Future Occurrence of Threshold-Crossing Seasonal Rainfall Totals: Methodology and Application to Sites in Africa

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(Manuscript received 12 April 2010, in final form 22 September 2010)

ABSTRACT

A statistical simulation framework is developed to explore the future frequencies of threshold-crossing events, focusing here on low seasonal rainfall totals. Global change (GC) is represented by a trend on the seasonal mean rainfall total. Natural decadal to multidecadal variability (MDV) is represented by an autoregressive process. Interannual variability (IV) of seasonal totals is represented by white noise with either a normal or skew normal distribution consistent with parameters observed in the historical record at the location being modeled. Monte Carlo simulations are undertaken for various combinations of the above components, and the authors evaluate the extent to which future event frequencies can be estimated from the statistics of previous years. The sample of four study locations used to illustrate the approach is drawn from the Millennium Villages Project in Africa, where the potential of index insurance as a development and adaptation tool has been considered, thereby bringing a targeted problem setting to the analyses. The simulations highlight a number of general principles. For example, it is shown that a 10% change in the mean rainfall can lead to a change of order times 2 in the number of threshold-crossing low seasonal rainfall totals, even without invoking any change in the characteristics of the IV. The magnitudes of change are also shown to be sensitive to the threshold studied, as well as to site-specific climate features (here, coefficient of variation and skewness). The framework developed permits quantification of how, especially in the near term (of order 30 years), MDV can strongly add to uncertainty about future event frequencies. Therefore, statistical treatment of estimated MDV magnitudes will often be a key input to optimal risk management, with further enhancements expected through explicit MDV forecasts. The results highlight the importance of finding optimal ways to update climate statistics such as event thresholds, in the presence of GC and MDV.

1. Introduction

Changes in the risk of extreme climate events pose one of the most serious challenges in adapting to an evolving climate. This has motivated considerable research into the nature and space–time patterns of climate extremes both in the historical period (Peterson and Manton 2008; New et al. 2006; Easterling et al. 2000a,b; Frich et al. 2002; Jenkinson 1955; Begueria and Vicente-Cerrano 2006) and projected forward through the twenty-first century by general circulation models (GCMs) under different emissions scenarios (Meehl et al. 2000; Kharin et al. 2007; Voss et al. 2002; Wehner 2004; Tebaldi et al. 2006; Paeth et al. 2009; Zwiers and Kharin 1998; Kharin and Zwiers 2000; Zhang et al. 2004). This research, underpinned by analysis of observations and GCM output, has built a knowledge base for general expectations on changes in climate extremes.

For many risk management applications, users need information tailored to their problem, and in many settings, information is needed on the expected future frequency of specified thresholds being crossed. In this paper, we develop a statistical simulation approach to provide baseline estimates of such risks, under specified assumptions of climate variability and change. It is intended to provide complimentary information to approaches that directly downscale and transform output from GCM...
scenarios (e.g., Li et al. 2009). This statistical approach permits the representation of natural decadal to multi-decadal variability (MDV) in a random stochastic sense, a feature sometimes not explicitly emphasized in GCM scenarios. In this paper, the methodology is applied to seasonal rainfall totals at four representative sites in Africa, and with a particular motivating problem of index insurance. The aim is to provide insights into the levels of bias and uncertainty in estimating event frequencies into the future, under specified assumptions of climate variability and change.

The framework introduced in this paper has attempted to maintain a simplicity that permits clear illustration of concepts. The focus is on low seasonal rainfall totals. One advantage of working with seasonal rainfall totals is that there is no clear consensus on whether greenhouse gases (GHGs) will systematically increase the variance of this variable (in contrast to daily rainfall variance, which is expected to systematically increase). Therefore, for seasonal rainfall totals, we make a simplifying assumption of no change in the interannual variance. It is nonetheless recognized that there is evidence that the variance of seasonal rainfall totals will modestly increase or decrease, in proportion to the increase or decrease in the seasonal rainfall climatology (Peng and Kumar 2005; Wu and Kirtman 2006). This systematic relation of mean rainfall to variance warrants further investigation, especially in its physical basis and implications for global change. However, in this paper, we focus on the fact that a shift in the mean total, with no change in the variance, can have nonlinear impact on the risk of given threshold events, as recognized in a general sense, in some previous studies (Katz and Brown 1992; Meehl et al. 2000). Therefore, one of the aims of this paper is to highlight and quantify this effect in a practical problem setting.

The presence of autocorrelation, as implied when MDV is present in a regional climate, adds further to the uncertainty of the expected change in the risk of given threshold events (Katz and Brown 1994). A few studies have considered the implication of such variability. For example, Colombo et al. (1999) studied the relationship between autoregressive processes and implications of extreme temperatures on power generation in Canada, while Karl et al. (1996) explored changes in extremes in U.S. climate indices in the presence of autoregressive moving average processes. The results in this paper also aim to provide insights into the quantitative implications of MDV in the chosen problem setting.

Informed by the above discussion, the simulation approach for seasonal rainfall totals in this study contains three components of variation: (i) a systematic percentage change in the mean, representing global change (GC); (ii) a stochastic low-frequency component, simulated through an autoregressive (AR) process, to represent the presence of MDV; and (iii) white noise interannual variability (IV). Two forms of IV are experimented with, sampled from the normal distribution and the skew normal distribution.

Drawing on Monte Carlo simulations containing different combinations of the three components of variation through 2100, the study assesses the extent to which statistics of the last 30 years (1979–2008) provide reliable estimates of the frequency of specified events throughout the twenty-first century. In the presence of GC, this approach is expected to yield substantial bias, and results serve as a quantification of the extent to which there is a change in the frequency of the specified event. Simulations with and without MDV provide an indication of the contribution such variations can make to uncertainty about the frequency of events, both in a stationary climate and in the presence of GC.

A further set of simulations is generated in which we update event thresholds each year as the twenty-first century unfolds. There is no attempt to optimize the updating period in this study, rather to provide a baseline estimate of improvement based on the widely used window of 30 years for updating climate statistics. The simulation framework developed here is well suited in the future to explore the properties of different optimizing systems (e.g., Livezey et al. 2007) under contrasting assumptions of variability and change.

The study is placed in the context of work that is considering development interventions, including the management of climate-related risks, in the Millennium Villages (MVs) in Africa (Sanchez et al. 2007). A survey was sent to each MV inquiring as to the primary sources of livelihood vulnerabilities, and in almost every case drought risk figured prominently in the results of the survey. In addition to undertaking general analysis of drought risks at the sites, one specific intervention that has been explored is index insurance (Ward et al. 2008). Index insurance is a tool that is being widely considered as a way of managing climate risks in both developing and advanced economies (Hellmuth et al. 2009; Berg et al. 2009). One of the emergent questions is the extent to which a changing climate will violate estimates of expected payout frequencies and render the tool unviable in such situations. Indeed, the sensitivity of insurance payouts under climate change is an emerging area of research (Iizumi et al. 2008). Therefore, the assessment of the reliability of “nowcasts” of the future probability of events frequencies under different variability and change scenarios has immediate relevance to index insurance. Discussion with development practitioners in the MVP highlighted two event thresholds (for low
seasonal rainfall) that were of particular interest: 1 in 8 (1/8) and 1 in 20 (1/20). These events could also be thought of as the seasonal low rainfall totals that have an 8- or 20-yr recurrence interval. This paper adopts these two thresholds for an initial exploration of sensitivities. This choice provides a sample of what may be considered a mild and a moderate event, while reserving for future work the issue of risks in the very extreme of the tail of distributions, such as 1-in-100-yr events. This includes reserving the implications of the application of extreme value theory. Previous climatological work has so far generally applied extreme value theory in the analysis of the more extreme tail of distributions (such as 1-in-100-yr events) as opposed to 1-in-20-yr events. Extreme value analyses often require estimation of the frequency of events that are beyond the obvious scope of the data, and therefore involve considerable interpolation assumptions (Coles 2001, chapter 1). A smaller number of simulations were undertaken with different event thresholds in the range of 1/8 to 1/20 to ensure results presented here were representative. The MV locations considered here are Tiby (Mali), Dertu (Kenya), Mbola (Tanzania), and Mwandama (Malawi) (see Fig. 1). These four locations were chosen because they span a range of the climatologies and environments found in Africa (ranging from near-desert to tropical humid), while still having vulnerability to drought.

Section 2 describes the data used and the overall methodology, including the simulation methods and some preparatory analysis. Section 3 presents simulations without GC and with GC (for normal and skew normal distributions), giving estimates of the magnitudes of changes of future event frequencies when thresholds are held constant and when thresholds are updated through the course of the twenty-first century. Section 4 explores the impact of MDV, both in a stationary climate and in the presence of GC. Again, results are presented with thresholds held constant and with thresholds updated. Conclusions are provided in section 5.

2. Data, preparatory analysis, and simulation methods

a. Rainfall data and indices

Monthly rainfall data are extracted from the Climate Anomaly Monitoring System—outgoing longwave radiation (OLR) precipitation index (CAMS–OPI) rainfall dataset (Xie and Arkin 1998; Janowiak and Xie 1999). This is a merged satellite–land station dataset, at resolution 2.5° latitude × 2.5° longitude, starting in January 1979 and updated to the present. It is recognized as one of the best complete, updated sets covering over 30 years. In considering practical risk management applications, its use has the advantage that the data are updated in near–real time. One aspect that would need to be evaluated for practical applications is the homogeneity over time given the merging of satellite and station data. However, for the purposes of illustrating the basic properties of the simulation framework proposed here, the dataset is considered suitable. Checks were made that station data contained the same basic properties for the four locations, using the Global Historical Climatology Network (GHCN) station dataset (the CAMS–OPI set is used for the analyses here because the GHCN data generally have many missing values). For all results reported here, the nearest grid box to the village locations (Fig. 1) is used.

The monthly CAMS–OPI rainfall data are averaged over the period of the rainy season at each location to form seasonal time series (Fig. 2). It was decided to not attempt to implement any preprocessing of these indices, such that, in the formulation adopted here, their standard deviation is deemed an estimate of the IV plus MDV. Establishing the best base statistics, drawing on the observed record, represents an important practical area for further exploration. The most challenging index here is Tiby, which, being located in the Sahel of Africa, exhibits the known partial recovery from the drought period of the 1970s and 1980s (Fig. 2a) (Nicholson 1980, 2005). However, even for this index, applying a least squares detrending algorithm only reduced the estimated standard deviation by about 15%.
b. Analysis procedure and verification statistics

For a given combination of GC, MDV, and IV, 100 stochastic simulations are run applicable to the period 2009–2100. This simulation size should be increased for more exact estimates, but is sufficient here to identify salient features in the results. For fixed threshold (FT) experiments, the observed series over the period 1979–2008 is used to define the thresholds that correspond to the 1/8 and 1/20 events. This is done by fitting either a normal distribution or skew normal distribution to the data for the 1979–2008 period. The normal distribution choice is the most widely used assumption for seasonal rainfall, and the skew normal distribution choice provides a comparison that focuses on a systematic feature of seasonal rainfall distributions that departs from normality (as discussed more in section 2c). A more substantial intercomparison would be needed to assess the advantages of various different distributions, and the purpose here is to provide an initial indication of sensitivity to the inclusion of the skew aspect. Since seasonal rainfall totals fall either above or below the given 1/8 and 1/20 threshold definitions, there is a binomial nature to the event frequency, described in appendix A. In a given 30-yr period, the expected number of 1/8 events is given by Eq. (A2) \( \frac{30}{8} \times 3.75 = 3.75 \), and 1/20 events is \( \frac{30}{20} \times 1.5 = 1.5 \). For presentation of results, a snap shot is taken usually for the near-term period 2011–40 and the later period 2071–2100, evaluating the extent to which there are departures from the expected number of 1/8 and 1/20 events.

The mean bias provides an indication of the extent to which there is a systematic tendency for more or less events than the expected value. In the index insurance context, the bias statistic corresponds to the mean change in payout frequency. For example, the bias in the 2011–40 period for the 1/8 event is calculated as

\[
\text{BIAS} = \left[ \frac{\sum_{i=1}^{100} (n_i - 3.75)}{100} \right],
\]

where \( n_i \) is the number of events over the period 2011–40 in simulation \( i \).

A second experiment type is termed updating threshold (UT). Here, the thresholds used to define the 1/8 and 1/20 events are always based on the previous 30 years of data. The procedure is exactly as before, fitting a normal
or skew normal distribution for the estimation of the 1/8 and 1/20 thresholds. Thus, UT experiments allow evaluation of the extent to which, under different variability and change scenarios, updating thresholds (using a 30-yr window) is able to reduce BIAS, relative to that found in FT experiments.

A second statistic consulted is the mean absolute error (ABSE). For example, the ABSE in the 2011–40 period for the 1/8 event is calculated

$$\text{ABSE} = \left[ \frac{100}{N} \sum_{i=1}^{100} \sqrt{(n_i - 3.75)^2} \right] / 100,$$

where \(n_i\) is the number of events over the period 2011–40 in simulation \(i\). ABSE gives an indication of the extent to which, across the 100 simulations, there are large positive or negative departures from the expected number of events. For some risk management applications, including index insurance, it is important to have estimates of the spread of possible outcomes. In the presence of MDV in the climate system, the risk of runs of anomalies of a given sign is increased, and this will translate in some simulations into a very large increase in the number of dry extremes, an increase that is much less common in simulations with white noise alone. Such simulations will be counterbalanced by others when the MDV introduces a wet phase, leading to very few or no dry extremes. Many risk management applications need to have an estimate of this volatility in expected extremes for a future period of time (such as the next 30 years). In terms of index insurance, a high ABSE indicates increased uncertainty in the expected number of payouts (even if not systematically shifted higher or lower). The increased uncertainty translates into a more uncertain and larger average risk cost (Hellmuth et al. 2009). In other applications settings, the ABSE can be a useful measure of the range of possible outcomes for which to plan on a decadal time scale. The ABSE is just one way of capturing this effect, and other tailored representations may be more useful in specified problem settings. Graphical presentations of this volatility are also made in this paper.

There are some systematic features of BIAS and ABSE that will influence results, in addition to the imposed climate scenarios and site-specific statistics. In the presence of a change in the mean, BIAS will be systematically sensitive to the height and slope of the probability density function (PDF) in the vicinity of the targeted thresholds (this effect is also discussed in Katz and Brown 1992). For example, the standard normal distribution shows that locally, in the vicinity of the 1/8 and 1/20 thresholds, the slopes of the PDF are about 0.24 and 0.17, respectively. A higher slope generally increases sensitivity and leads to higher BIAS values for a given change in the mean, so larger BIAS values are expected for 1/8 events than for 1/20 events. For the ABSE, there is sensitivity to the location of the threshold in the PDF under analysis. This is because the variance of the number of events in a specified x year period is a function of the threshold probability (see appendix A). The lower the threshold probability (i.e., the more extreme the event), the lower the expected value of the ABSE.

c. White noise simulation for the IV component

Table 1 gives some basic descriptive statistics of the rainfall indices. To give context to the studied sites, time series of average rainfall season precipitation (mm month\(^{-1}\)) at each gridbox location in Africa between 20°N and 20°S were also calculated. For 3.75°N to 3.75°S, series were calculated separately for April–June and October–December, to reflect the generally bimodal rainfall variations at these latitudes, while a single rainy season index was calculated to the north of 3.75°N (July–September) and south of 3.75°S (January–March). While this sample of series could be improved upon, it is deemed sufficient to place the indices studied here in context. Seasonal rainfall is greatest for Mwandama and Mbola, which are in the third quartile of all seasonal rainfall climatologies, while Tiby sits in the second quartile and Dertu, the driest location studied, is in the lower quartile of all seasonal rainfall indices. For a skewness normality test, the standard error of skewness is used, as given by Tabachnick and Fidell (1996):

$$\text{SES} = \sqrt{\frac{6}{N}},$$

where \(N\) is the sample size. The null hypothesis of 0 skew is only rejected in the case of Tiby, using a two-sided 5% statistical significance test (Table 1). However, there are systematic features that suggest skew is a general property of many seasonal rainfall time series in tropical Africa. Almost three-quarters of the skew values...
calculated for the 331 time series were positive. Furthermore, the approximate 25% of skew values that were negative were generally systematically located in moist humid environments, whereas the strongly positive skew tended to be found in drier climates. The correlation coefficient between skew and climatological rainfall was $0.46$ (Fig. 3). This skew–climatology relationship conforms with physical intuition: in arid environments, the low seasonal rainfall totals are bounded by 0 but the higher seasonal rainfall totals are unrestricted potentially leading to a strong positive skew. In contrast, a more humid climatology is likely to limit this effect, and in some cases the skew is actually reversed, with most years being at or above average precipitation, but with a few drier years serving as the strongest outliers.

Therefore the skew is considered a real feature of the IV for many locations. Of the 331 rainfall time series calculated, the skew values in Table 1 are spread quite evenly: Tiby ranked 60th, Mwandama 97th, Dertu 138th, and Mbola 315th. The skew is very relevant for estimating thresholds of $1/8$ and $1/20$ events, and so, for some of the experiments that follow, the skew in the observed record has been maintained by simulating white noise using the skew normal distribution (see appendix B). While there is limited climate literature to date on the use of the skew normal distribution, its capacity to capture the statistical characteristics of skewed distributions suggests its consideration here can help by at least adding an additional scenario for comparison with ones based on the normal distribution assumption. In terms of standardized units, Fig. 4 compares the fitted skew normal distributions for each study location and the associated thresholds for the $1/8$ and $1/20$ dry events. For comparison, the standardized normal curve and associated thresholds are also shown.

d. Addition of the trend component associated with GC

In the Intergovernmental Panel on Climate Change (IPCC) Fourth Assessment Report (AR4) (Solomon et al. 2007), projected change in rainfall was reported for different regions of Africa. In this study, the intention is not to make a best estimate of the trend to impose, but rather to investigate the implication of the magnitude of trends that are consistent with IPCC assessments. Inspection of trends reported in the IPCC for different regions in Africa (and other tropical continental regions) suggests that trends of $6-10\%$ of precipitation over the course of the twenty-first century are not unreasonable or inconsistent with the scale of the IPCC projections. Any trend shape could be imposed to capture this $6\%$ change of precipitation. However, we here impose a simple trend shape assumption, following the shape of a modeled twenty-first-century atmospheric CO$_2$ concentration [Fig. 5, based on Nakicenovic and Swart (2001) and consistent with typical scenarios used in Metz et al. (2007)]. This leads to the imposed trends here being very close to linear, and results in this paper would likely not be substantially altered by making a linear assumption. However, the results illustrate the methodology of adopting a best estimate of trend shape, which could be adjusted in more complex ways in the future according to specific scenarios for a study region.

The sites have contrasting coefficients of variation, and this has implications for the way the $+10\%$ and $-10\%$ trends impact on the series. For example, a $10\%$ change in the mean at Tiby corresponds to 0.7 standard
deviations, whereas at Dertu, it corresponds to 0.34 standard deviations (Table 1). The trend more dominates the variance in simulations for Tiby (example trace in Fig. 6a) as compared to Dertu (example trace in Fig. 6b), and this comes to have important implications for the behavior of changes in the 1/8 and 1/20 events in the simulations. In general, a higher mean–variance ratio will produce a larger shift in the frequency of extremes for a given percentage shift of the mean in skew normal distributions as well, although the magnitude of the response will be different because of the different shapes of the normal and skew normal distributions.

e. Adding the decadal to MDV

The nature and magnitude of natural MDV in the climate system differs across locations. For the simulations here, MDV is represented through a defined lag-1 (lag 1 here refers to a 1-yr lag) AR process, with $r_1$ always set to 0.6. This is at the upper end of magnitudes observed in regional climate indices (e.g., Sahel; Folland et al. 1991). This was chosen so that it is still in the range of realistic magnitudes, while being large enough for the effects to be clearly seen in the simulations (an example trace is in Fig. 7). There is therefore no attempt here to fit the most representative MDV for each location. Based on available literature, the imposed magnitude of AR MDV may be quite close to that appropriate to Tiby based on long historical and paleorecords (e.g., Shanahan et al. 2009), but likely substantially higher than is present in the climate system for the other three locations, (Nicholson and Entekhabi 1986). Nonetheless the intention is for the results to serve to illustrate the effects of MDV, which subsequently can be scaled to the best estimate of MDV magnitudes for different locations.

The simulations are undertaken by imposing the AR process (simulations for 2009–2100), and recalibrating the variance of each new series to be the same as the variance of the original series (Fig. 2, 1979–2008). This strategy can be modified in the future but serves here to highlight the way in which MDV introduces increased

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**Fig. 5.** Shape of trend imposed on the rainfall series based on the A1B CO$_2$ scenario for the twenty-first century, calculated here by taking the average of the Integrated Science Assessment Model and Bern model as given in Nakicenovic and Swart (2001).

**Fig. 6.** Illustration of a simulated trace for (a) Tiby and (b) Dertu under a scenario of a 10% decline in precipitation over the course of the twenty-first century.

**Fig. 7.** Illustration of a standardized simulated 500-yr trace for an autoregressive process with lag-1 autocorrelation = 0.6.
uncertainty in extremes, even under the variance normalization described above.

3. Experiments with and without GC

a. Baseline experiment (experiment 0)

First, to check the baseline properties of the simulation system, an experiment was run with white noise sampled from a normal distribution. There is no systematic persistence (MDV) or trend (GC) imposed. There is no uniqueness of the result expected for this experiment across the different village sites, since all locations have exactly the same normalized PDF. Therefore, 100 simulations for the period 2009–2100 were generated from the standard normal distribution with mean of 0 and standard deviation of 1. Since no systematic change is expected over time, results are calculated as the average of those found in the three snapshot periods of 2011–40, 2041–70, and 2071–2100. As expected, the mean BIAS is almost exactly 0. The average ABSE statistic for this experiment indicates a baseline uncertainty in the number of events to expect in any given 30-yr period with a stationary, white noise, normally distributed climate variable. The ABSE conforms to the expectations set by the binomial distribution analysis (appendix A), taking a value of 1.42 for the $1/8$ threshold and a value of 0.92 for the $1/20$ threshold. To better visualize the meaning of the ABSE statistics, Fig. 8 displays a count of the number of dry events in the 30-yr periods across the simulations. It can be seen how for the $1/8$ threshold, the spread of the number of events occurring in a 30-yr period is larger, as compared to the $1/20$ threshold. Subsequent simulations explore the change in BIAS and spread as GC and MDV are added into the simulations.

b. Impact of GC when thresholds are kept fixed (experiment 1)

In these simulations, the $+10\%$ and $-10\%$ trends in precipitation are gradually introduced to the series through the course of the twenty-first century (Fig. 5). Average BIAS and ABSE statistics for the periods 2011–40 and 2071–2100 are presented for simulations made with normally distributed IV (Table 2) and skew normal IV (Table 3). Various concepts are highlighted and patterns emerge from these results, with key salient features discussed below.

Given that thresholds are not updated in this set of experiments (FT experiment type), we expect considerable, and steadily growing through the century, systematic bias toward a lower number of dry events in the $+10\%$ scenario and a higher number of dry events in the $-10\%$ scenario. This is true throughout Tables 2 and 3, but with variations in magnitude attributable to various factors. First, results for the normal distribution simulations are focused upon (Table 2). An important contrast is highlighted and quantified between the $+10\%$ simulations and $-10\%$ simulations. This is visualized by focusing on the area to the left of each threshold, in the PDFs under the $+10\%$, no change, and $-10\%$ scenarios (Fig. 9). It can be seen how shifting the PDF to the left ($-10\%$ scenario) has a bigger impact on the area than does shifting the PDF to the right ($+10\%$ scenario). This means that the increase in dry events under a $-10\%$ scenario will have a greater magnitude than the reduction in dry events under a $+10\%$ scenario. This phenomenon occurs because the slope, as well as the height of the statistical distribution is greater to the right of the threshold than to the left of it. So when the $-10\%$ scenario is imposed, the height of the distribution increases markedly, whereas when the $+10\%$ scenario is imposed, the height of the distribution decreases more gently.
However, future work needs to assess the extent to which a change in mean rainfall in the real climate system, results on average in a change in variance, which in turn impacts the event frequency.

For the $\frac{1}{8}$ event in Table 2, the average BIAS for 2071–2100 across the four villages for the $+10\%$ scenario is $-2.1$, whereas the magnitude of the increase in the number of dry events under the $-10\%$ scenario is about twice as large, with the mean BIAS across the four villages at $+4.0$ (Table 2). Therefore, in the 2071–2100 period, rather than experiencing the expected average of 3.75 events, the simulations indicate an average of 7.75 events; this increase derives from just a 10% average of 3.75 events, the simulations indicate an average increase in spread in the $\frac{1}{8}$ events under the $+10\%$ scenario.

Substantial differences in the magnitude of BIAS exist across the villages in the simulations. For example, for the $\frac{1}{8}$ event and the $-10\%$ scenario, for the period 2071–2100, the average increase in $\frac{1}{8}$ events at Tiby is 5.28, whereas for Dertu it is 2.20. This difference occurs because the impact of the rainfall change on the area to the left of the threshold for Tiby (Fig. 9a) is much larger than that for Dertu (Fig. 9d). This is because a 10% downward shift in the rainfall total for Tiby corresponds to a larger shift in standard deviations. The key statistic is the ratio of the 10% change in rainfall (mm) to the standard deviation (as given in Table 1)—a ratio that is inversely proportional to the coefficient of variation statistic.

The ABSE statistics in these experiments are a function of both BIAS and spread, since the ABSE is calculated relative to the expected number of events [Eq. (2)]. This partly explains why ABSE grows more substantially through the century in Table 2 for $\frac{1}{8}$ events, because $\frac{1}{8}$ events generally have a larger BIAS than do $\frac{1}{20}$ events.

Table 3. As in Table 2, but for the skew normal distribution (these results are referred to as experiment 1, skew normal).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Village</th>
<th>$+10%$ scenario</th>
<th>$-10%$ scenario</th>
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<tr>
<td>ABSE $\frac{1}{20}$</td>
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<td></td>
<td>Mbola</td>
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<tr>
<td></td>
<td>Mwandama</td>
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<td>0.97</td>
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<tr>
<td></td>
<td>Dertu</td>
<td>0.88</td>
<td>0.92</td>
</tr>
<tr>
<td>ABSE $\frac{1}{8}$</td>
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<td>2.54</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td></td>
<td>Dertu</td>
<td>1.38</td>
<td>1.78</td>
</tr>
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</table>
(Table 3), as compared with +4.02 for the normal distribution simulation (Table 2). The skew normal simulations do reproduce the asymmetry result found in the normal distribution simulations, with a larger magnitude of BIAS and ABSE for the −10% scenario, as compared with the +10% scenario. Averaged across the sites, the magnitude ratio is quite consistent with that found in the normal distribution simulations.

Across the sites, the results show more variation than in the normal distribution case. This can be attributed to the varying and more complex shape of the skew distribution for each of the sites in the vicinity of the thresholds (Figs. 11a–d). For example, for Tiby (Fig. 11a) shifting the skew distribution 10% to the left can be seen to have a major impact on the area under the curve to the left of the 1/8 threshold, but not on the area to the left of the 1/20 threshold. This is reflected in Table 3 in the BIAS (4.22 for 1/8 versus 0.87 for 1/20) and ABSE (4.26 for 1/8 versus 1.42 for 1/20). For Tiby, the −10% shift actually results in the peak of the PDF being almost aligned with the 1/8 threshold value (Fig. 11a).

These variations serve to illustrate how different distributions will lead to variations in event frequency response when precipitation undergoes a specified percentage change. Nonetheless, there are also general tendencies that hold across the results. It is clear that a 10% downward shift in precipitation, across many realistic distributions, will tend to introduce on average a substantial increase in the frequency of 1/8 and 1/20 events in any given 30-yr period. For 1/8 events at the sites studied here, the average frequency in a 30-yr period rises to about 7.37 under normal distribution simulations, and to about 6.80 under skew normal simulations, both representing a near doubling of the expected value of 3.75.

c. Impact of GC when thresholds are updated (experiment 2)

This set of simulations differs from experiment 1 by applying the UT approach, as defined in section 2b. The concept is seen in Fig. 12, which shows a running average of the 30-yr event frequencies, as estimated using FT (experiment 1) and UT (experiment 2). With FT (Figs. 12a,c), the lines are nonstationary, summarizing the time evolution of the biases that were described in Table 2. With UT (Figs. 12b,d), the average event frequencies are
almost stationary. It is anticipated that there will still be a small bias of the same sign as in the FT experiments, because the updating procedure uses the previous 30 years to estimate the thresholds, which are applied to the following year. This implies the thresholds are slightly lagging behind the evolving trend. Nonetheless, it is hypothesized that BIAS can be substantially reduced by applying such a procedure, and this is visually apparent in Fig. 12.

To summarize the comparison, Table 4 shows, for each statistic (BIAS, ABSE), the ratio (experiment 2 divided by experiment 1) for the 10% precipitation scenario. All values are less than 1, indicating the expected improvements. For example, averaged over the four villages, and in absolute terms, BIAS for the 1/8 event for 2011–40 is reduced from 0.92 to 0.52 (i.e., with UT, the average number of events per 30 years is 3.75 + 0.52 = 4.27). Improvements are naturally stronger for the later period because the extreme event frequency is almost invariant when threshold determination is based on a sliding 30-year climatology. For example, for 2071–2100, BIAS is reduced from 4.02 to 0.82. The results in Table 4 and Fig. 12 support the idea that thresholds can be updated in real time to substantially reduce the expected threshold-crossing frequencies, and simulations such as the ones performed here can provide valuable insight to the magnitude of the expected under given background trend assumptions. Furthermore, no attempt here has been made to develop an updating procedure that maximizes the reduction in BIAS and ABSE.

4. Implications of MDV

a. A system with MDV and IV (experiment 3)

First, the properties of a system with MDV and IV are presented, using the AR process as described in section 2e. Simulations are performed using the FT approach. Compared to experiment 0 (white noise), we expect the MDV to add considerably to the ABSE. Compared to experiment 0 (white noise), we expect the MDV to add considerably to the ABSE.
As in experiment 0, there is no uniqueness in terms of event frequencies across the sites studied, since the process and distributions are identical. In the simulations, the ABSE increases substantially in experiment 3, as compared to experiment 0. The increase is of the order 50%, with the ABSE for the more extreme event increasing in percentage terms slightly less (45% compared to 58% increase for the $1/8$ event). The ABSE for the $1/8$ event in the presence of the MDV imposed here is comparable to the average ABSE that results from the $+10\%$ rainfall change simulation for 2071–2100 (Table 2) and from the $-10\%$ rainfall change simulation for midcentury (between the values for 2011–40 and 2071–2100 given in Table 2).

The significance of the MDV result is further emphasized in Fig. 13, which graphs the increase in spread of the event frequencies in a 30-yr period. The increased spread for the $1/8$ events is very marked, with considerable occurrences spanning from 0 events to as many as 11 events. It is also noticeable that for the $1/20$ event, there are simulations where 9 of the 30 years have crossed the $1/20$ threshold, and the presence of such a risk is of major consequence in evaluating costs for an index insurance scheme. In some settings, the change in the tail distribution characteristics for the $1/20$ event (i.e., the longer tail in the gray bars in Fig. 13b as compared with the black bars) may be more significant than the change in the tail distribution for the $1/8$ event (Fig. 13a).

### b. Impact of combined GC and MDV when thresholds are fixed (experiment 4)

The simulated traces contain a trend component, a slowly varying stochastic MDV component, and a stochastic white noise component that is normally distributed. The experiments can be compared to experiment 1 (normal distribution), which was identical except for the absence of MDV. In comparison with experiment 1 (Table 2), the mean BIAS is not expected to change, but the ABSE can be expected to increase because of the addition of the MDV. Table 5 provides a quantification of the general increase in ABSE that results when MDV is added to GC. It is noticeable that the fractional increases in ABSE are most pronounced in the first period (2011–40). This period is before the bias takes hold of the ABSE and provides an illustration of how MDV plays a strong role in contributing to the uncertainty in the first decades of any projection. This can be expected.
to be the case, regardless of initialization time in the twenty-first century, and emphasizes the importance of including uncertainty associated with the MDV, especially on the near-term time horizon. On time horizons that span longer into the future from any given initialization point, GC increasingly becomes the dominant factor in expectations of climate variables. This is clearly seen in Table 6, which shows average ABSE results across the different experiments. For example, averaged over the four villages for the −10% scenario with the $\frac{1}{8}$ threshold, the ABSE for 2011–40 is 1.79 in the presence of GC alone, 2.21 in the presence of MDV alone, and 2.34 with the combination of GC and MDV. In contrast, GC comes to dominate in the comparable statistics for 2071–2100, when GC alone yields an ABSE of 4.10 and GC combined with MDV yields only a relatively modest increase to 4.31. Nonetheless, the impact of adding MDV to GC can be seen graphically by comparing Figs. 14a,c (MDV and GC), with comparable graphics for GC alone (Figs. 10c,d). For example, for the −10% rainfall scenario and the $\frac{1}{8}$ events for Tiby and Mbola, the spread now clearly extends from 1 event in a 30-yr period out to even over 20 events in a 30-yr period (Fig. 14a). The next section will assess the extent to which the ABSE can be reduced by introducing the simple threshold updating system (UT approach).

### Table 4. The improvement found when event thresholds are continuously updated through the course of the twenty-first century. Experiment 2 simulation properties: −10% precipitation change scenario added to normally distributed interannual variability and event thresholds for year $i$ are based on the previous 30 years of the simulation. Values shown are the ratio: (results of experiment 2 divided by results of experiment 1 normal distribution). Therefore, ratios <1 indicate a reduction of BIAS and ABSE as a result of the updating procedure.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Village</th>
<th>2011–40</th>
<th>2071–2100</th>
</tr>
</thead>
<tbody>
<tr>
<td>BIAS $\frac{1}{8}$</td>
<td>Tiby</td>
<td>0.54</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>Mbola</td>
<td>0.54</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>Mwandama</td>
<td>0.60</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>Dertu</td>
<td>0.61</td>
<td>0.27</td>
</tr>
<tr>
<td>ABSE $\frac{1}{8}$</td>
<td>Tiby</td>
<td>0.76</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>Mbola</td>
<td>0.76</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>Mwandama</td>
<td>0.79</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>Dertu</td>
<td>0.84</td>
<td>0.63</td>
</tr>
</tbody>
</table>
c. Impact of GC and MDV when thresholds are updated (experiment 5)

These simulations apply the UT approach and are identical to experiment 2 except for the addition of the MDV. The ratio of ABSE (experiment 5/experiment 2) gives an indication of the additional uncertainty that is added by MDV even when thresholds are updated (Table 7). The ratios are generally around 1.3 and are quite constant over the simulation period. In absolute terms, Table 6 compares average ABSE values for experiment 5 with other experiments for the 1/8 threshold. One feature of the results is to emphasize, especially for information in the near term (here 2011–40), the importance of trying to find better ways of updating thresholds associated with MDV, as well as further exploring the improvement of updating systems in the presence of GC trends.

The rows in Table 6 corresponding to UT experiments have ABSE values that remain fairly constant through the twenty-first-century simulation. In contrast, the FT experiments show substantially increasing ABSE. For example, for 2071–2100 under the −10% scenario and 1/8 thresholds, the mean ABSE with UT is 2.16, compared to 4.31 for FT (Table 6). This improvement is visually seen comparing Fig. 14a with Fig. 14b (1/20 threshold) and Fig. 14c with Fig. 14d (1/20 threshold). For example, with UT it is fairly rare to get 5 events in a 30-yr period (Fig. 14d), whereas with FT (Fig. 14c) 5 or more events has a substantial likelihood of occurrence.

5. Conclusions

A statistical simulation framework for exploring changes in threshold-crossing events under different scenarios

<p>| Table 5. The increase in uncertainty when MDV is added to a precipitation change scenario, for fixed threshold experiments. Experiment 4 simulation properties: Precipitation change scenario, stochastic AR process with lag-1 autocorrelation = 0.6 and normally distributed interannual variability, and fixed event thresholds as estimated for 1979–2008. Values shown are the ratio: (results of experiment 4 divided by results of experiment 1 normal distribution). Therefore, ratios &gt;1 indicate an increase in ABSE through the addition of MDV variability. |</p>
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Village</th>
<th>+10% scenario</th>
<th>−10% scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSE 1/20</td>
<td>Tiby</td>
<td>1.55</td>
<td>1.40</td>
</tr>
<tr>
<td></td>
<td>Mbola</td>
<td>1.62</td>
<td>1.62</td>
</tr>
<tr>
<td></td>
<td>Mwandama</td>
<td>1.68</td>
<td>1.41</td>
</tr>
<tr>
<td></td>
<td>Dertu</td>
<td>1.38</td>
<td>1.55</td>
</tr>
<tr>
<td>ABSE 1/8</td>
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<td>1.57</td>
<td>1.19</td>
</tr>
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<td></td>
<td>Mbola</td>
<td>1.63</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>Mwandama</td>
<td>1.51</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>Dertu</td>
<td>1.59</td>
<td>1.34</td>
</tr>
</tbody>
</table>

<p>| Table 6. Summary of ABSE averaged across all four locations for the 1/8 event. Expt 0: normally distributed white noise; expt 3: AR (r = 0.6); expt 1: global change + normally distributed white noise, fixed event thresholds; expt 2: as in expt 1, but with updated thresholds; expt 4: global change + AR (r = 0.6); expt 5: as in expt 4 but with updated thresholds. For expt 0 and expt 3, since no systematic change is expected across the three periods, results shown are the average of results found for the three periods. |</p>
<table>
<thead>
<tr>
<th>Expt</th>
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<th>2011–40</th>
<th>2041–70</th>
<th>2071–2100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expt 0</td>
<td>0</td>
<td>1.42</td>
<td>1.42</td>
<td>1.42</td>
</tr>
<tr>
<td>Expt 3</td>
<td>0</td>
<td>2.21</td>
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<td>2.21</td>
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<td>Expt 1</td>
<td>−10%</td>
<td>1.79</td>
<td>2.25</td>
<td>4.10</td>
</tr>
<tr>
<td>Expt 2</td>
<td>−10%</td>
<td>1.40</td>
<td>1.58</td>
<td>1.62</td>
</tr>
<tr>
<td>Expt 4</td>
<td>−10%</td>
<td>2.34</td>
<td>3.35</td>
<td>4.31</td>
</tr>
<tr>
<td>Expt 5</td>
<td>−10%</td>
<td>2.05</td>
<td>2.28</td>
<td>2.16</td>
</tr>
<tr>
<td>Expt 1</td>
<td>+10%</td>
<td>1.43</td>
<td>1.76</td>
<td>2.23</td>
</tr>
<tr>
<td>Expt 2</td>
<td>+10%</td>
<td>1.27</td>
<td>1.34</td>
<td>1.37</td>
</tr>
<tr>
<td>Expt 4</td>
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<td>2.25</td>
<td>2.22</td>
<td>2.53</td>
</tr>
<tr>
<td>Expt 5</td>
<td>+10%</td>
<td>1.97</td>
<td>1.92</td>
<td>1.93</td>
</tr>
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</table>
a

\begin{figure}
\centering
\includegraphics[width=\textwidth]{distribution.png}
\caption{Distribution of the number of (threshold crossing) events in simulations for 2071–2100. The simulations contain both a precipitation change scenario (GC) and stochastic decadal to MDV, represented by an autoregressive process with lag-1 correlation $= 0.6$. (a) For $\frac{1}{8}$ event under $-10\%$ rainfall scenario with fixed thresholds; (b) for $\frac{1}{8}$ event under $-10\%$ rainfall scenario with updating thresholds; (c) as in (a), but for $\frac{1}{20}$ event; (d) as in (b), but for $\frac{1}{20}$ event.}
\end{figure}

Substantial increases in threshold-crossing events should be expected in the presence of a relatively modest trend, assuming no change in the characteristics of interannual variability. For example, for a downward trend of 10% through the course of the twenty-first century, and holding thresholds constant at 1979–2008 values, the frequency of $\frac{1}{8}$ events is approximately doubled by the 2071–2100 period. In the context of the primary motivating problem for these analyses, this corresponds to a doubling in payout frequency for an index insurance contract that, at the beginning of the century, is tuned to have a payout frequency of 1 in 8 years. A second important quantification is highlighted for the asymmetry in the above result: with an upward trend, the magnitude of the reduction in $\frac{1}{8}$ events is much less. These basic results can likewise be interpreted for changes in threshold-crossing seasonal wet events, since the distributions are

<table>
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<tbody>
<tr>
<td>ABSE $\frac{1}{20}$</td>
<td>Tiby</td>
<td>1.31</td>
<td>1.28</td>
<td>1.41</td>
<td>1.35</td>
</tr>
<tr>
<td>Mbola</td>
<td>1.50</td>
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<td>1.29</td>
<td></td>
</tr>
<tr>
<td>Mwandama</td>
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</tr>
<tr>
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<td>1.34</td>
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</tr>
<tr>
<td>ABSE $\frac{1}{8}$</td>
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</tr>
<tr>
<td>Mbola</td>
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<tr>
<td>Mwandama</td>
<td>1.52</td>
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<td>Dertu</td>
<td>1.79</td>
<td>1.38</td>
<td>1.22</td>
<td>1.48</td>
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</tr>
</tbody>
</table>
symmetric. Thus, for a +10% rainfall scenario, while the reduction of 1/8 dry events is relatively small, the increase of 1/8 wet events will also be of the order times 2 under the simulation framework introduced here.

Some implications of the presence of MDV have been illustrated. The main impact is change in the uncertainty of the number of events to expect. The ABSE statistic provides an integrated measure of bias in the number of events in a given 30-yr period, combined with the spread of the number of events about the expected value. MDV, while not changing BIAS in the formulation here, does increase the ABSE statistic because it increases the spread. MDV has been simulated as an AR process with lag-1 autocorrelation of 0.6. For GC and MDV magnitudes used here, MDV substantially increases the ABSE (in percentage terms) in the near term (results for 2011–40), relative to simulations with GC alone. By 2071–2100, the percentage increase in the ABSE is quite small from adding the MDV, since bias associated with the imposed GC is now dominating the ABSE statistic.

In practice, provision of information about climate statistics in the near term can, at the least, be improved upon by updating estimates in real time as the climate evolves. As a first exploration in this simulation framework, we have explored updating thresholds for year i based on the previous 30 years of data. It is demonstrated how BIAS and ABSE are substantially reduced when this simple system is applied, in the presence of the magnitudes of GC and MDV introduced here. For example, BIAS for the 1/8 event, in the presence of a downward trend of 10%, remains quite stationary through the course of the twenty-first century at the relatively modest levels of around +0.5–+1.0. In the future, the framework developed here could be deployed to explore the best updating procedures under different assumptions of variability and change. If too few years are selected, the limitations of the small sample size increase the size of the variance of the thresholds, and such thresholds will be less reliable, leading to increases in BIAS and ABSE. By contrast, when GC and MDV are present, using a base period that is too long will miss the opportunity to incorporate into the thresholds the changed state, as evidenced by the statistics of recent years. Concepts such as optimal climate normals and hinge fitting (Livezey et al. 2007; Huang et al. 1996) can be explored to find the best way to extract relevant information from the historical record, for estimating future climate statistics. Such approaches form a natural statistical complement to the merging of MDV and GC in dynamical prediction systems (Meehl et al. 2009).

Operational statistical simulations to estimate future extreme event frequencies could therefore be based on the type of methodology introduced here, with the simulations constrained by the future scenarios of seasonal rainfall totals associated with GC and/or MDV. For MDV, the future scenarios may be the full natural spectrum based on current knowledge of the climate system, or may be constrained by an ensemble forecast for the next 30 years for MDV (Meehl et al. 2009). For GC, the assumptions applied may be according to an ensemble with a systematic tendency, or with a distribution about a 0 tendency (as used illustratively here, with a +10% and −10% trend through the twenty-first century). When a systematic tendency and distribution is assumed for GC or MDV based on GCM simulations, the method could be viewed as representing an example of a statistical downscaling of GCMs.

The simulations have also highlighted the importance of addressing the shape of the distribution of the seasonal rainfall totals. Generally, the representative sample of skew distributions used here have tended to damp BIAS by about 25% on average, but with a considerable range that is specific to the skew distribution characteristic of each site studied. It has also been shown how results for different sites are very sensitive to the ratio of interannual standard deviation to the mean (i.e., coefficient of variation). A low coefficient of variation increases the magnitude of event frequency change, with Tiby the most sensitive site studied. The site-specific aspects (skew, coefficient of variation) have led to BIAS in fixed threshold experiments varying by more than a factor of 2, as compared with the average BIAS results across all sites.

Alternate distribution assumptions can be tested in this framework. For example, distributions such as the Box–Cox power transform may more efficiently capture the nonnormal aspects of seasonal rainfall totals. In particular, there is likely value in integrating extreme value theory and associated distributions (e.g., Hosking et al. 1985; Katz et al. 2002) into the simulation framework. For the seasonal rainfall totals explored in this example, extreme value distributions could be fit to the tail beyond a specified threshold value. Advantages are expected to be greatest when considering more extreme events than those considered here (1/8 and 1/20). Application of extreme value theory has been most widely applied in the climate literature to extremes of daily values in the target season (e.g., Kharin et al. 2007), and extension of the simulation scheme here to such daily extreme problems would likely also benefit from integration of extreme value theory.

Extension to variables that are closer to climate impacts is considered relatively simple in this framework. Sophistations beyond looking at low seasonal rainfall totals could be implemented, such as using simple water
requirement satisfaction indices, vegetation indices, or other drought indices, such as the Palmer drought severity index, that have been used in a number of climate variability and change impact studies (Li et al. 2009; Rind et al. 1997; Rosenzweig and Hillel 1993). These variables better account for the effects of temperature, or varying intraseasonal rainfall distributions on soil moisture content and other critical aspects of true hydrometeorological drought. Once an historical index is generated, it could be explored in the simulation framework developed here.

The representation of MDV in the simulations could be enhanced through a number of different statistical modeling approaches and methods, such as autoregressive moving-average models or wavelet analysis (e.g., Kwon et al. 2007). It may also become possible to represent skew in the MDV simulations. There is a growing literature in applied mathematics that explores the derivation of AR processes from nonnormal, and specifically skew normal distributions, replacing “normal” noise with noise sampled from a skew normal distribution (Pourahmadi 2007a,b). For the GC trends, analysis of GCM simulations and observations can be envisioned that leads to more sophisticated representations, including the assessment of trend-dependent IV characteristics (e.g., variance, skew, or coefficient of variation).

An overall aim has been to implement a relatively simple system that can allow assessment of the sensitivity of key climate statistics to a range of assumptions about climate variability and change. Evaluation of the sensitivity of key variables can inform risk management in a range of settings. A motivation for examples presented here has been the emerging area of index insurance. This problem needs evaluations of the sensitivity of key variables can inform risk management in the context of other risk management problem settings.

Acknowledgments. This work was supported through NOAA Grant NA05OAR4311004 and a research grant from SwissRe SWISS CU07-1995. The authors are grateful for discussions with Cheryl Palm on the context of the Millennium Villages and Dan Osgood on the framework of information needs for index insurance. Reviewer comments led to important improvements and clarifications in the text and are gratefully acknowledged.

APPENDIX A

Expected Value of the ABSE Statistic

For time series sampled from uncorrelated white noise, we expect the ABSE to conform to the expected variance given by the binomial distribution for discrete events. The probability \( p_k \) of \( k \) extreme events with frequency \( f \) in \( n \) years is shown:

\[
p_k = \binom{n}{k} f^k (1 - f)^{n-k}.
\]  

The expected value of the same binomial distribution is

\[
\mu = nf.
\]

Therefore, the expected ABSE of the binomial distribution is given by the sum

\[
\text{ABSE} = \sum_{k=0}^{n} |k - \mu| p_k.
\]

For the \( \nu/6 \) threshold \( (f = 0.125) \), this works out to \( \text{ABSE} = 1.45 \) and for the \( \nu/20 \) threshold \( (f = 0.05) \), this works out to \( \text{ABSE} = 0.98 \).

APPENDIX B

The Skew Normal Distribution

The practical advantage of the skew normal distribution is that it can have a positive, negative or zero skew. The PDF for a skew normal distribution is given by

\[
f(x) = \frac{2e^{-|x-x'|/2\omega^2}}{\omega\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt.
\]

This expression is equal to

\[
f(x) = \frac{e^{-|x-x'|/2\omega^2}}{\omega\sqrt{2\pi}} \left\{ 1 + \frac{1}{\sqrt{2}} \text{erf} \left( \frac{\alpha(x - \xi)}{\omega\sqrt{2}} \right) \right\}
\]

for values of \( x < \xi \), and

\[
f(x) = \frac{e^{-|x-x'|/2\omega^2}}{\omega\sqrt{2\pi}} \left\{ 1 - \frac{1}{\sqrt{2}} \text{erf} \left( \frac{\alpha(x - \xi)}{\omega\sqrt{2}} \right) \right\}
\]

for values of \( x > \xi \).

The parameters in Eqs. (B1)–(B3) are defined as follows:
\[ \delta = \frac{\alpha}{\sqrt{1 + \alpha^2}}; \]  

(B4)

by inversion, the relationship

\[ \alpha = \frac{\delta}{\sqrt{1 - \delta^2}} \]  

(B5)

also emerges.

The mean \( \mu \) is defined by

\[ \mu = \xi + \omega \delta \sqrt{\frac{2}{\pi}} \]  

(B6)

The variance \( \sigma^2 \) is defined by

\[ \sigma^2 = \omega^2 \left(1 - \frac{2\delta^2}{\pi}\right). \]  

(B7)

The skew \( \gamma \) is defined by

\[ \gamma = \frac{4 - \pi}{2} \left(\frac{\delta \sqrt{2/\pi}}{1 - 2\delta^2/\pi}\right)^{3/2}. \]  

(B8)

By inversion, the delta parameter can also be defined in terms of the skew

\[ \delta = \sqrt{\frac{\pi}{2}} \frac{\gamma^{1/3}}{\sqrt{\gamma^{2/3} + \left(\frac{4 - \pi}{2}\right)^{2/3}}} \]  

(B9)

(Azzalini 1985).

The parameters \( \alpha, \xi, \delta, \) and \( \omega \) are then inferred from the basic statistics (mean, variance and skew) of the empirical data, via Eqs. (B4)–(B9), and Eqs. (B1)–(B3) are computed and evaluated over a range of precipitation values to determine where the \( 1/8 \) and \( 1/20 \) thresholds lie.

REFERENCES


