Sensitivity of C-Band Polarimetric Radar–Based Drop Size Estimates to Maximum Diameter

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ABSTRACT

Estimating raindrop size has been a long-standing objective of polarimetric radar–based precipitation retrieval methods. The relationship between the differential reflectivity $Z_{dr}$ and the median volume diameter $D_0$ is typically derived empirically using raindrop size distribution observations from a disdrometer, a raindrop physical model, and a radar scattering model. Because disdrometers are known to undersample large raindrops, the maximum drop diameter $D_{max}$ is often an assumed parameter in the rain physical model. C-band $Z_{dr}$ is sensitive to resonance scattering at drop diameters larger than 5 mm, which falls in the region of uncertainty for $D_{max}$. Prior studies have not accounted for resonance scattering at C band and $D_{max}$ uncertainty in assessing potential errors in drop size retrievals. As such, a series of experiments are conducted that evaluate the effect of $D_{max}$ parameterization on the retrieval error of $D_0$ from a fourth-order polynomial function of C-band $Z_{dr}$ by varying the assumed $D_{max}$ through the range of assumptions found in the literature. Normalized bias errors for estimating $D_0$ from C-band $Z_{dr}$ range from $\pm 8\%$ to $15\%$, depending on the postulated error in $D_{max}$. The absolute normalized bias error increases with C-band $Z_{dr}$, can reach $10\%$ for $Z_{dr}$ as low as 1–1.75 dB, and can increase from there to values as large as $15\%$–$45\%$ for larger $Z_{dr}$, which is a larger potential bias error than is found at S and X band. Uncertainty in $D_{max}$ assumptions and the associated potential $D_0$ retrieval errors should be noted and accounted for in future C-band polarimetric radar studies.

1. Introduction

a. Background

The estimation of raindrop size distribution (DSD) parameters, including the central tendency of the DSD (mean or median drop size), has been a primary objective of polarimetric radar since the pioneering theoretical study of Seliga and Bringi (1976). Using surface disdrometer measurements of a heavy-rain event, Seliga et al. (1986) demonstrated that simulated differential reflectivity $Z_{dr}$ could provide reasonably accurate estimates of the median volume diameter $D_0$ when the $D_0 = F(Z_{dr})$ relation is derived from the disdrometer DSD observations and a radar scattering model. Observational studies focused on the estimation of $D_0$ or the mass-weighted mean diameter $D_m$ from radar measurements of $Z_{dr}$ alone and found reasonably good agreement with ground-based disdrometer (Goddard et al. 1982; Goddard and Cherry 1984; Aydin et al. 1987) and airborne particle imaging probe (Bringi et al. 1998) estimates. According to these empirical radar studies, $Z_{dr}$-based estimates of $D_0$ (or $D_m$) have an absolute normalized bias error $\leq 5\%$ and a normalized standard error of $7\%$–$15\%$ relative to disdrometer or probe measurements of $D_0$ (or $D_m$), which is consistent with the simulations of Seliga et al. (1986) and also Jameson (1994).

These early studies provide an analysis framework in which DSD parameters can be estimated directly from polarimetric radar observations using equations derived from disdrometer DSD measurements as input to a radar scattering model. Beyond DSD, the derived $D_0(Z_{dr})$ equation is dependent on other details of the rain model, including the assumed drop shape versus size relation (Goddard et al. 1982; Goddard and Cherry 1984; Bringi...
et al. 1998; Thurai and Bringi 2005). Drop oscillations and canting tend to bias the drop shape slightly toward a more spherical shape in a manner that is nonlinear with diameter (Chandrasekar et al. 1988; Beard et al. 1991; Brandes et al. 1998; Thurai and Bringi 2005; Thurai et al. 2009). The sensitivity of $D_0(Z_{dr})$ to uncertainty in the drop shape versus size relation led Brandes et al. (2002) and Gorgucci et al. (2002) to develop the so-called effective $\beta$ method for deriving DSD parameters ($D_0, N_w, \mu$) of an assumed gamma distribution (Ulbrich 1983) from the triplet of observed horizontal reflectivity $Z_b$, $Z_{dr}$, and specific differential phase $K_{dp}$. The method takes advantage of the combined use of $K_{dp}$ and $Z_{dr}$ to mitigate the effects of drop oscillation and canting by estimating the slope $\beta$ of an assumed linear relationship between drop shape and size. The effective $\beta$ method has been applied to polarimetric radar observations in a wide variety of climate regimes, and the retrieved DSD parameters were in general agreement with surface disdrometers (Brandes et al. 2002, 2003).

Brandes et al. (2004a) demonstrated that the estimation error for $\beta$ due to measurement error in $K_{dp}$ is very large for $K_{dp} < 1.5^\circ$ km$^{-1}$, thereby limiting the practical utility of the effective $\beta$ method for DSD retrieval to heavy rain (e.g., rain rate $R > 70$ mm h$^{-1}$ at S band or $R > 40$ mm h$^{-1}$ at C band). The retrieval of $D_0$ must often default to a $Z_{dr}$-only approach in many rainfall situations (Zhang et al. 2001; Brandes et al. 2002, 2006, 2009; Brandes et al. 2003, 2004a,b). Fortunately, significant progress has been made in the experimental measurement of drop shape over a wide range of sizes (Beard and Kubesh 1991; Andsager et al. 1999; Thurai and Bringi 2005; Thurai et al. 2007). The empirical results (e.g., Thurai and Bringi 2005; Thurai et al. 2007) are generally consistent with the range of axis ratios predicted by theory (Beard and Chuang 1987). Since there appears to be good agreement between recent empirical drop shape–size relations in the literature (Goddard et al. 1994; Thurai and Bringi 2005; Brandes et al. 2002; Thurai et al. 2007), the estimation of $D_0$ from $Z_{dr}$ can likely be accomplished without significant error because of drop shape assumptions (Brandes et al. 2003, 2004a,b; Brandes et al. 2006, 2009).

b. Motivation

Another long-standing issue in polarimetric radar rainfall retrieval methods is their potential sensitivity to DSD truncation, including assumptions regarding both maximum ($D_{\text{max}}$) and minimum ($D_{\text{min}}$) diameters (Ulbrich and Atlas 1984; Ulbrich 1985, 1992). Note that all drop diameters are in terms of the equivalent spherical diameter. Truncation of the DSD at the large diameter end of the spectrum can influence the accuracy of the gamma model parameters fit to the DSD using the method of moments (Ulbrich and Atlas 1998). Developing relations between radar observables and rainfall properties, which are both calculated from integral moments of the DSD, requires assumptions regarding the limits ($D_{\text{min}}, D_{\text{max}}$) of those rainfall integral parameters. Bias errors in the derived radar–rainfall relations can result simply from inappropriate assumptions regarding the limits of the DSD integrals involved (Ulbrich 1985). DSD moment errors (especially for high moments) are more sensitive to the DSD uncertainty for large raindrops (Cao et al. 2008; Cao and Zhang 2009). Therefore, the uncertain range of maximum raindrop diameter (including the truncation) could affect the accuracy of DSD retrieval. Since both $D_0$ and $Z_{dr}$ are calculated from high-order moments of the DSD, the development of an accurate $D_0(Z_{dr})$ relation also depends on acceptable DSD truncation assumptions, including the choice of $D_{\text{max}}$ (Ulbrich and Atlas 1984; Ulbrich 1992). Ulbrich and Atlas (1984) concluded that the relationship between $D_0$ and $Z_{dr}$ depends strongly on $D_{\text{max}}$ only when $D_0 \geq 2.5$ mm for an assumed gamma DSD. Since $D_{\text{max}}$ and $D_0$ appear to be proportional in DSD observations, Ulbrich and Atlas (1984) demonstrated that the relationship between $D_0$ and $Z_{dr}$ for a gamma DSD is relatively insensitive to changes in $D_{\text{max}}/D_0$ for values of $D_0 \leq 3.2$ mm provided $D_{\text{max}}/D_0 \geq 2.5$.

However, the measured ratio of $D_{\text{max}}/D_0$ rarely exceeds 2.5 in typical disdrometer sample volumes. In Keenan et al. (2001), the 95th percentile of $D_{\text{max}}/D_0$ in 1-min DSD data was 2.4 over Darwin, Australia. In a large sample of 1-min DSDs observed by 2D video disdrometers (2DVs) over Huntsville, Alabama, the mean $D_{\text{max}}/D_0$ was 2.0 and 92.5% of the data were characterized by $D_{\text{max}}/D_0 < 2.5$ (Fig. 1) when the number of drops $\geq 300$ and the rain rate $\geq 1$ mm h$^{-1}$. Using similar 2DVD instruments and thresholds, a similar result (i.e., mean $D_{\text{max}}/D_m = 1.9$) was found by Gatlin et al. (2015), who analyzed an order of magnitude more 1-min DSD samples from a variety of locations around the globe in a wide variety of precipitation conditions. As noted by Ulbrich (1992), the average $D_{\text{max}}/D_0$, which is observed by a single disdrometer, increases with the integration time (i.e., 1.9, 2.5, and 2.9 over 1, 10, and 30 min, respectively). Assuming the longer integration periods provide DSDs that are consistent with those found within a typical radar resolution volume, natural rainfall may typically meet the $D_{\text{max}}/D_0 \geq 2.5$ criteria required by Ulbrich and Atlas (1984) for $D_0(Z_{dr})$ estimates that are relatively insensitive to $D_{\text{max}}$ assumptions.

However, partially compensating volumetric sampling limitations of a single disdrometer with long integration periods likely mixes DSDs from a variety of
rainfall types and microphysical processes that may not be representative of the DSDs affecting an instantaneous measurement by radar. For example, one can consider the horizontal scale of advection given various sample times. Assuming a conservative relative motion of $5 \text{ m s}^{-1}$, a 1-km patch of rain will have moved by a disdrometer in 200 s (roughly 3 min of integration). One minute of integration results in a spatial scale of 300 m, which is about the size of a radar gate space. On the other hand, a 10–30-min integration period is equivalent to a horizontal scale of 3–9 km, which may be too large to be representative of instantaneous conditions in a radar sample. Hence, there is still some uncertainty regarding the appropriate $D_{\text{max}}/D_0$ in

![Diagram](image-url)

**FIG. 1.** Frequency histogram of (a) the ratio of the maximum diameter over the median volume diameter ($D_{\text{max}}/D_0$) and (b) the maximum diameter $D_{\text{max}}$ for 7678 one-minute drop size distributions collected in a variety of rain types over Huntsville using 2DVDs. Drop size distributions were utilized when total drop concentration $N_T \geq 300$ drops, rain rate $R \geq 1.0 \text{ mm h}^{-1}$, and no hail or other ice hydrometers were present in the sample volume. All diameters are equivalent spherical diameters.
a sample volume consistent with radar applications and hence regarding the sensitivity of $D_0(Z_{dr})$ to various $D_{\text{max}}$ assumptions.

It is important to note that the results of Ulbrich and Atlas (1984) are specific to S band ($\lambda = 10$ cm). More recent S band studies by Brandes et al. (2003, 2004a,b) have demonstrated good agreement between polarimetric radar–based and surface disdrometer-based retrievals of DSD parameters while evaluating the constrained-gamma approach of Zhang et al. (2001). Brandes et al. (2003) suggested that their DSD parameter retrievals from S-band $Z_{dr}$ were fairly insensitive to $D_{\text{max}}$ assumptions. However, Brandes et al. (2003) noted that large drop regions characterized by $Z_{dr} > 3$ dB ($D_0 > 3.2$ mm) were likely not well represented by the constrained-gamma model and were therefore ignored in their studies. In that respect, Brandes et al. (2003) conclusions regarding the robustness of S-band $D_0(Z_{dr})$ retrievals are similar to Ulbrich and Atlas (1984).

Several studies have noted the impact of Mie resonance associated with large raindrops on C band ($\lambda = 5$ cm) polarimetric radar observables and rainfall retrieval algorithms (Bringi et al. 1991; Meischner et al. 1991; Aydin and Giridhar 1992; Carey et al. 2000; Zrnić et al. 2000; Keenan et al. 2001). The behavior of $Z_{dr}$ in large (e.g., $D > 5$ mm) raindrops at C band in the Mie scattering regime is well known (e.g., Zrnić et al. 2000). Resonance occurs for drops larger than about 5 mm where C-band $Z_{dr}$ exhibits decidedly nonmonotonic behavior, especially relative to S-band (i.e., nonresonant) $Z_{dr}$ (Fig. 2). Deviations between C- and S-band $Z_{dr}$ reach 3.5 dB at drop diameters just below 6 mm. By comparison, X-band $Z_{dr}$ has a muted resonance response in raindrops larger than about 3 mm and is much closer to S-band $Z_{dr}$, except from 3 to 4 mm where deviations can reach up to 0.74 dB. Despite the obvious potential impact of resonance, the fundamental importance of measuring DSD central tendency, and the growing numbers of C-band polarimetric radars worldwide, no study to date has investigated the detailed sensitivity of C-band retrieval of $D_0(Z_{dr})$ to assumptions regarding $D_{\text{max}}$. We note that such $D_{\text{max}}$ sensitivity studies have been conducted on the C-band polarimetric retrieval of rain rate, attenuation, and differential attenuation (Zrnić et al. 2000; Keenan et al. 2001).

In situ aircraft probe, videosonde, and surface disdrometer observations have demonstrated that large raindrops in the range of 5–8 mm, and possibly up to 9–10 mm, occur in a wide variety of rainfall regimes (Beard et al. 1986; Rauber et al. 1991; Takahashi et al. 1995; Schuur et al. 2001; Hobbs and Rango 2004; Fujiyoshi et al. 2008; Gatlin et al. 2015). Nonetheless, large drop occurrence in disdrometer observations is rare. For example, 97% of all 1-min DSDs sampled by 2DVDs were characterized by $D_{\text{max}} < 5$ mm over Huntsville, Alabama (Fig. 1). In a 2DVD study of large ($D \geq 5$ mm) raindrop occurrence worldwide, Gatlin et al. (2015) found only 10,464 large raindrops in a total rain sample consisting of over 224 million drops (i.e., <0.004% occurrence on a per-drop basis). Because of sampling limitations, disdrometers likely undersample the number of large raindrops and hence $D_{\text{max}}$ (Ulbrich and Atlas 1984; Ulbrich 1992; Smith et al. 1993; Keenan et al. 2001; Brandes et al. 2003). As a result, there is still considerable uncertainty regarding the appropriate $D_{\text{max}}$ for any given rainfall situation. Reflecting this uncertainty, there is currently no consensus on the appropriate value of $D_{\text{max}}$ to use in a C-band radar analysis framework using disdrometer observations (e.g., Zrnić et al. 2000; Keenan et al. 2001).

In C-band radar studies, parameterized values of $D_{\text{max}}$ have been assumed to be constant in the range of 4–10 mm (Aydin and Giridhar 1992; Carey et al. 2000; Zrnić et al. 2000; Keenan et al. 2001; Tabary et al. 2009), a constant multiple $C$ of the disdrometer observed central tendency ($D_0$ or $D_m$) where the $C$ has ranged from 2.5 to 3.5 (e.g., $D_{\text{max}} = CD_0$) (Keenan et al. 2001; Bringi et al. 2002, 2003, 2006, 2009; Thurai et al. 2007), and the actual disdrometer-measured maximum diameter despite recognized sampling limitations (Ryzhkov and Zrnić 2005; Tabary et al. 2009). Several of these studies

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**Fig. 2.** The differential reflectivity ($Z_{dr}$; dB) of monodisperse raindrops of equivalent spherical diameter ($D$; mm) for X-band (red), C-band (blue), and S-band (dashed black) wavelengths. For the monodisperse simulations in this figure, the following assumptions were made: drop shape vs size relationship of Thurai et al. (2007), a drop temperature of 20°C, and a mean and standard deviation of the drop canting angle of 0° and 7.5° (Huang et al. 2008), respectively.
have developed and applied C-band $D_0(Z_{dr})_{fit}$ best-fit relations but no sensitivity test to the $D_{\text{max}}$ assumption has yet been conducted. As pointed out by Zrnić et al. (2000), the uncertainty in $D_{\text{max}}$ in the range of 5-8 mm is exactly where the maximum sensitivity to resonance is expected.

Because of the sampling limitations of disdrometers, $D_{\text{max}}$ is estimated or essentially parameterized in the studies above. In the instances where $D_{\text{max}}$ is parameterized as a multiple of $D_0 (D_{\text{max}} = CD_0)$, it is important to note that $D_0$ here is the disdrometer observed $D_0$. This approach was first taken by Keenan et al. (2001) to provide a physically realistic domain for $D_{\text{max}}$ in the polarimetric variable scattering calculations based on the observed DSD data even though the disdrometer typically underestimates $D_{\text{max}}$. Multiple studies (Bringi et al. 2002, 2003, 2006, 2009; Thurai et al. 2007) have since employed Keenan et al.’s approach. To the extent that the parameterized $D_{\text{max}}$ in the resulting rain model is different than the observed $D_{\text{max}}$, the resulting model $D_0$ will be different than the disdrometer observed $D_0$ even though the observed $D_0$ is sometimes used to parameterize $D_{\text{max}}$ in the first place. In the studies where $D_{\text{max}}$ is parameterized, it is the model $D_0$ that is used to develop the $D_0(Z_{dr})_{fit}$ equation to estimate $D_0$ from radar observations of $Z_{dr}$. More details will be provided in section 2.

c. Objectives

Because of their relative affordability, C-band polarimetric radars are in common use worldwide for both research and operations. As such, it is critical to assess errors associated with raindrop size retrievals at C band because of the uncertainty in the maximum raindrop diameter, as has been accomplished for other C-band rain algorithms such as rain rate and propagation correction (Zrnić et al. 2000; Carey et al. 2000; Keenan et al. 2001). The fundamental definition of maximum drop diameter, its impact to precipitation remote sensing algorithms, and more specific to this study, a complete characterization of potential errors in C-band radar drop size retrievals are important for a number of applications, including global physical and statistical ground validation (Chandrasekar et al. 2008) of satellite precipitation remote sensing methods such as for the NASA Global Precipitation Measurement (GPM) mission (Hou et al. 2014). Given the current uncertainty in parameterizing the large drop tail of the DSD (Zrnić et al. 2000), we conduct a $D_{\text{max}}$ sensitivity test to assess the potential errors inherent in the development of a C-band $D_0(Z_{dr})_{fit}$ relation using disdrometer data. Because $D_{\text{max}}$ variability is likely to have a more significant impact on $D_0$ retrievals at larger $Z_{dr}$ and because of the exacerbating influence of resonance scattering at C band, errors in the retrieval of $D_0$ as a function of $Z_{dr}$ are presented in addition to overall sample errors. Results at X and S band are also briefly compared with C band to highlight the important impact of resonance on these potential errors at C band. The potential effect of drop temperature is also explored.

2. Data and methodology

a. Raindrop model development

Data from the Colorado State University low-profile and NASA GPM Ground Validation compact 2DVDs (Schönhuber et al. 2008) located at the instrument berm of the National Space Science Technology Center (NSSTC) in Huntsville, Alabama, were utilized to develop an experimental raindrop model. Disdrometer data were collected in 1-min integration periods. To estimate robust DSD statistics and gamma fits to the DSD data, only 1-min periods with total drop concentration ($N_{\text{r}}$) $\geq$ 300 drops and $R \geq 1.0 \text{ mm h}^{-1}$ were used, providing 7678 one-minute-averaged samples of the binned DSD. DSD data from the low-profile and compact 2DVD units were available from 2007 to 2011 and from late 2009 to 2011, respectively. DSD data were binned at 0.25 mm through early 2010 after which time the bin size was reduced to 0.20 mm. A comparison of results showed no significant impact of the change in bin size to the goals of this study. A comparison of the side-by-side 2DVD units by Thurai et al. (2011) demonstrated excellent agreement in measuring DSD parameters. For example, the correlation coefficient and fractional standard error between the $D_m$ values measured by the collocated low-profile and compact 2DVDs was 0.95 and 5%, respectively.

The DSD dataset contained a wide variety of rainfall types, including convection and stratiform precipitation within ordinary thunderstorms, tropical storms, meso-scale convective systems, and severe storms occurring across all seasons of the year. One-minute-averaged DSD samples with likely hail, snow, or mixed-phase precipitation contamination were removed through manual inspection of the 2DVD fall speed data, surface temperature, available sounding data, Advanced Radar for Meteorological and Operational Research (ARMOR; Petersen et al. 2005, 2007) polarimetric observations, and NOAA Storm Data.

The method of truncated moments of Ulbrich and Atlas (1998) was utilized to fit a gamma distribution model (Ulbrich 1983) to the 1-min 2DVD DSD data. The method of truncated moments accounts for the finite $D_{\text{max}}$ in the retrieval of the gamma model parameters: $N_0$ (intercept parameter), $D_0$ (median volume diameter), and $\mu$ (shape parameter). The triplet of
gamma fit parameters along with the $D_{\text{max}}$ assumed for each sensitivity test and a constant $D_{\text{min}}$ fully characterized the gamma rain DSD for input into the radar scattering model, which is detailed in the next paragraph. The value of $D_{\text{min}}$ was fixed at 0.4 mm since the first bin of the 2DVD was not utilized in this study because of drop undercounting. Consistent with Ulbrich and Atlas (1998), the choice of $D_{\text{min}}$ had little impact on the outcome of this study. The parameterized $D_{\text{max}}$ was varied to encompass the variety of assumptions currently found in the literature, as discussed in more detail below. Note that all drop diameters (e.g., $D_{\text{min}}$, $D_{\text{max}}$, $D_0$) are in terms of equivalent spherical diameters. As found by Ulbrich and Atlas (1998), it is important to note that a change in DSD shape is associated with the selection of a parameterized $D_{\text{max}}$. In fact, that is why we derive the gamma DSD triplet of $(N_0, D_0, \mu)$ using the truncated method of moments after assuming the parameterized $D_{\text{max}}$. Other rain model assumptions required for input into the radar scattering model were 1) the recommended drop shape versus diameter relationship of Thurai et al. (2007), 2) a Gaussian canting angle distribution with mean of 0° and a standard deviation of 7.5° (Huang et al. 2008), and 3) a drop temperature of 20°C. The drop temperature $T$ of $T = 20°C$ was assumed throughout most of the study except when $T = 10°C$ or $T = 30°C$ was required for comparison.

b. Radar scattering model

The $T$ matrix model for oblate spheroids (Waterman 1969; Barber and Yeh 1975; Bringi and Chandrasekar 2001, appendix 3, 591–594) was used to calculate the individual scattering properties of each specified raindrop diameter (and hence shape), raindrop temperature and radar wavelength. The Mueller matrix model as implemented by Vivekanandan et al. (1991) was then used to calculate the polarimetric radar observables, including $Z_{dr}$, for each realization of the prescribed gamma rain DSD using the specified drop canting angle and radar elevation angle. The radar elevation angle was assumed to be 0°. The radar wavelength was set to C band (5.33 cm) for the bulk of the sensitivity study except when X-band (3.17 cm) or S-band (10.7 cm) $Z_{dr}$ was required for comparison.

c. Median volume diameter retrieval experiments

A series of experiments to estimate $D_0$ from $Z_{dr}$ [i.e., $D_0(Z_{dr})_{\text{fit}}$] were conducted by varying the parameterized $D_{\text{max}}$ through the range of assumptions found in the literature discussed in section 1, including 1) constant $D_{\text{max}} = 4, 5, 6, 7, 8, 9, \text{ and } 10$ mm; 2) actual 2DVD-measured $D_{\text{max}}$; and 3) $D_{\text{max}} = CD_0$ where $C = 2.0, 2.5, 3.0, 3.5, \text{ and } 4.0$ and $D_0$ here is the observed $D_0$ from the 2DVD observations of drop size and counts. Although likely physically unrealistic, the constant $D_{\text{max}}$ assumptions provide a direct way to explore the impact of resonance on the behavior of $D_0(Z_{dr})_{\text{fit}}$. In the latter case of $D_{\text{max}} = CD_0$, $D_{\text{max}}$ is capped at a maximum of 8 mm consistent with the practice of recent polarimetric studies (Bringi et al. 2006, 2009; Thurai et al. 2007) and with the idea that spontaneous raindrop breakup would typically occur around this diameter (e.g., Kamra et al. 1991). Note that constant $D_{\text{max}}$ of 9 and 10 mm were explored to understand the sensitivity of this assumption. All other rain and radar model characteristics were fixed for calculating the intrinsic $Z_{dr}$ associated with each DSD. To simulate the retrieval of $D_0$ from observed $Z_{dr}$ in the presence of radar measurement error, Gaussian noise was added to the simulated intrinsic $Z_{dr}$ values with a mean of 0 dB and a standard deviation of 0.25 dB. Vertically pointing scans (Gorgucci et al. 1999) can be used to mitigate bias error in $Z_{dr}$, thus justifying the 0 dB mean. The standard deviation of $Z_{dr}$ is based on vertically pointing ARMOR scans of drizzle (e.g., 0.20–0.25 dB).

For each experiment, a $(D_{\text{max}})_{\text{fit}}$ was assumed to develop a fourth-order polynomial fit to estimate $D_0$ (mm) from $Z_{dr}$ (dB) of the form

$$D_0(Z_{dr})_{\text{fit}} = aZ_{dr}^4 + bZ_{dr}^3 + cZ_{dr}^2 + dZ_{dr} + e,$$

where $a$, $b$, $c$, $d$, and $e$ are constants derived from the Levenberg–Marquardt algorithm (Levenberg 1944; Marquardt 1963) for nonlinear least squares curve fitting. It is important to point out that the $D_0$ used to regress Eq. (1) comes from the gamma DSD triplet of parameters $(N_0, D_0, \mu)$ derived from the truncated method of moments using the parameterized $D_{\text{max}}$ assumptions described above. The choice of a high-order polynomial for Eq. (1) was deemed necessary to provide a reasonable fit to $D_0(Z_{dr})_{\text{fit}}$ under a variety of $(D_{\text{max}})_{\text{fit}}$ assumptions that include the presence of large drops and resonant behavior in C-band $Z_{dr}$ (Fig. 3). As pointed out by Bringi et al. (2006), there is no physical reason for $D_0(Z_{dr})_{\text{fit}}$ to take the more common form of a power law. Bringi et al. (2009) also employed high-order polynomial fits to estimate $D_0$ from C-band $Z_{dr}$. In this study, the order of the polynomial was raised until the overall $D_0$ retrieval error was subjectively minimized. Note that this study did not quantitatively explore the relative performance of various functions, including other functions besides for polynomials, for estimating $D_0$ from C-band $Z_{dr}$.

Last, for each experiment, a $(D_{\text{max}})_{\text{truth}}$ was postulated as the true maximum diameter for developing
the truth dataset of \((D_0)_{\text{truth}}\) against which the polynomial fit in Eq. (1) was evaluated. The bias and standard errors of Eq. (1) were assessed as a function of the mismatch between the assumed \((D_{\text{max}})_{\text{fit}}\) for Eq. (1) and the postulated \((D_{\text{max}})_{\text{truth}}\). In this manner, the sensitivity of \(D_0(\text{Z}_{\text{dr}})_{\text{fit}}\) to \(D_{\text{max}}\) assumptions was assessed. For both values of \(D_{\text{max}}\), the truncated method of moments is used to develop the gamma triplet of parameters \((N_0, D_0, \mu)\). The quantity \((D_{\text{max}})_{\text{fit}}\) is used to derive \((N_0, D_0, \mu)_{\text{fit}}\), which are input to a radar scattering model to derive \(\text{Z}_{\text{dr}}(\text{fit})\) for the development of Eq. (1) or \(D_0(\text{Z}_{\text{dr}})_{\text{fit}}\). Meanwhile, \((D_{\text{max}})_{\text{truth}}\) is used to derive \((N_0, D_0, \mu)_{\text{truth}}\), which are also input to the radar scattering model to derive \(\text{Z}_{\text{dr}}(\text{truth})\). To determine bias and standard error as a function of \(D_{\text{max}}\) assumptions, the performance of the \(D_0(\text{Z}_{\text{dr}})_{\text{fit}}\) equation is evaluated against the \((D_0)_{\text{truth}}\) data.

The normalized bias (NB) and the normalized standard error (NSE) were used to evaluate the performance of the \(D_0(\text{Z}_{\text{dr}})_{\text{fit}}\) estimator in Eq. (1) relative to \((D_0)_{\text{truth}}\) according to

\[
\text{NB} = \left\langle \frac{\sum [D_0(\text{Z}_{\text{dr}})_{\text{fit}}] - (D_0)_{\text{truth}}]}{n} \right\rangle / (D_0)_{\text{truth}} \quad \text{and} \quad (2)
\]

\[
\text{NSE} = \left\langle \frac{\sum [(D_0(\text{Z}_{\text{dr}})_{\text{fit}}] - [D_0(\text{Z}_{\text{dr}})_{\text{fit}}] - (D_0)_{\text{truth}} + (D_0)_{\text{truth}}]}{n} \right\rangle^{1/2} / (D_0)_{\text{truth}}, \quad (3)
\]

where \(D_0(\text{Z}_{\text{dr}})_{\text{fit}}\) is the estimated \(D_0\) from polynomial fit in Eq. (1) that is associated with the assumed \((D_{\text{max}})_{\text{fit}}\), \((D_0)_{\text{truth}}\) is the postulated true \(D_0\) that is associated with the postulated \((D_{\text{max}})_{\text{truth}}\), the overbar indicates a mean, and \(n\) is the number of samples.

3. Results and discussion

The results of the \(D_0\) retrieval experiments are first overviewed by presenting the family of \(D_0(\text{Z}_{\text{dr}})_{\text{fit}}\) polynomials at C band associated with varying the \((D_{\text{max}})_{\text{fit}}\)
assumption through values typically found in the literature, as reviewed in section 1b. The potential overall bias and standard (i.e., scatter) errors of the $D_0$ retrievals and their sensitivity to $D_{\text{max}}$ assumptions are then assessed by assuming a $(D_{\text{max}})_{\text{fit}}$ and evaluating the associated $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomial against various truth datasets, $(D_0)_{\text{truth}}$ associated with a different postulated $(D_{\text{max}})$ truth. The $D_0$ retrieval errors associated with an incorrectly postulated $D_{\text{max}}$ are also evaluated as a function of $Z_{\text{dr}}$ and radar wavelength (X, C, and S band) to highlight the importance of resonant scattering. Finally, the potential impact of drop temperature is explored by conducting sensitivity tests at various drop temperatures.

### a. Sensitivity of polarimetric $D_0$ retrieval to maximum diameter at C band

Examples of simulated pairs of $(D_0, Z_{\text{dr}})$ data at C band and the associated $D_0(Z_{\text{dr}})_{\text{fit}}$ best-fit polynomials for the assumptions of $D_{\text{max}} = 3.5D_0$ and $D_{\text{max}} = 2D_0$ can be found in Figs. 3a and 3b, respectively. As expected, $D_0$ increases monotonically with increasing C-band $Z_{\text{dr}}$ with some scatter about a best-fit polynomial. The primary effect of increasing the assumed $D_{\text{max}}$ from $2D_0$ to $3.5D_0$ is to flatten the $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomial at moderate to large $Z_{\text{dr}}$. In other words, the simulated $D_0$ are systematically smaller at a given $Z_{\text{dr}}$ for $D_{\text{max}} = 3.5D_0$ relative to $D_{\text{max}} = 2D_0$, particularly at moderate-to-large $Z_{\text{dr}}$.

To better understand the effect of $D_{\text{max}}$ and the impact of resonant scattering at C band, the best-fit polynomials $D_0(Z_{\text{dr}})_{\text{fit}}$ assuming constant $D_{\text{max}}$ varying from 4 to 10 mm are shown in Fig. 4a. At most values of $Z_{\text{dr}}$, the $D_0$ inferred from the best-fit polynomials decreases with the assumed $(D_{\text{max}})_{\text{fit}}$. A significant transition in the functionality of $D_0$ with respect to $Z_{\text{dr}}$ occurs between the polynomials associated with $(D_{\text{max}})_{\text{fit}} = 5$ and 6 mm. The difference between the two polynomials is most obvious at moderate-to-large $Z_{\text{dr}}$ (e.g., $1.5 < Z_{\text{dr}} < 3$ dB) where the inferred $D_0$ from the polynomial assuming $(D_{\text{max}})_{\text{fit}} = 5$ mm becomes increasingly larger than the $D_0$ inferred from the polynomial with $(D_{\text{max}})_{\text{fit}} = 6$ mm. The $D_0(Z_{\text{dr}})$ polynomials for $(D_{\text{max}})_{\text{fit}} = 6$ mm tend to be much flatter with $Z_{\text{dr}} > 1.5$ dB than those for $(D_{\text{max}})_{\text{fit}} < 6$ mm. This significant transition in the $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomial behavior with $(D_{\text{max}})_{\text{fit}}$ is associated with resonance scattering and the rapid increase in $Z_{\text{dr}}$ for drops between 5 and 6 mm in diameter (Fig. 2).

Note that the $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomial for $(D_{\text{max}})_{\text{fit}} = 5$ mm is only strictly valid to about $Z_{\text{dr}} = 3$ dB as values larger than this do not occur when $(D_{\text{max}})_{\text{fit}} = 5$ mm (Fig. 4a). Extrapolation of the best-fit polynomial associated with $(D_{\text{max}})_{\text{fit}} = 5$ mm to larger values of $Z_{\text{dr}}$ would result in drastically larger estimated $D_0$ than the polynomial associated with $(D_{\text{max}})_{\text{fit}} = 6$ mm (or larger $D_{\text{max}}$). Similar conclusions can be drawn for the $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomial associated with $(D_{\text{max}})_{\text{fit}} = 4$ mm, which is much closer in behavior to the polynomial for $(D_{\text{max}})_{\text{fit}} = 5$ mm than to $(D_{\text{max}})_{\text{fit}} ≃ 6$ mm. Similarly, the $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomial for $(D_{\text{max}})_{\text{fit}} = 6$ mm is much closer to those polynomials associated with $(D_{\text{max}})_{\text{fit}} ≃ 7$ mm. In fact, there is very little difference in the estimated $D_0$ from the polynomials associated with $(D_{\text{max}})_{\text{fit}} ≃ 7$ mm except at very large $Z_{\text{dr}} > 4$ dB.

It is worth noting that the behavior of the $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomial for a $(D_{\text{max}})_{\text{fit}}$ given by the actual disdrometer-measured $D_{\text{max}}$ is very close to the polynomials for $(D_{\text{max}})_{\text{fit}} = 4–5$ mm for $Z_{\text{dr}} ≤ 2.25$ dB and then rapidly deviates as those polynomials curve upward to large $D_0$ for increasing $Z_{\text{dr}}$ (Fig. 4a). For $Z_{\text{dr}} > 2.25$ dB, the $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomial associated with the actual 2DVD-measured $D_{\text{max}}$ remains relatively flat similar to the polynomials for $(D_{\text{max}})_{\text{fit}} ≥ 6$ mm but falling at a noticeably larger $D_0$ for a given $Z_{\text{dr}}$ up to about 4.5 dB.

By assuming a constant $(D_{\text{max}})_{\text{fit}}$ varying from 4 to 10 mm, it is clear that 1) increasing $(D_{\text{max}})_{\text{fit}}$ results in generally smaller estimated $D_0(Z_{\text{dr}})_{\text{fit}}$ for a given $Z_{\text{dr}}$ up to about 4.5 dB, and 2) resonance scattering causes a dramatic decrease in the inferred $D_0(Z_{\text{dr}})_{\text{fit}}$ at $Z_{\text{dr}} > 1.5$ dB for all polynomials having $(D_{\text{max}})_{\text{fit}} ≥ 6$ mm. However, assuming a constant $D_{\text{max}}$ for all DSDs is not physically realistic, is not consistent with the 2DVD measurements in Fig. 1 and is not in keeping with many previous studies (section 1b).

To simulate more realistic $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomials and test a range of assumptions utilized in the literature, $(D_{\text{max}})_{\text{fit}}$ was started at $2D_0$, which is the mean value for the DSD dataset utilized in this study, and increased to 2.5$D_0$, then 3$D_0$ and finally 3.5$D_0$, which have all been utilized in the literature to retrieve polarimetric radar-based DSD equations (e.g., Fig. 1; Keenan et al. 2001; Bringi et al. 2002, 2003, 2006, 2009; Thurai et al. 2007). The resulting $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomials are provided in Fig. 4b along with the $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomials associated with $4D_0$ and the actual 2DVD-measured $D_{\text{max}}$ for reference. The coefficients of the corresponding fourth-order polynomials [Eq. (1)] can be found in Table 1.

As noted for constant $D_{\text{max}}$, the estimated $D_0$ from the $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomials for a given $Z_{\text{dr}}$ decreases with increasing $(D_{\text{max}})_{\text{fit}} = CD_0$ (i.e., with increasing C) (Fig. 4b). The differences between the various $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomials are particularly noticeable at $Z_{\text{dr}} > 1.5$ dB. As $(D_{\text{max}})_{\text{fit}} = CD_0$ (i.e., C) increases, the $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomials become increasingly flatter at $1.5 < Z_{\text{dr}} < 4$ dB, resulting in a significantly lower estimated $D_0$ for
a given $Z_{dr}$. The difference between adjacent $D_0(Z_{dr})_{fit}$ polynomials in the family of $(D_{\text{max}})_{\text{fit}} = CD_0$ curves becomes smaller as $C$ increases. In other words, the difference between $(D_{\text{max}})_{\text{fit}} = 2D_0$ and $2.5D_0$ is larger than the difference between $3D_0$ and $3.5D_0$. In fact, there is very little difference between assuming $(D_{\text{max}})_{\text{fit}} = 3.5D_0$ and $4D_0$. The $D_0(Z_{dr})_{fit}$ polynomial associated with a $(D_{\text{max}})_{\text{fit}}$ equal to the measured 2DVD $D_{\text{max}}$ is more similar to $(D_{\text{max}})_{\text{fit}} = 2D_0$ at small $Z_{dr}$ (although it is difficult to see that in Fig. 4b) and most similar to

FIG. 4. The simulated median volume diameter ($D_0$; mm) vs the simulated differential reflectivity ($Z_{dr}$; dB) at C band derived from observed DSD data. Each curve represents a fourth-order polynomial fit to simulated pairs of ($Z_{dr}$, $D_0$) with a different assumption regarding the maximum raindrop size ($D_{\text{max}})_{\text{fit}}$. (a) $D_{\text{max}}$ is assumed constant [($(D_{\text{max}})_{\text{fit}} = 4, 5, 6, 7, 8, 9,$ or $10$ mm as shown)]. For $(D_{\text{max}})_{\text{fit}} = 4$ and $5$ mm, markers not accompanied by a plotted curve represent an extrapolation of the polynomial fit beyond the maximum simulated $Z_{dr}$. (b) $D_{\text{max}}$ is assumed to be a multiple of $D_0$ [($(D_{\text{max}})_{\text{fit}} = 2D_0, 2.5D_0, 3D_0, 3.5D_0,$ and $4D_0$ as shown)]. In both (a) and (b), the dashed line represents the polynomial fit of $D_0$ vs $Z_{dr}$ for the actual observed 2DVD maximum. Polynomial coefficients for the curves in (b) can be found in Table 1. The remaining assumptions and methods for deriving the simulated pairs of ($Z_{dr}$, $D_0$) at C band from observed DSD data are discussed in section 2.
(D_{\text{max}})_{\text{fit}} = 2.5D_0 through a broad range of Z_{dr} from 1 to 4 dB.

b. Overall D_0 retrieval error

To assess the overall D_0 retrieval error, each D_0(Z_{dr})_{\text{fit}} polynomial associated with an assumed (D_{\text{max}})_{\text{fit}} was evaluated against a (D_0)_{\text{truth}} dataset associated with a postulated (D_{\text{max}})_{\text{truth}}. To visualize this kind of test in Fig. 5a, the pairs of (Z_{dr}, D_0)_{\text{fit}} data used for derivation of the (D_0)(Z_{dr})_{\text{fit}} polynomial assuming (D_{\text{max}})_{\text{fit}} = 3.5D_0 are accompanied by pairs of true (Z_{dr}, D_0)_{\text{truth}} assuming (D_{\text{max}})_{\text{truth}} = 2D_0, which is close to observed (Fig. 1). In this example, it is clear that the (D_0)(Z_{dr})_{\text{fit}} polynomial assuming (D_{\text{max}})_{\text{fit}} = 3.5D_0 is underestimating the true (D_{\text{max}})(Z_{dr})_{\text{truth}} at a given Z_{dr} assuming (D_{\text{max}})_{\text{truth}} = 2D_0.

The “fit” and “truth” datasets are then reversed in Fig. 5b. Correspondingly, it is easy to see how the (D_0)(Z_{dr})_{\text{fit}} polynomial assuming (D_{\text{max}})_{\text{fit}} = 2D_0 is overestimating the true (D_{\text{max}})(Z_{dr})_{\text{truth}} at a given Z_{dr} assuming (D_{\text{max}})_{\text{truth}} = 3.5D_0.

Normalized bias errors for various (D_0)(Z_{dr})_{\text{fit}} polynomials assuming constant (D_{\text{max}})_{\text{fit}} from 4 to 10 mm across the entire sample of (Z_{dr}, D_0)_{\text{truth}} assuming various (D_{\text{max}})_{\text{truth}} are provided in Fig. 6. For (D_{\text{max}})_{\text{fit}} = 6 mm and (D_{\text{max}})_{\text{truth}} = 6 mm, the absolute normalized bias errors of (D_0)(Z_{dr})_{\text{fit}} are small (<3%). For (D_{\text{max}})_{\text{fit}} = 6 mm and (D_{\text{max}})_{\text{truth}} < 6 mm, the normalized bias errors of (D_0)(Z_{dr})_{\text{fit}} range from −4.2% to −5.9%. Compared to a (D_{\text{max}})_{\text{truth}} equal to the 2D-measured (D_{\text{max}}), the normalized bias errors for (D_0)(Z_{dr})_{\text{fit}} assuming

### Table 1. Coefficients (a, b, c, d, and e) of the fourth-order polynomial for D_0(Z_{dr})_{\text{fit}} as shown in Eq. (1) for C band. The coefficients were obtained from a nonlinear least squares curve fit to pairs of simulated (Z_{dr}, D_0) for various D_{max} assumptions at C band. Methods and assumptions for developing the simulation dataset are discussed in section 2.

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![Fig. 5. As in Fig. 3, but (a) the pairs of (Z_{dr}, D_0)_{\text{fit}} data (black diamonds) used for derivation of the best-fit fourth-order polynomial \[D_0(Z_{dr})_{\text{fit}}; \text{asterisks}\] assuming (D_{\text{max}})_{\text{fit}} = 3.5D_0 are accompanied by pairs of true (Z_{dr}, D_0)_{\text{truth}} (red pluses) assuming (D_{\text{max}})_{\text{truth}} = 2D_0, and (b) the pairs of (Z_{dr}, D_0)_{\text{fit}} data (black diamonds) used for derivation of the best-fit fourth-order polynomial \[D_0(Z_{dr})_{\text{fit}}; \text{asterisks}\] assuming (D_{\text{max}})_{\text{fit}} = 2D_0 are accompanied by pairs of true (Z_{dr}, D_0)_{\text{truth}} (red pluses) assuming (D_{\text{max}})_{\text{truth}} = 3.5D_0. Note that the “fit” (black diamonds) and “truth” (red pluses) (Z_{dr}, D_0) datasets are simply reversed between (a) and (b).]
(D_{\text{max}})_{\text{fit}} \geq 6\, \text{mm} \text{ are approximately } -7\%. \text{ If } (D_{\text{max}})_{\text{fit}} < 6\, \text{mm and } (D_{\text{max}})_{\text{truth}} < 6\, \text{mm, then the absolute normalized bias errors of } D_0(Z_{\text{dr}})_{\text{fit}} \text{ are fairly small } (<4\%). \text{ If } (D_{\text{max}})_{\text{fit}} < 6\, \text{mm and } (D_{\text{max}})_{\text{truth}} \text{ is equal to the 2DVD-measured } D_{\text{max}}, \text{ then the normalized bias errors of } D_0(Z_{\text{dr}})_{\text{fit}} \text{ range approximately from } 3\% \text{ to } 9\%. \text{ On the other hand, if } (D_{\text{max}})_{\text{fit}} < 6\, \text{mm and } (D_{\text{max}})_{\text{truth}} > 6\, \text{mm, then the normalized bias errors of } D_0(Z_{\text{dr}})_{\text{fit}} \text{ are extremely large, ranging from } 26\% \text{ to } 75\%, \text{ and are largely the result of resonant scattering on the truth dataset and of extrapolating the } D_0(Z_{\text{dr}})_{\text{fit}} \text{ polynomials to larger values of } Z_{\text{dr}} \text{ that are not present in the fit dataset but are in the truth dataset (Fig. 4a). These very large values of normalized bias error are likely not realistic since } D_{\text{max}} \text{ is likely not constant for all DSD but Fig. 6 does emphasize the potential effect of resonance on the bias error associated with retrieving } D_0 \text{ using C-band } Z_{\text{dr}} \text{ associated with assuming an inappropriate } D_{\text{max}}.\text{ To provide a more realistic assessment of bias error, the normalized bias errors for various } D_0(Z_{\text{dr}})_{\text{fit}} \text{ polynomials assuming } (D_{\text{max}})_{\text{fit}} = CD_0 \text{ for } C = 2 - 4 \text{ across the entire sample of } (D_0)_{\text{truth}} \text{ assuming various } (D_{\text{max}})_{\text{truth}} = CD_0 \text{ for } C = 2 - 4 \text{ are provided in Fig. 7. Despite the known sampling limitations of disdrometers, the actual 2DVD-measured } D_{\text{max}} \text{ is also utilized as a potentially realistic } (D_{\text{max}})_{\text{fit}} \text{ and } (D_{\text{max}})_{\text{truth}}. \text{ As shown by the statistics of } D_{\text{max}}/D_0 \text{ in Fig. 1a and other studies (Keenan et al. 2001; Gatlin et al. 2015), this range of } C \text{ and hence } D_{\text{max}} \text{ should adequately represent a potential realistic range of } D_{\text{max}} \text{ when considering potential undersampling of large drops by the 2DVD. As a result, the range of normalized bias errors in Fig. 7 should bracket the overall bias errors potentially present in recent } D_0 \text{ retrieval studies using } Z_{\text{dr}} \text{ associated with potential misalignment of the assumed and actual } D_{\text{max}}. \text{ Because the } (D_{\text{max}})_{\text{fit}} = 2D_0 \text{ polynomial is the most different than the others (Fig. 4b), its range of possible normalized bias errors for } D_0(Z_{\text{dr}})_{\text{fit}} \text{ is the largest } (0\% - 16\%). \text{ The possible range of normalized bias errors for } D_0(Z_{\text{dr}})_{\text{fit}} \text{ decreases as } (D_{\text{max}})_{\text{fit}} \text{ increases from } 2D_0 \text{ to } 4D_0. \text{ For } (D_{\text{max}})_{\text{fit}} = 2.5D_0, \text{ the possible normalized bias error for } D_0(Z_{\text{dr}})_{\text{fit}} \text{ ranges from } -2\% \text{ to } 9\%. \text{ As expected from Fig. 4b, the range of possible bias error for } D_0(Z_{\text{dr}})_{\text{fit}} \text{ assuming a } (D_{\text{max}})_{\text{fit}} \text{ of the measured 2DVD } D_{\text{max}} \text{ falls between } (D_{\text{max}})_{\text{fit}} = 2D_0 \text{ and } 2.5D_0. \text{ For } (D_{\text{max}})_{\text{fit}} = 3D_0 \text{ (4D}_0), \text{ the possible...}
normalized bias error ranges from −6% to 4% (−8% to 0%). As summarized in section 1a, the absolute normalized bias error for $D_0$ in past studies has typically been estimated to be less than 5%. The results herein demonstrate that the absolute normalized bias error for estimating $D_0$ using C-band $Z_{dr}$ could be 2–3 times as large (i.e., 10%–15%) as previously estimated (section 1a) because of a potential error in the
assumed maximum drop diameter. Of course, the overall bias error depends on the degree of mismatch between the assumed and true $D_{\text{max}}$ and many (61%) of the tested scenarios in Fig. 7 have normalized bias errors falling within $\pm 5\%$.

Since the major effect of varying $C$ from 2 to 4 in $(D_{\text{max}})_{\text{fit}} = CD_0$ on the $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomials is to shift the $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomial upward and downward (Fig. 4b), it was hypothesized that the normalized standard error would not vary dramatically because of a mismatch between the assumed $(D_{\text{max}})_{\text{fit}}$ and $(D_{\text{max}})_{\text{truth}}$. Because there is a minor change in the shape of the polynomial with varying $(D_{\text{max}})_{\text{fit}}$ that is most apparent for $(D_{\text{max}})_{\text{fit}} = 2D_0$ relative to the others (Fig. 4b), it was anticipated that its potential range of normalized error would be the largest. Results for the normalized standard error generally confirm these expectations (Fig. 8). For $(D_{\text{max}})_{\text{fit}} = 2D_0$, the normalized standard error for $D_0(Z_{\text{dr}})_{\text{fit}}$ ranges from 14% to 22%, which is somewhat larger than found in past studies (7%–15%) as noted in section 1a. For the other tested $(D_{\text{max}})_{\text{fit}}$ in Fig. 8, the normalized standard error for $D_0(Z_{\text{dr}})_{\text{fit}}$ varied between 14% and 17%, which is on the high end but generally consistent with these past studies. In this study, the standard error of $Z_{\text{dr}}$ is assumed to be 0.25 dB, which is consistent with the standard deviation of $Z_{\text{dr}}$ in vertically pointing ARMOR scans of drizzle. While reasonable for ARMOR, this standard error of $Z_{\text{dr}}$ may be slightly higher than assumed in some prior studies, which could account for some of the difference in the normalized standard error for $D_0(Z_{\text{dr}})_{\text{fit}}$.

c. $D_0$ retrieval error as a function of differential reflectivity

Of course, bias and standard errors over the entire sample only tell part of the story. It is important to understand how errors in the estimated $D_0$ might vary as a function of the independently measured radar property,

FIG. 9. Normalized bias of the median volume diameter estimated from the best-fit polynomial, $D_0(Z_{\text{dr}})_{\text{fit}}$, as a function of binned $Z_{\text{dr}}$, assuming the $(D_{\text{max}})_{\text{fit}}$ of (a) $2D_0$, (b) $2.5D_0$, (c) $3D_0$, (d) $3.5D_0$, and (e) the actual 2DVD-measured $D_{\text{max}}$, relative to a true dataset, $(Z_{\text{dr}}, D_0)_{\text{truth}}$, with an assumed $(D_{\text{max}})_{\text{truth}}$ (colored lines as shown). The $Z_{\text{dr}}$ bins start at 0.25 dB, are separated by 0.5 dB, and end at 4.75 dB. Note that the first $Z_{\text{dr}}$ bin centered at 0.25 dB encompasses $Z_{\text{dr}} < 0.5$ dB, including some negative $Z_{\text{dr}}$ (e.g., Figs. 3 and 5), and the last bin centered at 4.75 dB encompasses $Z_{\text{dr}} > 4.5$ dB, including a few $Z_{\text{dr}}$ over 5 dB.
in this case $Z_{dr}$. As such, the normalized bias error is presented in Figs. 9a–e as a function of $Z_{dr}$ for the same range of assumptions for $(D_{\text{max}})_{\text{fit}} = CD_0$ ($C = 2\text{–}3.5$) or the measured 2DVD $D_{\text{max}}$. In each panel of Figs. 9a–e, the $(D_{\text{max}})_{\text{truth}}$, each represented by a different curve, is varied through a similar range of $D_{\text{max}}$ [i.e., $(D_{\text{max}})_{\text{truth}} = CD_0$ ($C = 2\text{–}4$) or the measured 2DVD $D_{\text{max}}$]. The largest normalized bias errors for $D_0(Z_{dr})_{\text{fit}}$ can be found at the two extremes of the assumed $(D_{\text{max}})_{\text{fit}}$, which are $(D_{\text{max}})_{\text{fit}} = 2D_0$ (Fig. 9a) and $3.5D_0$ (Fig. 9d). For $(D_{\text{max}})_{\text{fit}} = 2D_0$, the normalized bias error for $D_0(Z_{dr})_{\text{fit}}$ can exceed 0.1 for $Z_{dr}$ as low as $1\text{–}1.5$ dB and can increase from there to values as large as 0.23–0.47 for larger $Z_{dr}$, depending on the degree of mismatch between the assumed $(D_{\text{max}})_{\text{fit}}$ and $(D_{\text{max}})_{\text{truth}}$. Of course, if $(D_{\text{max}})_{\text{fit}}$ and $(D_{\text{max}})_{\text{truth}}$ are well aligned, then the bias errors with $Z_{dr}$ are generally smaller. Similarly, for $(D_{\text{max}})_{\text{fit}} = 3.5D_0$, the absolute normalized bias error for $D_0(Z_{dr})_{\text{fit}}$ can reach 0.1 for $Z_{dr}$ as low as $1\text{–}1.75$ dB and can increase from there to values as large as 0.17–0.29 for larger $Z_{dr}$ when the mismatch between the assumed $(D_{\text{max}})_{\text{fit}}$ and $(D_{\text{max}})_{\text{truth}}$ is large [e.g., see the curves for $(D_{\text{max}})_{\text{truth}} = 2D_0$, 2.5$D_0$, and the 2DVD-measured value in Fig. 9d].

Absolute normalized bias errors for $D_0(Z_{dr})_{\text{fit}}$ larger than 0.1 can be found at $Z_{dr} > 1.5$ dB for all assumed $(D_{\text{max}})_{\text{fit}}$ in Figs. 9a–e depending on the misalignment with $(D_{\text{max}})_{\text{truth}}$. Even if it is assumed that $D_{\text{max}}/D_0 \approx 2.5$ always, which may or may not be realistic as highlighted in Fig. 1a and earlier discussion, the absolute normalized bias error can still exceed 0.1 at $Z_{dr}$ as low as 1.75 dB (Fig. 9b), which is associated with a $D_0$ of 1.7–2.0 mm (Fig. 4b). Clearly, the relationship between $D_0$ and C-band $Z_{dr}$ for a gamma DSD is not relatively insensitive to changes in $D_{\text{max}}/D_0$ for values of $D_0 \leq 3.2$ mm provided $D_{\text{max}}/D_0 \geq 2.5$ as found by Ulbrich and Atlas (1984) for S-band $Z_{dr}$ in their study. This difference with the S band results in Ulbrich and Atlas (1984) is due in part to the effect of resonance on C-band $Z_{dr}$ in large drops greater than 5 mm in diameter (Fig. 2), although differences in methodology and data may also play some role.

As shown in Fig. 10, the normalized standard error for $D_0(Z_{dr})_{\text{fit}}$ tends to be a maximum at both small $Z_{dr} (<1.5$ dB) and very large $Z_{dr} (>4$ dB). In between a $Z_{dr}$ of 1.5 and 4 dB, the normalized standard errors of $D_0(Z_{dr})_{\text{fit}}$ tend to be <0.1. One exception is for an assumed $(D_{\text{max}})_{\text{truth}}$ equal to the 2DVD-measured $D_{\text{max}}$. The elevated values (>0.1) of normalized standard error at $Z_{dr} < 1.5$ dB are due in large part to the effect of random noise on $Z_{dr}$. For $Z_{dr} > 4$ dB, the sample size is relatively small and the parameterization error for $D_0(Z_{dr})_{\text{fit}}$ is likely larger (e.g., Fig. 3). The assumed $D_{\text{max}}$ does not generally have a large effect on the variation of the normalized standard error with $Z_{dr}$.

**d. Wavelength dependence of $D_0$ retrieval error**

To explore the wavelength dependence of these results while holding all else equal, the normalized bias of $D_0(Z_{dr})_{\text{fit}}$ as a function of $Z_{dr}$ for two different extreme $D_{\text{max}}$ mismatch scenarios is shown in Fig. 11 for X, C, and S band: (i) $(D_{\text{max}})_{\text{fit}} = 3.5D_0$ and $(D_{\text{max}})_{\text{truth}} = 2D_0$ (Fig. 11a), and (ii) $(D_{\text{max}})_{\text{fit}} = 2D_0$ and $(D_{\text{max}})_{\text{truth}} = 3.5D_0$ (Fig. 11b). In both scenarios, the absolute normalized bias
for $D_0(Z_{dr})_{fit}$ at X band slightly exceeds that of C and S band for $Z_{dr} < 1.5$ dB, which is associated with the slight impact of resonance on X-band $Z_{dr}$ between a drop diameter of 3 and 4 mm (Fig. 2). For $Z_{dr} \geq 1.5$ dB, the absolute normalized bias for $D_0(Z_{dr})_{fit}$ at C band exceeds X and S band (with one minor exception at $Z_{dr} = 3.75$ dB in Fig. 11a). In fact, the absolute normalized bias error for $D_0(Z_{dr})_{fit}$ at C band can be significantly larger than at S band by as much as 0.12–0.13 for $Z_{dr} > 3.75$ dB in Fig. 11b. This difference in the bias error of the $D_0$ estimate between C-band $Z_{dr}$ and S-band $Z_{dr}$ is due to the effect of resonance on C-band $Z_{dr}$ in large (> 5 mm) raindrops (Fig. 2).

e. Temperature dependence of $D_0$ retrieval error

The impact of $T$ was explored by conducting additional sensitivity tests at drop temperature of $T = 10^\circ C$ and $T = 30^\circ C$ in addition to the standard temperature of $T = 20^\circ C$. The values of $Z_{dr}$ at $T = 10^\circ C$ and $Z_{dr}$ at $T = 30^\circ C$ are now compared with $Z_{dr}$ at $T = 20^\circ C$ in Figs. 12a and 12b, respectively. At larger values of $Z_{dr}$, $(Z_{dr} at T = 10^\circ C) < (Z_{dr} at T = 20^\circ C)$ while it is reversed when the temperature is increased [i.e., $(Z_{dr} at T = 30^\circ C) > (Z_{dr} at T = 20^\circ C)$]. In other words, resonance has a larger impact on increasing C-band $Z_{dr}$ at warmer temperatures. The impact of varying temperature on $D_0(Z_{dr})_{fit}$ was explored for a variety of $D_{max}$ assumptions. The results are shown in Fig. 13 for $D_{max} = 3.5D_0$ since the results for other $D_{max}$ assumptions were comparable. As expected from Fig. 12, at larger $Z_{dr}$, there is a negative bias in $D_0(Z_{dr})_{fit}$ for a temperature that is colder in truth $(T_{truth} = 10^\circ C)$ than what was assumed in developing the fit equation $(T_{fit} = 20^\circ C)$ (Fig. 13) while there is a positive bias in $D_0(Z_{dr})_{fit}$ for a temperature that is warmer in truth $(T_{truth} = 30^\circ C)$ than what was assumed in developing the fit equation $(T_{fit} = 20^\circ C)$ (Fig. 13). For a fairly large temperature mismatch of 10$^\circ C$, the overall bias of $D_0(Z_{dr})_{fit}$ is negligible (∼0 dB) in the mean over the full range of $Z_{dr}$ (i.e., for all DSDs). For larger $Z_{dr}$ values, the $D_0(Z_{dr})_{fit}$ bias error can approach ±5%–10% for
a temperature bias of $\pm 10^\circ$C (Fig. 13). While these errors are not trivial, it is worthwhile to note that the impact of $D_{\text{max}}$ uncertainty is likely often larger than the impact of temperature uncertainty on the accuracy of the $D_0(Z_{\text{dr}})$ estimator (cf. Figs. 9 and 13).

4. Conclusions

Estimating the raindrop size, including median volume diameter, has been a long-standing objective of polarimetric radar–based precipitation retrieval methods, particularly those using the differential reflectivity. Theoretically, $Z_{\text{dr}}$ is a measure of the reflectivity-factor-weighted mean axis ratio and therefore indirectly the reflectivity-factor-weighted drop size via the assumption of a drop size relation (Jameson 1983; Bringi and Chandrasekar 2001, p. 398). The theoretical relationship between $Z_{\text{dr}}$ and $D_0$ (or $D_{\text{max}}$) is more complex and requires more assumptions regarding the DSD (e.g., Bringi and Chandrasekar 2001, p. 398). From a practical perspective, the relationship between $Z_{\text{dr}}$ and $D_0$ is typically derived empirically using rain DSD observations from a disdrometer such as the 2DVD, a raindrop physical model, and a radar scattering model. Because disdrometers are known to undersample large raindrops and therefore underestimate the maximum raindrop size, $D_{\text{max}}$ is often an assumed parameter in the rain physical model. Because there is remaining uncertainty regarding the appropriate $D_{\text{max}}$ for a given DSD (Fig. 1), there have been a wide variety of assumptions for $D_{\text{max}}$. Because $D_{\text{max}}$ affects the tail of the DSD and $Z_{\text{dr}}$ is reflectivity weighted (i.e., $D^6$ for Rayleigh–Gans scattering), variability in the $D_{\text{max}}$ assumption can affect the relationship between $Z_{\text{dr}}$ and $D_0$, as was noted in early studies such as Ulbrich and Atlas (1984) at S band.

Although there have been a number of DSD retrieval application studies at C band, the sensitivity of the relationship between $Z_{\text{dr}}$ and $D_0$ to $D_{\text{max}}$ and the associated potential error in the retrieved $D_0$ have not been investigated in any detail. C-band polarimetric radars are commonly used in research and operations worldwide because of their decreased cost relative to S band. Compared to S band, C band has some complicating factors to consider before analysis of raindrop properties, including increased propagation effects and resonance scattering (or non-Rayleigh–Gans scattering) in large raindrops (i.e., diameter $> 5$ mm). Resonance scattering can complicate the relationship between the $Z_{\text{dr}}$ and diameter of individual raindrops (Fig. 2) and between $Z_{\text{dr}}$ and $D_0$ (Fig. 4) of realistic DSDs with an assumed $D_{\text{max}}$

In this study, a series of experiments were conducted that postulate a $(D_{\text{max}})_{\text{fit}}$ for the fitting of a fourth-order polynomial to $(Z_{\text{dr}}, D_0)_{\text{fit}}$ data derived from 2DVD data, a rain model, and a radar scattering model at C band. The resulting $D_0(Z_{\text{dr}})_{\text{fit}}$ polynomial, which is associated with an assumed $(D_{\text{max}})_{\text{fit}}$, was then compared with a truth dataset, $(D_0)_{\text{truth}}$, which is associated with an assumed $(D_{\text{max}})_{\text{truth}}$ (Fig. 5). The normalized bias and scatter errors for $D_0(Z_{\text{dr}})_{\text{fit}}$ relative to $(D_0)_{\text{truth}}$ were then computed while varying both $(D_{\text{max}})_{\text{fit}}$ and $(D_{\text{max}})_{\text{truth}}$ through the range of assumptions found in the literature.

It is found that the overall absolute normalized bias errors for $D_0(Z_{\text{dr}})_{\text{fit}}$ can be as high as 10%–15% at C band depending on the degree of mismatch between the
The magnitude of the potential bias error in the estimated \(D_{\text{max}}\) associated with a given DSD is currently a tunable parameter in the radar retrieval model that can be adjusted to maximize agreement between polarimetric radar and independently observed (e.g., disdrometers, wind profilers) estimates of \(D_0\), assuming independent measurements of drop size are available. If adjustments to \(D_{\text{max}}\) are made to optimize agreement, then this uncertainty in \(D_{\text{max}}\) could be masking other potential sources of polarimetric radar bias error. Regardless of whether independent measurements of drop size are available or not, it would be highly desirable to reduce uncertainty in \(D_{\text{max}}\) associated with a given DSD to increase the robustness of the polarimetric radar estimate of \(D_0\), especially at C band. Ongoing efforts to reduce uncertainty in \(D_{\text{max}}\) include dense networks of many disdrometers distributed within a typical radar footprint (e.g., Jaffrain and Berne 2012; Petersen et al. 1993), estimating the appropriate maximum drop diameter of a DSD from a single disdrometer measurement over a period that is consistent with the spatial scales of radar data is difficult if not futile based on sampling limitations and may often underestimate \(D_{\text{max}}\). On the other hand, using larger \(D_{\text{max}}\) (e.g., \(\geq 2.5D_0\)) for all DSD that are exceedingly rare (i.e., occurrence at 95th to 99th percentile or larger) in single disdrometer measurements consistent with radar resolution volume spatial scales could potentially overestimate \(D_{\text{max}}\). As shown in this study, this uncertainty in \(D_{\text{max}}\) has implications for the estimated bias error in retrieved \(D_0\) using \(Z_{\text{dr}}\), especially at C band because of resonance.

Without reducing this uncertainty, \(D_{\text{max}}\) is currently a tunable parameter in the radar retrieval model that can be adjusted to maximize agreement between polarimetric radar and independently observed (e.g., disdrometers, wind profilers) estimates of \(D_0\), assuming independent measurements of drop size are available. If adjustments to \(D_{\text{max}}\) are made to optimize agreement, then this uncertainty in \(D_{\text{max}}\) could be masking other potential sources of polarimetric radar bias error. Regardless of whether independent measurements of drop size are available or not, it would be highly desirable to reduce uncertainty in \(D_{\text{max}}\) associated with a given DSD to increase the robustness of the polarimetric radar estimate of \(D_0\), especially at C band. Ongoing efforts to reduce uncertainty in \(D_{\text{max}}\) include dense networks of many disdrometers distributed within a typical radar footprint (e.g., Jaffrain and Berne 2012; Petersen et al. 1993), estimating the appropriate maximum drop diameter of a DSD from a single disdrometer measurement over a period that is consistent with the spatial scale of radar data is difficult if not futile based on sampling limitations and may often underestimate \(D_{\text{max}}\).

As noted by others (e.g., Ulbrich 1992; Smith et al. 1993), estimating the appropriate maximum drop diameter of a DSD from a single disdrometer measurement over a period that is consistent with the spatial scale of radar data is difficult if not futile based on sampling limitations and may often underestimate \(D_{\text{max}}\). On the other hand, using larger \(D_{\text{max}}\) (e.g., \(\geq 2.5D_0\)) for all DSD that are exceedingly rare (i.e., occurrence at 95th to 99th percentile or larger) in single disdrometer measurements consistent with radar resolution volume spatial scales could potentially overestimate \(D_{\text{max}}\). As shown in this study, this uncertainty in \(D_{\text{max}}\) has implications for the estimated bias error in retrieved \(D_0\) using \(Z_{\text{dr}}\), especially at C band because of resonance.

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![Graph](image-url)


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