Objective Cross Section Analysis

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ABSTRACT

The distribution of wind and of temperature in a vertical cross section through the atmosphere is derived from radiosonde and rawinsonde data by an objective analysis procedure suitable for automatic computers. The method of interpolation takes advantage of the fact that soundings in the plane of the cross section provide data along vertical lines rather than at randomly distributed points. The thermal wind equation is used to modify the interpolated distribution through a partial differential equation derived from the calculus of variations. Results indicate that the technique is capable of delineating not only the gross vertical structure of the atmosphere but also some significant layers of high stability and/or potential vorticity which are inadequately defined by analyses in quasi-horizontal planes at the standard pressure levels.

1. Introduction

A thorough synoptic study of atmospheric wind, geopotential and temperature distributions requires vertical as well as horizontal analyses. In most circumstances, however, meteorologists rely primarily on horizontal analyses with but a superficial reference to soundings. The amount of work necessary to construct exact vertical cross sections prohibits their use on an operational basis. This is unfortunate because the density of information in vertical planes greatly exceeds that in the horizontal. Furthermore, the scale of features encompassed in a synoptic analysis can be extended to much smaller dimensions if vertical cross sections are used. These smaller features are by no means unimportant in many meteorological problems.

The horizontal distribution of aerological variables is routinely analyzed by objective methods at several large meteorological centers. The procedures in use come under two general classifications which may be referred to as "the successive approximation method" and "the surface fitting method." The first (Cressman, 1959) has been adopted by the U. S. Weather Bureau National Meteorological Center and the second (Bushby and Huckle, 1957) by the British Meteorological Office. This paper presents an objective analysis procedure which has been devised for automatic construction of vertical cross sections through the atmosphere. It is based on Sasaki's (1958) generalization of the surface fitting method.

2. General procedure

In principle, the goal of any analysis should be to determine the complete three dimensional distribution of any given quantity. A stack of horizontal analyses approximates this ideal, provided that the analytical procedure relates the distribution at one level to that at adjacent levels, as, for example, by the hypsometric equation. In practice, only relatively gross aspects of the atmosphere's vertical structure can be defined by this approach. The problem arises, of course, from the big difference between the horizontal and the vertical dimensions of most atmospheric features. Data with the vertical resolution necessary to detect and identify atmospheric laminae are available from radiosonde and rawinsonde records (Danielsen, 1959), but the required detail could not be handled by objective analysis methods proposed until now.

Synoptic analysis of meteorological data, whether by subjective or objective means, involves two processes: 1) a continuum of some variable is defined by interpolation between isolated point values, and 2) inconsistencies in the data are removed by filtering (e.g., smoothing) and by interrelating the data through a predetermined relationship. Normally these two processes are carried out simultaneously but, following Sasaki, they were separated into distinct steps in the cross section analysis.

The interpolation in a vertical cross section may involve one step not necessary in isobaric analysis. Radiosonde data are processed to provide measurements at standard pressure levels but the radiosonde stations are not located in regular grids to provide measurements in standard cross section planes. If the analyst cannot select a line of stations it will be necessary to project data onto the cross section plane from radiosonde stations off the plane.

To a large extent, the human analyst uses pattern recognition as a guide for interpolation. Unfortunately
"Gestalt" is not suitable material for a computer program but a suitable assignment of weights to the data can ensure that a numerical procedure will "recognize" certain characteristics in the distribution of a variable. For example, the "streakiness" of atmospheric currents has been taken into account in determining the horizontal distribution of wind speed by the use of an elliptic weighting function with the major axis of the ellipse oriented along the direction of the observed wind. A similar tactic was employed for cross section analysis. Since stable layers in the atmosphere tend to be bound by isentropic surfaces, interpolation of potential temperature was influenced by a weighting function which was largest in the direction normal to the observed potential temperature gradient vector.

Under the geostrophic and thermal wind assumptions, the distribution of wind speed normal to a vertical plane (v) and the potential temperature (θ) distribution in that plane are redundant, i.e., either one implies the other. Since observed values of v and of θ (and, a fortiori, interpolated values) do not normally satisfy this mutual relationship, the results from the first steps in the analysis were blended into a compromise which does satisfy the assumptions. This final analysis was obtained as the solution to a partial differential equation based on the calculus of variations. The square of the weighted differences between the interpolated and the final v and θ distributions were integrated over the entire analysis area and the integral was minimized. Weighting factors were based on the relative reliability of data.

3. Equations

a. The shear-to-stability ratio vector. For convenience, the vertical coordinate of the cross section was taken to be the non-dimensional variable π defined by

$$\pi = \left( \frac{P}{P_0} \right)^{\kappa},$$

where P is the pressure in mb, P0, the fixed pressure of 1000 mb, and κ, the non-dimensional constant, 0.288.

In the x, π coordinate system the hydrostatic and geostrophic equations are

$$\frac{\partial v}{\partial \pi} = \frac{-R\theta}{\kappa},$$

and

$$v = -\frac{1}{f} \frac{\partial \phi}{\partial \pi},$$

where

ϕ is the geopotential in cm² sec⁻²,
R is the gas constant [2.87 × 10⁵ cm² (°K)⁻¹ sec⁻²],
θ is the potential virtual temperature in °K,
v is the component of the wind perpendicular to the x, π plane in cm sec⁻¹, and
f is the coriolis parameter in sec⁻¹.

If temperature is assumed to vary linearly with ϕ in some interval, the hypsometric equation [Eq. (2) integrated] becomes

$$\phi = \phi_0 + \frac{R}{\kappa} \left[ (\theta_0 - \gamma \pi_0) \ln \frac{\pi}{\pi_0} \right],$$

where γ is the lapse rate, (T₁ - T₀)/(π₁ - π₀), and the subscripts 0 and 1 refer to given values at two points.

The thermal wind equation obtained by combining Eqs. (2) and (3) is

$$\left( \frac{\partial v}{\partial \pi} \right) = -\frac{R}{f \kappa} \frac{\partial \theta}{\partial \pi}.$$

The horizontal component of the potential temperature gradient given by the observed wind shear through (5) and the vertical component of the gradient measured directly by the sounding uniquely define the magnitude and direction of the gradient vector.

At any point the value of θ and the vector \( \mathbf{v} \) imply the distribution of θ about the point. They provide a basis for a directionally weighted interpolation of the θ field. This is entirely equivalent to the use of observed heights and wind vectors at a number of points to analyze the height of an isobaric surface. The only differences are: 1) the two pieces of information needed to define the \( \mathbf{v} \) vector are not given at a single point as they are in the case of the wind vector, and 2) the vertical alignment of sounding data means that there is a greater overlap in the information provided by successive pairs of data points in the sounding than there is in the information provided by the randomly distributed height and wind data in an isobaric surface.

Upon substitution of the identity

$$\left( \frac{\partial v}{\partial \pi} \right) = \left( \frac{\partial v}{\partial \theta} \right) \left( \frac{\partial \theta}{\partial \pi} \right)_v,$$

we may derive from Eq. (5) the following expression for the direction of the vector:

$$\tan \psi = \frac{f v}{R \theta},$$

where the angle ψ is measured from the vertical.

The significance of the shear-to-stability ratio in analyzing the potential temperature distribution was pointed out by Spillmann (1940). The principle advantage of using a π vertical coordinate is derived from the simplicity of Eqs. (5) and (6). The same equations with height, pressure or log pressure as vertical coordinates involve additional terms and/or variable factors that must be evaluated at each point.

Since θ values at each grid point should be obtained by solving Eqs. (5) and (6) for several data pairs in soundings on both sides of the grid point, it is apparent
that simplicity in (5) and (6) is an important practical consideration.

b. The analysis equation. If a primed quantity denotes the difference between the final and the interpolated value of that quantity at any point, i.e.,

$$\theta' = \theta - \theta_\ast; \quad v' = v - v_i,$$

(7)

where $i$ subscript indicates the interpolated value, then the objective of the second stage of the analysis is to obtain fields of $\theta$ and $v$ which satisfy Eq. (5) and which minimize the quantity $I$ defined by

$$I = \int_A \left( \alpha_i^2 v'^2 + \alpha_2^2 \theta'^2 \right) ds,$$

(8)

where $\alpha_i$'s are weights determined by data reliability. The integration should include the entire analysis area. Substituting (7), (2) and (3) into (8) yields

$$I = \int_A \left[ \frac{\partial \phi}{\partial x} \left( - \frac{1}{f} + \frac{v_i}{x} \right)^2 + \frac{\partial \phi}{\partial \pi} \left( \frac{\kappa}{R} - \frac{\theta_i}{\partial \pi} \right)^2 \right] d\pi dx$$

$$= \int_A FdS.$$  

(9)

In (9) the only unknown quantity is the geopotential. Although the distribution of geopotential is not desired explicitly, all desired quantities can be derived from it. Note that the geopotential is a derived parameter not an independently interpolated quantity. To this point the reported geopotentials have not been used in the analysis. They will be used only to specify boundary conditions.

The minimum $I$ is obtained when

$$\frac{\partial}{\partial x} \left( \frac{\partial F}{\partial \phi_x} \right) + \frac{\partial}{\partial \pi} \left( \frac{\partial F}{\partial \phi_\pi} \right) = 0$$  

(10a)

in the interior of the analysis area and

$$\left[ \frac{\partial F}{\partial \phi_x} \right]_{x_1} = \left[ \frac{\partial F}{\partial \phi_\pi} \right]_{\pi_1} = 0$$  

(10b)

along the boundaries of the area, where $\phi_x$ and $\phi_\pi$ are the partial derivatives of $\phi$ with respect to $x$ and $\pi$, $\delta \phi$ is the change of $\phi$ at boundary points ($\delta \phi = \psi'$) and $x_1$, $x_2$ and $\pi_1$, $\pi_2$ represent the end points of boundary segments. Substituting for $F$ in (10a) yields the "analysis equation"

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial}{\partial x} \left( \frac{1}{\alpha_i^2} \right) \frac{\partial^2 \phi}{\partial x^2} = - \left[ \frac{\partial v_i}{\partial x} + \frac{\partial}{\partial x} \left( \frac{1}{\alpha_i^2} \right) \frac{\partial \theta_i}{\partial \pi} \right].$$  

(11a)

where the right hand side is a function of known (i.e., interpolated) quantities only.

In Eq. (11a) the term $\frac{\partial \phi}{\partial x} \left( \frac{1}{f} \right)$ has been neglected in comparison with $\frac{1}{f^2} \frac{\partial v_i}{\partial x}$ and $\frac{\partial}{\partial x} \left( \frac{1}{f} \right)$ has been neglected in comparison with $\frac{1}{f^2} \frac{\partial \theta_i}{\partial x}$. For a wind speed of 10 m sec$^{-1}$ and a shear of as little as 10 m sec$^{-1}$ per 1000 km at 45° latitude, the neglected terms are an order of magnitude smaller than the terms retained.

Eq. (11) may be solved numerically by standard techniques such as relaxation. The boundary conditions are satisfied either by $\delta \phi = 0$, i.e., by retaining the original $\phi$ values on the boundaries, or by satisfying

$$\frac{\partial \phi}{\partial x} = -fv_i \quad \text{and} \quad \frac{\partial \phi}{\partial \pi} = -\frac{R}{\kappa}$$  

(11b)

on the lateral and horizontal boundaries, respectively.

4. Practical considerations

a. Interpolation. It was mentioned earlier that two distinct steps were used to interpolate the analysis variables and to establish consistency between the variables. Interpolation itself also was divided into a logical sequence of separate steps.

In the first of these steps, wind and potential temperature were interpolated along the vertical line at which the observations were made. The virtual temperature was assumed to vary linearly with $\pi$ between observed values, while wind speed and direction were assumed to vary linearly with height. The virtual temperature at each level of the cross section was found first and these figures inserted into the hypsometric equation gave the height values needed to find the wind at each level.

The horizontal scale of the cross section was fixed by the position of the first and last soundings to be used. These were taken as the lateral boundaries of the analysis area. At each interior grid point the interpolated $\theta$ value was defined as a weighted mean of 20 values extrapolated from the two adjacent soundings. The ten values from each sounding are generated by data from 5$\pi$ levels above and below the level of the grid point. The following finite difference equation was used to obtain each of the 20 values:

$$\theta_\pi = \frac{\theta_\pi + \theta_\pi}{2} + \frac{\theta_\pi - \theta_\pi}{2} \cos \psi$$

$$\times \left[ (M \Delta k)^2 + (N \Delta f)^2 \right] \cos (\psi' - \psi),$$  

(12)

$2$ A $\pi$ increment of 0.01 between levels was selected for all cross sections shown in this paper. This arbitrary choice can be modified at will.
where

$$\psi = \pi - \tan^{-1}\left(\frac{N\Delta j}{M\Delta k}\right)$$

and

$$\psi = \tan^{-1}\left(-\frac{\frac{1}{2} D_2 u_2 - u_1}{R \frac{D_2}{D_1} \theta_2 - \theta_1}\right).$$

The notation is best explained by reference to Fig. 1. The quantities $D_x$ and $D_z$ are the grid increments in the horizontal and vertical, respectively.

The weight applied to each of these values was

$$W = \frac{\tan^2(\psi')}{(M\Delta k)^2 + (N\Delta j)^2}. \quad (13)$$

Since this weight becomes infinite when $\psi' - \psi = \pi/2$ (the isentropes pointing directly at the interpolation point), an upper limit was imposed on $W$ for $\psi' - \psi$ within a few degrees of $\pi/2$.

Occasionally the $\nabla \theta$ vectors at adjacent soundings form a particular combination which imply the existence of a superadiabatic layer somewhere between the soundings. The situation is illustrated schematically in Fig. 2. All superadiabatic layers were replaced by adiabatic layers having a potential temperature equal to the mean of the original values.

The wind between soundings was obtained by linear interpolation along isentropes. In certain critical areas of the cross section this procedure gave much better results than interpolation along isobars; in other regions the two methods performed about equally. The difference occurred in strongly baroclinic regions where isentrope and isotach surfaces are both inclined to isobaric surfaces at relatively large angles (cf. illustrations under Section 6). Since it is the delineation of precisely these regions that is of greatest interest in a study of atmospheric structure, the extra complication involved in interpolation along isentropes was very much worthwhile.

In summary, the first stage of the analysis consisted of the following sequence of operations:

1. Vertical interpolation of $\theta$ and $v$ along the line of ascent of each sounding.
2. Interpolation of $\theta$ using Eq. (12).
3. Removal of superadiabats (if any).
4. Interpolation of $v$ along isentropes.

b. Establishing consistency. The profile of geopotential along the lowest pressure level in the cross section was determined from the interpolated wind speeds which were assumed to be in geostrophic balance at that level. The geopotential at all interior points and along the lateral boundaries were built up from the lower level through the hypsometric equation and the interpolated $\theta$ values. Considering the number of approximations made in the analysis up to this point, an automatic geostrophic balance certainly could not be expected between the interpolated winds along the upper boundary and the geopotential difference across the boundary (i.e., the difference between the geopotential at the topmost points of the lateral boundaries). The effect of this inconsistency appeared to be smallest when it was spread out over the boundary by allocating the total geopotential difference in proportion to the interpolated wind speed at each point.

In the few particular cases used to test the analysis procedure, interpolation between each sounding, rather than between those at the end points only, did not improve the specification of upper and lower boundary conditions. Instead, it tended to increase the total
absolute value of the discrepancy between the geographic gradient implied by the interpolated winds and the difference in geopotential across adjacent grid points. It is obvious, however, that in many instances the additional information available at each sounding point would be essential to a proper assignment of boundary geopotential.

Two terms in the analysis equation include a parameter which determines the relative weight to be assigned to the interpolated fields of wind and of temperature. This weighting factor need not be a constant but the form of the analysis equation presented above has been derived on that assumption. The examples of the next section were obtained using a weighting factor which was arbitrarily chosen so that a difference of 1C between the interpolated potential temperature at a point and the final value at that point had the same significance as a difference of 1 m sec\(^{-1}\) between the interpolated and final wind speed.

**c. Potential vorticity.** If the cross section plane is nearly perpendicular to the wind direction and if the curvature of the flow is small, the component of horizontal wind shear which is defined by the cross section will be a good approximation to the total horizontal shear. At the conclusion of the objective analysis the given shear component and the potential temperature provide the wherewithal for an estimate of the potential vorticity distribution. Relatively little additional work is required to obtain this by-product since all the necessary information is already given at each grid point.

If the relative vorticity is taken to be \(-\frac{\partial \psi}{\partial x}\) (the component \(v\) was defined as positive blowing out of the cross section plane), the potential vorticity on an isentropic surface,

\[ \tilde{\psi}_{\theta} = \alpha \nabla \theta \cdot (\nabla \times V + 2\Omega), \]

is approximated by

\[ \tilde{\psi}_{\theta} = \frac{\alpha^2}{p} R f \pi \left[ \frac{\partial^2 \phi}{\partial \varphi^2} \left( \frac{\partial \phi}{\partial \varphi} + f^2 \right) - \left( \frac{\partial \phi}{\partial \varphi} \right)^2 \right], \tag{14} \]

where \(\tilde{\psi}_{\theta}\) is the potential vorticity and other symbols are as defined previously. Potential vorticity on an isobaric surface is measured by the first term in the square brackets while conversion to an isentropic surface is accomplished by the inclusion of the second term.

5. Limitations and sources of error

No provision has been made to include soundings that do not span the vertical extent of the cross section. The method described also includes no provision for the use of either temperature or wind information when only one of the two types of data is available.

In the "analysis equation" (11a), interpolated stability values and interpolated wind shear values enter as a weighted sum which acts as a "forcing function" for the equation. Since this sum, rather than its individual components, determines the solution of the equation, it is possible to obtain an analysis in which stability and wind shear differ greatly from their interpolated values even though their sum has been nearly conserved. This will happen particularly if the wind shear or stability along the boundary are in serious disagreement with interpolated values near the boundary. The difficulty is inherent in Sasaki's method. Just as our analysis equation does not distinguish between stability and wind shear, the equation corresponding to (11a) which is applicable to analysis in horizontal planes does not distinguish between shear and curvature vorticity. An example of the effect of this ambiguity appears in the cross section for 0000 GMT, 19 April 1963 which will be discussed later.

The analysis equation also has the effect of smoothing the interpolated fields since any initial imbalance between isotachs and isentropes at one grid point is removed by altering the analysis at surrounding points as well as at the point where the imbalance was found.

6. Results

Three examples of objectively analyzed cross sections will illustrate the capabilities and limitations of the method outlined above. In each case the objective analysis will be compared to the best subjective analysis available. This comparison must be made with the following points in mind: 1) The isothach distribution in the subjective analysis represents actual winds whereas the objective version depicts the geostrophic wind in accord with the analyzed temperature distribution; 2) The subjective analyses are based on all available information including horizontal isentropic charts before and after the time of the cross section. This additional information allows non-linear interpolation between soundings on the basis of continuity and conservation laws; 3) The subjective analysis utilized information obtained from the original strip charts produced by the radiosonde recorders but only a limited number of discrete point values of temperature taken from these soundings were read into the machine for the objective analysis.

a. 21 January 1959, 12 GMT; Cape Hatteras to Fort Churchill. During the 24 hours following 0000 GMT, 21 January 1959, an intense cyclone developed over the middle of North America.\(^3\) At 1200 GMT, 21 January, the flow ahead of the upper level trough associated with this cyclone was nearly perpendicular to a line from Cape Hatteras, N. C., to Fort Churchill, Manitoba. An objectively analyzed cross section\(^4\) along

\(^3\) This storm was chosen for a study of vertical velocities and latent heat release by Danard (1964).

\(^4\) Since no automatic curve following device was available to the authors, the isopleths in all cross sections shown in this paper were drawn by hand from the 1710 (57×30) point values given by the computer.
Fig. 3. Objectively analyzed cross section for 21 January 1959 at 0000 GMT. Isentropes (5°C intervals) are solid lines while geostrophic wind (10 m sec⁻¹ intervals) are dashed lines. A positive wind component is one flowing toward the viewer from the cross section.

Fig. 4. Subjectively analyzed cross section for 21 January 1959 at 0000 GMT. Note that isotachs (dashed lines at 5 m sec⁻¹ intervals) are drawn for the observed wind component normal to the cross section plane rather than for the geostrophic wind component.
this line is shown in Fig. 3. A subjective analysis of the same situation is shown in Fig. 4.

In both Figs. 3 and 4, the $\theta$ lines were drawn for the same 5C interval, however the isotachs in the hand analysis (Fig. 4) were drawn for a 5 m sec$^{-1}$ interval rather than the 10 m sec$^{-1}$ interval of the machine analysis. The machine analysis also extends to 50 mb while the hand analysis terminates at 100 mb. Taking cognizance of these differences in the two figures it is apparent that the overall patterns of stability and baroclinity are similar. Both show that the troposphere is most strongly baroclinic below the jet and the stratosphere most baroclinic above the jet. They differ however in the position of the jet. This difference is due partly to the large separation between Sault Sainte Marie and Trout Lake (approximately twice the distance between the other soundings) and partly to the difficulty of interpolating when the maximum is located between observing stations.

In the hand analysis the position of the jet and the isotach distribution about the jet were transferred from analyses of quasi-horizontal isentropic charts. The lack of this information in the machine affected the magnitude of the isotach maximum as well as its position. It should be noted, however, that analyses of the quasi-horizontal charts indicated that the trajectory of the actual wind had sufficient anticyclonic curvature to account for an observed speed fifty per cent larger than the 60 m sec$^{-1}$ geostrophic maximum.

The radiosonde data from Washington, D. C., which was withheld from the machine, can be compared with the analyzed temperature distribution (Fig. 5). The greatest temperature difference is about 5C. It occurs
at 160 mb where a shallow but strong inversion observed just above the tropopause is not reproduced in the analysis. This failure can be attributed to the absence of such an inversion in the soundings on either side of Washington. Elsewhere in Fig. 5 the greatest difference is about $3^\circ$C at 230 mb. Below this level there is a nearly adiabatic layer which was reported at Pittsburgh but not at Norfolk. In this case the analysis erred by assigning too much weight to the stratification observed at the station nearer to Washington. The low level inversion over Washington was not reproduced by the analysis partly because it straddles the lower boundary of the cross section (943 mb), and partly because the analysis smooths such inversions. The smoothing effect will be discussed in more detail in connection with the next example.

The objective analysis can be evaluated further through the potential vorticity distribution. This is shown in Fig. 6. Since two orders of magnitude separate the highest and lowest values of potential vorticity, it was convenient to draw isopleths at logarithmic intervals. The isopleths are labeled in cgs units [cm$^3$ ($^\circ$C) gm$^{-1}$ sec$^{-1}$] multiplied by 10$^7$. The potential vorticity in the subjective analysis was not determined quantitatively. However, it can be readily deduced qualitatively by examining the stability and the shears on isentropic surfaces. For example, the baroclinic layer which slopes from the lower stratosphere at Trout Lake to the lower troposphere at Flint contains two regions of cyclonic shear separated in the vicinity of Sault Sainte Marie by a region of anti-cyclonic shear. Because the stability is relatively high throughout this layer the two regions of cyclonic shear should be regions of relatively high potential vorticity and they should be separated by a region of intermediate or low potential vorticity.

Fig. 6 does, in fact, show one vorticity maximum in the lower troposphere between Pittsburgh and Flint and another relative maximum extends from the lower stratosphere north of Sault Sainte Marie. This second feature is obviously too far south and the reason is the erroneous objective analysis of the jet which was discussed previously.

The very strong potential vorticity gradient at about 175 mb from Cape Hatteras to Flint reflects the abrupt change in stability from the troposphere to the stratosphere in this region. The stratospheric maximum in potential vorticity occurs where the shear becomes cyclonic between Pittsburgh and Flint. There is much larger cyclonic shear in the stratosphere over Trout Lake but no corresponding vorticity maximum appears in Fig. 6 again because the jet was erroneously analyzed by the objective method.

**b. 21 April 1963, 1200 GMT; San Diego to Seattle.**

A large cyclone dominated the upper troposphere and lower stratosphere over the western United States on 21 April 1963. The cross section shown in Figs. 7 and 8 describes the distribution of potential temperature and the normal wind components along a vertical plane that follows the west coast stations. Located west of the cyclone center, the cross section plane curves in the

![Fig. 7. Objectively analyzed cross section for 21 April 1963 at 1200 GMT.](image-url)
same sense as the wind flow about the cyclone. Therefore, the tangential components of the wind, although quite large between Seattle and Oakland, did not contribute to the analysis. This line of stations served to test the capability of the objective analysis method when the cross section was not oriented perpendicular to the flow.

Within the pressure interval (100 to 850 mb) common to both analyses we find a large-scale similarity between the objective and hand analyses. The objective analysis, as one might expect, has smoothed out some of the short wavelength oscillations and generally reduced the baroclinity. The reduction of baroclinity in the stable layer that slopes from the 500-mb level at Point Arguello to the lower stratosphere north of Oakland is serious. It destroys the spatial continuity of the stable layer and significantly alters the distribution of potential vorticity.

Since the discrepancies exist between the observation stations perhaps the machine analysis is correct and the hand analysis is wrong. In this particular case the accuracy of the objective analysis can be tested against two independent effects. When the interpolated wind and temperature fields are brought into adjustment both fields are altered. If the interpolated fields are close to adjustment the alterations are minor but if a large maladjustment exists in the interpolated values the changes in both fields spread outward from the point of maladjustment as the integral in Eq. (8) is minimized. Although the maladjustments are most probable between observing stations, their influence can spread beyond the stations. In this case the sounding at Point Arguello was considerably altered, as can be seen by comparing the 315 and 320K isentropes in Figs. 7 and 8. The Oakland sounding was also altered between 300 and 310K. At both stations the changes spread the isentropes and decreased the stability and baroclinity in the sloping stable layer. The objective analysis is therefore suspect.

Aircraft sampling of radioactivity offers a second method for testing the accuracy of the two analyses. This cross section was chosen to be upwind from one of the flight operations of Project Springfield (Danielson, 1964). Air in the plane of the cross section between Los Angeles and Oakland moved eastward, passing Nevada and Arizona between 1800 and 2400 GMT. During this time WB-50 and RB-37 aircraft were flying across the stream flow taking temperature, dew point, Doppler wind and radioactivity measurements at altitudes ranging from 18,000–42,000 ft. Their measurements show that the stable layer was continuous from the lower stratosphere to the mid troposphere and
that it contained radioactivity as concentrated as in the lower stratosphere. Also the aircraft wind and temperature measurements show that the horizontal wind shears and temperature gradients were as large and as concentrated as shown in the hand analysis, not spread out as in the objective analysis. Furthermore the potential vorticity computed from the aircraft data shows a continuous tongue of high values extending downward and southward from the cyclonic stratosphere which was not broken as in Fig. 9.

The combined evidence supports the subjective analysis not the objective analysis. To isolate the source of the error and to eliminate the error has been challenging and somewhat irritating. Interpolating winds along the $\theta$ surfaces rather than the pressure surfaces improved the analyses but has not eliminated the above discrepancy. It is now clear that the difficulty stems from three sources:

1. When a stable zone has a very steep slope between radiosonde stations, the interpolated $\theta$ field tends to spread the zone, i.e., to reduce both the stability and horizontal temperature gradient in the zone.

2. Stable zones with the steepest slopes are often located beneath a wind maximum or jet core. Thus for proper analysis a wind maximum must be interpolated between stations.

3. The reported wind shears are weakened and extended beyond the limits of a baroclinic zone because they are two or four minute averages computed from data points one minute apart. Since these shears enter the initial $\theta$ interpolation they also act to spread the zone.

A survey of the $\theta$ and $v$ fields both before and after applying the analysis equations indicates that all three error sources contribute in the case under discussion. It is difficult to assess their relative importance but it seems likely that the failure to interpolate the wind maximum is the most serious error. To eliminate this error will require extrapolating errors from the adjacent stations.

It should be noted, incidentally, that the rawin observations for Los Angeles at the time of this cross section terminated at 5000 m. The winds used in both the subjective and objective analyses above this level were interpolated from surrounding stations. At 18 km the Los Angeles wind was estimated as $235^\circ$ at 10 m sec$^{-1}$, which was observed at San Nicolas. At the same time and height, however, the Point Arguello wind was $223^\circ$ at 23 m sec$^{-1}$. The anticyclonic shear implied by these two winds exceeded the coriolis parameter and produced an anomalous cell of negative potential vorticity at 75 mb (not shown in Fig. 9). A similar feature appeared in the potential vorticity distribution of the case to be discussed next in which actual wind data was used throughout.

c. 19 April 1963, 0000 GMT; Midland to Boise. The cross section shown in Figs. 10 and 11 was made perpendicular to the flow. Fig. 11 was prepared for the Project Springfield report which accounts for the differ-
Fig. 10. Objectively analyzed cross section for 19 April 1963 at 0000 GMT.

Fig. 11. Subjectively analyzed cross section for 19 April 1963 at 0000 GMT. Note that the isotachs (dashed lines) are labeled in knots and that the isentropes are drawn for every 2°C. The numbers in heavy print refer to radiometric observations and are disintegrations per minute per standard cubic foot of air for total \( \beta \) radiation (after Danielsen, 1964).
Before we compare the analyses in Figs. 10 and 11, we must stress that Fig. 11 was prepared from both aircraft measurements and radiosonde data. The flight paths were about 100 miles west of Denver, i.e., west of the continental divide. When the aircraft crossed and recrossed the stable layer at 27,000 ft, the layer was strongly baroclinic. East of the divide, on the Denver sounding, the stable layer was compressed vertically and exhibited no vertical shear of wind. It is quite probable that the mountains induced standing waves in the layer and the lack of vertical shear would imply a transit by the radiosonde at a ridge in the wave. Whether this assumption is correct or incorrect, the lack of shear at Denver must have been due to a short wave.

This cross section is a strong challenge to an objective analysis method because it contains a deep layer of almost zero stability. The adiabatic layer extends upward from the earth’s surface to varying depths but across the entire section. Although the stability is very small the layer is not barotropic. This, in itself, should cause no trouble because the thermal wind only depends on the horizontal $\theta$ gradient. However, the thermal wind relation might be invalidated by the vertical mixing which would be expected and was observed in this baroclinic adiabatic layer.

Actually the objective analysis depicts this feature quite well. In certain regions it produced slight superadiabatic lapse rates but they are not objectionable. The wind field in the adiabatic layer was also resolved.

Fig. 12. Observed and analyzed temperature profiles above Denver, Colorado, 19 April 1963 at 0000 GMT. The line joining the circles is the observed data, that joining the crosses is the analyzed profile.

Fig. 13. The approximate potential vorticity distribution derived from the objective analysis shown in Fig. 10.
well by the objective analysis. In the hand analysis isentropic charts and sounding stations up and down wind from the cross section were used to resolve the wind field. Considering the complexity of the adiabatic layer and the fact that data from only the five stations were utilized, the objective analysis performed most satisfactorily.

As in the previous example the stable layer sloping from 600 mb at Midland to the lower stratosphere at Lander has been broadened and weakened.

The magnitude of the error can be seen by comparing the observed and the analyzed temperature profiles above Denver (Fig. 12). In this case, however, the isotachs and isentropes have the proper alignment. North of Denver the wind shears on the isentropic surfaces are cyclonic, south of Amarillo the shears are anticyclonic. In the region between, the shears are approximately zero. The corresponding potential vorticity distribution is shown in Fig. 13.

7. Conclusions

The success achieved in this first attempt to analyze cross sections objectively appears to be at least as good as that attained in the early work on objective isobaric analysis (Panofsky, 1949). From the results obtained this far several ways of improving the analyses are suggested:

1. The most glaring defect in the present method would certainly be removed by some form of non-linear interpolation of wind components. This might be based on the geostrophic wind gradients implied by the first estimate of the potential temperature field.

2. The pattern recognition aspect of the analysis should be reinforced. For example, soundings could be segmented into layers of relatively uniform stability and the layers could be interpolated between adjacent soundings by altering the potential temperature distribution. Correspondence between layers on two soundings could be established on the basis of similar potential temperatures within the layers. Such a procedure would improve the continuity in the stability field.

3. Adjustment between the interpolated wind and temperature fields should be made in such a way that the analysis is most affected where the interpolation is least reliable, i.e., modifications of the interpolated values should be proportional to distance from soundings. The most desirable relationship between the wind and temperature fields in the final analysis is an open question and depends, at least in part, on the purpose of the analysis.

The use of an analysis equation based on the calculus of variations (Sasaki's method) may not be the most suitable way to obtain a cross section. The theoretical elegance of an analysis equation is offset in practice by the need for boundary conditions which generally cannot be determined with satisfactory accuracy. A method based on iterative interpolation (successive approximation) would be free from this requirement and would, in addition, allow more flexibility in the choice of "smoothness" of the analysis.

The initial data provided to the objective analyses which have been shown above did not include any information "projected" from stations lying outside of the cross section plane. A successful analysis scheme could incorporate such information by analyzing secondary cross sections linking pairs of stations on opposite sides of the main cross section. Such a procedure was judged to be inappropriate for use with the present scheme in view of the errors discussed above.

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