Vertical Velocities and Available Potential Energy Generated by Landscape Variability—Theory

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ABSTRACT

It is shown that landscape variability decreases the temperature in the surface layer when, through mesoscale flow, cool air intrudes over warm patches, lifting warm air and weakening the static stability of the upper part of the planetary boundary layer. This mechanism generates regions of upward vertical motion and a sizable amount of available potential energy and can make the environment of the lower troposphere more favorable to cloud formation. This process is enhanced by light ambient wind through the generation of trapped propagating waves, which penetrate into the midtropospheric levels, transporting upward the thermal perturbations and weakening the static stability around the top of the boundary layer. At moderate ambient wind speeds, the presence of surface roughness changes strengthens the wave activity, further favoring the vertical transport of the thermal perturbations. When the intensity of the ambient wind is larger than $5 \text{ m s}^{-1}$, the vertical velocities induced by the surface roughness changes prevail over those induced by the diabatic flux changes. The analysis is performed using a linear theory in which the mesoscale dynamics are forced by the diurnal diabatic sensible heat flux and by the surface stress. Results are shown as a function of ambient flow intensity and of the wavelength of a sinusoidal landscape variability.

1. Introduction

It is well known that mesoscale processes can act to provide adequate moisture and instability for convection to initiate, and surface processes such as those driven by surface heterogeneity or soil moisture gradients can play a fundamental role in the development of convection. Once initiated, the interaction of convection with shear can enhance storm evolution and lead to severe weather (Chang and Wetzel 1991). In this perspective, the surface conditions (dry versus wet soil; presence of heterogeneities in surface conditions such as dry land adjacent to wet land or alternating dry and wet patches) are important since they affect the partitioning between latent and sensible heat fluxes. Roy and Avisser (2002) found that coherent mesoscale circulations were triggered by surface heterogeneities in Amazonia. Otterman (1977, 31–41) speculated that anthropogenic changes from bare soil to a complex vegetated surface in regions of marginal rainfall can favor convective precipitation. His hypothesis was supported by an analysis of the rainfall patterns in Israel (Otterman and Sharon 1979; Otterman et al. 1990). Moreover, specific humidity in the convective boundary layer is increased for wet surfaces, leading to larger convective available potential energy (CAPE). In addition, model simulations have shown that a realistic vegetated soil has an impact on the formation of drylines, since the vegetation is a source of moisture, and since the heterogeneous distribution of the vegetation enhances the gradients of the surface fluxes providing the solenoidal forcing to local frontogenetic flows (Shaw et al. 1997). Moreover, observations show that in dryline regions the probability of deep convection initiation is high in the late afternoon when the updraft forced by a mesoscale secondary circulation can locally break the boundary layer capping, and that the wind shear can modulate the convective initiation in the mesoscale updrafts at
the dryline (Ziegler and Rasmussen 1998). The development of thermally forced secondary circulations is favored by the absence of ambient flow, since they are suppressed by ambient flows with a wind speed exceeding 6 m s\(^{-1}\) for surface inhomogeneities larger than 50 km, or by weaker winds for smaller inhomogeneities (Segal and Arritt 1992).

From a climatic point of view, land-use changes have an impact on the regional and global scale, since spatially heterogeneous land-use effects may be at least as important in altering the weather as changes in climate patterns associated with greenhouse gases (Pielke et al. 2002; Pielke 2005), and, while Pielke et al. (2007) discussed the diverse role of land-use/land-cover change on precipitation, Feddema et al. (2005) showed model results indicating that land use and land cover will continue to have an important influence on climate for the next century. Analyzing the relation between the changes in the vegetation and the rainfall in the Sahelian region, Taylor et al. (2002) have demonstrated, through general circulation model (GCM) simulations, how these changes could cause substantial reductions of the precipitation and examined the hypothesis that anthropogenic changes to the land surface caused the drought conditions that have persisted since the late 1960s; however, the recent historical land-use changes are not large enough to conclude that they are principal cause of the Sahel drought.

On a regional scale, the climate of the United States has changed with vegetation in modern times, with significant changes of temperature and humidity distribution up to 500 hPa (Bonan 1997). Model simulations confirm that significant atmospheric effects have been caused by human modification of the landscape over the U.S. central plains (Weaver and Avisser 2001). A strong influence of the land use and land cover on the diurnal temperature range, with the greatest range over bare soil and the smallest range over forest-covered soil, has been observed by Gallo et al. (1996). In the U.S. central plains there is observational and model evidence of seasonal temperature differences of 5°–10°C between patches of dry land and patches of cultivated land with horizontal dimensions of the order of 20–50 km, which can drive mesoscale flows similar to sea breezes (Segal and Arritt 1992), while Shaw and Doran (2001) observed that, in the U.S. central plains, the nonseasonal divergence patterns are related to gentle topographic features rather than land-use features.

On a more local scale, Anthes (1984) found that bandwidths of about 20 km or wider are needed in order to produce vertical circulations extending to heights of 1 km or more in the atmosphere. Banta and White (2003) observed that the mixing height difference caused by small-scale landscape variability decreases from 600 m to negligible values as the ambient wind increases from 1 to 6 m s\(^{-1}\). De Ridder and Gallée (1998) found that, when the soil moisture availability increases in a semiarid region like southern Israel, there is a reduction of the thermal diurnal amplitude, and an enhancement of the moist convection and precipitation. Perlin and Alpert (2001) conclude, through the analysis of two convective rain studies in this semiarid region done using model simulations, that there is “a positive influence of the anthropogenic land use changes on the enhancement of thermal convection and associated rainfall.” There are several studies on the regional importance of spatial and temporal variations in soil moisture and vegetation coverage, and several authors studied the influence of landscape heterogeneity on cumulus convection including Segal et al. (1989a,b), Chang and Wetzel (1991), Fast and McCorcle (1991), Chen and Avisser (1994a,b), Li and Avisser (1994), Clark and Arritt (1995), Cutrim et al. (1995), Lynn et al. (1995a,b, 1998), Rabin et al. (1990), Rabin and Martin (1996), and Wang et al. (2000). For the sake of completeness, Cotton and Pielke (2007) list papers regarding how regional weather patterns are affected by land-use and land-cover change.

The results found by Anthes (1984) using a linear model indicate that planting bands of vegetation in semiarid regions can, under favorable large-scale atmospheric conditions, enhance convective precipitation. Motivated by the above studies and further elaborating on the work done by Anthes, we examine the impact of mesoscale landscape variability on the tropospheric parameters that may favor convective development. Since the work done by Anthes (1984) is focused on the vertical velocities forced by diabatic fluxes in the planetary boundary layer (PBL) in the absence of ambient flow, we extend our analysis to the mechanism through which weak ambient flow can favor development of convection. Since most of the above studies are observational, numerical, or mixed observational and numerical, with the exception of Anthes’ work, we approach the problem using an analytical linear theory developed by Dalu and Pielke (1993) and Dalu et al. (2003) for the thermally forced secondary flows, where, in the present paper, we have added the effects induced by spatial gradients of surface roughness, which can be important in the presence of an ambient flow, since surface roughness changes contribute to the vertical fluxes up to midtropospheric levels (Kustas et al. 2005).

The main motivation for using a theoretical approach is the apparent contradictions found in the results of previous authors. To illustrate, the thermally forced
secondary circulations are weakened or suppressed by the ambient flow (Segal and Arritt 1992), the ambient flow modulates the convective initiation (Ziegler and Rasmussen 1998), the surface roughness plays no role in the development of the convection (De Ridder and Gallé 1998), the surface roughness changes contribute to the vertical fluxes up to midtropospheric levels (Kustas et al. 2005), the mixing height difference decreases as the ambient wind increases (Banta and White 2003). To establish the range of validity of the above results as a function of wavelength and ambient flow intensity, a very large number of numerical simulations would be necessary. We therefore found it useful to approach the problem with a theory where the results were expressed in terms of readable equations, showing, in fact, that there is no contradiction in the above results, but ranges of validity in terms of patch size and wind intensity.

In terms of simple solutions we revisit the main results found in previous experimental and numerical studies; namely, we evaluate the decrease of the temperature induced by the ambient flow and by the secondary mesoscale flow in the lower layers of the PBL, and, in order to explore the environment modifications that may increase the chances of deep convection development, we study the intensity and the phase of the updraft at the top of the PBL, the weakening of the static stability around the top of the boundary layer, and the amount of the potential energy available in the PBL after sunset.

We show the results as a function of the wavelength of the landscape variability and of the intensity of the ambient flow, focusing on a landscape variability with wavelengths between 10 and 100 km in the presence of a wind intensity up to 10 m s$^{-1}$, since the atmospheric response is generally weak outside this range. At larger wavelengths the gradients are small, and at smaller wavelengths (100 m–10 km), diffusion homogenizes the temperature gradients above the surface layer, killing the vertical velocities in the upper part of the boundary layer (Anthes 1984; Dalu and Pielke 1993).

2. Governing equations

We use a linear theory to study the impact on the secondary mesoscale flow induced by changes of the diabatic sensible heat source $Q$ and by the surface stress $\tau$. We assume that the vertical intensity is proportional to the gradient of the surface stress, $w \propto \partial_z \tau$, then, since $u = -\partial_x w \, dx$ [second equation in Eq. (2)], $w \propto \partial_z \tau$ [first equation in Eq. (1)]. In addition we assume that over the nonvegetated patches $Q$ is positive and $\tau$ is small, and that over the vegetated patches $Q$ is negative and $\tau$ is large. In the time operator $\mathcal{L}$, we adopt a horizontal diffusion coefficient $K = 100 \, m^2 s^{-1}$ and we neglect the vertical diffusion term $K \partial_z$, simplifying the mathematics (Dalu and Pielke 1993). We have verified that the weakening of the horizontal gradients at wavelengths smaller than 20 km are as those shown by Anthes (1984), who keeps both diffusion terms. For simplicity, the Rayleigh friction coefficient $\lambda$ is assumed the same in the momentum and in the thermodynamics equations; $U$ is the ambient flow intensity; $u$, $v$, and $w$ are the momentum components; $\phi$ is the geopotential; and $b$ is the buoyancy perturbation. When the hydrostatic approximation can be made (Dalu et al. 2003), the fourth equation can replace the third equation in 1. Here, $N = 10^{-2} \, s^{-1}$ is the Brunt–Väisälä frequency and $f = 10^{-4} \, s^{-1}$ is the inertia frequency.

We prescribe that the vertical divergence of the heat flux is constant through the depth of the convective boundary layer ($h_Q$) and vanishes above it:

$$Q = Q_0 \text{He}(h_Q - z) \exp(\omega_0 t + k_0 x),$$  \hspace{1cm} (4)

with $k_0 = \frac{2 \pi}{L}$; $\omega_0 = \frac{2 \pi}{1 \text{day}}$; $Q_0 = \omega_0 N^2 h_Q$;

$$N^2 = \frac{g}{\Theta_z}; \quad h_Q = \mu_Q^{-1} = 1000 \, m.$$  \hspace{1cm} (5)

In Eq. (4), $\text{He} (h - z)$ is the Heaviside function, where $\text{He}(z < h) = 1$ and $\text{He}(z > h) = 0$; $L$ is the horizontal wavelength of the landscape variability, $k_0$ is the horizontal wavenumber; $\Theta_z$ is the vertical gradient of the ambient potential temperature; $\Theta$ is the average potential temperature of the PBL; and $g$ is the gravitational acceleration.
We do not adopt an exponential decaying diabatic source as in Anthes (1984), because, with this choice, about one-third of the total sensible heat would be above the PBL, which is about half of the heat within the PBL, and, since the secondary flow is driven by the horizontal gradients of the heat source [Eq. (21)], the intensity of this flow above the PBL would be arbitrarily enhanced:

\[
Q \propto \exp\left(-\frac{z}{h_Q}\right) \Rightarrow \int_{0}^{\infty} Q \frac{dz}{h_Q} = \frac{1}{e-1} \approx \frac{1}{2}, \quad \text{or}
\]

\[
\int_{0}^{\infty} Q \frac{dz}{h_Q} = \frac{1}{e-1} \approx \frac{1}{2}.
\]

In the absence of ambient flow and of secondary circulation \((U, u, v, w = 0)\), the potential temperature perturbation is proportional to the time integral of the forcing \((\delta \theta_{wa} \propto \int Q dt \approx \hat{L}_{\omega}^{-1} Q)\), where \(\hat{L}_{\omega}\) is the Fourier transform of the operator in Eq. (3):

\[
\hat{L}_{\omega} = [i \omega_0 + (\lambda + K k_0^2)].
\]

The temperature perturbation is vertically distributed assuming that the mixing due to the convective adjustment makes the PBL isentropic (Green and Dalu 1980):

\[
\delta \theta_{wa} = \left(\frac{h_Q - z}{h_Q}\right) \Delta \omega \theta_0 He(h_Q - z) \exp(i\omega_0(t - t_0)).
\]

\[
\frac{\Delta \theta_{wa}}{\Delta \theta_{\omega_0}} = \frac{\omega_0}{\hat{L}_{\omega}} = \frac{\omega_0}{\sqrt{(\lambda + K k_0^2)^2 + \omega_0}}; \quad \Delta \theta_{\omega_0} = h_Q \Theta_z,
\]

and \(t_0 = \frac{1}{\omega_0} \tan^{-1}\left(\frac{\omega_0}{\lambda + K k_0^2}\right)\).

The time lag \(t_0\) decreases as the Rayleigh friction increases. In addition, small-scale features, smaller than 20 km, are quicker in responding to the forcing, since the time lag is further reduced by diffusion processes at high wavenumbers [Eq. (10) and Fig. 1a]. In the absence of diffusion and Rayleigh friction \(t_0 = \tau / (2 \omega_0)\), the time lag is 6 h; that is, if the maximum of the diabatic heat occurs at noon, the maximum of the diabatic temperature occurs at 1800 LT. Therefore, in order to reduce the time lag to a more reasonable value, we adopt a value of the Rayleigh friction coefficient equal to the inertia frequency, \(\lambda = f\). With this choice the maximum of the diabatic temperature occurs at about 1430 LT \((t_0 = 2.4)\) over patches with a wavelength larger than 20 km [Eq. (11) and Fig. 1a]:

\[
t_0 = \frac{1}{\omega_0} \tan^{-1}\left(\frac{\omega_0}{\lambda}\right) = 2.4 \quad \text{when} \quad \lambda = f = 10^{-4} \text{ s}^{-1}.
\]

In Eqs. (8)–(11) we used and hereinafter we will use the definition of a complex number \(Z\):

\[
Z = a + ib = \rho \exp(i\alpha) \quad \text{with}
\]

\[
\rho = |Z| \quad \text{and} \quad \alpha = \tan^{-1}\left(\frac{b}{a}\right),
\]

where \(\rho\) is the modulus and \(\alpha\) is the angle (or argument) of the complex number \(Z\).

\[\text{a. Adopted wind stress and diabatic forcing}\]

Since the deep convection is most likely to occur in the late afternoon (Ziegler and Rasmussen 1998), in order to evaluate the integrated daytime impact of the dynamics of the secondary flow on the atmospheric parameters, we average in time the diabatic forcing in Eq. (4). The average is done through the half day of the diurnal time; this operation flattens the diabatic source to \(2/\pi \approx 2/3\) of its maximum. The mathematics is simplified without changing the inherent nature of this impact:

\[
\bar{Q} = Q_0 \text{He}(h_Q - z) \exp(i\omega x) \frac{2}{\text{day}} \int_{0}^{0.5 \text{ day}} \text{Im}[\exp(i\omega t)] dt.
\]

We study the dynamics of the secondary flow forced by \(\bar{Q}\) and by \(\tau\), with the assumption that they are a reasonable representation of the afternoon conditions:

\[
\bar{Q}(x, z) = \frac{2}{\pi} Q_0 \text{He}(h_Q - z) \exp(i\omega x)
\]

and

\[
\bar{\tau}(x, z) = \mu_\tau \tau_0 \text{He}(h_r - z) \exp(i\omega x),
\]

with \(\tau_0 = C_D U^2\); \(C_D = 1.5 \times 10^{-5}\); \(h_r = \mu_\tau^{-1} = 100\) m; and

\[
\tau(x, z) = -\tau_0 [\mu_\tau(h_r - z)] \text{He}(h_r - z) \exp(i\omega x).
\]

Here \(C_D\) is the drag coefficient, and \(\tau_0\) is the amplitude of the surface stress. In Eq. (16) we prescribed that the surface stress linearly decreases through the depth \(h_r\) of the surface layer (SL), where the vertical divergence of the stress is constant [Eq. (14)], and that both vanish.
above the SL. Since we assumed that the diabatic source is positive over the nonvegetated patches and negative over the vegetated patches, and that the surface stress is large over the vegetated patches and small over the nonvegetated patches, $Q(x, z)$ in Eq. (14) and $\tau(x, z)$ in Eq. (16) are taken with opposite sign. The potential temperature perturbation in the absence of secondary flow is the balance between the stationary diabatic forcing [Eq. (14)] and the dissipation due to the Rayleigh friction and diffusion ($\delta\theta \approx -\hat{L}_\lambda^{-1}\overline{D}$), where $\hat{L}_\lambda$ is the Fourier transform of the operator in Eq. (3) for stationary forcing:

$$\hat{L}_\lambda = (\lambda + K k_0^2 + ik_0 U),$$  \hspace{1cm} (17)

$$\delta\theta_\lambda = \frac{2}{\pi} \left( \frac{h_Q - z}{h_Q} \right) \Delta\theta_0 \Delta\omega \text{He}(h_Q - z) \exp ik_0(x - x_0),$$  \hspace{1cm} (18)

with

$$\frac{\Delta\theta_\lambda}{\Delta\theta_0} = \frac{\lambda}{\sqrt{(\lambda + K k_0^2)^2 + (k_0 U)^2}}$$

and

$$x_0 = \frac{1}{k_0} \tan^{-1} \left( \frac{k_0 U}{\lambda + K k_0^2} \right).$$  \hspace{1cm} (19)

Here $\Delta\theta_0$ is the amplitude of the thermal contrast between the vegetated and the nonvegetated patches.

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**Fig. 1.** (a) Time lag $t_0$ between the diabatic source $Q$ and the temperature perturbation $\delta\theta_\omega$. (b) Downwind displacement $x_0$ of the diabatic temperature perturbation $\delta\theta_\omega$ (solid line); $x_Q$ displacement of the vertical velocity $w_Q$ in the SL (dashed line); $x$, displacement of the vertical velocity $w$, in the SL (dotted line). (c) Decrease of the thermal contrast between the vegetated and the nonvegetated patches, $\Delta\theta_\omega$. (d) Maximum of the updraft at the top of the boundary layer. Combined updraft induced by the diabatic flux and the surface roughness, $w_{Q,\omega}$ (solid line); updraft induced by the diabatic flux only, $w_Q$ (dashed line); updraft induced by the surface roughness only, $w$, (dotted line).
when \( U = 0 \) and \( K = 0 \). The ambient flow reduces \( \Delta \theta_{a} \) as shown in Fig. 1b. The reduction of the thermal contrast is strongly enhanced by the diffusion processes when the patches are smaller than 20 km as in Anthes (1984), Fig. 1b, and Eq. (19). In addition, the ambient flow displaces the temperature perturbations downstream a distance, \( x_{o} \). The displacement of the temperature perturbation is a function of the ambient flow intensity and of the wavelength as shown in Fig. 1b [Eq. (19)]. At high wind speed the advection time is much smaller than the thermal equilibrium time, \((k_{0} U)^{-1} \ll \lambda^{-1}\), so we have the maximum displacement of the temperature perturbation, \(x_{o} \), does not exceed one-fourth of a wavelength \( L \) (Fig. 1b).

\[
\frac{k_{0} U}{\lambda} \gg 1 \Rightarrow x_{o} \leq \frac{\pi}{2k_{0}} = \frac{L}{4}.
\]  

(20)

The maximum displacement, \( x_{o} \), does not exceed one-fourth of a wavelength \( L \) (Fig. 1b).

b. Streamfunction equation and vertical wavenumber

Using the mass continuity [second equation in Eq. (2)] to define the streamfunction \( \psi (\partial_{z} \psi = u \) and \( \partial_{t} \psi = -w \), we derive from the Navier–Stokes equations the streamfunction equation (Dalu et al. 2003):

\[
\hat{\lambda}_{\lambda} \partial_{z} \left[ \text{first equation in Eq. (1)} \right] + f \partial_{z} \left[ \text{second equation in Eq. (1)} \right] - \hat{\lambda}_{\lambda} \partial_{z} \left[ \text{third equation in Eq. (1)} \right]
\]

\[ - \partial_{z} \left[ \text{first equation in Eq. (2)} \right] \Rightarrow \]

\[
(\hat{\lambda}_{\lambda}^{2} + f^{2}) \partial_{zz} \psi + (\hat{\lambda}_{\lambda}^{2} + N^{2}) \partial_{x} \psi = -(-\hat{\lambda}_{\lambda} \partial_{z} \tau + \partial_{z} \psi) \]

(21)

\[
\hat{\lambda}_{\lambda} \psi(z) - \mu_{1}^{2} \psi(z) = F(z) \quad \text{with} \quad \psi(x, z) = \varphi(z) \exp(i k_{0} x),
\]

(22)

with \( \mu_{1}^{2} = k_{0}^{2} \left( \frac{\hat{\lambda}_{\lambda}^{2} + N^{2}}{\hat{\lambda}_{\lambda}^{2} + f^{2}} \right) \),

(23)

and \( F(z) = \frac{1}{\hat{\lambda}_{\lambda}^{2} + f^{2}} \left[ \mu_{1}^{2} \tau_{0} \hat{\lambda}_{\lambda} \delta(h_{z} - z) - i k_{0} \frac{2}{\pi} Q_{0} \text{He}(h_{Q} - z) \right] \),

(24)

Here \( \hat{\lambda}_{\lambda} \) is defined in Eq. (17). \( \mu_{1}^{2} \) is the vertical wavenumber squared, and \( \delta(h_{z} - z) \) is the Dirac function. Since \( \tau_{0} \propto U^{2} \) [Eq. (15)] the contribution of the surface stress rapidly increases with the wind speed. In addition, since \( \hat{\lambda}_{\lambda} = (\lambda + K k_{0} + i k_{0} U) \), when the ambient flow is sufficiently large \((U \gg \lambda k_{0}^{-1})\), the main contribution to the secondary flow comes from the horizontal gradients of the surface stress \( \partial_{x} \tau \):

When \( U \gg 1.5 \text{ m s}^{-1} \) and \( L \approx 100 \text{ km} \Rightarrow \hat{\lambda}_{\lambda} \tau \approx i k_{0} U \tau \).

(25)

The streamfunction \( \varphi(z) \) is computed through the convolution between the Green function, \( g(z, \xi) \), and the forcing \( F(\xi) \):

\[
\varphi(z) = \varphi_{Q}(z) + \varphi_{I}(z) = [g(z, \xi)(F(\xi) - G(\xi))], \quad g(z, \xi) = \frac{\exp(\mu_{1}|z - \xi|)}{2\mu_{1}}
\]

(26)

In Eq. (26), the boundary condition of vanishing vertical velocity at the surface has been achieved by introducing in the lower semiplane a mirror image \( G(z) \) of the nonhomogeneous term \( F(z) \) as in Dalu and Pielke (1989):

\[
\varphi_{Q}(z) = -\frac{i Q_{0}}{\pi k_{0} \left( \hat{\lambda}_{\lambda}^{2} + N^{2} \right)} \left[ 2[\exp(\mu_{1} z) - \text{He}(h_{Q} - z)] - \text{sign}(z - h_{Q}) \exp(\mu_{1} |z - h_{Q}|) - \exp[\mu_{1} (z + h_{Q})] \right].
\]

(27b)
ambient flow intensity: $\log_{10}(|\mu_1|)$ (solid line); $\log_{10}(|\mu_1|)$ (dashed line); $\log_{10}(|\mu_1|)$ (dotted line).

Fig. 2. Logarithm of vertical wavenumber and of its real and of its imaginary part, as a function of the wavelength and of the ambient flow intensity: $\log_{10}(|\mu_1|)$ (solid line); $\log_{10}(|\mu_1|)$ (dashed line); $\log_{10}(|\mu_1|)$ (dotted line).

To have the correct confinement and propagation of the energy, in the exponential of the Green function, the real part of the vertical wavenumber is chosen with a negative sign and the imaginary part with a positive sign (Smith 1979):

$$\mu_1 = -|\mu_1| + i|\mu_1|. \quad (28)$$

The modulus of the vertical wavenumber, $|\mu_1|$, of the real part, $|\mu_r|$, and of the imaginary part, $|\mu_i|$, are shown in Fig. 2. When the ambient flow is weak, the real part of the vertical wavenumber is much larger than the imaginary part ($|\mu_1| \approx |\mu_1|$), and the secondary flow is trapped and confined in the PBL. When the ambient flow is strong, the imaginary part of the vertical wavenumber is much larger than the real part ($|\mu_1| \approx |\mu_1|$), and the propagating inertia–gravity waves fill the entire troposphere. In the presence of a moderate ambient flow, the secondary flow is made of propagating trapped waves, which fills the lower half of the troposphere. In the absence of ambient flow, Rayleigh friction, and diffusion, the ratio between the vertical wavenumber and the horizontal wavenumber equals the ratio between the Rossby radius ($R_0$) and the depth of the convective layer (Dalu and Pielke 1989):

$$\text{when } U = 0 \Rightarrow \mu_r = 0 \text{ and } \frac{\mu_i}{k_0} = \frac{R_0}{h_Q} = \frac{N}{f}. \quad (29)$$

In the presence of ambient flow with no dissipation, the vertical wavenumber is equal to the Scorer parameter ($l$) (Scorer 1949):

$$\text{when } U \neq 0 \Rightarrow \mu_r = 0 \text{ and } \mu_i = \frac{N}{U} = \frac{N}{f}. \quad (30)$$

3. Atmospheric response to a landscape variability

The vertical velocity forced by a sinusoidal diabatic source in the absence of surface stress,

$$\overline{Q}(x,z) = \frac{2}{\pi} Q_0 \text{He}(h_Q - z) \cos(k_0 x), \quad (31)$$

is obtained by multiplying by $i k_0$ the streamfunction $[\overline{\psi} = \varphi_Q \exp(i k_0 x)$ in Eq. (22), where $\varphi_Q$ is defined in Eq. (27b)], and taking the real part

$$w_Q(x, z) = -\text{Re}[ik_0 \psi_Q(x, z)] = -\text{Re}[ik_0 \varphi_Q(z) \exp(i k_0 x)] = \frac{Q_0}{\pi p_Q^2} \{ \cos[k_0(x - x_Q) + \mu_i(z + h_Q)] \exp(-\mu_r(z + h_Q)) \}
+ \text{sign}(z - h_Q) \cos[k_0(x - x_Q) + \mu_i(z - h_Q)] \exp(-\mu_r(z - h_Q))
+ \frac{2 Q_0}{\pi p_Q^2} \{ \text{He}(h_Q - z) \cos[k_0(x - x_Q)] - \cos[k_0(x - x_Q) + \mu_i z] \exp(-\mu_r z) \}. \quad (32)$$

The ambient flow $U$ displaces the secondary flow through the imaginary part $i k_0 U$ of the operator $\hat{L}_h$ [Eq. (17)]. The displacement $x_Q$ is computed using the argument of the complex part of the amplitudes of $-i k_0 \varphi_Q$.

$$Z_Q = \hat{L}_h^2 + N^2 = p_Q^2 \exp(k_0 x_Q); \quad \rho_Q^2 = |Z_Q| \text{ and } x_Q = \frac{1}{k_0} \tan^{-1} \left[ \frac{\text{Im}(Z_Q)}{\text{Re}(Z_Q)} \right]. \quad (33)$$

The vertical velocity forced by the surface stress is obtained by multiplying by $i k_0$ the streamfunction $[\psi = \varphi_r \exp(i k_0 x)$ in Eq. (22), where $\varphi_r$ is defined in Eq. (27a)], and taking the real part.
\[w_r(x, z) = -\text{Re}[ik_0\psi_r(x, z)] = -\text{Re}[ik_0\varphi_r(z) \exp(i k_0 x)]\]

\[= \frac{\mu_r^2 \tau_0}{2k_0 \varphi_r} \left\{ \cos(k_0(x - x_c) + \mu_r(z - h_c)) \exp(-\mu_r(z - h_c)) - \cos(k_0(x - x_c) + \mu_r(z + h_c)) \exp[-\mu_r(z + h_c)] \right\},\]

with \(\partial_x \tau = \mu_r \tau_0 \text{He}(h_r - z) \cos(k_0 x)\)

and \(Z_r = -\frac{Z_Q}{ik_0 \varphi_r} = \rho_r^2 \exp(i k_0 x_c); \quad \rho_r^2 = |Z_r| \quad \text{and} \quad x_r = \frac{1}{k_0} \tan^{-1}\left[\frac{\text{Im}(Z_r)}{\text{Re}(Z_r)}\right].\)

When the wavelength is sufficiently large, the hydrostatic approximation can be made, and \(\varphi_r^2 + N^2\) can be replaced by \(N^2\) in the vertical equation:

when \(h_Q k_0 \ll 1\), then \(x_Q = 0\) and

\[x_r = \frac{1}{k_0} \tan^{-1}\left(\frac{\lambda + K k_0^2}{K_0 U}\right),\]

where \(x_Q\) and \(x_r\) are the displacements of the vertical velocities \(w_Q\) and \(w_r\) near the ground \((z \ll h_Q)\). Therefore, in the surface layer, the thermally forced hydrostatic vertical velocity, \(w_Q\), is always in phase with the diabatic source \([x_Q = 0 \text{ in Eq. (37)}]\) and, in addition, also \(w_r(x, z)\) is in phase with the diabatic source when \(k_0 U \gg \lambda + K k_0^2\). When the advection time equals the dissipation time due to the Rayleigh friction and diffusion processes \((k_0 U = \lambda + K k_0^2), w_r\) is in phase with the diabatic thermal perturbation \(\partial_\lambda\) in Eq. (18).

When \(U \neq 0\) and \(\mu_r \neq 0\), then above the surface layer the vertical velocities are a composition of backward tilted waves [Eqs. (32) and (34)]. The zero phase equations for the diabatic forced waves and for the surface stress forced waves are, respectively,

\[x = x_Q - \frac{\mu_l}{k_0} z \quad \text{and} \quad x_r = x_r - \frac{\mu_l}{k_0} z, \quad \text{(38)}\]

The horizontal momentum components are the vertical derivative of the streamfunction. These components are obtained multiplying by \(\mu_1\) [Eq. (28)] the streamfunction and keeping the real part:

\[u_Q(x, z) = \text{Re}[\mu_1 \psi_Q(x, z)] \quad \text{and} \quad u_r(x, z) = \text{Re}[\mu_1 \psi_r(x, z)].\]

When the diabatic source and the surface stress are both present, the momentum components are, respectively,

\[w_{Q, r}(x, z) = w_Q(x, z) + w_r(x, z) \quad \text{and} \quad u_{Q, r}(x, z) = u_Q(x, z) + u_r(x, z).\]

\[a. \text{ Vertical velocity at the top of the convective boundary layer}\]

Since substantial updrafts can favor the formation of clouds, we study the behavior of the vertical velocity at the top of the convective boundary layer (CBL) as a function of the wavelength and of the ambient flow intensity; the results are shown in Fig. 1d. When the ambient wind is weak, the vertical velocity is mainly diabatically forced. When the ambient wind is strong, the vertical velocity induced by the surface roughness prevails; the latter occurs when the ambient flow intensity exceeds 5 m s\(^{-1}\). Since \(w_Q\) and \(w_r\) do not have the same phase \((x_Q \neq x_r)\), the maximum of \(w_{Q, r}\) is not simply the sum of the maximum of \(w_Q\) with the maximum of \(w_r\).

At very high wavenumber, the vertical velocities are small because of the diffusion as in Anthes (1984). At low wavenumber, the horizontal gradients are small, and the thermally forced vertical velocities are small and less sensitive to the ambient flow. At intermediate wavenumber \((10 < L < 30 \text{ km})\), the thermally induced vertical velocities at the top of boundary layer are larger in the presence of a moderate ambient flow than in the absence of an ambient flow.

In the absence of ambient flow \((U = 0)\), \(w_Q\) is positive over the warm nonvegetated patches and negative over the cool vegetated patches. In the presence of ambient flow \((U \neq 0)\), \(w_Q\) is in phase with the diabatic forcing only in the surface layer [Eq. (37)], while above the surface layer the updraft shifts toward the vegetated patches as the altitude increases [Eq. (38)]. At the top of the PBL \(w_Q\) becomes positive over the vegetated patches when

\[\mu_l h_Q = n \pi, \quad n = 1, 3, 5, \ldots \quad \text{or} \]

\[\frac{N}{U} h_Q \approx \pi \Rightarrow U = \frac{N h_Q}{\pi} \approx 3 \text{ m s}^{-1}. \quad \text{(41)}\]

Therefore, the presence of moderate winds are beneficial to cloud formation since they enhance the vertical velocity at the top of the PBL (Fig. 1d), shifting, in
addition, the updraft from the nonvegetated patches toward the vegetated patches. In the approximation in Eq. (41) we assumed \( n = 1 \) and \( \mu_v \approx 1 \), where \( l \) is the Scorer parameter [Eq. (30)].

**b. Decrease of the temperature in the surface layer and increase of the temperature in the upper half of the boundary layer**

The real part of the diabatic temperature perturbation in Eq. (18) is the thermal response to the sinusoidal diabatic heat in Eq. (31):

\[
\delta \theta_\lambda = \frac{2}{\pi} \left( \frac{h_Q}{h} \right) \Delta \theta \lambda \left( \Theta \delta \theta_\lambda \right) \Delta \theta_\lambda \cos k_0 (x - x_0).
\]

(42)

The thermal contrast between the vegetated and the nonvegetated patches, \( \Delta \theta_\lambda \), is reduced and advected a distance \( x_0 \) downstream by the ambient flow [Eq. (19); Figs. 1b,c].

The diabatic temperature perturbation is further perturbed by the advection of the secondary flow forced by the diabatic source,

\[
\delta \theta_Q = \delta \theta_\lambda - \frac{1}{\hat{L}_\lambda} \left[ w_\lambda (\Theta + \partial_\lambda \delta \theta_\lambda) + u_\lambda \delta \theta_\lambda \right],
\]

(43)

which horizontally averaged over one wavelength becomes

\[
\overline{\delta \theta_Q} = \frac{1}{L} \int_{-0.5L}^{0.5L} \delta \theta_Q \, dx.
\]

(44)

In Fig. 3a we show this temperature vertically averaged through the lower part of the boundary layer (\( \overline{\delta \theta_{SL}} \)), and averaged through the upper half of the boundary layer (\( \overline{\delta \theta_{CBL}} \)).

\[
\overline{\delta \theta_{SL}} = \frac{5}{h_Q} \int_0^{0.2h_Q} \left( \frac{\overline{\delta \theta_Q}}{h} \right) \, dz \quad \text{and}
\]

\[
\overline{\delta \theta_{CBL}} = \frac{2}{h_Q} \int_{0.5h_Q}^{h} \left( \frac{\overline{\delta \theta_Q}}{h_Q - h} \right) \, dz.
\]

(45)

The cool air, advected from the vegetated patches toward the nonvegetated patches, lifts and replaces the warm air above nonvegetated patches. It results in a substantial cooling of the horizontally averaged temperature of the surface layer and a warming of the boundary layer above it, as shown by \( \overline{\delta \theta_{SL}} \) and \( \overline{\delta \theta_{CBL}} \) in Fig. 3a. When the wavelength is smaller than one Rossby radius \( (L < R_0) \), at low wind speed, the warm air in the SL above the nonvegetated patches is entirely replaced by the cool air advected from the vegetated patches, efficiently decreasing the temperatures in the SL. At high wind speed \( \delta \theta_Q \) becomes negligible, since \( \Delta \theta_\lambda \) is negligible (Fig. 1c).

**c. Weakening of the static stability around the top of the convective boundary layer**

The diabatic temperature perturbation caused by the advection of the secondary flow forced by the diabatic source and by the surface stress is

\[
\delta \theta_{Q,\tau} = \delta \theta_\lambda - \frac{1}{\hat{L}_\lambda} \left[ w_\lambda (\Theta + \partial_\lambda \delta \theta_\lambda) + u_\lambda \delta \theta_\lambda \right],
\]

(46)
and its horizontal average over one wavelength is

\[ \overline{\delta \theta_{Q,\tau}} = \frac{1}{L} \int_{-0.5L}^{0.5L} \delta \theta_{Q,\tau} \, dx. \]  

(47)

The vertical profile of the horizontal Brunt–Väisälä frequency squared and averaged over one wavelength is

\[ \overline{N^2} = \frac{g}{\Theta} \{ \Theta_z + \partial_x \delta \theta_{Q,\tau} \}. \]  

(48)

The perturbed Brunt–Väisälä frequency around the top of the convective boundary layer (\( \overline{N_{\text{CBL}}} \)) is shown in Fig. 3b:

\[ \overline{N_{\text{CBL}}} = \frac{1}{0.4 h_Q} \int_{0.9 h_Q}^{1.2 h_Q} \overline{N} \, dz. \]  

(49)

At low wind speed when the cool air over the vegetated patches intrudes under the warm air above the nonvegetated patches, the static stability around the top of the convective boundary layer weakens. At high wind speed, \( \delta \theta_Q \) is small (Fig. 3a), the perturbation of the static stability around the top of the convective boundary layer is mainly due to the wave action induced by the changes of surface stress.

In Figs. 4a,b we show the perturbed static stability around the top of the CBL, averaged over a nonvegetated patch and averaged over a vegetated patch, re-
respectively, when the secondary flow is forced by the \( \text{diabatic source} \) and by the surface stress. At low wind speed the static stability over the nonvegetated patches and over the vegetated patches is about the same. At high wind speed the static stability over the nonvegetated patches increases, while the static stability over the vegetated patches decreases. This behavior is due to the change of the phase of the waves with altitude in the presence of ambient flow [Eq. (38)].

To emphasize the role of the waves generated by the surface stress and the change of the phase of these waves with the wind speed, we adopt \( \tau \neq 0 \) and \( Q = 0 \), in order to avoid the interference between the waves generated by the surface roughness and the waves generated by the diabatic source. We show the resulting perturbed static stability around the top of the CBL averaged over a nonvegetated patch and averaged over a vegetated patch in Figs. 4c,d. At low wind speed the perturbation is negligible except at very high wavenumber \( [w_z \propto \hat{L}_a \tau = (\lambda + K \hat{k}_0 + i k_0 U) \tau] \) [Eqs. (17), (34), and (36)], and \( \tau \approx U^2 \) [Eq. (15)], in agreement with results by De Ridder and Gallée (1998), who found that the convective processes are insensitive to the surface roughness. While at moderate wind speed, we find that there is a 10% perturbation of the static stability, which is in agreement with the results by Kustas et al. (2005), who found that surface roughness changes contribute to the vertical fluxes up to midtropospheric levels. In addition, concerning the phase of these waves, we find that, when \( U \approx 3 \text{ m s}^{-1} \), the perturbation of the static stability becomes positive over the smooth nonvegetated patches and negative over the rough vegetated patches because of the tilt of the waves [Eq. (41)], with an increase of the stability over the nonvegetated patches equal to the decrease over vegetated patches.

\[ d. \text{ Energetics} \]

Since in summer after sunset the wind intensity often rapidly decreases \([(Q, U) \to 0] \), in this paragraph we evaluate the intensity of the updraft after sunset from the residual potential energy at sunset. Assuming that at this time of the day the temperature perturbation is \( \delta \theta_{(Q,r)} \), the available potential energy (APE) is

\[
\text{APE} = \frac{1}{h_{0L}} \int_{-0.5L}^{0.5L} \delta \theta_{(Q,r)} \left[ \int_0^{h_0} \left( \frac{g z' \delta \theta}{\Theta} \right) dz \right] dx = \frac{1}{2} (u^2 + v^2 + w^2),
\]

where \( z' \) is the vertical distance of the air particle with a temperature perturbation, \( \delta \theta_{(Q,r)} \), between its actual level and the level of neutral buoyancy, and where \( u, v, \) and \( w \) are the modulus of the momentum components averaged in the PBL over a wavelength \( L \). From the continuity equation and from the second momentum equation we have

\[
w = -\frac{ik_0}{\mu_1} u \quad \text{and} \quad v = -\frac{f}{L} u, \quad \text{or}
\]

\[
\bar{w} = \frac{k_0}{\mu_1} \bar{u} \quad \text{and} \quad \bar{v} = \frac{f}{L} \bar{u};
\]

\[
\text{APE} = \frac{1}{2} \bar{u}^2 \left[ 1 + \left( \frac{f}{|L|} \right)^2 + \left( \frac{k_0}{|\mu_1|} \right)^2 \right] = \frac{1}{2} \bar{u}^2 \left[ 1 + \left( \frac{|\mu_1|}{k_0} \right)^2 \left( 1 + \left( \frac{f}{|L|} \right)^2 \right) \right].
\]

Results shows that after sunset there is still enough potential energy to drive substantial updrafts: \( \pi_Q \) and \( \bar{\pi}_Q \) computed using \( \delta \theta_{(Q)} \) in Eq. (43) are shown in Figs. 5a,b, respectively, and \( \bar{w}_{(Q,r)} \) and \( \bar{w}_{(Q,r)} \) computed using \( \delta \theta_{(Q,r)} \) in Eq. (47) are shown in Figs. 5c,d, respectively. The intensity of the updraft, \( \bar{w}_{(Q,r)} \), decreases as the wind speed increases, since the temperature contrast between the patches weakens [Eq. (19) and Fig. 1c]. While the contribution of the surface stress to the updraft, \( \bar{w}_{(Q,r)} \), is negligible at low wind speed, it becomes important at moderate wind speed.

\[ 4. \text{ Conclusions} \]

To summarize the results by different authors, we developed a theory in which the results are formulated in terms of mathematical solutions in which the processes, through which landscape variability modifies the environment, are easily identified, and in which the range of validity of the results are established as a function of the size of the patches and of the intensity of the wind.

Our results show that, when the patches are a small fraction of the Rossby radius, the reduced thermal diurnal amplitude is due to the intrusion of cool air in the surface layer, which lifts and replaces the warm air above the warm nonvegetated patches. Concerning intentional changes of the surface characteristics in order to modify the environment of the lower troposphere to enhance the chances of convective precipitation, the following considerations can be useful.

The ambient wind greatly reduces the thermal contrast and the thermally driven updraft; however, at the top of the boundary layer, this updraft is more intense in the presence of a light ambient wind.

When the wind speed is moderate, the wave activity weakens the static stability at the top of the boundary
layer, and, since these waves are tilted, a significant updraft is displaced over the vegetated patches at the top of the boundary layer.

The contribution of the wind stress becomes relevant in the presence of a moderate wind speed, and, when the ambient flow is stronger than \(5 \text{ m s}^{-1}\), the vertical velocity is mainly due to the mechanical effects induced by the changes of surface roughness.

The residual available potential energy in the PBL can drive significant secondary mesoscale flow after sunset.

To summarize the main findings, a moderate ambient flow (3–4 m s\(^{-1}\)) in the presence of a landscape variability modifies the environment making it favorable to cloud formation through the enhancing of the updraft at the top of the PBL (Fig. 1d) and the shifting of this updraft from the nonvegetated patches toward the vegetated patches (Fig. 4d), where the static stability is weakened as a result (Fig. 4b). In addition, after sunset, when the ambient flow and the diabatic forcing weaken, the residual potential energy can drive evening deep convection (Fig. 5). The results are shown as a function of the ambient flow intensity and of wavelength of a sinusoidal landscape variability.

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REFERENCES


—, W. E. Schreiber, G. Kallos, J. R. Garratt, A. Rodi, J.


