The Effect of Curve Fits for the Disdrometer Calibration on Raindrop Spectra, Rainfall Rate, and Radar Reflectivity

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(Manuscript received 22 January 1992, in final form 29 May 1992)

ABSTRACT
Peaks in number density observed by many authors using Joss–Waldvogel disdrometers are almost certainly due to the manner in which a best-fit curve was chosen to represent the calibration data. The extreme sensitivity to small changes in the calibration curve was demonstrated with extensive observations of Malaysian tropical rain, which for best-fit calibrations showed multiple peaks; however, when a linear interpolation between calibration points was applied, only single peaks were seen in this first, recalibrated dataset. Hence, field evidence for multiple peak equilibrium distributions obtained in numerical models should be reconsidered. On the other hand, the rainfall and radar reflectivity, calculated from best fits, differ by less than 4% from that based on linear interpolation between calibration points.

1. Introduction
The Joss–Waldvogel disrometer (Joss and Waldvogel 1967) is a ground-based instrument used to measure raindrop spectra. Several field observations made with this instrument have indicated shoulders or peaks at specific diameters in number density or number density per logarithmic diameter (e.g., Steiner and Waldvogel 1987; List 1988; Zawadzki and de Agostinho Antonio 1988; Asselin de Beaville et al. 1988; Levin et al. 1991; List et al. 1991). Sheppard (1990) recalibrated the signal-processing electronics of a disdrometer and showed that the actual boundaries of the channels or size bins into which the drops were sorted differed from the channel boundaries specified by the manufacturer. This discrepancy is sufficient to produce maxima at diameters between 0.6–0.7, 1.0–1.2, and 1.8–2.1 mm when a Marshall–Palmer distribution is input to the disdrometer signal-processing unit. Interestingly, the locations of these peaks are similar to that of peaks observed in the field.

Previous calibrations of disdrometers have related the measured voltage to the drop size by means of a geometric fit. Here such a best-fit curve is compared to a linear interpolation between calibration points, and both are applied to give different approximations of drop size distributions measured by a disdrometer during phases I and II of the Joint Tropical Rain Experiment of the Malaysian Meteorological Service and the University of Toronto in Penang, Malaysia, during the fall seasons of 1986 and 1990. This represents the first extensive set of drop size distribution measurements presented using the recalibration of the disdrometer. Sheppard (1990) had previously shown how the data from one rain event of 1-h duration changed due to the recalibration. This work also extends that of Sheppard by comparing the magnitudes of the peaks observed in the field to those that are instrument related. The total rainfall amount and total radar reflectivity are calculated from the different spectra generated by using different fits. The rainfall is then compared with that measured by a raingage adjacent to the disdrometer, and finally a Z–R relation is calculated on the basis of different calibration fittings of raindrop spectra.

2. The Joss–Waldvogel disdrometer
The Joss–Waldvogel disdrometer consists of an electromechanical transducer and an electronics processing box. When a raindrop hits a styrofoam cone mounted on the transducer, the cone and two attached coils within the transducer are driven downwards through a magnetic field, thereby inducing a voltage in the sensing coil that is amplified and applied to the driving coil. The amplitude of this pulse is not directly proportional to the mechanical momentum or to the force of an impacting drop (Kinnell 1976), hence, a parameterized relation between the output pulse $U_L$ and the drop diameter $D$ must be used. Joss and Waldvogel (1977) reported that

$$U_L = k D^n, \quad 3.1 < n < 4.3, \quad (1)$$

where $D$ is in millimeters and $k$ is a coefficient of the fit; the value of $n$ varies with $D$ due to varying impact
time, collision mechanics, and a nonlinear variation of terminal velocity with diameter. Kinnell (1976) and Sheppard (1990) used a value of 3.7 for \( n \), and Sheppard used a value of 0.02586 V for \( k \). Equation (1) is assumed valid for all disdrometer transducers and is called the standard transducer characteristic.

Signal-processing electronics are used to compress the output voltage range from 0.3 mV–10 V to 0.16–10 V. Our disdrometer operation manual gives a parameterization of the compressed voltage \( U_C \) in volts in terms of drop diameter in millimeters as

\[
U_C = 0.94 D^{1.52}.
\]  

(2)

Compression of the voltage data by the signal-processing electronics produces deviations from Eq. (2) that produce artificial peaks in the raindrop spectra.

Sheppard (1990) investigated how much the actual voltage boundaries of the 20 standard raindrop sorting bins differed from the voltage boundaries specified by the manufacturer. He then determined the effect of the differences of these boundaries on a Marshall–Palmer, henceforth MP, distribution (Marshall and Palmer 1948) of the form

\[
N(D) = N_0 e^{-\Delta D},
\]  

(3)

with \( N_0 = 0.08 \text{ cm}^{-4} \), \( \Delta = 4.1 R^{-0.21} \text{ mm}^{-1} \), and \( R \) the rain rate (mm h\(^{-1}\)). Sheppard's data are presented in Fig. 1. The ordinate is number density per logarithmic diameter interval, \( a(l) \), rather than number density per linear interval, \( N(D) \), as Sheppard used, so that the peaks are more prominent. The two scales are related by

\[
a(l) = N(D) D,
\]  

(4)

where \( l = \log D \). Maxima not found in the original MP distribution are clearly seen at diameters between 0.6–0.7, 1.0–1.2, and 1.8–2.1 mm, with a lesser maxima between 3.0 and 3.3 mm. Marshall–Palmer distributions, however, have broad single peaks that shift to larger diameters with increasing rain rates. The average drop size distribution measured in 1990 in Penang, Malaysia, during phase II of the Joint Tropical Rain Experiment, denoted MII (List et al. 1991), multiplied by 20, is also shown in Fig. 1 to illustrate that these artificial peaks have similar relative magnitudes to those observed in the field. All drop size distributions are displayed in logarithmic coordinates so that the magnitudes of the peaks may be compared easily; this could not be done using linear diameter coordinates. An MP distribution is plotted for comparison.

The curves of Sheppard were also compared to several field measurements of raindrop spectra made with disdrometers (Steiner and Waldvogel 1987; Zawadzki and de Agostinho Antonio 1988; Asselin de Beauville et al. 1988; List et al. 1991; Levin et al. 1991). These spectra, obtained by using a hand scanner on the appropriate published figure and converting \( N(D) \) to \( a(l) \), are plotted in Fig. 2. For comparison, each spectrum was normalized by dividing by the maximum value of \( a(l) \) for that spectrum; Sheppard's spectrum was divided by a larger value, 3.1, since his case contained many more small drops. With the exception of Levin et al.'s (1991) spectrum, all observations show evidence of Sheppard's (1990) peaks at similar diameters and with similar relative heights. These similarities led to a more detailed disdrometer calibration.

3. Calibration of the disdrometer

A recalibration of the disdrometer, type RD-69, was performed at the Laboratory of Atmospheric Physics at the Eidgenössische Technische Hochschule (ETH) in Zurich, Switzerland. Linear voltages \( U_L \) corresponding to various diameters \( D \) [Eq. (1)] were fed into the processing electronics, and the corresponding compressed output voltage \( U_C \) was measured (Table 1). The calibration curve obtained by linear interpolation between nearest calibration points is referred to as the standard. Figure 3 shows the calibration points together with curves representing the best geometric and polynomial fits to the data, labeled g-fit and p-fit, respectively. The curves are only shown in the diameter range 0–3 mm so that small differences are more easily visible; the complete calibration data is included in Table 1. The interpolation or calibration standard is not shown but would be represented by straight lines.
Fig. 2. Number density per logarithmic diameter, \( n(l) \), against diameter for drop size distributions recorded with a disdrometer by Steiner and Waldvogel (S&W), Zawadzki and de Agostinho Antonio (Z&A), Asselin de Beauville et al. (APML), Levin et al. (LFTW), and Liu et al. (MII).

connecting neighboring points. The geometric fit is given by

\[
D = \left( \frac{U_C}{0.933} \right)^{1/1.47},
\]

where \( \chi^2 = 0.037 \) for 75 degrees of freedom. A tenth-order polynomial fit was also performed, with the diameter given by

\[
D = \sum_{i=0}^{10} a_i U_C^i,
\]

Table 1. Calibration for the drop diameters against \( U_C \) using the standard transducer characteristic [Eq. (1)] for the University of Toronto processor RD-69, S/N: 21170.

<table>
<thead>
<tr>
<th>( U_C ) (V)</th>
<th>( D ) (mm)</th>
<th>( U_C ) (V)</th>
<th>( D ) (mm)</th>
<th>( U_C ) (V)</th>
<th>( D ) (mm)</th>
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<tr>
<td>0.160</td>
<td>0.313</td>
<td>0.170</td>
<td>0.323</td>
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<td>0.334</td>
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<td>0.190</td>
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<td>0.354</td>
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<td>0.230</td>
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<td>0.465</td>
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<td>0.320</td>
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<td>0.495</td>
<td>0.350</td>
<td>0.509</td>
</tr>
<tr>
<td>0.365</td>
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<td>0.380</td>
<td>0.535</td>
<td>0.400</td>
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<td>0.440</td>
<td>0.585</td>
<td>0.460</td>
<td>0.604</td>
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<tr>
<td>0.480</td>
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<td>0.654</td>
<td>0.540</td>
<td>0.685</td>
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<td>0.748</td>
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<td>0.850</td>
<td>0.937</td>
<td>0.900</td>
<td>0.967</td>
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<td>1.000</td>
<td>1.041</td>
<td>1.050</td>
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</tr>
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<td>1.100</td>
<td>1.128</td>
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<td>1.196</td>
<td>1.260</td>
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<td>1.500</td>
<td>1.406</td>
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<td>1.840</td>
<td>2.600</td>
<td>1.940</td>
</tr>
<tr>
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<tr>
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<td>5.100</td>
<td>3.186</td>
<td>5.400</td>
<td>3.327</td>
</tr>
<tr>
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<td>6.000</td>
<td>3.585</td>
<td>6.400</td>
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<td>9.000</td>
<td>4.769</td>
</tr>
<tr>
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<td>4.956</td>
<td>10.000</td>
<td>5.134</td>
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</tr>
</tbody>
</table>

with the coefficients of the fit listed in Table 2. For this fit, \( \chi^2 = 0.015 \) for 67 degrees of freedom. This fit is valid only over the range of calibration data, but it is rare for a drop to register a voltage outside this range. The original manufacturer’s fit to different calibration data [Eq. (2)], labeled m-fit, is also shown. The corresponding \( \chi^2 = 0.279 \) for 75 degrees of freedom as compared with the standard. The \( \chi^2 \) values indicate that all curves provide good statistical fits to the data.

Table 2. Coefficients of the fit for Eq. (6) giving \( D \) versus \( U_C \) for the disdrometer calibration performed in 1990.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( a_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2153 mm (^{-3})</td>
</tr>
<tr>
<td>1</td>
<td>0.4936 mm (^{-3})</td>
</tr>
<tr>
<td>2</td>
<td>1.4631 mm (^{-3})</td>
</tr>
<tr>
<td>3</td>
<td>-1.9828 mm (^{-3})</td>
</tr>
<tr>
<td>4</td>
<td>1.2368 mm (^{-3})</td>
</tr>
<tr>
<td>5</td>
<td>-0.4382 mm (^{-3})</td>
</tr>
<tr>
<td>6</td>
<td>0.09484 mm (^{-6})</td>
</tr>
<tr>
<td>7</td>
<td>-0.01279 mm (^{-7})</td>
</tr>
<tr>
<td>8</td>
<td>0.001049 mm (^{-8})</td>
</tr>
<tr>
<td>9</td>
<td>-4.796 \times 10^{-9} mm (^{-9})</td>
</tr>
<tr>
<td>10</td>
<td>-9.360 \times 10^{-10} mm (^{-10})</td>
</tr>
</tbody>
</table>
The manufacturer’s fit does not apply to the present status of the University of Toronto instrument since the processing electronics were replaced just before the 1986 Malaysian experiment; but it may give an idea about possible variations between different instruments. The differences between the present fits and the original calibration might, however, also include errors caused by simulating drops electronically. The original calibration was obtained using many real drops, which may give deviations from the electronically expected behavior due to variations in terminal fall speed, state of the cone, location of splashing on the cone, and many others.

Following and extending Sheppard’s (1990) work, analysis was performed to investigate how the calibration might affect the measurement of an ideal MP distribution. For an MP distribution, the number of drops in each of 5000 evenly distributed small size bins was calculated. The compressed voltage corresponding to the mean diameter of each bin was calculated using the standard and was then reconverted to an apparent diameter using Eq. (2). The number of drops in each of the 20 larger size classifications normally used by the disdrometer were calculated using the apparent diameters of the 5000 bins. The resulting distributions (figure not shown) were almost identical to those found by Sheppard (1990). There were peaks at diameters between 0.6–0.7, 1.0–1.2, and 1.8–2.1 mm. This verified that our processing electronics worked similarly to Sheppard’s and suggested that the peaks were caused by the processing electronics.

4. Calculation of raindrop spectra

The University of Toronto disdrometer was utilized at the Bayan Lepas International Airport from 15 October to 1 November 1986 and from 4 October to 4 November 1990 during phases I and II of the Joint Tropical Rain Experiment (List 1988; List et al. 1991). The compressed voltages and the raindrop arrival times at which they were measured were stored without assigning size bins, allowing an investigation of how different calibration curves affected the raindrop spectra. In 1986, there were 12 distinct rain events having precipitation greater than 0.1 mm comprising 23 h of data and, in 1990 14 distinct rain events ($R > 0.1$ mm) comprising 38 h. To establish $N(D)$, the number of drops in bins were counted (bins 0.25–0.35, 0.35–0.45, …, 1.85–1.95, 1.95–2.1, 2.1–2.3, …, 4.1–4.3, 4.3–4.5 mm). This number was then divided by the sampling area, duration of collection, and bin width. Size-specific number flux was converted to specific number density by dividing by the terminal velocity of each drop as it was added to a bin. The terminal velocity $V_T$ (cm s$^{-1}$) for the average bin diameter $D$ in millimeters was obtained using the Best (1950) approximation and is given by

$$V_T = 970.5 \left[ 1 - \exp \left( - \left( \frac{D}{0.177} \right)^{1.147} \right) \right].$$ (7)

The number density was converted to $a(l)$ using Eq. (4). The raindrop spectra for each file was then calculated and plotted using each of the four calibration options.

To examine the data, the Malaysian raindrop spectra measured in 1990 were sorted into four different groups according to the 1-min-averaged rain rates (0.0–1.0, 1.0–5.0, 5.0–25.0, and greater than 25.0 mm h$^{-1}$). Figure 4a shows the average number densities $a(l)$ for each group using the g-fit, whereas Fig. 4b displays the spectra using the standard. Figures 5a and 5b show the

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![Fig. 4. Number density per logarithmic diameter, $a(l)$, versus diameter for average drop size distributions sorted according to rain rate measured in Malaysia in 1990: (a) using the geometric fit to the calibration data and (b) using the calibration standard.](image)
average raindrop spectra measured in 1986 for the same rain-rate ranges according to the g-fit and the calibration standard, respectively.

Spectra obtained from the g-fit and the standard calibration differ substantially. Using the g-fit, definite peaks with varying height exist at more or less fixed diameters of approximately 0.7, 1.1, and 1.8 mm (Fig. 4a and Fig. 5a), whereas there is basically only one peak at a diameter that increases with rain rate when the standard calibration by interpolation is used (Fig. 4b and Fig. 5b). It may be added that the peaks around 0.5 mm in Fig. 5 are caused by temporary instrument-related effects because the peak was only seen in some cases in 1986.

Figures 4b and 5b show no evidence of shape conservation, a strict requirement for equilibrium distributions (List et al. 1987), because the spectra change with R. There are fewer and fewer small raindrops for higher rain rates. At high rain rates, however, most smaller raindrops are not measured by the disdrometer because of a higher low-diameter cutoff (List 1988) that is dependent on rainfall rate; hence, shape conservation would not be expected.

It is desirable to fit a curve through the calibration points so that the voltage measured by a disdrometer can be converted to a diameter without linearly interpolating between calibration points. The 1990 average spectral data (MIM) from all cases with 1-min-averaged rain rates between 5 and 10 mm h\(^{-1}\) using the standard, the best geometric and polynomial fits to the calibration data, and the manufacturer's fit to a different calibration are presented in Fig. 6. The peak at 1.8 mm is present for all calibration options except for the case of the linear interpolation (the standard); the peak at 0.7 mm present for the fits is also negligible for the standard. The middle peak is present at 1.1 mm for all calibrations but it is not nearly as big for the standard; the standard gives an ordinate of 280 m\(^{-3}\), while the statistically good p-fit gives 360 m\(^{-3}\). In Fig. 3, it can be seen that the slope of the p-fit begins to differ from that of the standard around 1.1 mm, explaining this discrepancy. Similar figures were examined for all different rain rates from both 1986 and 1990 Malaysian datasets (figures not shown). In particular, multiple peaks existed for the spectra obtained using the geometric and polynomial curve fits to the calibration.

Fig. 5. As in Fig. 4 except that the disdrometer data was obtained in Malaysia in 1986.

Fig. 6. The a(I) versus diameter relation for drop size distributions measured in Malaysia in 1990 with rain rates between 5 and 10 mm h\(^{-1}\). The different curves represent the different spectra generated using the indicated calibrations.
points of Table 1. This suggests that neither fit is ade-
quate, even though they seem to match the data very
closely.

When the field data is processed using the standard,
then unimodal distributions, similar to an MP distri-
bution, are evident where the diameters of the peaks
also increase with rainrate. This suggests that the mu-
tiple-peak scenarios previously reported by List et al.
(1991) were perhaps even exclusively due to instru-
mental errors and that the peaks seen by other authors
were also instrument related. Because of the calibration
sensitivities involved, it is not possible to correct the
data of others without having their actual information
on drop sizes (not bin associated drops).

5. Error considerations

The errors associated with the calibration determine
the accuracy of the fits. After the electronic calibration,
933 drops of an average diameter 0.834 ± 0.002 mm,
having a fall velocity of greater than 95% of their termi-
nal speed, were impacted on the disdrometer sensor
head at ETH in Zurich. The average drop diameter
measured by the disdrometer using the standard trans-
ducer characteristics [Eq. (1)] was 0.838 ± 0.008 mm,
with a standard deviation of 5.2% or 0.044 mm. This
single-point data and errors cannot be extrapolated to
other calibration points. Without knowing the error
bars, the goodness of the fit cannot be determined. If
the error bars are large, then all curve fits are equivalent
and it cannot be determined within errors if the spectra
have single or multiple peaks. In this case, the disdrome-
ter cannot be used to measure raindrop size distribu-
tions with any accuracy. If the error bars are so small
that the fits lie well outside the errors (i.e., two error
bars), then the spectra obtained from curve fits are un-
acceptably inaccurate. In such a situation, the linear
interpolation between data points is the only acceptable
 calibration, and the resulting spectra may be realistic.
It is possible that the spectra may not be realistic be-
due to measuring different wind conditions, impact angles, or colli-
sion dynamics may give different compressed voltages
than those suggested by the standard.

The exact size of the error bars of both the calibration
points and the spectral data have not been determined.
There may also be errors due to varying drop shape at
impact, impact time, and terminal velocity (Kinnell
1976), but Joss and Waldvogel (1977) pointed out that
disdrometer measurements would not be made during
such extreme conditions. There was a deviation of 5%
in the drop diameters measured in the laboratory. It
can be numerically shown, however, that raindrop
spectra are not affected by small random variations of
drop sizes about their mean value if a sufficient number
of drops is measured.

The previous discussion shows that the use of the stan-
standard transducer characteristic [Eq. (1)] in the cali-

bration is not desirable because the drop size distri-
bution is very sensitive to small changes in the cali-
bration curve. The use of an approximate relation for
$U_L$ versus $D$ may also produce inaccuracies that may
change the appearance of the spectra. There are most
probably no sudden slope changes in $U_L(D)$, however,
contrary to the currently used $U_C(D)$, because the ex-
ponent $n$ in Eq. (2) would smoothly change with di-
ameter as the nature of the drop collision with the
transducer gradually changed. The sudden jumps in
the calibration in Fig. 3 are produced as the gain of
the compression amplifier quickly changes slope at
certain voltages.

Until a complete calibration of the disdrometer is
performed, where drops of varying known sizes falling
at their terminal velocities are dropped on the trans-
ducer, it will be difficult to determine the exact size
distributions of drops reaching the ground and the asso-
ciated errors. Pushing accuracy, however, will not
necessarily make sense because of the natural variability
of shapes, impact angles, impact points, and ambient
wind.

It may be useful, however, to replace the signal-pro-
cessing box of the disdrometer by an analog-to-digital
converter that could handle in one stage the range of
linear output voltages produced by the transducer.
Thus, future models of the transducer could bypass the
processing box, and direct calibrations of the variation
of $U_L$ with $D$ may improve spectra measurements.
Many such models would be needed to devise new ways
of using $U_L$ for more accurate measurements, however,
since problems may arise when measuring the peak of
$U_L$ because the filter in the processor makes an inte-
gration over the waveform (Joss 1992, personal com-
munication).

6. Rain spectra evolution models

Numerical models of the time evolution of steady
rain have predicted the evolution of drop size distribu-
tions to three-peak equilibrium distributions (3PED)
below cloud base (e.g., Valdez and Young 1985; List
et al. 1987; Brown 1987; Feingold et al. 1988) using
parameterizations of laboratory raindrop collisions
(Low and List 1982) that simulated natural events.
Models of nonsteady rain using a pulsed input of rain-
drops at the shaft top (McFarquhar and List 1991)
found three-peak distributions with peak locations
similar to those of the 3PED but with differing peak
heights. The location of the peaks at 0.284, 0.8, and
2.0 mm coincide well with the peak locations of the
modified MP distribution and of the observed spectra
using the various fits to the calibration data. The nu-
merical models are in no way affected by the recal-
ibration of the disdrometer, however, because the ex-
periments on which they were based are totally inde-
pendent of disdrometer performance and data. Hence,
field evidence for the equilibrium distributions previ-
ously quoted (McFarquhar and List 1991) must now
be reconsidered in view of this arbitrary coincidence.

The suggested agreement between model results and
field observations was somewhat puzzling since the evolution times were not large enough to produce equilibrium. For a rain rate of 5 mm h\(^{-1}\), more than 5 min, corresponding to about 3 km of free fall, is required for a 3PED to begin to develop from an arbitrary MP distribution (List et al. 1987). The presence of updrafts or downdrags, however, might considerably modify these expectations concerning the time it takes drops to fall. But, given the frequent presence of the ice phase and cloud heights often observed above the \(-4^\circ\)C level, a three-peak distribution might not be expected in Malaysia in the “average.” For rain from clouds without ice, continuing interactions between drops should achieve and maintain equilibrium quicker. That may require more warm-rain data than is presently available.

7. Rain-rate data from the disdrometer

The disdrometer appears to be limited in its ability to measure raindrop size distributions. This may not be surprising since this instrument was originally developed to allow calculation of rain rate and radar reflectivity. Rain rate, weighted \(D^{3/2}\), and radar reflectivity, weighted \(D^6\), are more influenced by the population of larger raindrops than is the number density, weighted \(D^0\). Thus, the omission of smaller raindrops due to the cutoff should not affect, as significantly, estimates of rain rate and radar reflectivity. Yet, the problems with the nonlinear processing of the electronics still exist.

Figure 7 shows the average rain-rate distributions with size for all spectra having a total rain rate between 5 and 10 mm h\(^{-1}\), as measured in 1990 and for the different calibrations. The distribution shape is quite sensitive to the calibration fits; the spectra using the standard lacks the sharp peak around 1.9 mm that all other spectra have. Differences in the spectra are especially prominent between the diameters of 1.2 and 2.5 mm. The spectra obtained from the two fits to the calibration data obtained at ETH are similar to the spectra based on the standard for drops larger than about 2.5 mm, with the original manufacturer’s fit spectra differing much more.

The total rain rates, calculated from the area under the curves in Fig. 7, were determined for all Malaysian disdrometer files from both 1986 and 1990 using all calibration options. Figure 8 shows that the rain rate obtained by using the geometric fit of the calibration data is similar to that obtained by linearly interpolating between the calibration points, with an average difference over all cases of 1.4%. The correlation using the polynomial fit is even better (figure not shown), with an average difference of 0.8%. The rain rate obtained from the m-fit is also shown; the m-fit rain rate is typically lower by about 7%.

The time-integrated rain rates predicted by the disdrometer data were also compared with rainfall amounts measured by a raingage adjacent to the disdrometer during both phases I and II of the Joint Tropical Rain Experiment. All disdrometer files corresponding to rainfalls greater than 0.1 mm were examined for overlap and then compared with raingage data. Figure 9 shows the total rainfall calculated from the disdrometer data using the standard as a function of the rainfall measured by the gauge. The reading error for total rainfall measured with the raingage was 0.2 mm for every 10 mm of rain, while the error associated
with the disdrometer rainfall amount is unknown. With the exception of one heavy-rain case, where the disdrometer significantly underestimated the amount of rain, the disdrometer and raingage data are in reasonable agreement. In total, over 1986 and 1990 for the 18 compatible cases, the raingage measured 153.9 mm of precipitation and the disdrometer measured 137.8 mm of rain, representing a 10.5% difference. The average of the difference of the measurements was 14.6%, with a standard deviation of 12.2%. Joss and Waldvogel (1977) state that the disdrometer rain rate should be at least 90% of the raingage if the disdrometer is functioning correctly. Thus, while the agreement is not as good as would be desired, the disdrometer provides, in general, good measurements of the rain rate.

There may be a problem in the rain-rate comparison caused by the dead time of the disdrometer. After the disdrometer records a raindrop, it is prevented from recording another for a certain time period until the oscillation of the transducer styrofoam cone is sufficiently damped. The quantitative way in which to correct results for this problem is not known, however.

8. Radar reflectivity calculated from the disdrometer

The radar reflectivity calculations from the disdrometer data are examined in the same way as the rainrate measurements. Figure 10 shows the differential reflectivity distributions $Z(D)$ for all spectra having a rain rate between 5 and 10 mm h$^{-1}$ as measured in 1990 in Malaysia and for the different calibrations. The distribution obtained from linearly interpolating between the calibration points (the standard) differs from that obtained from the fits to the calibration data because it lacks the sharp peaks at 1.8 and 3.3 mm.

The total reflectivities were calculated for all disdrometer files from both the 1986 and 1990 Malaysian field project phases using all calibrations. The reflectivity obtained using the geometric fit to the calibration data is quite similar to that obtained using the standard, and there is even a better match up using the polynomial fit (figure not shown). The average difference between the geometric fit and the standard is 4%, and for the polynomial fit and the standard it is 1%. Thus, the reflectivity may be calculated using a calibration curve that is a good fit to the calibration data. Again, the deadtime is not accounted for.

9. $Z-R$ relations

If the drop size distribution is known, a corresponding $Z-R$ relation can easily be calculated. To avoid overpowering the $Z-R$ relation by the many light rain events, the rain rates and radar reflectivities were calculated and averaged for the following ranges: 0.0–0.5, 0.5–1.0, 1.0–2.5, 2.5–5.0, 5.0–10.0, 10.0–25.0, and greater than 25.0 mm h$^{-1}$. A $Z-R$ relation, calculated from the seven resulting data points by performing a least-squares fit of $\log Z$ versus $\log R$, was written in the form

$$Z = AR^b, \quad (8)$$

with $Z$ in its standard units (mm$^6$ m$^{-3}$) and $R$ in millimeters per hour. For the 1986 data and the standard calibration, $A = 340 \pm 24$ and $b = 1.33 \pm 0.03$, and for the 1990 data, $A = 403 \pm 16$ and $b = 1.31 \pm 0.02$.

The $Z-R$ relations were also calculated using the different calibration options. No substantial differences
in the $A$ and $b$ values existed even when the old manufacturer's fit was used. Thus, the $Z-R$ relations can be considered representative for the Malaysian data for the precipitation mix observed, independent of the chosen curve fits.

10. Conclusions and future prospects

The results can be summarized as follows:

1) A recalibration of a disdrometer showed that the raindrop spectra were very sensitive to the manner in which a curve was fit to the calibration data. Using a new calibration curve obtained by linearly interpolating between data points, the previously predicted multiple peaks of List et al. (1991), based on smooth curve fits, disappeared when the uneven working characteristic of the disdrometer processing unit was corrected. The peaks measured by other authors are most likely also related to smooth curve fits of calibration points because they had similar locations and heights as those of List et al. (1991) and the instrument-related ones of Sheppard (1990).

2) The degree of accuracy of measured raindrop size distributions remains unknown, and practical limitations may always limit conclusions about the exact shape of distributions, especially for drops smaller than 2 mm. Redesigning the electronics may help; however, it is not a panacea.

3) It cannot be categorically stated that raindrop size distributions with multiple peaks do not exist. They have been measured by Willis (1984) using an optical spectrometer. Emphasis should be placed on obtaining intercomparison of the same rainfall with several different instruments.

4) The calculations of rain rate and radar reflectivity based on disdrometer measurements are unaffected by the type of fitting curves to the calibration data. The rainfall amount was also in reasonable agreement with that made by a raingage adjacent to the disdrometer during the Joint Tropical Rain Experiment in Malaysia.

5) A relation $Z = 403R^{1.33}$ was found to characterize the corrected average drop spectra measured by the disdrometer in Malaysia in 1990. A wide variation of the multiplication and the power factor between different events was noted.

The results presented here represent the first extensive dataset obtained using the recalibration of the disdrometer. Their differences from the data obtained using the older calibration suggest that other disdrometer datasets should be reexamined subject to the availability of the measured diameters.

Acknowledgments. This paper is dedicated to Dr. Pat Squires who continuously provided leadership in cloud physics. One of the authors (RL) remembers with pleasure the stimulating discussions with his longtime friend. We thank Donat Högl from the Institute for Atmospheric Physics (ETH) in Zurich for performing the recalibration of the disdrometer and commenting on the manuscript and acknowledge Brian Sheppard for supplying us with his data and for his discussions with us. We acknowledge Stewart Cober and Blair Greenan for their discussions and help. This research was supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada and the Canadian Atmospheric Environment Service (AES). One of the authors (GM) is indebted to the NSERC and the AES for postgraduate fellowships.

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