The Response of an Open Stratospheric Balloon to the Presence of Inertio–Gravity Waves

P. ALEXANDER, J. CORNEJO, AND A. DE LA TORRE

Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Buenos Aires, Argentina

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ABSTRACT

Analytic solutions for the vertical response of an open stratospheric balloon to the presence of inertio–gravity waves during its ascent and descent are obtained. Monochromatic waves with simultaneous variations in density, velocity, and temperature are considered. Two extreme cases for the thermal conductivity of the balloon are analyzed: perfect and null. In the first case the velocity variations of the wave dominate the balloon’s behavior, but in the second one the air density oscillations also become significant. It is concluded that open stratospheric balloons may behave more adiabatic than perfect conducting.

1. Introduction

The advantages in the use of balloons in atmospheric soundings, versus other techniques based on rocketborne instruments or ground-based measurements, are at present better recognized. In situ measurements of the principal atmospheric parameters may be obtained at constant levels of pressure or as a function of altitude with unimprovable spatial and temporal resolutions, considering the moderate balloon velocities and the high performance of the instruments used at the present time. Some types of balloons, like the open stratospheric ones, are able to raise up to 2 tons of payload to a height of over 40 km. One important obstacle in the interpretation of balloon data of winds is given by the need of an accurate knowledge of the velocity of the balloon with respect to a fixed frame, particularly in the vertical direction.

Superpressure constant-density balloons have been successfully used as horizontal quasi-Lagrangian tracers of air parcels in the upper troposphere and lower stratosphere (Morel and Bandeen 1973) as well as in the tropical boundary layer (Cadet and Ovarlez 1976). It has been remarked that the horizontal behavior of large-volume balloons also indicates the horizontal velocity of the air (Talagrand and Ovarlez 1984). In the vertical direction the problem becomes more complicated because the drag force of the air on the balloon coexists with the buoyancy force. Vertical air motions have been inferred up to the stratosphere from changes in the rate of ascent of radiosonde-type balloons launched from the ground (Kitchen and Shutt 1990). On the other hand, from observations performed by high-resolution sonic anemometers onboard open stratospheric balloons, it has been observed recently that the measurements of vertical wind oscillations (relative to the gondola) correspond mainly to the effect of the buoyancy force on the balloon. This forcing has no relationship with the drag exerted on the balloon by the vertical motion of the air and it is basically induced by gravity wave density variations (de la Torre et al. 1996).

In the last three decades, a considerable number of studies has been reported in relation to the response of sondes type and superpressure balloons to the propagation of gravity waves (see, e.g., Hirsch and Booker 1966; Hanna and Hoecker 1971; Reynolds 1973; Levanon and Kushnir 1976; Massman 1978; Lalas and Einaudi 1980; Nastrom 1980; Kitchen and Shutt 1990; Gardner and Gardner 1993). Nastrom (1980) studied the response of superpressure balloons, considering simultaneous variations of vertical wind velocity and air density. Assuming a phase quadrature between both variables, he found solutions for the balloons’ responses that depend on the static stability, the wave period, and the corresponding density and velocity oscillations. As Nastrom (1981) later pointed out, a more complete formulation of the problem should not ignore the thermodynamic properties of balloons. When they oscillate in the vertical direction, they contract and expand continuously. The reluctance of the gas inside a balloon to adjust its temperature to the surrounding air originates an exchange of heat, which in turn varies the volume of the balloon, implying that the force balance is modified. The dynamical response of balloons to the action of atmospheric perturbations represents by itself an essential feature to be explored in detail in order to properly interpret the experimental data obtained. Also, the balloon trajectories themselves can give us information about air parameters. Recent balloon soundings

Corresponding author address: Dr. P. Alexander, Departamento de Física, Facultad de Ciencias Exactas y Naturales, Ciudad Universitaria, 1428 Buenos Aires, Argentina.

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(Kitchen and Shutts 1990; Barat and Cot 1992) have also contributed to the evaluation of the importance of gravity waves in both micro- and mesoscales in the lower and middle atmosphere. In this subject, numerous theoretical and observational studies have focused on the inertio–gravity waves (see, e.g., Fritts and Yuan 1989; Falkovich 1992), which involve characteristic times of several hours.

In the present work, the vertical dynamical response of a typical open stratospheric balloon to monochromatic, inertio–gravity waves is studied for two extreme cases of the balloon’s thermal conductivity: perfect and null. It is sufficient to consider single waves because data indicate (Barat and Cot 1992; de la Torre et al. 1994) that one dominant mode with a wavelength between 1 and 2 km is likely to be present in both the troposphere and the lower stratosphere. The inertio–gravity waves propagate almost vertically, and their period may be very long when compared with the duration of a flight, so we shall consider them to be stationary. An expression for the vertical velocity oscillation of the balloon is found in both cases as a function of the altitude and parameters of the balloon, the undisturbed atmosphere and the wave.

2. Equation of vertical motion

The equation of vertical motion for a balloon may be written as (see, e.g., Nastrom 1980; Nastrom and VanZandt 1982)

\[
\left( M_b + C_m M_a \right) \frac{d w_b}{d t} = M_a \left[ (1 + C_m) \frac{\partial w_a}{\partial t} + (w_a + C_m w_b) \frac{\partial w_b}{\partial z} \right] - \frac{1}{2} \rho_a C_d A_b (w_b - w_a) |w_b - w_a| - g(M_b - M_a),
\]

with \( A_b \) the cross-sectional area of the balloon, \( C_d \) the drag coefficient, \( C_m \) the added mass coefficient, \( g \) the acceleration of gravity, \( M_a \) the mass of displaced air (= \( \rho_a V \)), \( M_b \) the mass of the balloon system (skin + gas + gondola), \( \rho_a \) the ambient air density, \( V \) the volume of displaced air (= gas volume), \( w_a \) the vertical air velocity, \( w_b \) the vertical balloon velocity, \( z \) the vertical coordinate (earth surface; \( z = 0 \)), and \( d/dt \) the time derivative following the balloon motion from a ground-fixed reference system; hereinafter, the subindices \( a \) and \( b \) refer, respectively, to the air and the balloon. The forces on the right-hand side of (1) are called the dynamic, form drag, and buoyancy terms, respectively. Their origin and interpretation can be found in several publications (see, e.g., Taylor 1928; Prandtl 1952; Landau and Lifshitz 1959; Batchelor 1970). It should be emphasized that the above expression becomes valid for large Reynolds numbers (characteristic values for the balloon diameter and velocity relative to the air and for the atmospheric kinematic viscosity in the altitude range of the flights respectively are 20 m, 5 m s\(^{-1}\), and \( 10^{-4} \) m\(^2\) s\(^{-1}\), so typically \( Re = 10^6 \)), because in that case both the viscous stress and the history term (Basset 1888) may be discarded. However, that condition also implies that the drag force cannot be replaced by Stokes’s law of resistance. Skin friction drag, aerodynamic lift, and possible self-induced motions or very small scale turbulence were assumed negligible. We have discarded possible coupling of the horizontal and vertical motions through the drag term, because the difference between the balloon and the air velocities in the horizontal plane is negligible as compared to the vertical difference, for the types of balloons that are driven by the horizontal wind like the open stratospheric ones. (The approximation is meaningful for ascents or descents, although previous works have considered it for floatations, where it probably becomes fair.) Rewriting (1), we obtain for the ascent (minus sign in the drag term) and descent (plus sign in the drag term) of the balloon

\[
\left( M_b + C_m M_a \right) \frac{d w_b}{d z} =
M_a \left[ (1 + C_m) \frac{\partial w_a}{\partial t} + (w_a + C_m w_b) \frac{\partial w_b}{\partial z} \right] + \frac{1}{2} \rho_a C_d A_b (w_b - w_a)^2 - g(M_b - M_a),
\]

where \(|w_b|\) is an order of magnitude larger than \(|w_a|\).

To consider the action of a gravity wave on the motion of the balloon, we write the following variables in the usual form of a mean value plus a perturbation:

\[
\rho_a = \bar{\rho}_a + \rho'_a, \quad V = \bar{V} + V', \quad w_a = \bar{w}_a + w'_a, \quad w_b = \bar{w}_b + w'_b, \quad T_a = \bar{T}_a + T'_a,
\]

where \( T_a \) is the air temperature. Formally, different notations should be used for balloon and atmospheric averages and deviations, but there is no need to do so if the wave is stationary and if the balloon response, as will be found below, has a spatial periodicity of one wavelength. Assuming that \( \bar{w}_a = 0 \) and that \( \bar{w}_b \) is approximately constant for this type of balloon during both the ascent and descent stages of sounding (de la Torre et al. 1996), we replace (3) in (2). If we separate the resulting equation in zero- and first-order parts, and then apply the Boussinesq condition (in computing rates of change of momentum, density variations are taken into account only when they give rise to buoyancy forces), we obtain

\[
\frac{1}{2} \rho_a C_d A_b \bar{w}_b^2 - g M_b + g \bar{\rho}_a \bar{V} = 0
\]
and

\[(M_b + C_m \rho_a \ddot{V}) \frac{d \dot{w}_b}{dz} + C_m \rho_a \ddot{w}_b \frac{\partial w'_a}{\partial z} = 0\]

\[\pm \rho_a \ddot{w}_b (w'_a - w'_b) + g(\ddot{w}_a V' + \rho'_a \ddot{V}).\]  

(5)

Combining the last two equations with

\[M_a = \rho_a \ddot{V}\]  

(6)

\[M'_a = \rho_a V' + \rho'_a \ddot{V}\]  

(7)

yields

\[(M_b + C_m M_a) \frac{d \dot{w}_b}{dz} = C_m M_a \ddot{w}_a \frac{\partial w'_a}{\partial z} + 2g \frac{(M_a - M_b)}{w_b} (w'_a - w'_b) + gM'_a.\]  

(8)

The last expression is valid for the ascent and the descent. The change in the limits of integration, balloon conditions, and the sign and magnitude of \(\dot{w}_b\) will yield different numerical results for both stages.

3. Solutions

We are seeking solutions for Eq. (8) in order to find \(w_b(z)\) for soundings performed between the earth's surface and the lower stratosphere (about 25 km). To accomplish this task, we need to know \(M_a(z)\), \(M'_a(z)\), and \(w'_a(z)\), which implies that we must give further details about the balloon, atmosphere, and wave conditions. We also have to specify the parameter values in order to find the results below.

The starting positions \(z_s\) for the ascent and the descent will respectively be at 0 and 25 km. We shall assume that the gas pressure inside the balloon equals the air pressure at any altitude, so \(\rho_a = \rho_p = \rho\) (hereinafter, the index \(g\) will refer to the gas). This may be considered to be an excellent approximation near the large opening in the bottom part of the balloon. A calculation of a representative atmospheric pressure decrease between the lower and upper extremes of the balloon shows that the superpressure in the top part will not exceed 0.1% of the atmospheric value (superpressure balloons typically have a 5% difference). The perfect gas law for the air and the balloon gas

\[p = \rho_a R_{a,g} T_{a,g}\]  

(9)

is taken to be valid, where \(R_{a,g} = R/W_{a,g}\), \(R\) being the universal gas constant and \(W_{a,g}\) the molecular weight of the air or the gas. The same law holds of course for the undisturbed values \(\rho_{a,g}\) and \(T_{a,g}\). The maximum volume of the considered balloons (typically 12,000 m³) is large enough to ensure that no gas is lost during the whole journey, so the variations in the volume depend essentially upon the pressure and the transport of heat through the skin. As this last parameter is usually unknown, we shall consider two opposite cases, which place the constraints for any real situation: perfect and null conductivity. In the first case the gas temperature reproduces exactly the external atmospheric temperature, so \(T_a = T_p = T\), and in the second one the gas undergoes an adiabatic evolution. We consider that the choice of an isothermal atmosphere (this implies that we are neglecting departures of no more than 15% from the constant value) will reveal the basic characteristics of the balloon's response and will simplify the interpretation of the results.

Let us consider a monochromatic stationary wave

\[w'_a = \beta \sin kz.\]  

(10)

From the energy equation for internal gravity waves including a linear damping rate \(a\) (Lindzen 1990), keeping in mind that in the Boussinesq approximation temperature and density variations are in phase opposition and taking into account an isothermal atmospheric profile, we may write

\[\rho'_a = \rho_a \frac{\zeta}{1 + (a^2/\omega^2)^{1/2}} \sin(kz + \theta)\]  

(11)

with

\[\zeta = \frac{(\gamma - 1)g}{\gamma \omega R_a T_a} \beta\]  

(12)

\[\theta = \arctan \frac{\omega}{a},\]  

(13)

where \(\gamma = 1.4\) is the ratio of specific heats for the air, \(k = 2\pi/\lambda\), and \(\omega = 2\pi/\tau\), \(\lambda\) and \(\tau\) being, respectively, the wavelength and the period of the wave. Recent studies (see, e.g., Fritts 1989; Walterscheid and Schubert 1990) indicate that diverse mechanisms may strongly limit the growth of the velocity amplitude of gravity waves with height. To avoid unnecessary complications in the derivation and interpretation of results, we shall take a constant value for \(\beta\) in the region of interest. In the appendix it is shown that representative values of \(a\) are about one or two orders of magnitude smaller than \(\omega\) (we acknowledge an outline of this proof to an unknown referee). As we may then consider \(a/\omega \ll 1\), Eq. (11) will be replaced by

\[\rho'_a = \rho_a \zeta \cos kz.\]  

(14)

Notice that Eqs. (10), (12), and (14) are the same expressions that would be obtained from the polarization relations by Hines (1960), where the atmosphere is taken to be isothermal and the waves are assumed to occur adiabatically.

The typical parameter values that will be used below to find the results are given in Table 1. They have been selected in accordance with the Ports sounding campaign (de la Torre et al. 1996), which was supported by the French Balloon Programme of the Centre National d'Etudes Spatiales, and consisted of the launching of four balloons near the Andes Mountains (32°S, 68°W) in 1990. The open stratospheric balloons are nearly spherical, so \(C_m = 1/2\). For the given values of
Table 1. Parameter values for a typical sounding.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ) (cm s(^{-1}))</td>
<td>10.0</td>
</tr>
<tr>
<td>( \lambda ) (km)</td>
<td>2.00</td>
</tr>
<tr>
<td>( \tau ) (h)</td>
<td>18</td>
</tr>
<tr>
<td>( W_a )</td>
<td>29.0</td>
</tr>
<tr>
<td>( W_g )</td>
<td>2.02</td>
</tr>
<tr>
<td>( M_a ) (kg)*</td>
<td>50.0/5.00</td>
</tr>
<tr>
<td>( M_g ) (kg)*</td>
<td>150/105</td>
</tr>
<tr>
<td>( \omega_g ) (m s(^{-1}))*</td>
<td>7.00/-3.00</td>
</tr>
</tbody>
</table>

* Values separated by the slash refer respectively to the ascent and the descent.

where \( F \) is found from the condition \( w_b'(z_d) = 0 \). The parameter values for the ascent and the descent that have been calculated according to the data of Table 1 are exhibited in Table 2.

Notice that if \( F \neq 0 \) the exponential term of Eq. (21) reflects the brief transitory response of the balloon after the start of any of both stages. The value of \( F \) could become more significant if \( k z_s \approx \pi /2 + j \pi \) with \( j \in \mathbb{Z} \). For the parameter values that have been found, the above equation may be rewritten as

\[
w_b'(z) = \beta \sin(kz).
\]

This means that the balloon instantaneously reproduces the wave velocity variations found along its ascensional or descendent path. The density has no effect on the balloon because the latter always expands or contracts so as to counterbalance the density variations of the mean atmosphere and the wave, keeping the mass of displaced air constant [see Eqs. (15) and (17)]. Notice also that we did not need to specify the temperature of the undisturbed atmosphere to find the above expression.

An interesting feature is that an expression may be found in the present case for the drag coefficient. In fact, taking into account that the balloon is spherical and that inside and outside the balloon the pressure and the temperature are identical, bearing in mind Eqs. (4), (9) for air and gas, and (15), we obtain that for fixed balloon parameters

\[ C_d \propto \rho_a^{-1/3}. \]

Although we do not need to know this quantity to find the above solutions, the result might be useful in other problems.

b. Null conductivity

The gas inside the balloon (usually H\(_2\)) now follows the adiabatic equation

\[
\frac{p}{\rho_g^2} = \text{const.}
\]

If we differentiate with respect to \( z \) and equate the relative pressure variation of the gas thus obtained with

Table 2. Perfect conductivity: calculated parameter values for a typical sounding.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q ) (m(^{-1}))</td>
<td>0.446/0.513</td>
</tr>
<tr>
<td>( S ) (m(^{-1}))</td>
<td>2.21 \times 10^{-3}/8.00 \times 10^{-4}</td>
</tr>
<tr>
<td>( F ) (cm s(^{-1}))</td>
<td>0.0209/0.0456</td>
</tr>
</tbody>
</table>

* Values separated by the slash refer respectively to the ascent and the descent.
that of the air, which might be found from the perfect gas law and the hydrostatic condition

$$-\frac{dp}{dz} = \rho_a g,$$  \hspace{1cm} (24)

we may write

$$\frac{d\bar{\rho}_g}{\bar{\rho}_g} = -\frac{g}{\gamma R_a T_a} \frac{dz}{dz}.$$  \hspace{1cm} (25)

From the integration of the last equation we obtain

$$\bar{\rho}_g = \rho_{g0} e^{-gz/\gamma R_a T_a},$$  \hspace{1cm} (26)

where \( \rho_{g0} \) is the gas density at \( z = 0 \). From the hydrostatic condition and the perfect gas law for the air we obtain

$$\bar{\rho}_a = \rho_{a0} e^{-gz/\gamma R_a T_a}$$  \hspace{1cm} (27)

with \( \rho_{a0} \) the air density at \( z = 0 \). Moreover, if we consider that the gas has been left in the balloon enough time before launch to ensure that it will initially have the same temperature as the air,

$$M_a = \rho_a \bar{V} = \rho_a \frac{M_g}{\rho_g} = M_{a0} e^{-az}$$  \hspace{1cm} (28)

with

$$M_{a0} = M_g \frac{W_a}{W_g}$$  \hspace{1cm} (29)

$$\alpha = \frac{(\gamma - 1) g}{\gamma R_a T_a}.$$  \hspace{1cm} (30)

From Eq. (23)

$$\frac{p'}{\bar{p}} = \gamma \frac{\rho'_a}{\rho_a},$$  \hspace{1cm} (31)

and being \( p' \) of second order because \( T'_a \) and \( \rho'_a \) are in phase opposition, we conclude that

$$V' = 0,$$

so

$$M'_a = \rho'_a \bar{V} = \rho'_a \frac{M_g}{\rho_g} = \rho_{a0} \xi \cos k z \frac{M_g}{\rho_g} = M_{a0} \xi \cos k z.$$  \hspace{1cm} (32)

Contrarily to the first case, we may foresee that the atmospheric density variations now will become significant, because Eqs. (28) and (32) show that the volume variation does not counterbalance the changes in the density, so as to keep the mass of displaced air constant.

With the aid of expressions (10), (14), (28), and (32), Eq. (8) may now be written in the following form:

$$\frac{dw'_b}{dz} + \frac{2g}{w_b^2} \frac{M_{a0} e^{-az} - M_b}{0.5(M_{a0} e^{-az}) + M_b} w'_b = -\frac{2g \beta}{w_b^2} \frac{M_{a0} e^{-az} - M_b}{0.5(M_{a0} e^{-az}) + M_b} \sin k z$$

$$+ \left( \frac{g \xi}{w_b} + \frac{k \beta}{2} \right) \frac{M_{a0} e^{-az}}{0.5(M_{a0} e^{-az}) + M_b} \cos k z,$$  \hspace{1cm} (33)

and the general integral is

$$w'_b(z) = \left[ \int_{z_0}^{z} \left( e^{\xi (2N_1 e^{-az} - N_2)} \sin k z + (e^{\xi (2N_1 e^{-az} + N_2)} - N_2) e^{-az} \right) dz \right] e^{-az}$$

$$+ w'_b(z_0) \left( \frac{e^{-az_1}}{N_1 e^{-az_1} + N_2} \right) e^{az} (N_1 e^{-az} + N_2)^{3/\alpha}$$  \hspace{1cm} (34)

with

$$\epsilon = \frac{2g}{w_b^2}$$  \hspace{1cm} (35)

$$N_1 = \frac{0.5 M_{a0}}{0.5 M_{a0} + M_b}$$  \hspace{1cm} (36)

$$N_2 = \frac{M_b}{0.5 M_{a0} + M_b}.$$  \hspace{1cm} (37)

Notice that \( N_1 + N_2 = 1 \) and that they have been defined as dimensionless parameters to avoid the need of carrying unnecessary mass units.

Equation (34) has no general analytical solution. The homogeneous part of the solution to Eq. (33) is given by the last term of Eq. (34), so

$$w_{bH}(z) = G e^{e^{az-az}} \left( \frac{N_1 e^{-az} + N_2}{N_1 e^{-az} + N_2} \right)^{3/\alpha},$$  \hspace{1cm} (38)

where \( G \) is found from the condition \( w'_b(z_0) = 0 \). If Eq. (33) was of the type

$$\frac{dw'_b}{dz} + L_1 w'_b = L_2 \sin k z + L_3 \cos k z$$  \hspace{1cm} (39)
with $L_1$, $L_2$, and $L_3$ constants, then the particular solution would be [see Eqs. (18) and (21)]

$$w_{bp}(z) = A \sin(kz + \varphi), \quad (40)$$

where

$$A = \left( \frac{L_2^2 + L_3^2}{L_1^2 + k^2} \right)^{1/2} \quad (41)$$

$$\varphi = \arctan \left( \frac{L_1 L_3 - k L_2}{L_1 L_2 + k L_3} \right). \quad (42)$$

We try equivalent expressions for the present case, that is to say

$$w_{bp}(z) = A(z) \sin(kz + \varphi(z)) \quad (43)$$

with

$$A(z) = \left[ \beta^2 \left( \frac{2N_1 e^{-\alpha z} - N_2}{N_1 e^{-\alpha z} + N_2} \right)^2 + \left( \frac{k \beta}{\epsilon} \right)^2 \left( \frac{N_1 e^{-\alpha z}}{N_1 e^{-\alpha z} + N_2} \right)^2 \right]^{1/2} \quad (44)$$

$$\varphi(z) = \arctan \left[ \frac{\left( \frac{k \beta}{\epsilon} \right)^2 \left( \frac{N_1 e^{-\alpha z} - 2N_1 e^{-2\alpha z} - N_2}{N_1 e^{-\alpha z} + N_2} \right) - \frac{k \beta}{\epsilon} (2N_1 e^{-\alpha z} - N_2)}{\beta \left( \frac{2N_1 e^{-\alpha z} - N_2}{N_1 e^{-\alpha z} + N_2} \right)^2 + \frac{k}{\epsilon} \left( \frac{N_1 e^{-\alpha z}}{\epsilon} \right)^2} \right]. \quad (45)$$

According to the considered conditions $k \ll \epsilon$, and we get

$$A(z) = \left[ \beta^2 + \left( \frac{\bar{w}_b}{N_1} \right)^2 \left( \frac{N_1 e^{-\alpha z}}{2N_1 e^{-\alpha z} - N_2} \right)^2 \right]^{1/2} \quad (46)$$

$$\varphi(z) = \arctan \left[ \frac{\bar{w}_b N_1 e^{-\alpha z}}{\beta (2N_1 e^{-\alpha z} - N_2)} \right]. \quad (47)$$

For $\alpha, k \ll \epsilon$, which applies to our description, the particular solution satisfies Eq. (33). The parameter values for the ascent and the descent calculated according to the data of Table 1 are given in Table 3. We have taken for the isothermal atmosphere $T_a = 250$ K.

A brief transient response also occurs in this case when $G \neq 0$. In our example $\varphi(z)$ is not located in the vicinity of $0^\circ$, so the value of $G$ becomes more significant than the value of $F$ in the first case. However, the contribution of the homogeneous part is again negligible. The above results are not as straightforward as in the first case and they need some further discussions.

For $\varphi = 0^\circ$ there is no phase difference with the wave's velocity, and for $\varphi = 90^\circ$ the balloon follows the density oscillations. Equation (47) shows that the departure of $\varphi$ from $0^\circ$ will increase, as expected, with stronger $\beta$ and weaker $\beta$. Notice that $N_1$ and $N_2$ play an important role in the value that $\bar{w}_b$ acquires, so we cannot draw any conclusions about the dependence on these parameters unless we have an experimental curve on how the three are linked for this type of balloon. From (46) we can see that, due to the density oscillations, the response of the balloon is always larger than the wave's velocity amplitude, but we are again not able to discuss the relation with $N_1, N_2$, and $\bar{w}_b$.

In Figs. 1 and 2 we have respectively shown the response of the balloon during the ascent and the descent according to Eqs. (43), (46), and (47), including the air vertical velocity and density oscillations as a background reference. We observe considerable differences with the first case because the buoyancy variations drastically affect the response of the balloon. Variable amplitude and phase with height during both stages become evident. It may be seen that the evolution of the response during the ascent is not opposite, but similar, to the one for the descent. In Figs. 3 and 4 we have plotted the amplitude and phase of the balloon's response for both stages. Notice that the two parameters are always increasing functions of time and that the variation of $\varphi$ is larger for the descent than for the ascent. At first sight some of these facts seem to be contradictory, because for decreasing air density oscillations with height one would expect the amplitude and phase to be reduced during the ascent and to vary in the opposite sense for the descent. It is also interesting that $A$ may become much larger than $\beta$. The clue to these matters lies in the factor

$$\frac{N_1 e^{-\alpha z}}{2N_1 e^{-\alpha z} - N_2} = \frac{0.5M_a}{\bar{M}_a - M_b}, \quad (48)$$

because for the ending part of both stages $\bar{M}_a \rightarrow M_b$. However, $A$ and $\varphi$ in Eqs. (46) and (47) will not grow out of bounds for $\bar{M}_a = M_b$, because in that case we should consider Eqs. (44) and (45), as the terms containing $k/\epsilon$ would become significant when compared with those without it. It should be also taken into ac-
Table 3. Null conductivity: calculated parameter values for a typical sounding.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{\alpha} ) (m(^{-1}))</td>
<td>0.0400</td>
</tr>
<tr>
<td>( \alpha (\text{m}^{-1}) )</td>
<td>3.90 \times 10^{-5}</td>
</tr>
<tr>
<td>( \epsilon (\text{m}^{-1}) )*</td>
<td>0.400/2.18</td>
</tr>
<tr>
<td>( \delta_1 )</td>
<td>0.705/0.255</td>
</tr>
<tr>
<td>( \delta_2 )</td>
<td>0.295/0.745</td>
</tr>
<tr>
<td>( G (\text{cm s}^{-1}) )*</td>
<td>-17.7/2.09</td>
</tr>
</tbody>
</table>

* Values separated by the slash refer respectively to the ascent and the descent.

count that the perturbation method we have applied would break down if our solutions yielded \( w_b \) on the order of or larger than \( w_k \). The reason for which the \( M_a \rightarrow M_b \) effect acts on the density but not on the velocity part becomes clear in (33). The second term on the left-hand side may be thought of as being due to a frictional force, and the right-hand side may be interpreted as an oscillatory forcing. The friction term, which includes \( w_k \), tends to 0 for \( M_a \rightarrow M_b \) in the same way that the first term on the right-hand side, which is linked to \( w_k \), does. Both expressions contain the same factor with \( M_a \) and \( M_b \) because the drag acts on the relative velocity. However, the term related to the density in that equation does not tend to 0 because it is proportional to \( M_a \). Therefore, when a balloon is ending a stage and \( M_a \) is approaching \( M_b \), the damping effect of the drag is reduced, and one might observe significant oscillations caused by the density variations. If we notice that \( Q \rightarrow 0 \) when \( M_a \rightarrow M_b \), some of the above arguments also may be applied for perfect conductivity in Eq. (18) but in that case there are no density effects.

One cause for the difficulty in finding an exact solution to the present problem is the interaction of three very different space scales. The parameter \( 1/\alpha \) defines an atmospheric thermodynamic scale, \( 2\pi/k \) gives a typical wave distance, whereas \( 1/\epsilon \) takes part in the definition of a kinematic scale on which transients act.
stratospheric balloon in the presence of monochromatic, inertia-gravity waves during its journey. The mass of displaced air is a fundamental parameter of the problem and it is a function of the heat exchanged between the balloon and the surrounding air. As this last physical magnitude is usually unknown, we focused on two extreme cases, perfect and null balloon thermal conductivity, which place the limits for any real situation. In the first case the balloon velocity variations sharply follow the fluctuations in the wind velocity, but in the second one the density oscillations become very important in the resulting response of the balloon. According to the height and the conductivity of the balloon, its phase difference with the wave velocity may vary in almost the whole range between 0° and 90°, so the air velocity oscillations cannot be directly inferred from the rate of ascent of these balloons. Therefore, a sound knowledge of the balloon’s conductivity or accurate measurements of the balloon’s vertical position and the air velocity relative to the gondola become essential for an adequate interpretation of data. For a further analysis of the balloon-response dependence on the parameters, it would be also convenient to obtain experimental curves of \( \bar{w}_b \) against \( N_1 \) and \( N_2 \). The present study and the reference of de la Torre et al. (1996) would also imply that open stratospheric balloons behave more adiabatic than perfect conducting. The dependence of the drag coefficient on air parameters and on the heat exchange of the balloon with the surroundings was also outlined.

APPENDIX

An Estimation of the Wave Energy Damping Rate

Let us consider

\[
\frac{dE}{dt} = -aE
\]  
(A1)

\[
E = \frac{1}{2} \rho_a \left( \frac{\beta}{\cos \psi} \right)^2,
\]

(A2)

where \( E \) is the mean total energy of the wave and \( \psi \) the angle of wave propagation with respect to the ground. From Eq. (27) we know that

\[
\rho_a = \rho_a e^{-z/H},
\]

(A3)

where \( H \) is the scale height. We may assume that the wave vector direction remains constant in the range 0–25 km, and then (A1), (A2), (A3), and the constancy of \( \beta \) yield

\[
\frac{dE}{dt} = \frac{\partial E}{\partial z} \frac{dz}{dt} = -\frac{E}{H} c = -aE,
\]

(A4)

where

\[
c = \frac{\omega}{k}
\]

(A5)
is the vertical phase speed. We get
\[ a = \frac{\omega}{kH} \quad (A6) \]
and according to the values of the present problem
\[ \frac{a}{\omega} \approx 0.04 \ll 1. \]
We finally notice that \( a \) is at least about one order of magnitude smaller than \( \omega \) if \( \lambda < H \).

**REFERENCES**


