 1600 MST on 20 January. Darker regions in Fig. 8a are receiving less energy as a result of shadowing. Figure 8b is a cross section of solar radiation values running southwest to northeast and bisecting CAMP, taken from Fig. 8a. The three sharp peaks correspond to the exposed walls of buildings, and the valleys, that are equal to 0.2 MJ, correspond to shadowed areas. CAMP is located on the boundary of a shadowed region.

b. Horizontal cross section of the temperature structure

In order to analyze changes in the vertical structure in and out of the shadow, three profiling sites were chosen on the basis of model results (Figs. 1 and 7): (A) northeast of CAMP, CAMP, and (B) southwest of CAMP near the Federal Building. Profiles were obtained, in succession, at each of the three locations to provide a nearly "simultaneous" (within 20 minutes) horizontal picture of the lower atmosphere (Table 1). Figures 9, 10, and 11 show three such vertical cross sections taken on 13 January.

At 1415 MST (Fig. 9), the local inversion layer caused by the shadowing is visible in the profiles at site (B) and CAMP only. The upper limit of the inversion is about 50 m. Site (A) was in sun. Possible heat island effects are also visible in Fig. 9 as seen in the 1°–2°C warmer temperatures above 50 m at site (B), which is closer to downtown Denver than CAMP.

At 1520 MST (Fig. 10), the sun is again shining at CAMP. The inversion is visible only near the Federal Building (B) where shadowing exists. The upper limit

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![Figure 9](https://example.com/fig9.png)

**FIG. 9.** Temperature profiles taken at site A, CAMP, and site B at 1415 MST 13 January 1989. Sites B and CAMP are both shadowed at this time.
of the inversion is now 70–80 m. The suggested heat island effect appears to be still present.

At 1635 MST (Fig. 11), just before sunset and with all three sites in shadow, we do not see an inversion. All three profiles are similar up to about 125 m except for a 0.5 °C warming between (A) and CAMP and between CAMP and (B). Again this offset in the profiles is believed to be due to the heat island effect.

To help analyze the horizontal spatial variability of shadowing over downtown Denver, two instantaneous model simulations were run. At 1415 MST (Fig. 12), the extension of the shadow is short, but the contrast between shadowed and unshadowed areas is high: >300 W m⁻². This situation corresponds to a strong inversion at (B) and inversions which appear to be decreasing at CAMP and disappearing at (A) (Fig. 9). At 1520 MST (Fig. 13), a unique situation exists in which the direct incoming solar radiation beams are parallel to the SW–NE running streets. This short period of sun is enough to modify the atmospheric stability and strongly weaken the inversion at CAMP (Fig. 10).

c. Effects on carbon monoxide concentrations

Figure 14 depicts a time series of CO at CAMP for 13 and 20 January from 1100 until 2200 MST; shadowed periods on 13 January are highlighted. These plots represent 5-min averages recorded specifically to analyze small-scale changes in the CO. In a general sense the fluctuations represent locally produced evening rush hour CO collecting on the windward side of downtown and trapped beneath the local inversion. Taking into account the time lag, there is excellent correlation between shadow of the snow-covered surface and increasing CO on 13 January (Segal et al. 1989). On 20 January the lack of snow cover is evident during the first shadowed period; there is no significant increase in CO and a more gradual increase for the second shadow compared with 13 January. Because evening rush hour starts around this time, it is not possible to say conclusively whether the increase in CO is due to the increase in traffic or to the formation of the inversion. Comparison of the traffic counts made at CAMP show that the rush hour increase in traffic on the two days was generally the same, but 20 January had 5% fewer cars each hour than the peak count on 13 January.

The sharp increase in CO after sunset is believed to be the result of a combination of the strong shallow surface inversion (Fig. 7) and stagnation, especially for 20 January when the 5-min average winds were less than 1 m s⁻¹. The onset of the drainage winds blowing from the southwest down the South Platte River Basin.
causes the sharp drop in CO on both days after 2030 MST (Neff 1989b; Wolfe and King 1990).

6. Summary

This study is an initial step in attempting to analyze the effects of building shadowing on inversion formation in downtown Denver. Large-scale meteorology was nearly identical on both days. A weak synoptic pattern produced light winds that allowed us to study the inversion formation under ideal conditions and without significant turbulent mixing. Temperature profiles show that a low-level inversion forms when the afternoon shadows first reach the CAMP site. Clear skies, light winds, timing of the shadows and surface albedo are factors that control the formation, strength, and duration of the inversion. The inversion on 13 January is slightly stronger, as expected, because of the snow cover. Profiles taken in and out of shadow, at nearly the same time, show that the inversion structure, when it exists, varies over short distances.

Model simulations provide very useful information on the timing and positioning of the shadows around CAMP. These simulations also indicate that for the model domain there are large areas of shadowing. Local inversion formation and solar radiation differences in and out of shadow in the early afternoon (1300–1600 MST) correlate to observed results. Both the model and temperature profile observations suggest that stronger and longer lasting inversions may exist within the more shadowed regions of the city’s core.

We were able to examine the effects of shadowing on CO concentrations measured at CAMP when there was little advection. The sharp increases in CO clearly correspond to the shadowing at CAMP for snow-covered conditions. Shadowing may also be important for a variety of other surface conditions. However, we cannot conclude this from just two cases. This study was limited to a very small area, and additional work is needed to fully understand the interactions between meteorology, shadowing, and vehicle emissions and their relative contributions to Denver’s pollution. It is clear, however, that these studies on thermal structure can have application in other large urban or complex terrain regions.

Acknowledgments. This study was supported by the Swiss National Science Foundation and in part by the Colorado Department of Health.

APPENDIX

Main Equations Used in the Model of Simulation of Direct and Diffuse Anisotropic Radiation

1. Initialization

a. Solar time

\[ ST = MST + \text{Longitude correction} + \left( \frac{\text{EQ} \times 12}{\pi} \right) \]  
(A1)

where

- \( ST = \) Solar time
- \( MST = \) Mountain Standard Time
- \( \text{EQ} = \) Equation of time in radians
- \( \text{EQ} = 0.000075 + 0.001868 \cos \theta_0 - 0.032077 \sin \theta_0 - 0.014615 \cos 2 \theta_0 - 0.040849 \sin 2 \theta_0 \)
- \( \theta_0 = \frac{2 \pi}{365} (dn / 365) \)
- \( dn = \text{Julian Day} \)

Longitude correction = ±4 min/degree of latitude

b. Position of the sun

\[ \theta = \text{asin} \left[ (\sin \delta \times \sin \text{lat}) + (\cos \delta \times \cos \text{lat} \times \cos dh) \right] \]  
(A2)

\[ \alpha = \text{acos} \left[ \frac{(\sin \delta \times \cos \text{lat}) - (\cos \delta \times \sin \text{lat} \times \cos dh)}{\cos \theta} \right] \]  
(A3)

where

- \( \theta = \) elevation of the sun in degrees
- \( \alpha = \) azimuth of the sun in degrees
- \( \text{lat} = \) latitude in degrees
- \( \delta = \) declination in degrees = \(-23.4683 \times \cos \left[ \left( 0.9856 \times dn \right) + 9.3 \right] \)
- \( dh = \) hour angle in degrees = \( \left\langle (\text{STH} \times 100) + \left[ (\text{STM} \times 100) / 60 \right] / 100 \right\rangle - 12 \) 15.0

STH = solar time hours
STM = solar time minutes
c. Energy at the top of the atmosphere

\[ E_s = I_0 R^{-2}, \tag{A4} \]

where

\[ I_0 = \text{solar constant on the top of the atmosphere} = 1353 \text{ W m}^{-2} \]
\[ R^{-2} = 1.000110 + 0.034221 \cos \theta_0 + 0.001280 \sin \theta_0 + 0.000719 \cos 2 \theta_0 + 0.000077 \sin 2 \theta_0 \]
\[ R = \text{ratio of earth–sun distance and mean distance} \]

2. Direct radiation

\[ F_d = E_s \times t \times \cos js; \quad \text{if} \quad \cos js > 0 \]
\[ F_d = 0; \quad \text{if} \quad \cos js \leq 0, \tag{A5} \]

where

\[ F_d = \text{direct beam on the ground in W m}^{-2} \]
\[ t = \text{total atmospheric transmittance} = \exp(-Tkn) \]
\[ k = \text{extinction factor for pure and dry atm. (pure Rayleigh scatter)} \]
\[ k = 0.024 + 0.01 \exp[\exp(-0.0647m, \pm 0.7653)] \]
\[ m_r = \text{relative optical air mass} \]
\[ m_r = [1 - (0.1H/1000)] \times \{1/\sin \theta + 0.15(\theta + 3.885)^{-1.253} \} \]
\[ T = \text{aerosol and water scattering process} \]
\[ T = [(\theta + 85)/(39.5e^{-w} + 47.4) + 0.1] + (16 + 0.22w)\beta \]
\[ e = \text{constant of Neper} = 2.7182818 \]
\[ H = \text{altitude in meters} \]
\[ \beta = \text{coefficient of turbidity (Dogniaux 1970)} \]
\[ w = \text{precipitable water in g m}^{-3} \]
\[ js = \text{angle between the "sun vector" and the perpendicular} \]
\[ \text{to the slope} \]

3. Diffuse anisotropic radiation

a. General equation

\[ D = L_0 \int_{0}^{90°} \int_{0}^{360°} L' \cos jc \times \cosh dc \text{dhc,} \tag{A6} \]

where

\[ D = \text{diffuse anisotropic radiation in W m}^{-2} \]
\[ L_0 = \text{zenithal luminance of the sky} \]
\[ L' = \text{relative luminance of a point of the sky} \]
\[ hc = \text{elevation of computed point} \]
\[ ac = \text{azimuth of computed point} \]
\[ jc = \text{angle of incidence of radiation in a point of sky on an inclined surface with coordinates p (slope) and ap (orientation)} \]

b. Regression equation

\[ D = 0.0076154 L_0 \sum_{hc=2.5°}^{87.5°} \sum_{ac=2.5°}^{357.5°} L' \cos jc \times \cosh dc, \tag{A7} \]

where

\[ L_0 = (0.00236^2 + 0.202 + 1.3513) T \times R^{-2} \]
\[ L_c = L_0 \{[0.910 + 10 \exp(-3\gamma) + 0.45 \cos^2 \gamma] \times 1 - \exp(-0.32 \cosh \gamma) \} \times \{0.27385 (0.910 + 10 \exp[-3(90 - \theta)] + 0.45 \sin^2 \theta) \} \]
\[ L' = L_0/L_0 \]
\[ \cos \gamma = \sin hc \times \sin \theta + \cosh \theta \cos \theta \cos (ac - \alpha) \]
\[ \cos jc = \cos p \times \sin hc + \sin p \times \cosh \theta \cos (ac - \alpha) \]
\[ L_c = \text{luminance at a point of the sky} \]
\[ \gamma = \text{angle between sun and point of the sky with coordinates hc (elevation) and ac (azimuth)} \]

REFERENCES


