Improved Radar Ice Water Content Retrieval Algorithms Using Coincident Microphysical and Radar Measurements

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ABSTRACT

Airborne radar reflectivity measurements at frequencies of 9.6 and 94 GHz, with collocated, in situ particle size distribution and ice water content measurements from the Cirrus Regional Study of Tropical Anvils and Cirrus Layers (CRYSTAL) Florida Area Cirrus Experiment (FACE) in Florida in July 2002, offer one of the first opportunities to evaluate and improve algorithms for retrieving ice water content from single-wavelength spaceborne radar measurements. Both ice water content and radar reflectivity depend on the distribution of particle mass with size. It is demonstrated that single, power-law, mass dimensional relationships are unable to adequately account for the dominating contribution of small particles at lower reflectivities and large particles at higher reflectivities. To circumvent the need for multiple, or complex, mass dimensional relationships, analytic expressions that use particle ensemble mean ice particle densities that are derived from the coincident microphysical and radar observations are developed. These expressions, together with more than 5000 CRYSTAL FACE size distributions, are used to develop radar reflectivity–ice water content relationships for the two radar wavelengths that appear to provide improvements over earlier relationships, at least for convectively generated stratiform ice clouds.

1. Introduction

Spaceborne cloud radars offer an opportunity to quantitatively evaluate representations of clouds and cloud processes in global climate models and also provide global surveys of vertical profiles of cloud bulk microphysical properties. This information, used in climate models, can reduce the uncertainty in model predictions of the earth’s current and future climates (Stephens et al. 2002). Proper retrieval algorithms, however, are needed to reduce the equivalent radar reflectivity factor ($Z_e$; for definition see Smith 1984) to fundamental physical variables that are used in, and central to, climate model simulations. If spaceborne radar data are to reach their potential usefulness, a clear understanding of the factors influencing $Z_e$, and the range of validity and error bars associated with retrieval algorithms, are needed.

Ice water content (IWC), a variable that is central to cloud bulk properties and is indirectly important to the earth’s radiation budget, is directly prognosticated by modern climate models (Tiedtke 1993). In our study, we demonstrate the use of radar reflectivity measurements to accurately retrieve IWC and to improve understanding of how $Z_e$ and IWC are related. Ideally, $Z_e$–IWC relationships would be developed on the basis of direct, coincident measurements of $Z_e$ from radar and from IWC measured in situ from an aircraft located

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within or near the radar beam volume. Virtually no relationships have been developed in this way, because coordinated radar and in situ measurements are difficult to make, and instruments that directly measure IWC have only recently become available. Furthermore, homogeneous cloud layers, in which the 10\(^6\), or so, possible difference between radar and in situ probe-sampling volumes is unimportant, are difficult to find. This is especially true when considering the need to cover the range of cloud scales from single cells to mesoscale convective complexes and the microphysical and other properties that are introduced by regional, seasonal, and temperature variations.

Almost all relationships between \(Z_e\) and IWC have been developed from calculations. This has probably been the simplest way to approach this problem because multiple relationships can be developed from measurements over a broad range of cloud types and conditions. Both factors depend upon the following particle size: for IWC, the particle size distribution (PSD) and the particle mass dimension relationship \(m(D)\), and for \(Z_e\), the PSD and square of the particle mass with size \(m^2(D)\). Until recently, because the PSD and \(m(D)\) relationship data were disparate and sparse, certain assumptions were required in their calculation. This uncertainty is partly responsible for the development of a number of \(Z_e\)–IWC relationships containing significant differences (Sassen et al. 2002), even though most are developed for the same cloud type and geographical domain. Recent progress has allowed PSD measurements over a wide range of sizes. These data, used with direct measurements of IWC, have led to the development of power-law \(m(D)\) relationships for particle ensembles (PSD) that are measured in situ in the same volume (Heymsfield et al. 2004). There remain a number of considerations in developing \(Z_e\)–IWC relationships that are general enough for global radar retrievals of IWC. Without knowledge of the characteristic size of the particle population (i.e., median volume or mass diameter), some method to specify the characteristic particle size is needed (Atlas et al. 1995; Liu and Illingworth 2000; Matrosov et al. 2002). Another concern is the response of a particular wavelength radar to particles of a size that are comparable to its wavelength (Mie scattering; Matrosov 1998). The ice bulk density must be known for all particles. A fourth issue, identified here, is whether an \(m(D)\) power-law-type relationship that is derived from a combination of PSD and IWC measurements, or from earlier \(m(D)\) measurements developed from surface observations, can be used to derive both IWC and \(Z_e\). The development of \(m^2(D)\) relationships is problematic, because, although IWC measurements may place constraints on the integral of \(m(D)\) across the size distribution, there is no simple way to place constraints on the \(m^2(D)\) relationship, other than through the use of collocated \(Z_e\) measurements, with the attendant mismatch in radar versus in situ probe-sampling volumes.

To evaluate whether \(m(D)\) relationships can be used to accurately derive both the IWC and \(Z_e\), this study uses measurements at two radar wavelengths, together with collocated in situ measurements of the PSD and direct IWC measurements. Section 2 develops analytic relationships between IWC or \(Z_e\) and the PSD, and identifies the crucial variables that are not well known from observations. Section 3 describes the dataset that is used, and section 4 presents the observations. Parameterizations are developed in section 5. The results of the study are summarized and conclusions are drawn in section 6.

2. Analytic \(Z_e\)–IWC relationships

We consider analytic relationships between IWC or \(Z_e\) and the properties of the PSD, and identify issues that are related to the calculation of each of these variables, thereby facilitating the interpretation of the results presented in section 4.

The IWC can be derived from

\[
\text{IWC (g m}^{-3}\text{)} = 10^6 \sum_{i=1}^{n} N_i(D) D_i m_i(D),
\]

where \(N_i(D)\) is the size-dependent concentration (in cgs units) and \(n\) is the number of size bins. In integral form,

\[
\text{IWC (g m}^{-3}\text{)} = 10^6 \int_0^{D_{\text{max}}} N(D) m(D) \, dD,
\]

where \(N(D)\) is the concentration of particles per unit diameter interval with maximum size \(D\). The mass \(m\) of an ice particle (in grams) of the maximum dimension \(D\) can be represented by one of the following:

\[
m = \frac{\pi}{6} \rho_l D_{\text{ml}}^3 = \frac{\pi}{6} D^3 \rho_e = aD^b,
\]

where \(\rho_l\) is the density of liquid water, \(D_{\text{ml}}\) is the melted liquid equivalent diameter, \(\rho_e\) is the particle effective density (see Heymsfield et al. 2004a), \(D\) is the maximum particle dimension, and \(a\) and \(b\) are coefficients in a power-law \(m(D)\) relationship. The values of \(a\) and \(b\) can be specific to given ice particle habits, or they may be general for particle populations (e.g., Brown and Francis 1995, hereinafter BF; Mitchell 1996; Heyms-
field et al. 2004). Inserting the right two equations of Eq. (3) into Eq. (1) yields the relation

$$\text{IWC (g m}^{-3}) = \frac{\pi}{6} \times 10^6 \rho_{\text{em}} \sum_{i=1}^{n} N_i(D_i)D_i^3$$

$$= a \times 10^6 \sum_{i=1}^{n} N_i(D_i)D_i^b,$$

where $\rho_{\text{em}}$ is the population mean mass-weighted ice particle effective density (Heymsfield et al. 2004). Matrosov (1999) has also estimated ensemble mean ice particle densities.

If $N(D)$ is represented by a single gamma distribution of the form

$$N(D) = N_0 D^\mu e^{-\lambda D},$$

where $N_0$ is a parameter of the size distribution, $\lambda$ is the slope, and $\mu$ is the dispersion, then

$$\text{IWC} = \frac{\pi}{6} \times 10^6 N_0 \frac{\Gamma(4 + \mu)}{\lambda(4 + \mu)}.$$  

Alternatively,

$$\text{IWC} = a \times 10^6 N_0 \Gamma(b + 1 + \mu) \lambda(b + 1 + \mu).$$

In this dataset, fewer than 5% of the size distributions were bimodal, and, therefore, a single gamma distribution adequately describes the PSD. Equations (6) and (7) integrate the equation from 0 to $\infty$ and ignore the truncation effect on the size distribution at $D_{\text{max}}$—here a reasonable assumption. The right side of Eq. (7) ignores the part of the mass dimension relationship where the mass exceeds the mass of solid ice spheres. This occurs in sizes below about 100 $\mu$m, which, in the situations considered here, amounts to a mean overestimate of less than 2% of the IWC.

An expression that is similar to Eq. (7) can be generated through a gamma distribution that represents the PSD in terms of the melted equivalent diameter, the second-from-the-left term in Eq. (3). This method implicitly assumes values for coefficients $a$ and $b$ but, because they are incorporated into the development of the size spectrum $N(D_{\text{ml}})$, the original PSD may only be traced backward through several steps, and may not be retrievable if several different ice crystal habits are involved; the method is not used here.

If we assume that the ice scatterers are in the Rayleigh regime where the wavelength $\Lambda \gg D$—valid here for the radar frequency of 9.6 GHz ($\Lambda = 3.1$ cm), but not generally at 94 GHz ($\Lambda = 0.32$ cm)—the equivalent radar reflectivity for the particle population can be represented by

$$Z_e (\text{mm}^6 \text{ m}^{-3}) = \left(1.09 \frac{K_i}{K_w}\right)^2 \times 10^{12} \sum_{i=1}^{n} N_i(D_i)D_{\text{ml}}(\text{cm})^6,$$

where the factor 1.09 in the bracket represents the ratio of the density of liquid water to solid ice, and $K_i/K_w$ converts the radar reflectivity with respect to solid ice to the equivalent radar reflectivity. The factor of $10^{12}$ is a conversion from cm$^2$ to mm$^6$ m$^{-3}$. It follows from Eq. (3) that for an individual particle of size $i$

$$D_{\text{ml}}(\text{mm}) = \left(\frac{\rho_{\text{em}} Z_e}{\rho_0}\right)^2 D_i(\text{cm})^6 \times 10^6$$

$$= \left(\frac{6}{\pi \rho_0}\right)^2 \times 10^6 \rho_{\text{ml}} D_i(\text{cm})^{2b},$$

then

$$Z_e = \left(\frac{\rho_{\text{em}} Z_e}{\rho_0}\right)^2 \left(1.09 \frac{K_i}{K_w}\right)^2 \times 10^{12} \sum_{i=1}^{n} N_i D_i^6$$

$$= \left(\frac{6}{\pi \rho_0}\right)^2 \times 10^6 \rho_{\text{ml}} \frac{N_i \Gamma(7 + \mu)}{\lambda(7 + \mu)} \left(1.09 \frac{K_i}{K_w}\right)^2 n_i D_i^{2b}.$$  

The term $\rho_{\text{em}} Z_e$ is the reflectivity-weighted population mean density, and follows from the method used in Heymsfield et al. (2004) to represent the mass-weighted population mean density. For gamma-type PSDs,

$$Z_e = \left(\frac{\rho_{\text{em}} Z_e}{\rho_0}\right)^2 \times 10^{12} \left(1.09 \frac{K_i}{K_w}\right)^2 n_i \frac{N_i \Gamma(7 + \mu)}{\lambda(7 + \mu)}$$

$$= \left(\frac{6}{\pi \rho_0}\right)^2 \times 10^6 \rho_{\text{ml}} \frac{N_i \Gamma(2b + 1 + \mu)}{\lambda(2b + 1 + \mu)}.$$  

Equation (11) ignores the effects of truncation of the size distribution at some maximum particle size. This effect is minor for the cases investigated. The right side of the equation again ignores the part of the mass dimension relationship where the mass exceeds the mass of solid ice spheres.

For Mie scattering, as in the 94-GHz radar measurements, the Bohren and Huffman (1983) equations for spheres can be used in conjunction with particle density information to produce approximate estimates of the
backscatter coefficients for the larger particles, although more elaborate but unproven methods have been developed (see Mishchenko et al. 2000). We now evaluate the relationship between IWC and \( Z_e \) by dividing Eq. (6) by Eq. (11), leading to

\[
\frac{\text{IWC}}{Z_e} (\text{g mm}^{-6}) = \left[ \frac{\pi \lambda^4}{6 \times 10^6 (1.09 K/K_w)^2} \right] \frac{\rho_{cm}}{(\rho_{Z/e} \theta_0)^2} \times \left[ \frac{\Gamma(4 + \mu)}{\Gamma(7 + \mu)} \right] \\
\times \left[ \frac{\Gamma(b + 1 + \mu)}{\Gamma(2b + 1 + \mu)} \right]. \tag{12a}
\]

A relationship that is similar to Eq. (12b) has been developed recently by Mitchell et al. (2004). Approximate values for \( \rho_{cm} \) have been deduced from direct measurements of the IWC (Heymsfield et al. 2004). However, \( \rho_{Z/e} \) cannot be deduced directly from the IWC measurements, because \( Z_e \) at the aircraft position has rarely been measured and it is unclear whether the assumption of the \( m(D) \) relationship follows a power law over all sizes of a PSD. This issue is addressed in section 4.

3. Data

Coincident airborne radar and in situ microphysical measurements that were collected during the Cirrus Regional Study of Tropical Anvils and Cirrus Layers (CRYSTAL) Florida Area Cirrus Experiment (FACE) in southern Florida during July 2002, can be used to test whether we can accurately predict both IWC and \( Z_e \) and if the two parameters are related by Eq. (12). In 41 instances, spanning the temperature range from \(-25^\circ \) to \(-52^\circ \) on 19, 23, 28, and 29 July, vertical profiles of \( Z_e \) were obtained at frequencies of 9.6 (wavelength \( \lambda = 3.1 \) cm) and 94 GHz (\( \lambda = 0.32 \) cm) from the National Aeronautics and Space Administration (NASA) ER-2 aircraft flying at a height of about 20 km. At the same time, the University of North Dakota Citation aircraft, directly underneath the ER-2, made ice cloud microphysical measurements. These instances were nearly coincident—the mean horizontal spatial distance between the ER-2 and Citation was 0.69 \( \pm \) 1.13 km, with a maximum difference of 4 km, and the mean time difference between the remote sensing and in situ measurements at these positions was 33 \( \pm \) 52 s, with a maximum of 197 s. To minimize the ER-2 and Citation spatial and temporal sampling differences, \( Z_e \) values were obtained at two points within a few seconds before and two points within a few seconds after what was deemed to be the Citation latitude, longitude, and height. Therefore, for each PSD and IWC value, there are four corresponding values for the radar reflectivity. Heights were collocated to within 37.5 m, which was the vertical resolution of each radar. At the Citation height, the minimum detectable \( Z_e \) of the 9.6-GHz radar is about \(-20 \text{ dBZ} \) (Heymsfield et al. 1996), and for the 94-GHz cloud radar it is 5–10 dB lower (Li et al. 2004).

Direct measurements of the IWC were obtained from a counterflow virtual impactor (CVI) probe on the Citation. These measurements are composed of sizes above about an 8-\( \mu \)m diameter and of IWCs above 0.01 g m\(^{-3}\). The uncertainty in the condensed water content is about 11% at 0.2 g m\(^{-3}\), increasing from baseline errors to 23% at 0.01 g m\(^{-3}\). As the IWC increases above about 1.0 g m\(^{-3}\), the CVI measurement becomes saturated. There are also instances in high-liquid-water regions in which the exhaust port of the CVI may become iced, reducing flow through the probe and the IWC measurement. Particle size distributions in nonuniform size bins covering the range from about 30 \( \mu \)m to above 1 cm in maximum dimension were obtained by two-dimensional imaging probes—a 2D-C (cloud probe), and a high-volume particle spectrometer (HVPS). The size distributions were converted to IWC values using Eqs. (1) and (3), and to \( Z_e \) values using \( m(D) \) relationships given in the next section. Qualitative information was obtained in sizes from about 2 to 50 \( \mu \)m from a forward-scattering spectrometer probe (FSSP), although breakup of particles in the probe’s inlet can lead to overestimates of the particle concentrations. Heymsfield et al. (2004) describe the development of appropriate density relations for the CRYSTAL FACE observations and how this information and the size distributions are used to derive IWC.

4. Observations

The primary focus of this section is to assess whether accurate calculations of the IWC and \( Z_e \) can be made from the PSD, given the same PSD and \( m(D) \) relationship. First, the observations and analysis of the 41 sets of collocated radar and IWC data are presented. The data are then analyzed and interpretations are drawn.

a. Data

The equivalent radar reflectivity factor used here ranged between \(-20 \text{ and } 12 \text{ dBZ} \) (Fig. 1a). With in-
Fig. 1. Coincident measurements from the ER-2 and Citation aircraft on 19, 23, 28, and 29 Jul: (a) radar reflectivities at two wavelengths from ER-2; (b) IWC measurements from CVI probe coincident with ER-2 radar reflectivity measurements at 9.6 GHz.
creasing $Z_e$, the $Z_e$ values at 9.6 GHz increased relative to those at 94 GHz, because Mie scattering effects progressively reduced $Z_e$ at 94 GHz relative to $Z_e$ at 9.6 GHz.

The measured IWC generally increased with $Z_e$ (Fig. 1b), although over the range of the measurements $Z_e$ and IWC exhibit considerable scatter, especially below −10 dBZ or 0.1 g m$^{-3}$, respectively. Approximately 40% of the CVI IWCs fall below 0.05 g m$^{-3}$; 30% fall in the range of 0.05−0.1 g m$^{-3}$; and 25% range between 0.1 and 0.5 g m$^{-3}$ (Fig. 2). Therefore, a wide range of IWC was sampled. Note that the horizontal bars representing the measured dBZs in Fig. 1b are the standard deviation of the dBZ values bounding the Citation location (section 3) rather than the range of values, which are omitted to improve the clarity of the plot.

The IWCs were calculated for each measured (binned) PSD by using the coefficients $a = 0.0061$ and $b = 2.05$ in the relationship $m = aD^b$ deduced for the Citation CRystal Face dataset (Heymsfield et al. 2004). It is found that in the large particle (2D probe) sizes (>50 μm), the mean of the ratio $r = \text{IWC(calculated)}/\text{IWC(CVI)} = 0.867$, and the median = 0.885 for the collocation periods. The value of $r$ is reasonable, given the possible contributions by small particles discussed in the appendix. Where the calculated IWC at several points exceeded 1 g m$^{-3}$, there may have been instances where the CVI exhaust port was blocked. The calculated IWC were 3 times those measured.

The calculated $Z_e$ values for both frequencies are too large by an average of 5 dB. For this reason, in the analysis that follows, we will assume that we have no a priori knowledge of the $a$ and $b$ coefficients. We use the IWC as determined by the CVI as a constraint to retrieve the $a$ and $b$ coefficients [from Eq. (4)].

Initially, we neglect the IWC that is contained in the small-FSSP-sized (<50 μm) particles. First, $b$ is taken to be a constant value within the previously established range from 1.8 and 2.2 (see Mitchell 1996). From Eq. (4), and for a given value of $b$, $a = \text{IWC(CVI)} \times 10^{-5}j\left(\sum_{i=1}^n N_iD_i^b\right)$. (When the implied densities exceed 0.91 g cm$^{-3}$, they are fixed at this value.) For each value of $b$,
the derived values of $a$ for the 41 PSD are averaged to obtain a single average value $\bar{a}$.

In Fig. 2, the IWCs that are measured by the CVI during the collocations have been sorted according to increasing value and are plotted with square symbols. The corresponding calculated IWC that are obtained by using values of $b$ from 1.8 to 2.2 in increments of 0.1, taking $\bar{a}$ as derived from above, are also plotted in the figure. Changing the coefficient $b$ to fall within the range from 1.8 to 2.2 has little effect on the calculated IWC, with the lower values of $b$ having slightly higher IWCs at lower IWCs and lower at the higher IWCs; the reverse is true for the higher $b$ coefficients. There is no way to assess which value or values of $b$ are correct from this analysis.

The four instances of high-CVI IWCs that are substantially below those derived from the PSDs in Fig. 2 could be biased low for reasons that are noted earlier.

b. Sensitivity to small ice particles

The IWCs in particles below the imaging probe’s size-detection threshold of about 50 $\mu$m may be an important contribution to the IWC because IWC = function($m$) = $f(D^2)$ (see the appendix, Fig. A2), but much less so to $Z_e$ because $Z_e$ = function($m^2$) = function($D^4$) (see the appendix, Fig. A3). Because small particles may affect the IWC calculation, they must, therefore, be considered because they will influence the deduced values of $\bar{a}$. If we take $f$ to be the fraction of the IWC in sizes >50 $\mu$m, then $\bar{a}$ is reduced by $f$ for given values of $b$.

To assess the contributions to the IWC and $Z_e$, by small particles, we use the Citation CRYSTAL FACE FSSP data. Unfortunately, the FSSP PSD is a combination of real data and artifacts produced by breakup, in unknown proportions. In the appendix, we estimate upper limits to the IWC and $Z_e$ contributions from small particles by assuming that all FSSP particles are real, and taking the mass of each particle to be the same as those of solid ice spheres. This analysis suggests that, on average, 90% of the IWCs could have been in large particles when the IWCs < 0.1 g m$^{-3}$, and 80% for larger IWCs. The actual IWC in large particles is expected to be greater for the larger IWCs, because the problem with artifacts in the FSSP data increases as particle sizes, and IWCs, increase. Furthermore, our estimates of IWC in larger particles from the density estimates in Heymsfield et al. (2004) is, on average, 0.87 (from earlier discussion), and the densities from this technique are relatively insensitive to the IWC contribution by the small particles for reasons cited in that article. We also show in the appendix that for $Z_e$ above $-20$ dB, the FSSP-sized particles add less than 0.1 dB to $Z_e$. In the discussion that follows, we evaluate the effects of $f$ values ranging from 0.6 to 1.0 on $\bar{a}$ and ignore the contributions by the FSSP particles on $Z_e$.

c. Sensitivity of IWC and $Z_e$ to mass dimension relationships

The radar reflectivities as calculated at 9.6 GHz—a frequency at which Mie scattering effects and attenuation by the ice between the radar on the ER-2 and the Citation location in cloud are minimal—are compared with the measured values for $b$ of 1.8 and 2.0 and $f$ ranging from 0.4 to 1.0 in Figs. 3a and 3b. For each point, the difference in the calculated and measured dBZ$_e$ values is taken. The data are sorted according to the measured reflectivities, with measured values shown with vertical lines at key points. The number of data points is about a factor of 4 times the number of collocations (41), because $Z_e$ values for two points either side of the Citation are usually used. This procedure avoids the possibility that the mean reflectivity values may not be representative of the actual values at the Citation location. Taking $f$ to lie somewhere in the range of 0.7–1.0, the calculated values of $Z_e$ are larger than those measured for all reflectivities. For $Z_e$ values near $-10$ dBZ$_e$, this difference is 3 and 6 dB for $b = 1.8$ and 2.0, respectively, assuming that $f = 0.7$. The calculated difference in $Z_e$ values is larger for values between 5 and 10 dBZ$_e$, where larger particles are present. The discrepancies between the observations and measurements point to the possibility of masses that are too large in large-particle sizes and, further, that the value of $b$ may change with increasing particle size. These points are elaborated upon below.

A similar set of calculations for a frequency of 94 GHz shows discrepancies that are similar to those found at 9.6 GHz, but the differences between the calculated and measured reflectivities are smaller, by about 2–3 dB, and there is little difference between the estimates when $b$ is 1.8 or 2.0 (Figs. 3c and 3d). Because weighting at this frequency as compared with 9.6 GHz is to smaller sizes because of Mie scattering effects, we surmise that there is a better match to the particle mass at the smaller sizes (but larger than the mass-weighted mean diameter).

Aspects of the discrepancies that are noted between the calculated and measured reflectivities, and the lack

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1 The IWC calculated for a given $\bar{a}$, $b$ pair (IWC$_{cal}$) to that measured by the CVI (IWC$_{meas}$) is derived for each of the 41 samples. The average of IWC$_{cal}$/IWC$_{meas}$ for the 41 samples is forced to be 1.0 by scaling $\bar{a}$.
Fig. 3. Difference between calculated and measured radar reflectivities for (top), (bottom) the two radar wavelengths, and for (left), (right) two powers of \( b \). The calculations are for fractions of particles \( \geq 50 \) \( \mu \text{m} \), ranging from 0.6 to 1.0. The abscissa represents increasing measured reflectivity, with dashed vertical lines showing key reflectivity divisions.
of such discrepancies between the calculated and measured IWCs, can be elucidated by examining the median mass or reflectivity-weighted diameters $D_{med}$, which can be calculated from the binned size distributions. These are diameters that split in half the distribution of IWC or $Z_e$ with size. We use the nomenclature $D_{mm}$ to represent $D_{med}$ for IWC and $D_{mZ}$ to represent $D_{med}$ for reflectivity. The basic idea is to show, using $D_{med}$, that successfully deriving the IWC does not necessarily imply success in deriving $Z_e$. The $D_{med}$ values are derived for each combination of $b$ and $f$ values.

In Fig. 4, we show the $D_{med}$ values for $b = 2.0$ and $f = 0.7$, plotted in terms of the calculated reflectivity (at 9.6 GHz) rather than the measured reflectivities, to produce results that provide insight into the dependencies noted in Fig. 3. The highest $D_{mm}$ values are only about 0.03 cm and are relatively constant with increasing $Z_e$. In contrast, the relative difference between $D_{mZ}$ and $D_{mm}$ becomes smaller with decreasing $Z_e$, although $D_{mZ}$ is always larger than $D_{mm}$. Direct measurements of the IWC, therefore, can only provide information on $peZ$ [Eqs. (10)–(12)] at low reflectivities, where there is the greatest scatter (see Fig. 1b). An $n(D)$ relationship that accurately predicts the IWC will also accurately predict $Z_e$ only for less than about $-10$ dBZ$_e$. A mass dimension relationship that works for IWC will, therefore, not necessarily be valid for $Z_e > -10$ dBZ$_e$, because IWC and $Z_e$ are weighted by different moments. The results shown in Fig. 4 take $f$ to be 0.7; larger values of $f$ would have caused larger discrepancies between $D_{mm}$ and $D_{mZ}$.

We now examine the significance of the overestimates in the calculated $Z_e$ values for higher reflectivities. Increases in $Z_e$ correspond to increases in $D_{mZ}$ (Fig. 4), and the $D_{mZ}$ values at higher reflectivities become larger. For gamma distributions, $D_{mm} = (b + 1 + \mu)/\lambda$ (Mitchell 1996) and $D_{mZ} = 2b + 1 + \mu/\lambda$. By examining the values for $\mu$ and $\lambda$ from the dataset, we find that $b$ probably decreases with increasing reflectivity and particle size. This result is demonstrated in Fig. 5, by comparing the difference between the calculated and measured $Z_e$ values for $b$ of 1.6 and 1.8, sorted along the abscissa by reflectivity, as in Fig. 3. It is noted in Fig. 5 that $b = 1.6$ provides a better estimate of $Z_e$ at the higher reflectivities than $b = 1.8$. Therefore, $b$ changes with increasing reflectivity and the mass dimension relationship is not represented well by a single power law. Experimentation with the various types of curves suggests that a good one to use is a gamma-type mass dimension relationship. It can be easily integrated.
Fig. 5. As in Fig. 3, except shown as the difference between the calculated and measured reflectivities at the two wavelengths, and for $b$ values of 1.6 and 1.8.
in combination with a gamma-type size distribution and fits the data reasonably well.

The discrepancies identified in sections 4a–c are summarized in Fig. 6. The solid lines in the figure show the mass dimension relationships deduced from the CVI data. They are obtained by plotting the coefficient a versus the power b for the mass dimension relationship that is deduced from the CVI and probe data, assuming 40%–100% of the mass to be in large particles detected by the 2D probe. As inferred from the FSSP data, for \( f = 0.8 \), the coefficients should lie above the bold solid line. The values for a and b that are deduced by Heymsfield et al. (2004) and BF lie close to it. Mitchell’s (1996) values for aggregates of side planes and bullets, shown by the large circle, lie somewhat below it. Values for a and b that match the radar data may also be deduced using PSD and Mie scattering calculations for the two-wavelength radar, and are given by the dashed lines in the figure. These estimates are essentially insensitive to small particle data and, at a frequency of 9.6 GHz, the Mie effects are negligible. For b at 1.6–2.2, the lines that are deduced from the radar data fall considerably below the thick solid line, although they approach it as values for b decrease. Figure 6 illustrates the different power-law mass dimension relationships for IWC and \( Z_e \).

d. Population mean reflectivity-weighted ice density

To circumvent the need for a complex \( m(D) \) relationship and to derive the population mean effective density \( \bar{\rho}_{em} \) [Eq. (6)], we use the method that is outlined in Heymsfield et al. (2004). This method, deriving \( \bar{\rho}_{em} \) from the ratio of the IWC to the sum of the spherical particle volumes of the particle populations in imaging probe and FSSP sizes, is used here to develop \( \bar{\rho}_{em} \) relationships for calculating \( Z_e \) from Eq. (11). The FSSP particles contribute insignificantly to these estimates because the technique uses the spherical particle volume and not their mass. Estimates of \( \bar{\rho}_{em} \) that are derived this way, and are represented in terms of the slope of the size distributions \( \lambda \) for coincident times, are given in Fig. 7a with a curve fitted to the results. This curve conforms closely to the relationship presented in Heymsfield et al. (2004) for the CRYSTAL FACE Citation dataset. Using the square of the particle vol-
ume with measured reflectivities at two wavelengths [Eq. (10)], we have derived $\bar{\rho}_e$ values and represented them in terms of $\lambda$, the slope of the size distribution, with curve fits indicated (Figs. 7b and 7c). The $\bar{\rho}_e$ values are lower than those for $\bar{\rho}_{\text{em}}$, because the weighting is to larger, lower-density particle sizes. The values of $\bar{\rho}_e$ at 94 GHz are artificially low for smaller values of $\lambda$ (larger particle sizes) to account for the Mie scattering effects, but its use produces accurate values of $Z_e$ without the need to employ Mie scattering calculations.

5. Parameterizations

Expressions presented in section 2, and the density formulations derived in section 4, can be used to develop $Z_e$ versus IWC relationships and related algorithms that are independent of the assumption of a power-law-type mass dimension relationship. The goal of this section is to derive such relationships and to assess whether they offer improvements over earlier relationships.

We first examine whether the complete set of in situ ice cloud data that are collected by the Citation aircraft during CRYSTAL FACE, and converted to IWC and $Z_e$ values using several approaches, are consistent with the measurements obtained during the ER-2/Citation collocations. We will assess consistency by examining whether the ratio IWC/$Z_e$ that is derived from this larger dataset conforms to the trends that are observed in the measurements.

In Fig. 8, the gray squares in each panel show the
Fig. 8. Estimation of the ratio of IWC to $Z_e$ as a function of the PSD slope $\lambda$. The gray squares in each panel are the measured values from section 4c, with $Z_e$ for 9.6 GHz. Black points show the ratio of (a) IWC(CVI) to $Z_e$ calculated from the PSD using $m(D)$ relationship from Heymsfield et al. (2004), (b), (c) the ratio of IWC and $Z_e$ derived from the $m(D)$ relationship, (c) the ratio of IWC and $Z_e$ using the new parameterization. (d)–(i) The ratio of IWC and $Z_e$ is derived from Eq. (14), using a number of earlier IWC–$Z_e$ relationships reported in Sassen et al. (2002).
measurements from section 4c, giving the ratio of the IWC from the CVI to the corresponding $Z_e$ value from the 9.6-GHz measurement at the aircraft position, plotted in terms of the slope $\lambda$ derived from the corresponding PSD. The ratio generally increases with increasing $\lambda$, signifying a shift to narrower size distributions where IWC becomes increasingly important relative to $Z_e$. There is obvious scatter noted in this relationship.

The black dots in Fig. 8a show the ratio of CVI IWC to $Z_e$ calculated from the PSD, assuming Rayleigh scatterers and the Citation mass dimensional relationship from Heymsfield et al. (2004), where $b = 2.1$. In this figure, a dataset consisting of 5700 5-s binned-averaged size distributions from the Citation particle probes during CRYSTAL FACE are used. From Fig. 3, we find that dBZ, is up to 4 dB too high, $Z_e$ is up to 2.5 too large, and IWC/$Z_e$ is up to 40% of the actual value. This is borne out in the plot: the black points fall below most of the gray squares. However, there is good correspondence for large $\lambda$s. This suggests that the mass dimensional relationship works well for small particle sizes (high $\lambda$), but overpredicts for the larger ones. Figure 8b uses IWC and $Z_e$ that are calculated using the CRYSTAL FACE mass dimension relationship, omitting small particle data. The primary difference from the data in Fig. 8a where CVI IWCs are used is that the peak ratios (at large $\lambda$) are smaller. The absence of small particles contributes to lower ratios. The data in Fig. 8b show little scatter because of IWC, and $Z_e$ is calculated using the same $m(D)$ relationship.

In Fig. 8c, IWC and $Z_e$ are each calculated from the 5700 PSDs using the density representations shown in Figs. 7a and 7b and the parameterization given by Eqs. (7) and (11). The IWC/$Z_e$ values conform closely to the observations, indicating that the new approach provides results that agree well with the observations. Note that the IWCs derived from this approach agree well with the CVI measurements (Heymsfield et al. 2004) with the median ratio of IWC from the parameterization to CVI being 0.91]; therefore, the $Z_e$ values are likely to be quite accurate as well.

We now evaluate whether the $Z_e$–IWC relationships that are developed in earlier studies fit the measured IWC/$Z_e$ trend with $\lambda$. Eq. (12a) relates the relationship IWC/$Z_e$ to the properties of the PSD. In Eq. (12a), $\mu$ and $\lambda$ are unknown. Because $\mu$, $\text{F}_{\text{em}}$, and $\text{F}_{\text{ed}}$ can be approximately deduced from $\lambda$ (Heymsfield et al. 2002), IWC/$Z_e$ can be represented entirely in terms of $\lambda$, that is, in terms of the PSD slope parameter. This is similar to the Bartnoff and Atlas (1951) relation $Z_e = G \times \text{IWC} \times (D_0)^{3}$ (see Atlas et al. 1995), where $D_0$ is the median volume diameter that characterizes a size distribution and is inversely related to $\lambda$, and $G$ is a dimensionless parameter dependent upon the PSD. The Bartnoff and Atlas (1951) approach was used by Matrosov et al. (2003), from which Fig. 8d is derived. These results are slightly offset from the measurements (gray symbols) and are remarkably similar to those shown for our approach in Fig. 8b. (Matrosov et al. use a different mass dimension relationship.)

A number of earlier studies have related $Z_e$ and IWC by power-law equations of the form

$$IWC = cZ_e^d,$$  \hspace{1cm} (13)

where $c$ is the coefficient and $d$ is the exponent (see Sassen et al. 2002), and it is usually, but not always, assumed that the ice scatterers are in the Rayleigh regime. Therefore,

$$\frac{IWC}{Z_e} = c^{1/d}(\text{IWC})^{(d-1)/d}. \hspace{1cm} (14)$$

In Figs. 8e–i, IWC/$Z_e$ ratios that are derived from Eq. (14) from the 5700 Citation data points are plotted.\textsuperscript{2} In these plots, the scatter is much greater and, over the full range of $\lambda$, the results do not conform to the observations. Where $\lambda$ is small, with both methods, the ratios exceed the measured values, also suggesting that using power-law mass dimension relationships for IWC and $Z_e$ lead to this result.

We can further assess whether the use of our new parameterization represents improvements over earlier approaches. In Fig. 9a, we relate $Z_e$ measured at 9.6 GHz to the ratio of IWC derived from the PSD fit parameters to the CVI IWC. It is noted that except for the highest $Z_e$ values, where the CVI measurements may be underestimated, the values from the parameterization are, on average, within about 10% of the measured values, with a relatively small standard deviation about the mean. Also shown in Fig. 9a are points where $Z_e$ and IWC are each derived from the PSD fit parameters. The calculated $Z_e$ values are close to the measured values, with a median difference of $-0.12 \pm 1.86$ dB. The ratio of IWC derived from the parameterization to the CVI IWC remains unchanged. Therefore, if the PSD fit parameters are known, our parameterization closely approaches the measured values.

In Figs. 9b–f, we use several of the IWC–$Z_e$ relationships from Fig. 8 and the $Z_e$ measurements at 9.6 GHz to calculate IWC and relate these values to the CVI

\textsuperscript{2} In Eq. (14), we use IWC derived from our parameterization rather than that measured from the CVI, because the CVI IWCs are limited to greater than 0.01 g m$^{-3}$, and are saturated above 1.0 g m$^{-3}$. Our parameterization appears to give good results.
IWC; median values of the calculated to measured IWC and the standard deviations are shown. The Sassen (1987), Sassen and Liao (1996), and Liu and Illingworth (2000) relationships provide good results overall, but with more scatter than is observed here. It is not known how well these relationships (or ours, for that matter) will perform for the lower-$Z_e$ values found in cirrus. Because the $Z_e$ values from the parameterization are

Fig. 9. Evaluation of the parameterization and earlier IWC–$Z_e$ relations against the observations. The abscissa is $Z_e$ measured at 9.6 GHz, except where noted. The denominator in the ordinate is IWC(CVI). (a) Symbols differentiate where the parameterization and PSD fit parameters are used to calculate IWC from where both variables are derived from it; (b)–(f) $Z_e$ measured at 9.6 GHz is used to calculate the IWC; (g), (h) as in (b)–(f), but using the $Z_e$–IWC relationship.
probably quite accurate, we have developed a synthetic dataset. It consists of 5700 $Z_e$ values at 9.6 and 94 GHz, and corresponding IWC values, using the PSD fit parameters and density expressions from sections 2 and 4. These data are plotted in Figs. 10a and b, with curves fitted to the data. Contrary to most earlier IWC–$Z_e$ relationships using a single equation [Eq. (13)] based on a single power-law mass dimension relationship, our approach requires more than one IWC–$Z_e$ equation that accounts for the deviation from a single power law in the mass dimension relationship. In Figs. 9g and h, the new IWC–$Z_e$ relationships, and the $Z_e$ measurements at the two radar wavelengths to derive IWC, are compared with the CVI measurements. Overall, the new relationships produce good fits to the data and represent improvements over the earlier approaches, at least for the dataset evaluated here.

Because our parameterization for IWC and $Z_e$, represented in terms of $\lambda$, seems to give reliable and self-consistent results (Fig. 8c), our parameterization should provide consistent results in cloud models and radar retrieval algorithms. Some means of specifying $\lambda$ are required. There are four approaches by which this can be done. The first of these uses temperature, for example, Liu and Illingworth (2000). The second uses the dual-wavelength ratio (dWR), the logarithmic difference in reflectivities at 9.6 and 94 GHz (Matrosov 1998; Hogan and Illingworth 1999). The third approach uses

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**Fig. 10.** The $Z_e$–IWC relationship derived using 5700 size distributions from the Citation aircraft during CRYS TAL FACE, together with the density representations from Fig. 7: (a) 9.6 and (b) 94 GHz.
the moments (mean and standard deviation) of the Doppler velocity spectrum (Matrosov et al. 2002; Mace et al. 2002), and the fourth uses radar reflectivity at each wavelength. As a first approximation, λ can be derived from temperature, with different relationships that are suitable for stratiform and convectively generated ice clouds (see Heymsfield et al. 2005). An attempt to use it with this dataset was unreliable and produced too much scatter, and therefore is not recommended. The second and third approaches are obviously inapplicable to a single wavelength, non-Doppler radar (CloudSat). The fourth method is examined in Heymsfield et al. (2005) from which it was found statistically that, to a first approximation, calculated values of $Z_e$ and $\lambda$ are related. Figures 11a,b relate the $Z_e$ values that are derived from the PSD fit parameters to the corresponding $\lambda$ for the CRYSTAL FACE Citation aircraft data. It should be noted that the curves shown in Fig. 11 fit the data reasonably well. Obviously, more data are needed to establish the generality of these results, perhaps using approaches one to three, to establish their applicability to other cloud types and locations.

Fig. 11. Radar reflectivity represented in terms of the slope of the particle size distribution for wavelengths of (a) 9.6 and (b) 94 GHz. Curve fits to the data are shown.
6. Summary and conclusions

This study reports calculations of the ice water content and radar reflectivity at two frequencies from in situ measurements of particle size distributions. The size distribution measurements were taken at temperatures from −25°C to −52°C in convectively generated Florida stratiform ice cloud layers. The size distribution and single power-law-type mass dimension relationships were used to calculate IWC and $Z_e$. We evaluate the accuracy of the calculations approach with respect to coincident IWC and $Z_e$ measurements at wavelengths where the ice scatterers are in Rayleigh and Mie regimes. The observations covered a wide range of IWCs from 0.01 to more than 1 g m$^{-3}$, and reflectivities from below −15 to above 12 dBZ$_e$. The analyses disclosed that IWC can be accurately calculated from particle size distributions and a single mass-dimension relationship, with particle mass being roughly proportional to the square of the particle maximum dimension. This is because the relatively small sizes in narrow ranges that contribute most of the IWC occur where a power-law does appear to be valid. For the conditions sampled, the range of particle sizes that contribute most to the radar reflectivity is much wider, because the radar reflectivity is roughly proportional to the fourth power of the physical particle maximum dimension. Except for $Z_e$ values of −10 dBZ$_e$ and below, single power-law relationships that fit the IWC observations do not extend to the larger particle sizes that contribute most to radar reflectivity. To further delineate the extent of the disparity, observations at higher reflectivities are required. There is a strong indication that the exponent “$b$” in the mass dimension relationship decreases as particle size increases.

An approach that uses separate population mean effective densities for ice water content and for radar reflectivity is suggested. It will avoid requirements for complex or multiple power-law relationships in calculating IWC and $Z_e$ and the development of IWC–$Z_e$ algorithms. Analytic expressions for densities, for IWC and $Z_e$, are derived in terms of the slope of the particle size distribution, which can be estimated from the radar reflectivity. Relationships between IWC and $Z_e$ could then be derived for each of the two radar wavelengths without assuming a single power-law-type mass dimension relationship. It is clear that larger sets of coincident IWC and $Z_e$ data are required to evaluate the general applicability of this method.

It has also been demonstrated that a strong requirement exists to quantify ice water content for those particle sizes smaller than 50 μm that are presently below those measured by imaging probes. These measurements are necessary for the interpretation of mass dimension relationships from particle size distributions. The absence of this data makes interpretations of the suitability of this and other algorithms ambiguous.

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APPENDIX

Fraction of Ice Water Content in Small Particles

The purpose of this appendix is to estimate the fraction of the IWC that is contained in large- (>$50$ μm) and small- (≤50 μm) particle diameters (maximum dimension). We use the FSSP data from the University of North Dakota Citation aircraft collected during CRYSTAL FACE, and assume that all FSSP particles are solid ice spheres of their measured diameters. With these assumptions about the FSSP data, Fig. A1 shows the ratio IWC(FSSP)/IWC(CVI) for 19, 25, and 28 July 2002. The data for 29 July are not considered because the FSSP failed to collect data for much of the flight. There is large variability noted in the fraction of ice in small (FSSP) particle sizes for each flight (Fig. A1). The collocation periods (points and horizontal bars at the top of the plots) contain values that are far from the mean for each day. In Fig. A2, the values for the fraction of ice mass in small sizes are shown for all days combined in terms of the measured IWC. There does appear to be a weak dependence of this fraction on the IWC, with the magnitude increasing from 30% to 50% as the IWC increases. This increase may signify particle breakup, because increasing IWC is usually associated with increasing particle sizes.

The FSSP size distributions are used to calculate an upper limit to the reflectivity that is contained in the small particles. For each of the 3 days identified above, the PSDs that are measured by the imaging probes are used to calculate $Z_e$, assuming that these particles are Rayleigh scatterers. The binned size distributions and the mass dimension relationship based on the CRYSTAL FACE dataset, $m = 0.0061D^{2.05}$, are used for these calculations. The FSSP particles are assumed to be solid ice spheres, and the combined FSSP and imaging probe $Z_e$ are derived. Small particles contribute
decreasing $\Delta Z_e$ with increasing $Z_e$ coming from the large particles (Fig. A3). Given that the $\Delta Z_e$ values are upper limits, the FSSP-sized particles contribute insignificantly when $-20 \text{ dB} < Z_e$.

The following calculations form the basis of a sensitivity study that is used in the rest of this section to examine the influence of small particles on the calculated values of IWC and $Z_e$. For a given value of the fraction of ice in large particle sizes, $f = [\text{IWC(CVI)} - \text{IWC(FSSP)}]/\text{IWC(CVI)}$ and for a fixed value for the
Fig. A2. Same as Fig. A1, except for the 3 days combined, represented in terms of IWC(CVI).

Fig. A3. Additional reflectivity added by FSSP particles for the 3 days (as in Fig. A1) combined, represented in terms of the reflectivity calculated from the size distributions of large particles using the $m(D)$ relationship given for the CRYSTAL FACE clouds in Heymsfield et al. (2004).
coefficient $b$, for each PSD, $a = f \times \text{IWC(CVI)} \times 10^{-6} / (\sum n_i N_i D_i^3)$. For values of $f$ between 0.4 and 1.0 and with $b$ ranging from 1.8 to 2.2, mean values of $\bar{a}$ are derived as indicated in footnote 2 (see section 4). As shown in Fig. A4 with $b = 2.0$ (for illustration purposes), the calculated IWCS scale linearly with $f$ (not surprisingly). From Fig. A4 there is no way of assessing which value of $f$ is appropriate for the particular observation.

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