Analysis of Rain Types and Their Z–R Relationships at Different Locations in the High Andes of Southern Ecuador

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ABSTRACT

Information on the spatiotemporal rainfall occurrence, its microphysical characteristics, and its reflectivity–rainfall (Z–R) relations required to provide rainfall mapping based on rain radar data is limited for tropical high mountains. Therefore, this study aims to analyze rainfall types in the Andes cordillera to derive different rain-type Z–R relations using disdrometer observations at three study sites representative for different geographic positions and elevations (2610, 3626, and 3773 m MSL). Rain categorization based on mean drop volume diameter ($D_m$) thresholds \[0.1 \leq D_m \leq 0.5; 0.5 \leq D_m \leq 1.0; 1.0 \leq D_m \leq 2.0\] was performed using drop size distribution data at a 5-min time step over an approximate 2-yr period at each location. The findings are as follows: (i) Rain observations characterized by higher (lower) $D_m$ and rain rates are more frequent at the lower (higher) site. (ii) Because of its geographic position, very light rain (drizzle) is more common at higher altitudes with longer-duration events, whereas rainfall is more convective at the lower range. (iii) The specific spatial exposition regarding cloud and rain formation seems to play an important role for derivation of the local Z–R relationship. (iv) Low A coefficients (≤60) for the first rain type resemble typical characteristics of orographic precipitation. (v) Greater values of A (lowest and highest stations for $D_m > 1.0$ mm) are attributed to transitional rainfall as found in other studies. (vi) Rain-type Z–R relations show a better adjustment in comparison with site-specific Z–R relationships. This study is the first contribution of Z–R relations for tropical rainfall in the high Andes.

1. Introduction

Mountain ranges have a major impact on global and regional climate. Their topographic complexity strongly affects rainfall processes, and this in turn makes quantitative precipitation estimates in space and time very difficult. Unfortunately, operational rainfall monitoring in highlands is scarce and unevenly distributed, and therefore their rainfall dynamics is significantly underexplored in comparison with other areas of the world (Wohl et al. 2012). This situation particularly holds for the tropical Andes, where high spatiotemporal rainfall variability has been demonstrated (Espinoza Villar et al. 2009; Scheel et al. 2011; Célleri et al. 2007; Pulwarty et al. 1998; Rollenbeck and Bendix 2011) mainly because of complex rainfall formation processes (Poveda et al. 2005). In addition, rainfall microphysical characteristics [e.g., drop size distribution (DSD)] behind its generation are still unknown. Rainfall has countless implications for human activities, water resource management, and biological and ecological processes, just to name a few, and a tremendous impact on the socioeconomic development.
of the Andean region. Irrigated and rain-fed agricultural areas are still very large (De Bièvre et al. 2003) and spatially distributed, and hydropower generation relies on rainfall and its variability (Céllerí and Feyen 2009). In Ecuador, hydropower generation accounts for approximately 45% of energy production (Buytaert et al. 2006; Céllerí and Feyen 2009) and is projected to reach 80% by 2020. In addition, the occurrence of extreme rainfall events (e.g., in the scope of the ENSO phenomenon) may cause hazards for populations and human activities. In consequence, knowledge of the spatiotemporal rainfall distribution is of major importance for economic issues. Therefore, reliable rainfall data and their related microphysical characteristics are greatly needed.

Given the difficulties in deploying rain gauges due to the complex topography of mountain regions, the use of rain radars is highly beneficial in providing spatially well-distributed rainfall data (Yoon et al. 2012). Nevertheless, the hydrological visibility as a result of beam blockage is often a limiting factor (Pellarin et al. 2002). Depending on the technology used, radar implementation might be too expensive [e.g., polarimetric systems as used in Next Generation Radar (NEXRAD); Cunha et al. 2015]. Recently, cost-effective scanning X-band radars with one plan position indicator (PPI) have emerged on the market as a good supplement to sparsely distributed rain gauge networks. This is particularly helpful for developing countries such as Ecuador, where the Cajas X-band Selex (CAXX) single-polarized weather radar (Rainscanner; Selex ES GmbH) was installed at 4450 m above mean sea level (MSL) as part of a novel weather radar network developed in tropical high mountains (RadarNet-Sur; Bendix et al. 2017), making it arguably the highest radar in the world.

Quantitative precipitation estimation (QPE) can be extremely difficult and challenging. In this context, converting native radar variables—normally reflectivity (Z)—into reliable rainfall rates (R) is one of the most important steps, but it is particularly difficult in mountain regions (Rollenbeck and Bendix 2006, 2011; Germann et al. 2006; Savina 2011). One possible way to overcome this issue is to use an adapted climatic or, even better, a rain-type-based Z–R relation. For the latter, a quantitative rain-type classification is needed. Approaches for classifying rain types generally rely on (i) an application-oriented discrimination based only on rain rates (Cerro et al. 1997; Sen Jaiswal et al. 2009), (ii) DSD information (Rosenfeld et al. 1993; Atlas et al. 2000; Uijlenhoet 2001; Uijlenhoet et al. 2003; Nzeukou and Sauvageot 2004; Caracciolo et al. 2008; Ochou et al. 2011), or (iii) polarimetric rainfall retrieval algorithms, as proposed by L’Ecuyer et al. (2004) and Zhang et al. (2011). Unfortunately, most studies in mountain regions have been carried out without fully considering rainfall spatial variability and, consequently, the effects of altitudinal gradients on Z–R relation parameters (Tokay et al. 2009). Moreover, few Z–R relations adapted to tropical lowlands (Rao et al. 2001; Russell et al. 2010; Ramli and Tahir 2011; Kumar et al. 2011; Tenório et al. 2012; Bamba et al. 2014) have been provided thus far and may not represent the rainfall characteristics at high altitudes very well. In consequence, neither rainfall microphysical characteristics such as DSD nor climatic and/or rain-type-based Z–R relations are available to date for tropical high mountains, such as the south Ecuadorian Andes.

With the objective of improving our understanding of tropical rainfall structure and its spatiotemporal variability (Céllerí et al. 2007; Rollenbeck et al. 2011) in the Andean mountain range of south Ecuador, in this study we aim to derive Z–R relations for different rainfall types occurring at three terrain elevations with different topographic location characteristics regarding its synoptic forcing. The relations were derived depending on the drop size distribution through a mean volume diameter [\(D_m\) (mm)] characterization that may account for tropical rainfall variability at high altitudes.

2. Materials and methods

a. Study sites

Figure 1 shows the CAXX radar situated at the upper limit of the Cajas National Park (2°45’S, 79°16’W) on Cerro Paragüillas and extent of its 100-km maximum measurement range. Three disdrometer sites were selected to represent different altitudinal positions in the CAXX radar range and beyond, in the western Andean cordillera. A summary of study sites information is shown in Table 1. These sites were located within the Macizo del Cajas, part of the United Nations Educational, Scientific and Cultural Organization’s (UNESCO’s) World Network of Biosphere Reserves.

Two of the disdrometer study sites, Balzay and Virgen, were located along an altitudinal gradient within the Quinuas Ecohydrological Observatory. The lowest site is the Balzay meteorological station, at 2610 m MSL, which is in the Cuenca city outskirts. Climate at Balzay is characterized by a mean annual precipitation of 969 mm and a mean temperature of ~14°C (Córdova et al. 2016).

The second station, Virgen, is located at 3626 m MSL in a shrub-dominated (subpáramo) transition zone from upland forest to páramo (Hofstede et al. 2014) within a basin at a maximum elevation of 4400 m MSL with slopes that drain toward the east (i.e., the Atlantic Ocean).
Average annual precipitation in this area is approximately 998 mm, average air temperature is 6.5°C and relative humidity is ~90%. For a more detailed climatic characterization of the site, the reader is referred to Carrillo-Rojas et al. (2016) and Córdova et al. (2016).

The third and highest disdrometer site is situated in the páramo ecosystem at an elevation of 3773 m MSL in the Zhurucay Ecological Observatory within a basin at a maximum altitude of 3900 m MSL on the slopes draining to the west (i.e., the Pacific Ocean). Climatologic conditions are characterized by an average temperature of 6°C, a relative humidity of 91% (Córdova et al. 2015), and annual precipitation ranging from 900 to 1600 mm with weak seasonality (Padrón 2013). The climatology of the area is influenced by the Pacific coastal regime from the west, and continental and tropical Amazon air masses from the east (Vuille et al. 2000). As a consequence, prevalent convective and orographic cloud formation occurs (Bendix et al. 2006). Close-to-freezing temperatures cause moist air condensation resulting in drizzle that accounts for approximately 30% of annual rainfall volume in the area (Padrón et al. 2015).

**b. Instruments and database**

The instruments used for this study were two high-resolution disdrometers [Thies Clima laser precipitation monitor (LPM); Thies Clima 2007] based on a laser sensor that produces a horizontal light strip. When a precipitation particle falls through the light beam, the receiving signal is attenuated. The particle diameter is calculated from the reduction in amplitude. Moreover, the fall speed of the particle is determined from the duration of the reduced signal. The sensor operates at a wavelength of 785 nm and has a reference measurement area of 45.6 cm² with a resolution of 0.005 mm h⁻¹. Each drop is assigned to one of 22 size bins and one of 20 velocity bins ranging over 0.125–8 mm diameter and 0–10 m s⁻¹, respectively. The range of size and velocity classes is finer for smaller and slower particles and coarser for larger and faster ones. A data telegram is transmitted every 60 s indicating the numbers of drops detected in each of the total 440 (2² × 2²) different possible classes. Assessments of the Thies optical disdrometer have been carried out by Frasson et al. (2011) and Sarkar et al. (2015), and many other studies have used this instrument (e.g., Fernández-Raga et al. 2010; Jameson et al. 2015; Chen et al. 2016). Despite its very good performance, the instrument is known to show some difficulties at very high wind speeds (Frasson et al. 2011), which, however, were not present at our sites.

The first LPM disdrometer (LPM-1) was located at Zhurucay station from 2012 to 2014. After, it was moved...
to Balzay station while a second Thies disdrometer (LPM-2) was acquired. The LPM-2 was calibrated next to LPM-1 for a one-month period and installed after at the Virgen site. Data used in this study were collected at 5-min sampling resolution over different time periods. Because of the data integration time, a result similar to the sequential intensity filtering technique (SIFT) proposed by Lee and Zawadzki (2003) is obtained to overcome the drop sorting effect on the disdrometer database. Data from March 2015 to August 2016 were available at Balzay, from April 2015 to August 2016 at Virgen, and from November 2012 to August 2014 at Zhurucay. Altogether, a total of 8142, 25804, and 18019 sampling observations were obtained at Balzay, Virgen, and Zhurucay stations, respectively. A summary of available data is provided in Table 1.

c. Methods

We implemented our methodology systematically as follows (Fig. 2). First, we derived the rain rate and reflectivity from DSD observations. Then, we split the available disdrometer data from each study site into rainfall events by defining properties of a time series subset. A detailed explanation of these properties is described below [section 2c(3)]. However, because of each study area’s rainfall variability, we selected a rainfall categorization based on the mean volume diameter ($D_m$). After we identified the events and classified the 5-min rainfall observations, we calculated the $A$ and $b$ parameters of the power law $Z = AR^b$ for each categorized observation subset. Later, we evaluated predicted rain rates $R$ from reflectivity $Z$ values and type-specific $A$ and $b$ parameter estimates. Finally, we validated the rain-type-based $Z$–$R$ relations through statistical measures of goodness of fit between observed and predicted rain rates. The complete data processing chain is shown in Fig. 2 and a more detailed explanation is provided in the subsequent sections.

1) RAINDROP SIZE DISTRIBUTION AND RAINFALL INTEGRAL PARAMETERS

Integral parameters such as rain rate $R$ (mm h$^{-1}$) and reflectivity $Z$ [mm$^6$ mm$^{-3}$; or dBZ = $10 \log_{10}(Z)$] are defined as

$$R = \frac{6\pi}{10^5} \int D^3 \nu(D) N(D) dD \quad \text{and} \quad (1)$$

$$Z = \int D^6 N(D) dD, \quad (2)$$

where $D$ is the raindrop diameter (mm), $N(D)dD$ is the number of drops per unit volume (mm$^{-3}$) with diameters between $D$ and $D + dD$, and $\nu(D)$ is the terminal raindrop fall velocity (m s$^{-1}$). The number concentration of raindrops per unit volume per unit size $N(D)$ was calculated from the Thies disdrometer counts according to

$$N(D_i) = \sum_{j=1}^{20} \frac{n_{ij}}{AtV_jA\Delta D_i}. \quad (3)$$

Here, $N(D_i)$ is the drop size distribution in $i$th size class (mm$^{-3}$ mm$^{-1}$), $D_i$ is the midsize diameter of the $i$th class (mm), $A$ is the cross-sectional area of the sensor (m$^2$), $t$ is the measuring time (s), $n_{ij}$ is the number of drops within the $i$th size and $j$th velocity class, and $V_j$ is the fall speed of the $j$th velocity class (m s$^{-1}$) in agreement with Beard (1976). Thus the finite difference equivalences for $R$ [Eq. (1)] and $Z$ [Eq. (2)] were derived from the distribution of all particles over class binning as follows:
\[
R = \frac{6\pi}{10} \sum_{i=1}^{20} \sum_{j=1}^{20} \frac{n_{ij} D^3}{A_{ij}}, \quad \text{and}
\]
\[
Z = \sum_{i=1}^{20} \sum_{j=1}^{20} \frac{n_{ij} D^6}{A_{ij}}.
\]

Usually for long wavelengths \(\lambda\) (e.g., C-band radars) the Rayleigh approximation is used [as in Eq. (2)], where the effective backscattering cross section of the raindrop is roughly proportional to the sixth power of its diameter. X-band-related studies in literature have used both the Rayleigh (Lo Conti et al. 2015) and Mie regimes (Delrieu et al. 1997; van de Beek et al. 2009). Nevertheless, several investigations (e.g., Wexler and Atlas 1963; Löffler-Mang et al. 1999; Tokay et al. 2001; Maki et al. 2005; Dolan and Rutledge 2009) documented the impact on the derivation of the \(Z-R\) relationship parameters due to the scattering regime used and highlight the importance of considering the Mie scattering in short wavelengths such as X band. Thus, for the purposes of this study, reflectivity was calculated in both Rayleigh and Mie scattering regimes in order to learn about the effect of the drop size spectra on the subsequent \(Z-R\) relationship derivation. For this, the effective backscattering cross section was calculated using the implementation in MATLAB by Mätzler (2002). A detailed explanation of the Mie theory is well described in van de Hulst (1957) and Horvath (2009).

Observed DSDs are commonly represented by theoretical distribution approximations. Two models for the drop size distribution were used to describe and compare the observed DSDs at the study sites: (i) The gamma drop size distribution (Ulbrich 1983) in the form of \(N(D) = N_0 D^\lambda \exp(-\Lambda D)\) where \(D\) is the raindrop diameter and \(N_0\), \(\mu\), and \(\Lambda\) are the intercept, shape, and slope parameters, respectively. (ii) The Marshall–Palmer (Marshall and Palmer 1948) distribution, defined as \(N(D) = N_0 \exp(-\Lambda D)\), where \(N_0 = 8000 \text{ m}^{-2} \text{ mm}^{-1}\) and \(\Lambda = 41R^{-0.21}\). For the former, the truncated-moment method (Ulbrich and Atlas 1998) was used for the estimation of the three parameters.

2) QUALITY CONTROL

In a recent study, Padrón et al. (2015) compared the LPM-1 with a corresponding adjacent 0.1-mm-resolution rain gauge at different rain-rate intensities and velocities. Because of the sensitivity of the disdrometer to small drops, differences occurred mainly during low-intensity events, where the rain gauge produced underestimations of approximately 15%. Therefore, we base our data quality on the detailed study of Padrón et al. (2015) regarding LPM-1 operation, which was used afterward to perform the LPM-2 intercalibration.

In addition, we applied the quality control procedure as described in Friedrich et al. (2013) in order to remove measurement inaccuracies related to (i) misclassification of particles, (ii) margin fallers, and (iii) splashing effects. For a detailed explanation of this procedure, the reader may refer to Friedrich et al. (2013). The particle classification scheme (Fig. B1 in Friedrich et al. 2013) was adapted by considering changes in fall velocity on high-altitude pressure conditions (Beard 1976). This quality control process confirmed the very low presence of solid precipitation (i.e., hail, snow, graupel). In our dataset, only four observations showed solid precipitation at the lowest site (Balzay). Neither Virgen nor Zhurucay presented solid precipitation observations.

Besides, we compared time series of 30-min rain totals of a collocated Texas Electronics, Inc., TE-525 tipping-bucket rain gauge (0.1-mm resolution) at each location with its corresponding disdrometer data for the study period. We found correlation coefficients of 0.93, 0.96, and 0.94 for Balzay, Virgen, and Zhurucay, respectively. As expected, systematic errors were found. Relative bias was 5% (Zhurucay), 18% (Virgen), and −6% (Balzay). It can be largely explained by the climatology at all sites and their rainfall regimes. The differences in relative bias are related to the differences in measurements of very small (high) rain rates Virgen (Balzay) and a mixture of both at Zhurucay. It should be stressed that we only retained observations with more than 100 droplets and a rain rate higher than 0.1 mm h\(^{-1}\) for further analyses to disregard noisy observations. Furthermore, regular maintenance was performed on both LPMs to avoid contamination of the light beam.

3) RAINFALL EVENT DETECTION

To obtain representative data that accounts for the rainfall regime of each study site while avoiding isolated observations, we first identified rainfall events that occurred during the observation periods. In this context, Dunkerley (2008) provided a well-documented review of rain event properties found in the literature. He concluded that they differ widely from one study to another (e.g., event durations from 3 min to 24 h were reported). Unfortunately, no standard methodology for rain event selection exists. Nevertheless, the author strongly recommends a more detailed reporting of guidelines used in investigations related to rain events. According to Dunkerley (2008), a rain event is identified by a continuous time interval of rain during which there are no rainless gaps with a duration exceeding the minimum interevent time. Usually this definition is augmented with some additional criteria that any rain event must meet (e.g., minimum total accumulation and minimum duration). In this study, we used the following.
three criteria to extract events from 5-min rain-rate time series. (i) Minimum duration: keeping in mind that most convective events are short duration, a rain event is characterized by at least two continuous single rainy records. (ii) Minimum interevent time: we performed several experiments modifying this time criterion while, in general, the results remained unchanged. Therefore, the interevent time was selected according to previous rainfall disdrometer-related studies (Larsen and Teves 2015; Padrón et al. 2015; Tokay et al. 2003) that define half an hour—30-min interevent—as the minimum time to separate one event from another. (iii) Minimum rain-rate accumulation: this value was fixed to 5 mm, providing a good trade-off by reducing the bias of short and extremely light rain events.

4) RAINFALL CLASS SELECTION

Observations of selected rainfall events were classified according to an adequate tailor-made categorization that accounts for rainfall variability at each site. Some studies have used the most common classification schemes based on nonoverlapping rain-rate (Llasat 2001; Varikoden et al. 2011; Sen Jaiswal et al. 2009) or reflectivity thresholds. However, the major drawback of these methods is that they do not consider DSD variability of rain (i.e., different DSDs can arise at similar rain rates). This is the case for the drizzle rain type in the Andean highlands, whose occurrence and importance is noted by Padrón et al. (2015), Rollenbeck and Bendix (2011), and Muñoz et al. (2016). Other studies use a DSD parameterization (i.e., assuming a gamma or an exponential distribution) to obtain a rainfall separation criterion, as in Tokay and Short (1996). Thus, to objectively compare and account for local rainfall characteristics in the study areas, we selected a rainfall categorization using the mean volume diameter \( D_m(\text{mm}) \):

\[
D_m = \frac{\int_{0}^{\infty} N(D)D^4 \, dD}{\int_{0}^{\infty} N(D)D^3 \, dD},
\]

defined by Testud et al. (2001) from a normalized distribution, where \( N(D) \) is the drop size distribution and \( D \) is the particle diameter. For each spectrum, the third and fourth moments are calculated, and \( D_m \) is subsequently determined. Finally, we defined four rainfall categories according to \( D_m \), which may explain rainfall structure differences: 1) \( 0.1 < D_m (\text{mm}) \leq 0.5 \); 2) \( 0.5 < D_m (\text{mm}) \leq 1.0 \); 3) \( 1.0 < D_m (\text{mm}) \leq 2.0 \); 4) \( D_m (\text{mm}) > 2.0 \). This tailor-made classification was based on the findings of Uijlenhoet et al. (2003) in northern Mississippi that showed that mean raindrop sizes > 1.0 mm characterized convective rainfall, while transitional and stratiform rainfall was mainly associated by mean raindrop sizes of 0.6–0.9 mm. The classification was also supported by the observation of the local \( D_m \) frequencies at all study sites as shown in section 3a.

5) Z–R RELATION CALCULATION

After classifying individual rain observations, we calculated the Z–R relation \((A \text{ and } b \text{ coefficients})\) from the classified observation rain rates and corresponding Z values provided by the instruments. For this, we used the well-known Z–R power-law relationship defined by Marshall and Palmer (1948) for rain-rate estimation based on radar reflectivity records:

\[
Z = AR^b, \tag{7}
\]

where \( Z \) (mm\(^6\) mm\(^{-3}\)) is the reflectivity value, \( R \) (mm h\(^{-1}\)) is rain rate, and \( A \) and \( b \) are empirical parameters and are referred to as coefficient and exponent, respectively. The commonly adopted method of a linear regression of \( \log(R) \) on \( \log(Z) \) has been demonstrated to lead to suboptimal parameters (Alfieri et al. 2010). Thus, we performed a nonlinear (power law) regression by fitting least squares with \( Z \) as the independent variable and compared both regression approaches in similar fashion with van de Beek et al. (2016). Values of \( A \) and \( b \) were calculated by applying Eq. (7) to all rainy records \((Z, R)\) as well as to the subset of each rainfall category independently.

d. Validation of rain-type Z–R relations

The validation of rain-type Z–R relations was performed over the 5-min observations corresponding to 20% of rainfall events. These observations were not used for the Z–R parameter derivation process and covered the entire Z reflectivity range. The performance of our type-specific Z–R relations for each study site was assessed using statistical measures for goodness of fit between observed and predicted rain rates. Thus, we calculated the predicted rain rates \((R)\) through the Z–R relation using the obtained \( A \) and \( b \) parameters per rainfall category and observed reflectivity \((Z)\) records of the 5-min sampling time series as the independent variable. We applied the formula to \( Z \) according to the respective class. Statistical measures derived were the coefficient of determination \( r^2 \), root-mean-square error (RMSE; mm h\(^{-1}\)), and bias (mm h\(^{-1}\)).

3. Results

a. Event detection and 5-min rainfall categorization

We found 488, 594, and 887 rainfall events at Balzay, Virgen, and Zhurucay, respectively, that met the rainfall
event criteria previously defined. They correspond to a total of 5451, 17096, and 13582 sampling observations. Figure 3 illustrates the frequency distributions of rainfall event duration, rain rate, and $D_m$ for each study site. While the event duration has a minimum of 10 min at all sites, it increases to 745 min (Balzay), 2480 min (Virgen), and 750 min (Zhurucay). We also found that mean event duration varies between stations, yielding 180 min at Balzay, 470 min at Virgen, and 225 min at Zhurucay (Fig. 3), altogether pointing to longer-duration events at higher altitudes. Approximately half the events are shorter than 125, 335, and 170 min.

From those rainfall observations, only 129 (Balzay), 34 (Virgen), and 115 (Zhurucay) rainfall observations were categorized as $D_m \geq 2.0$. Thus, we disregarded this rainfall category from our study because of lack of representation. Relative frequencies of rainfall categories are illustrated in Fig. 4. Observations categorized with the smallest $D_m$ are more frequent at the higher sites, Zhurucay and Virgen, than at the lowest site, Balzay, which may be related to more common drizzle rainfall at higher altitudes. The next rain category $[0.5 < D_m (\text{mm}) \leq 1.0]$ seems to occur at slightly different relative frequencies at all sites. In contrast, rain observations characterized by bigger droplets $[1.0 < D_m (\text{mm}) \leq 2.0]$, most related to moderate–heavy rain, are high in number at Balzay and very uncommon at Virgen.

Figure 5 illustrates the distribution of rainfall categories at the study sites. The upper limit of rain rates produced by the lowest range of $D_m [0.1 < D_m (\text{mm}) \leq 0.5]$ is relatively low, with values of approximately $1.1 \text{ mm h}^{-1}$ at Virgen and $3.2 \text{ mm h}^{-1}$ at Zhurucay. On the other hand, few low rain rates are characterized by the biggest $D_m [1.0 < D_m (\text{mm}) \leq 2.0]$ at all locations in comparison with the preceding classes. At first glance, Fig. 5 appears to hinder real discrimination between classes. In Fig. 6, however, it is shown to what extent the rain categories are overlapping: rain rate and reflectivity systematically increase according to rain type. It should be noticed that the distributions, although roughly similar at a specific rain class, correspond to a different number of observations at each site. Median values of both rain rate and reflectivity are lower for the rain types $0.1 < D_m (\text{mm}) \leq 0.5$ and $0.5 < D_m (\text{mm}) \leq 1.0$ at Virgen in comparison with the other sites.

b. Drop size distribution

The drop size distribution of three different rain rates (1, 10, and $15 \text{ mm h}^{-1}$) is shown in Fig. 7 for all sites. At the highest rain rate, the observed DSD at the lowest station (Balzay) begins to exceed that of the high-altitude stations, revealing rain-rate–droplet number combinations not observed at higher altitudes. On the other hand, at the highest station (Zhurucay), droplets bigger than 4 mm are registered, which are not present at the other sites. This points to extended droplet growth at lower altitudes leading to higher rain rates at the expense of smaller droplets. The theoretical model of Marshall–Palmer better fits the data at the highest station, Zhurucay, while Virgen consistently presents a lower number of droplets per diameter. The gamma

![Fig. 3. Distributions of rainfall event duration, rain rate, and mean volume diameter $D_m$ for all disdrometer study sites.](image1)

![Fig. 4. Relative frequency of $D_m$ in 5-min-interval observations.](image2)
model (Ulbrich 1983) seems to be suitable for the lower stations, Virgen and Balzay. Figure 8 illustrates the average raindrop size spectra at all study sites. The more isolated subpáramo station, Virgen, shows a lower droplet concentration in comparison with the other sites (Fig. 8a). When data are divided according to the \(D_m\) classification, it can be observed that the drop size spectrum is more similar within classes (Fig. 8b) in comparison with the rain-rate discrimination (Fig. 7), particularly at low droplet diameters.

c. Estimation of Z–R parameters

Once we identified the rainfall events, we calculated parameters \(A\) and \(b\) of the Z–R relation [Eq. (7)] for all observations at each site. For illustration purposes, Fig. 9 shows the differences that arise from applying a linear \(\log(R)\) on \(\log(Z)\) and nonlinear (power law) regression on the calculation of Z–R parameters as well as the calculation of reflectivity (Z) by using both scattering regimes, Mie and Rayleigh. It can be seen that the nonlinear approach outperforms its counterpart and also departs from the 1-by-1 line at the lowest rain rates. It seems to be an effect of the predominance of least squares fitting on higher rain rates. The scattering regime calculation leads to slight differences in terms of goodness-of-fit statistics. Nevertheless, as only the big droplets are affected by Mie scattering in X band, as expected, the greatest difference arises from the station with the most frequent high \(D_m\) (Balzay), which deviates mostly between Mie and Rayleigh regimes. The opposite occurs in the subpáramo station (Virgen) where the lower \(D_m\) mostly occur. Thus the Mie scattering regime is used in this article hereinafter. Parameters on these site-specific Z–R relations were similar in terms of exponent \(b\) values (2.05 < \(b\) < 2.25); however, coefficient \(A\) at the highest station Zhurucay (76) was lower than at the other sites Balzay (103) and Virgen (104).

Afterward, we calculated parameters \(A\) and \(b\) of the Z–R relation [Eq. (7)] for each subset of 5-min rainfall observations that belongs to the corresponding rainfall category (Fig. 10). Generally, the coefficient and exponent increase through the rain categories. The Z–R relation becomes more nonlinear at higher \(D_m\) values, namely, moderate–heavy rainfall. Balzay shows the higher (lower) coefficients (exponents) for each rainfall category. Hereafter, rainfall types are referred indistinctly by terms of light and heavy rain, which are directly associated with the \(D_m\) range: lower \(D_m\) values are related to light rain and higher \(D_m\) values are related to heavy rain.

d. Evaluation of Z–R parameters

For all observed reflectivity values, we calculated the rain rate based on the Z–R relations found per rainfall classification by means of the nonlinear (power law) regression with Z calculated in the Mie scattering regime. The goodness of fit between observed and predicted rain rates is illustrated in Fig. 10. In addition to the coefficient of determination \(r^2\), statistical values of RMSE and bias are shown. We found that the lowest station (Balzay) showed the highest \(r^2\) in all rain categories. Rain rates characterized by the highest range of \(D_m\) \([1.0 < D_m \text{ (mm)} \leq 2.0]\) are slightly overestimated (i.e., bias = 0.187) at Balzay and underestimated (i.e., bias = −0.198) at Virgen. Finally, we found a stable high \(r^2 \sim 0.8–0.9\) for most rain categories for all sites; however, the lower \(r^2 = 0.728\) was found at Zhurucay for the lowest \(D_m\) range \([1.0 < D_m \text{ (mm)} \leq 2.0]\).

4. Analyses and discussion

In light of the results, there is clearly a different contribution of rainfall events with \(0.1 < D_m \text{ (mm)} \leq 0.5\)
and $1.0 < D_m (mm) \leq 2.0$ through the study sites, which is directly related to their altitudes and type of cloud formation. Our findings indicate the presence of significant amounts of drizzle, characterized by small droplets, particularly in the high and subpáramo (stations Zhurucay and Virgen). It is worthwhile to mention that had all rain-rate observations been considered, drizzle occurrence would still have been greater than rain. However, after the threshold used as quality control for rain rate (i.e., $R > 0.1 \text{ mm h}^{-1}$), the number of lower $D_m$ observations decreased remarkably. Low $A$ coefficients in rain type $0.1 < D_m (mm) \leq 0.5$ are in good agreement with orographic rain $Z$–$R$ relations ($Z = 16.6 R^{1.58}$ and $Z = 31 R^{1.71}$) found at high altitudes in Hawaii (Blanchard 1953).

A noticeable difference in the rainfall regime with largest $D_m$ is observed between the three sites. Heavy rainfall events, here associated with $1.0 < D_m (mm) \leq 2.0$, mostly occur at the lower elevation (Balzay), while, in contrast, higher-elevation sites have a lower relative frequency of heavy rain. Although Virgen and Zhurucay have similar altitudes, they differ mainly in the $A$ coefficient of the corresponding $Z$–$R$ relation. In particular, Zhurucay is associated with a higher reflectivity at a given rain rate because of a higher droplet concentration. Both sites are located in different areas of the western Andes cordillera, with Virgen located on the eastern draining slopes (i.e., toward the Atlantic Ocean) and Zhurucay on the western draining slopes (i.e., toward the Pacific Ocean). Zhurucay is also closer to the lower areas of the adjacent Yunguilla Valley. Advection of moist air masses from the coastal Pacific lowlands through this valley, local orographic uplift, and stronger thermal upslope breezes might affect rain formation processes in this area.

As expected, DSDs according to rain rates (Fig. 7) are diverse at all locations and clearly divergent at a particular rain rate. Differences in drop size and number concentration at all sites (Fig. 8a) are more evident in the $D_m$ categorization (Fig. 8b). From the microphysical interpretation of $Z$–$R$ relations (Fig. 3 in

![Fig. 6. Distribution of rain-rate and reflectivity values associated with each rainfall category according to $D_m$ binning.](image-url)
Steiner et al. (2004), there is no evidence of number-controlled (i.e., $D_m$ constant) or size-controlled (i.e., drop number density constant) rain types. Thus, it is suggested that rainfall types are controlled by variations of mean drop size and number concentration. Regarding the overall and rain-type $Z$–$R$ relations, the latter clearly outperform the adjustment at all sites (e.g., at Balzay, $r^2$ increases from 0.729 to 0.867, RMSE decreases from 1.609 to 0.738 mm h$^{-1}$, and bias decreases from 0.286 to 0.027 mm h$^{-1}$). Thus, variations of coefficients and exponents of $Z$–$R$ relations through the different study sites and rain categories suggest that not only the altitude but also the specific spatial exposition regarding cloud and rain formation seems to play a key role in determining local $Z$–$R$ relationships.

For comparison purposes, Table 2 shows a summary of similar $Z$–$R$ relation studies performed at equatorial and high-altitude locations. It should be stressed that these investigations are diverse in the use of methods to fit $Z$–$R$ relations to DSD data, which are not described in all of the studies. Our purpose henceforth is to contextualize our results with those of remarkable proximity, similar geographical location, and/or influence of complex terrain. For instance, the study conducted by Germann et al. (2006) in an alpine mountainous region only provides a single $Z = 316R^{1.5}$ relation. We obtained lower coefficients and higher exponents for all rainfall categories than the $Z$–$R$ relation reported there. A single/unique $Z$–$R$ relation would not consider rainfall variability in the Andean cordillera.

The exponents found in this study are roughly similar within each rain category at all sites (1.27–1.44, 1.34–1.57, and 1.57–1.68). The lowest values per rain-type class were obtained at Balzay. Uijlenhoet et al. (2003) also found that exponent $b$ varied slightly through the rain classes. However, coefficients increase in a different order in our study (i.e., from smaller to larger $D_m$) and they are also of lower magnitude. This is
most likely due to the wide range of rain rates obtained in our results for each rain category, in contrast to other studies that use rain-rate thresholds as well as intra-event mean drop size variability (Rao et al. 2001; Uijlenhoet et al. 2003). Nevertheless, using only the $D_m$ parameter as a rain classification criterion, evaluation of independent data shows good measurements of goodness of fit ($r^2 = 0.8$, RMSE = 1.7 mm h$^{-1}$, and bias = 0.2 mm h$^{-1}$) relative to those reported by Ramli and Tahir (2011) in a low-altitude tropical climate. According to our validation, RMSE values vary from 0.038 to 1.656, whereas these values increase from 3.65 to 16.97 in Ramli’s study. A similar behavior was presented in the bias statistic. These results indicate that our study is reliable in terms of accuracy of predicted rain rate based on rain-type $Z$–$R$ relations. Furthermore, indications of an upper limit of 2.0 mm for warm tropical rain, as suggested by Atlas et al. (2000), are in agreement with this study.

There are few studies that have reported similar low magnitudes of $A$ and $b$ (Atlas et al. 2000; Rosenfeld et al. 1993) in lowland tropical equatorial regions (Table 2). Those from American equatorial areas seem to be more consistent with our findings. The lower coefficient values ($A = 196$, $b = 230$) reported by Bamba et al. (2014) correspond to the geographically nearest station, located in French Guyana, with respect to our study sites. These values are in better agreement with those calculated in this study for the highest $D_m$ class ($A = 205$, $b = 1.68$), mainly related to transitional rain types with lower coefficients. The exponents reported by Bamba are lower ($1.34$ and $1.35$) than those calculated in this study ($1.57$ and $1.68$), which may compensate for differences in the coefficients between both investigations. Interestingly, Tenório et al. (2012) found lower exponent values ($1.27$ and $1.28$) in Northeast Brazil than those usually reported for other equatorial zones. In contrast, our

FIG. 9. The $Z$–$R$ parameters derivation by considering Mie and Rayleigh scattering regime calculation for $Z$, as well as two regression methods: linear fitting on logarithmic transformation of $R$ and $Z$ ($\log R$–$\log Z$) and nonlinear (power law) fitting $[R = (Z/A)^{1/b}]$. No classification has been performed. Correlation of observed 5-min-time-step rain-rate measurements and predicted rain-rate values on the independent validation dataset is illustrated. The bisector line is shown in gray for all stations.
exponent $b$ values are higher in general ($b \sim 1.6$) but associated with lower $A$ coefficient values. We presume that higher values found in the literature (Table 2) mainly characterize the monsoon nature, associated with bigger droplets and higher rain-rate intensities, of the West African (Russell et al. 2010; Bamba et al. 2014) and Asia–Australian (Rao et al. 2001; Kumar et al. 2011) rainfall that would differ from the South American equatorial zone.

5. Conclusions

Specific rain-type $Z$–$R$ relations were derived for different mountain altitudes and locations, from a lower inter-Andean valley to higher páramo sites, representing the rainfall condition in the Andean cordillera, Ecuador. These $Z$–$R$ relations were obtained based on the climatic conditions but also included the precipitation variability at each location using a mean volume diameter ($D_m$) classification analysis. This is the first study of this kind, performed at tropical high altitudes and simultaneously considering rainfall variability. The following conclusions were drawn from the analysis:

1) The $D_m$ classification approach used in this study points out to a better adjustment of rain-type $Z$–$R$ relationships in comparison with a single $Z$–$R$ relationship. The parameters of the rain-type $Z$–$R$ relationships increase according to $D_m$ categorization. Our results confirm that at a given rain rate, a light rain category [$0.1 < D_m (\text{mm}) \leq 0.5$] tends to be associated with smaller $D_m$ and larger raindrop concentrations, whereas heavier rainfall categories tend to exhibit larger $D_m$ and smaller droplet concentrations.

2) The influence of the sites’ spatial exposition on the derivation of rain-type $Z$–$R$ relationship is confirmed, where both location and elevation play a key role in characterizing the $Z$–$R$ relationship. Thus, the spatial variability of rainfall in tropical high mountains should be analyzed before only using a single $Z$–$R$ relation.

3) In the Andes of Ecuador, the $Z$–$R$ relations mainly show a transitional rainfall nature, which differs from most studies in the equatorial zone. This result was particularly found at the lowest station (Balzay) and the station oriented toward the Pacific coastline.
(Zhurucay) in the highest radius class. Here, the presence of moist air advection, higher thermal breeze, and better droplet growth conditions seems to produce more convective activity than in the remote subpáramo station. The results reveal a clear difference in rainfall types at different terrain exposures, with drizzle being most dominant at the most isolated station, Virgen.

4) Tropical high-elevation rainfall structure in terms of DSD, under the influence of steep and complex terrain in the high Andes, has been documented for the very first time. Comparison with theoretical distribution models has shown that the exponential Marshall–Palmer model better fits the data at the highest station, Zhurucay, while the gamma distribution is more suitable for the lower stations, Virgen and Balzay.

5) To our knowledge, this study constitutes the first contribution of tropical high mountain radar reflectivity determination by optical disdrometers and related rain-type Z–R relationships.

The Z–R relations found in this study will be used as starting point for rainfall mapping from raw reflectivity data of CAXX radar in southern Ecuador (refer to Bendix et al. 2017) in areas where the influence attenuation can be overcome. Despite the attenuation issue, the advantages of the higher spatial resolution provided by X-band systems allow for more detailed studies of urban hydrology in the near range as planned for the city of Cuenca (30-km range for CAXX) to support water management. For this purpose, the disdrometers remain operational and will be used in the future. Consequently, further work will focus on exploring techniques to classify each pixel with the proper Dm category by using as inputs to the model the reflectivity values, the distance from the radar, and the attenuation along the current bin.

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REFERENCES


