Determination of the Radiative Properties of Stratiform Clouds from a Nadir-Looking 95-GHz Radar

A. Guyot and J. Testud

CETP/CNRS/UVSQ, Centre d’Étude Environnements Terrestre et Planétaires, Velizy, France

T. P. Ackerman

Department of Meteorology, The Pennsylvania State University, University Park, Pennsylvania

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ABSTRACT

Several space agencies are presently considering missions with active instruments (radar, lidar), which are able to document cloud stratification and cloud microphysical properties on the global scale. The objective of this paper is to develop an algorithm to derive as much information as possible from a single-frequency, nadir-looking cloud radar operating from an airborne or spaceborne platform. It is impossible to derive all parameters of interest in the radiative budget of a cloud from only the radar reflectivity profile, unless some a priori knowledge of cloud processes is introduced in the formulation of the algorithm itself. The a priori knowledge considered here (total concentration of particles invariant with altitude, adiabatic liquid water content) does not apply to all cloud types but only to warm stratiform clouds where entrainment is weak.

The algorithm concept and inversion procedure, including a stable scheme for correcting the radar reflectivity for attenuation, are first described. A test of the algorithm is then performed using numerical simulations in order to investigate the sensitivity of the retrieval to measurement noise, degradation of the range resolution, shape of the cloud droplet distribution, and presence of entrainment. In the realistic conditions of an airborne experiment, the retrieval of cloud base \( h_b \), total number concentration of particles \( N_T \), profiles of the liquid water content, and effective radius \( r_e \) can be performed with good accuracy (provided the entrainment coefficient is below 1 km\(^{-1}\)). With the sampling characteristics of a spaceborne radar, retrievals of the cloud base and liquid water content remain reasonably accurate, but the estimates of \( N_T \) and \( r_e \) are degraded to a level where they become meaningless.

A test of the algorithm is performed using a dataset from the zenith-pointing ground-based 94-GHz radar of The Pennsylvania State University, obtained during the Continental Stratus Experiment. The algorithm is found to be successful in 43% of cases. An attempt of evaluation of the retrieval is made by comparison with ceilometer data. Most failure cases are probably due to the presence of drizzle.

1. Introduction

The earth’s climate is determined to a large extent by radiation fluxes. Cloud cover greatly influences the earth’s radiative budget, although its role in influencing climate change remains to be quantified (Senior and Mitchell 1993). In the past years, passive observations of clouds from space helped to improve cloud parameterization in large-scale atmospheric models, but clouds are still crudely represented, especially regarding their vertical structure. Indeed, although passive observations from space help to monitor the cloud top reasonably well, they do not provide much information about cloud base and cloud layering, which are essential to compute the complete radiative budget. For this reason, several space agencies are presently considering missions with active instruments (radar, lidar), which are able to document cloud stratification and cloud microphysical properties on the global scale. The European Space Agency, in particular, is considering the Earth Radiation Mission (ERM) as one of the priority candidate missions for its Earth Explorer Program. ERM anticipates deploying a cloud radar and a lidar on the same space vehicle.

In this context, the objective of this paper is to develop an algorithm to derive as much information as possible from the single-frequency, nadir-looking cloud radar of ERM. The parameters of interest in the radiative budget are cloud top, cloud base, and the profile of cloud water content and effective cloud droplet radius. It is impossible to derive all these parameters from only the radar reflectivity profile delivered by the radar unless some a priori knowledge of cloud processes is intro-
duced in the formulation of the algorithm itself. This is the approach followed in this paper.

The a priori knowledge considered here does not apply to all cloud types but only to warm stratiform clouds where entrainment is weak. The present algorithm does not apply for example to ice clouds as cirrus where sedimentation plays an important role, or to cumulus where entrainment is stronger.

In this paper we first describe the algorithm concept and inversion procedure; then we perform some robustness tests of the algorithm from numerical simulations. Sampling strategies corresponding to spaceborne or airborne operations are considered. Then the method is applied to real data from a vertically pointing ground-based radar.

2. Algorithm concept

A nadir-looking, single-frequency cloud radar only measures the equivalent radar reflectivity profile \( Z(h) \), where \( Z \) exceeds the detection threshold. In fact when the along-path attenuation is significant, the radar does not measure the “true” reflectivity \( Z \) but an attenuated reflectivity \( Z_a \), related to \( Z \) through

\[
10 \log[Z_a(h)] = 10 \log[Z(h)] - 2 \int_h^\infty K(s) \, ds, \tag{1}
\]

where \( Z \) and \( Z_a \) are expressed in \( \text{mm}^6 \, \text{m}^{-3} \) and \( K \) is the specific attenuation (one way) in decibels per kilometer.

In this paper we consider warm clouds, that is, clouds composed of only liquid water droplets. For such clouds, even with a radar frequency of 95 GHz, the Rayleigh approximation is valid; that is \( Z \) is the sixth moment of the particle size distribution, while \( K \) is proportional to its third moment.

The interesting parameters to be derived in order to characterize cloud radiative properties are the cloud water content LWC, and the effective droplet radius \( r_e \) (Hansen and Travis 1974):

\[
LWC = \frac{\pi}{6} \rho_w \int N(D)D^5 \, dD \quad [\text{g m}^{-3}] \quad \text{and} \quad (2)
\]

\[
r_e = \frac{\int N(D)D^3 \, dD}{2 \int N(D)D^2 \, dD} \times 10^6 \quad [\mu \text{m}], \tag{3}
\]

where \( D \) is the droplet diameter (in m), \( N(D) \) is the droplet size distribution (in \( \text{m}^{-1} \)), and \( \rho_w \) is the density of liquid water (1000 \( \text{g m}^{-3} \)).

Without precise knowledge of \( N(D) \), it is impossible to derive the two parameters LWC and \( r_e \) from only the measurement of \( Z(h) \), and a fortiori from \( Z_a(h) \), when significant along-path attenuation occurs. Moreover, while we expect the cloud top to be detected by the nadir-looking radar, this may not be the case for cloud base because (i) the droplets are small at cloud base, bringing the radar reflectivity below the detection threshold, and (ii) the along-path attenuation may also contribute to the deterioration of the signal-to-noise ratio (SNR).

The subsequent approach takes into account in the retrieval algorithm some a priori knowledge of the cloud characteristics based on hypotheses of the cloud processes. These assumptions cannot be the same for all cloud types. In this paper we concentrate on warm clouds, and more specifically on warm stratiform clouds in which entrainment is weak or negligible.

a. Assumptions

In principle, the algorithm considers a simplified model of stratocumulus in which the two basic assumptions are (i) lateral entrainment is negligible and (ii) all the condensation nuclei are activated at cloud base. These assumptions were made by Fox and Illingworth (1997) in their study of warm stratiform clouds. The consequence of these assumptions is that the total droplet concentration \( N_r \) is conserved between cloud base and cloud top (more precisely, \( N_r \) decreases slightly with altitude due to the decrease in air density), and that the cloud water content LWC follows the wet-adiabatic law.

Another assumption is that the cloud droplet size distribution is well represented by a gamma distribution as

\[
N(D) = N_o D^\mu \exp(-\Lambda D) \quad [\text{m}^{-4}], \tag{4}
\]

where \( D \) is the particle diameter; and \( N_o, \mu, \) and \( \Lambda \) are the parameters of the distribution. For stratocumulus, we use the value \( \mu = 2 \) given by Shettle (1990). Note that (4) implies that drizzle drops have negligible contribution to the radar reflectivity. The validity of this assumption will be discussed later.

b. Mathematical formulation

The \( n \)th moment of the distribution is defined as \( M_n = \int N(D)D^n \, dD \), so it may be shown that

\[
N_r = M_6
\]

\[
= N_o \Gamma(\mu + 1)10^{-6}/\Lambda^{\mu+1} \quad [\text{cm}^{-3}] \tag{5}
\]

LWC = \( (\pi \rho_w/6)M_3 \)

\[
= (\pi \rho_w/6)N_o \Gamma(\mu + 4)/\Lambda^{\mu+4} \quad [\text{g m}^{-3}] \tag{6}
\]

\[
Z = M_6
\]

\[
= N_o \Gamma(\mu + 7)10^{19}/\Lambda^{\mu+7} \quad [\text{mm}^6 \, \text{m}^{-3}] \tag{7}
\]

These three parameters will always be expressed in these units in the paper.

By combining (5), (6), and (7) \( Z \) may be expressed as a function of LWC and \( N_r \) (assuming that \( \mu \) is an integer):
where $r$ is the cloud base:

where $\{0 \leq r \leq 1\}$ denotes all the radar range gates in which $Z_s$ is detected, that is, those values that minimize the following cost function.

\[ S = \frac{1}{N} \sum_{i=1}^{N} \left[ 10 \log[Z_{\text{measured}}(h_i)] - 10 \log[Z_{\text{theoretical}}(h_i; h_b, T_b, N_f)] \right]^2 \] (12)

\[ [\text{dB} \ Z^2], \]

where $(i)$ denotes all the radar range gates in which $Z$ data are available.

Once these parameters are determined, the LWC can be calculated inverting (8) as

\[ LWC(h) = \frac{\pi \rho_e}{6} \left( \frac{(\mu + 3)(\mu + 2)(\mu + 1)}{(\mu + 6)(\mu + 5)(\mu + 4)} Z(h) N_f^2 \right)^{1/2} \times 10^{-6} \] (13)

\[ [\text{g} \ m^{-3}]. \]

Similarly the effective radius $r_e$ profile may be calculated as

\[ r_e(h) = \frac{(\mu + 3)}{2\Lambda} \] (14)

\[ = \frac{(\mu + 3)}{2} \left[ Z(h) \left( \Gamma(\mu + 1) \right)^{1/6} N_f \left( \Gamma(\mu + 7) \right)^{1/2} \right] \times 10^{2} \] [\mu m].

\[ c. \text{ Processing} \]

A quasi-Newton algorithm is used to find the minimum of this multiparameter function. This routine is given domain limits for each searched variable, so that the retrieved parameters are physically realistic and also requires input values to start its search. In the domain fixed by the limits of the parameters, there is only one minimum value of the cost function.

We observed that this algorithm encountered considerable difficulty in determining correctly the three parameters, so we decided to introduce a relationship between the cloud-base altitude and temperature. For this, the simple assumption that the temperature at cloud base is close to the wet-bulb temperature was made; consequently, $T_b(h_b)$ is defined by the local wet-bulb temperature profile. This reduction of the number of degrees of freedom ensures an improved robustness of the algorithm. This hypothesis is not very constraining since an error in the determination of $T_b$ of $3^\circ C$ modifies the estimation of LWC by less than 10%.

The influence of the initial parameters is important because the algorithm may diverge. Such a situation can be identified by checking the final value of the cost function $S$, which determines the goodness of the fit between experimental and theoretical data. A too-high value ($S > 1 \text{ dBZ}^2$ for the simulations) detects a failure of the algorithm. We generally use initial values of $h_b$ of a few hundred meters (a low value) and $N_f$ of about 100 cm$^{-3}$. These values are modified if the algorithm diverges.

The accuracy in the retrieval of effective radius and liquid water content obviously depends on the quality of the retrieval of the adjusted parameters.

\[ d. \text{ Improved version of the algorithm: Correcting for along-path attenuation} \]

The along-path attenuation tends to bias the measured reflectivity and thus to degrade the quality of the comparison between the experimental and ‘theoretical’ $Z$. In this section we propose an improved version of the algorithm that includes a correction for attenuation. In the framework of the Rayleigh approximation, it can be shown that the specific attenuation $K$ is proportional to the third moment of the DSD, as is LWC. Both parameters are related to $Z$ and $N_f$ through $K \approx \text{LWC} \approx N_f^{0.5} Z^{0.5}$. The exact value of the coefficient at $0^\circ C$ is

\[ K = 1.06 N_f^{0.5} Z^{0.5} \] [dB km$^{-1}$].

\[ (15) \]
where \( N_T \) is still expressed in \( \text{cm}^{-3} \) and \( Z \) in \( \text{mm}^6 \text{ m}^{-3} \).

Under the assumption of a power-law \( K-Z \) relationship \((K = aZ^b)\), Hitschfeld and Bordan (1954) have shown that the “true” (corrected) reflectivity \( Z \) expressed with respect to the “attenuated” (measured) reflectivity is

\[
Z(r) = \frac{Z_a(r)}{1 - 0.46ab \int_0^r Z_a(s)^b \, ds} \quad [\text{mm}^6 \text{ m}^{-3}] \quad (16)
\]

Introducing this formulation in the cost function (12), and taking account of (15), leads to the following new formulation:

\[
S = \frac{1}{N} \sum_{i=1}^{N} \left[ 10 \log \left( \frac{Z_{\text{measured}}(h_i)}{\left( 1 - 0.24N_T^{0.5} \int_{h_i}^{r} Z_{\text{measured}}(s)^{0.5} \, ds \right)^2} \right) \right] - 10 \log \left( Z_{\text{theoretical}}(h_i; h_b, T_b, N_T) \right)^2 \quad (17)
\]

The Hitschfeld and Bordan correction is known to be very unstable in the condition of a forward integration and with a fixed \( a \) coefficient of the \( K-Z \) power law. In the present algorithm the formulation is stable because the \( a \) coefficient is adjusted by the algorithm itself through the adjustment of \( N_T \). This version of the algorithm uses the same numerical approach as the uncorrected algorithm for the determination of \( N_T \) and \( h_b \), and, subsequently, \( \text{LWC} \) and \( r_c \).

3. Testing the algorithm with simulated data

In this section, we first describe the cloud model and the sampling process used to generate simulated data. Then we evaluate the robustness of the algorithm by examining its sensitivity with respect to the simulated measurement noise and to the cloud model assumed in the inversion.

a. Cloud model

Our cloud model is intended to simulate a stratocumulus cloud in various conditions of entrainment. To describe the vertical profile of the liquid water content, \( \text{LWC}(h) \), we use the formulation proposed by Brenguier (1991), which determines \( \text{LWC} \) from the adiabatic \( \text{LWC}_a \) as

\[
\frac{\partial \text{LWC}}{\partial h} = \frac{\partial \text{LWC}_a}{\partial h} - \alpha \text{LWC}, \quad (18)
\]

where \( \alpha \) (\( \text{km}^{-1} \)) is the entrainment coefficient.

The total number of particles per unit volume \( N_T \) is assumed to be unaffected by entrainment because we consider that this process adds as many condensation nuclei as it removes. In stratocumulus, \( \alpha \) ranges from 0 to 10 \( \text{km}^{-1} \); while in cumulus \( \alpha \) may reach between 5 and 10 \( \text{km}^{-1} \) (Brenguier 1991).

In the simulation model we considered values of the entrainment coefficient ranging from 0 to 10 \( \text{km}^{-1} \) and values of the shape parameter \( \mu \) of the cloud droplet distribution ranging from 0 to 6, while the cloud in the inverse model is characterized by \( \alpha = 0 \) and \( \mu = 2 \). Given \( \alpha \) and \( \mu \), the steps to generate the apparent reflectivity profile are the following.

1) Calculate \( \text{LWC}(h) \) from \( \text{LWC}_a \) using (18).
2) derive \( Z(h) \) from \( \text{LWC}(h) \) and the value of \( N_T \) from (8).
3) derive \( K(h) \) from \( Z(h) \) and \( N_T \) through (15), and
4) derive \( Z(h) \) from \( Z(h) \) and \( K(h) \) from (1).

b. Sampling scheme—Measurement noise

Two different sampling schemes are used, one aimed at representing an airborne experiment with a 33-m range resolution, and the other adapted to the spaceborne radar of the Earth Radiation Mission with degraded sampling (500-m range resolution, with oversampling). Realistic noise is introduced in the simulated data as in Testud et al. (1996). In the “airborne radar” simulation, the simulated measurement noise is characterized by the detection threshold at 0-dB SNR and at 3-km range, \( D_{\text{sn}} \approx 3 \text{ km} \) (the detection threshold increases with range as \( r_c^2 \)), and by the number of independent samples \( N_p \).

In the spaceborne simulation, the detection threshold \( (D_{\text{sn}}) \) is approximately constant (because the radar is very far away). In order to simulate the oversampling (one sample each 100 m, while the resolution is 500 m), the following procedure was applied. The reflectivity \( Z_a \) was sampled every 100 m with a noise characterized by \( \text{[DTh, } N_p/5] \). We then averaged over five consecutive 100-m samples. The result of this procedure is that the output data reflects the 500-m range resolution and the \( \text{[DTh, } N_p] \) noise characteristics, but, in addition, the correlation of the measurement noise between two adjacent samples (100 m spaced) is simulated in a realistic way. The values of the parameters used in the two simulations are given in Table 1.

One consequence of introducing realistic noise is that, at the level of the analysis, a criterion should be applied to reject too-noisy data. At the cloud top, we eliminate data that correspond to entrainment, so only reflectivities increasing with altitude are kept (under 1.8 km with the 33-m resolution and under 2 km with the 500-m resolution). At the base, we reject the data when the SNR gets below \(-12 \text{ dB} \). This threshold is determined from the data of Table 1, by considering that a measurement cannot be exploited when the relative standard fluctuation (or effective SNR; Meneghini and Kozu 1990) of the signal exceeds 0.7.
c. Results of the simulation

1) Algorithm verification

In this section, the direct and inverse model for the cloud are identical. Our goals are to evaluate (i) to what extent the scheme to correct for attenuation actually improves the retrieval, (ii) how the algorithm behaves in the presence of simulated measurement noise, and (iii) how the retrieval is degraded by the poorer range resolution anticipated for the spaceborne cloud radar.

For the simulated data in Figs. 1 and 2, which display good configurations for the algorithm, the parameters used in the direct model are $\alpha = 0, \mu = 2$ and $N_r = 250 \text{ cm}^{-3}$; the pressure and temperature at cloud base are $P_b = 900 \text{ hPa}$ and $T_b = 274.65 \text{ K}$ (i.e., 1.5°C). True reflectivity and fitted profiles are both plotted for the total height of the graph.

Figure 1 displays results for the airborne simulation (range resolution of 33 m). It illustrates, for a particular realization of the simulated measurement noise, the performance of the retrieval achieved with the first version of the algorithm (Fig. 1a, without correction for attenuation) and with the improved version (Fig. 1b, with attenuation correction). It is clear that the effect of attenuation is not negligible and that the improved version of the algorithm leads to a fitted reflectivity profile much closer to the original reflectivity than with the first version. The improvement in the retrieval of the three parameters—$N_r$, $P_b$, $T_b$—is also significant: $N_r = 376.7 \text{ cm}^{-3}$, $P_b = 894.1 \text{ hPa}$, and $T_b = 274.2 \text{ K}$ in the first version and $N_r = 272 \text{ cm}^{-3}$, $P_b = 901.3 \text{ hPa}$, and $T_b = 274.7 \text{ K}$ in the improved version. Note that the accuracy in $N_r$ is not as good as for $P_b$ and $T_b$: this is because the shape of the reflectivity profile is less sensitive to $N_r$.

Figure 2 corresponds to the 500-m resolution. Qualitatively we observe the same as in Fig. 1: that is, correcting for attenuation produces a fitted reflectivity much closer to the original profile. The distance between the fitted and original $Z$ profiles, however, increased, when compared with Fig. 1. The retrieval of $P_b$ and $T_b$ is not affected very much by the sampling degradation, but the error in $N_r$ gets larger, especially using the first version of the algorithm (without correcting for attenuation) where the retrieved $N_r$ is more than twice its original value.

Table 2 displays an evaluation of the algorithm retrieval (in its improved version) of $N_r$ and $h$, based on 100 runs corresponding to statistically independent realizations of the simulated measurement noise. In the airborne simulation, the mean value of the retrieved $N_r$ is within 10% of the input value and its standard deviation is ±25%. The mean value of $h$ (using the altitude/pressure conversion) is within 15 m of the input value (1000 m) with a standard deviation of ±78 m. For the spaceborne simulation, the degradation of the estimate is quite significant for $N_r$, which has a mean value (700 cm$^{-3}$) almost three times the input value and standard deviation of ±500 cm$^{-3}$. This effect is again related to the poor sensitivity of the adjustment to $N_r$, as mentioned previously. But the degradation of the estimate of the cloud base remains modest with a bias of 40 m and a standard deviation of ±90 m.

Table 3 shows the statistics (based on the same 100 independent runs) corresponding to the $LWC_{max}$ and effective radius $r_{e-max}$ at cloud top, as deduced from (13) and (14). The input values are $LWC_{max} = 1.59 \text{ g m}^{-3}$ and $r_{e-max} = 14.7 \mu m$. In the airborne simulation, the biases in $LWC_{max}$ and $r_{e-max}$ are quite small, and the standard deviation of the estimate is ±8.5% for both parameters. In the spaceborne simulation, the bias and standard deviation in the estimate of $LWC_{max}$ are of the same order as in the airborne simulation (though slightly degraded), but the $r_e$ estimate shows such a large bias and standard deviation (4 μm, ±4 μm) that it is essentially unusable.

Table 4 displays the cross-correlation coefficients between the four retrieved parameters $N_r$, $h$, $LWC$, and $r_e$. The above diagonal values are the cross-correlation coefficients for the airborne simulation, and below are those for the spaceborne. For both simulations, the maximum cross-correlation coefficient is obtained between LWC and $h$. This is due to the fact that the liquid water content is defined as the adiabatic value and thus is largely determined by the cloud-base altitude. In the airborne simulation, the cross-correlation coefficient is also close to 1 between $N_r$ and LWC, resulting from (13) (linking LWC and $N_r$). Cross-correlation coefficient values are smaller in the spaceborne simulation because of the deterioration of the $Z$ profile retrieval. Since $r_e$ is not directly proportional to a moment of the distribution, its cross-correlation coefficients with all the other parameters are weaker.

Thus the statistics confirm the fact that the cloud base is generally well determined in both airborne and spaceborne simulations, whereas the accuracy in $N_r$, while acceptable in the airborne simulation, is severely degraded in the spaceborne one, leading to large biases and statistical fluctuations. As a consequence, the estimate of the liquid water content, which is highly correlated with cloud base, is satisfactory in both airborne and spaceborne simulations, whereas the estimate in the

| Table 1. Parameters used in the simulations of airborne and spaceborne radar sampling. |

<table>
<thead>
<tr>
<th></th>
<th>Airborne simulation</th>
<th></th>
<th>Spaceborne simulation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dθh</td>
<td>3 m</td>
<td>$N_r$</td>
<td>33 m</td>
<td>$N_r$</td>
</tr>
<tr>
<td></td>
<td>−18 dBZ</td>
<td>Range res.</td>
<td>Sampling</td>
<td>Range res.</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>33 m</td>
<td>1000</td>
<td>500 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>33 m</td>
<td>100 m</td>
<td></td>
</tr>
</tbody>
</table>

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Fig. 1. Results of the algorithm applied to simulated data with a 33-m range resolution (a) without correcting for attenuation and (b) after correcting for attenuation.

Fig. 2. As in Fig. 1 except at a 500-m range resolution.
TABLE 4. Cross-correlation coefficients between retrieved parameters (improved version) based on 100 runs with independent noise generation. Mean and standard deviation values are given for both \( N_T \) and \( h_b \).

<table>
<thead>
<tr>
<th>( X )</th>
<th>( N_T )</th>
<th>( h_b )</th>
<th>LWC</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_T )</td>
<td>X</td>
<td>-0.9951 [500 m]</td>
<td>0.9900 [33 m]</td>
<td>-0.7037 [33 m]</td>
</tr>
<tr>
<td>( h_b )</td>
<td>-0.6391 [500 m]</td>
<td>X</td>
<td>-0.9996 [33 m]</td>
<td>0.7430 [33 m]</td>
</tr>
<tr>
<td>LWC</td>
<td>0.6417 [500 m]</td>
<td>-0.9976 [500 m]</td>
<td>X</td>
<td>-0.7266 [33 m]</td>
</tr>
<tr>
<td>( r )</td>
<td>0.0171 [500 m]</td>
<td>0.0783 [500 m]</td>
<td>-0.0781 [500 m]</td>
<td>X</td>
</tr>
</tbody>
</table>

TABLE 3. Same as Table 2 but for retrieval of maximum liquid water content \( \text{LWC}_{\text{max}} \) and effective radius \( r_{\text{max}} \).

<table>
<thead>
<tr>
<th>( X )</th>
<th>( \text{LWC}_{\text{max}} )</th>
<th>( r_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_T )</td>
<td>1.622</td>
<td>14.49</td>
</tr>
<tr>
<td>( h_b )</td>
<td>0.140</td>
<td>1.240</td>
</tr>
<tr>
<td>( \text{LWC}_{\text{max}} )</td>
<td>1.59 g m(^{-3})</td>
<td>14.7 ( \mu ) m</td>
</tr>
</tbody>
</table>

The effective radius, which is very sensitive to \( N_T \), is good in the airborne simulation, but totally degraded in the spaceborne one.

2) Algorithm Robustness with Respect to Cloud Model

In this section our aim is to study how the algorithm retrieval is affected when the cloud used in the direct model differs from the cloud assumed in the inverse model. We examine separately the influence of the shape parameter \( \mu \) and of the entrainment coefficient \( \alpha \). Because the noise is set to zero in order to avoid confusion of interpretation, there is only a single run of the algorithm for each different value of \( \mu \) or \( \alpha \).

The influence of the shape parameter \( \mu \) is illustrated in Fig. 3, which corresponds to a simulation in which the inverse cloud model remains defined by \( \alpha = 0, \mu = 2 \), while in the direct cloud model \( \alpha \) is set to 0, but \( \mu \) is 1, 2, 3, 4, 5, or 6. Figure 3a displays the variation of the errors in the retrieval of the cloud parameter \( \Delta N_T \) and \( \Delta h_b \), as functions of \( \mu \), for airborne (33-m range resolution) and spaceborne (500-m range resolution) simulations. The corresponding cost function is plotted in Fig. 3b as a function of \( \mu \). The cost function with the 33-m resolution remains smaller than 0.6 (dBZ\(^2\)) at all \( \mu \) of the investigated range and shows a minimum very close to zero for \( \mu = 2 \), which is expected since the inverse and direct cloud models then fit. The cost function is higher with the 500-m resolution because of the sampling degradation, but stays below 1.5 (dBZ\(^2\)).

The errors in the retrieved parameters \( \Delta N_T \) and \( \Delta h_b \) remain within acceptable limits with 33-m resolution, especially if one considers that the probable range of uncertainty of \( \mu \) in stratocumulus is from 1 to 4. In these cases \( \Delta N_T \leq 70 \text{ cm}^{-3} \) and \( \Delta h_b \leq 40 \text{ m} \). With 500-m resolution, the results are much degraded. We obtain \( \Delta N_T \leq 700 \text{ cm}^{-3} \) and \( \Delta h_b \leq 150 \text{ m} \) for \( 1 < \mu < 4 \).

The influence of entrainment is illustrated in Figs. 4 and 5, where, in the direct model, \( \alpha \) is equal to 2, while \( \alpha = 0, 0.5, 1, 2, 5, \) or 10 km\(^{-1}\). Figures 4a and 4b compare the original (true), attenuated (apparent), and corrected for attenuation reflectivity profiles with the Z profile adjusted by the algorithm; both figures correspond to a 33-m resolution sampling with \( \alpha = 1 \) km\(^{-1}\) (Fig. 4a) and \( \alpha = 5 \) km\(^{-1}\) (Fig. 4b). For \( \alpha = 1 \) km\(^{-1}\), the deformation of the Z profile due to entrainment is quite moderate, and the inverse model provides a satisfactory adjustment of the “corrected reflectivities.” An entrainment of \( \alpha = 5 \) km\(^{-1}\) leads to a much stronger deformation of the Z profile that the inverse model barely handles because it does not include a representation of entrainment.

The impact of entrainment in the retrieval of the cloud parameters \( N_T \) and \( h_b \) is shown in Fig. 5a for \( \alpha \) ranging from 0 to 10; the correlative variation of the cost function is shown in Fig. 5b. With the 33-m resolution, the error \( \Delta N_T \) is acceptable (\( \leq 150 \text{ cm}^{-3} \)) as long as \( \alpha \leq 1 \) km\(^{-1}\). With the 500-m resolution, \( \Delta N_T \) falls within the acceptable threshold only for \( \alpha = 0.5 \) km\(^{-1}\). For both resolutions, \( N_T \) diverges (the fixed maximal value of \( N_T \) in the algorithm is 1000 cm\(^{-3}\) when \( \alpha = 5 \) km\(^{-1}\), which is an entrainment typical of convective clouds). The cloud-base retrieval \( h_b \) is much more stable: \( \Delta h_b \) stays within \( \pm 200 \text{ m} \) through the full range of \( \alpha \) at both resolutions. The cost function (Fig. 5b) increases with entrainment (which indicates that the inverse model finds it increasingly difficult to match the “data”). In the range \( 0 < \alpha < 5 \) km\(^{-1}\), \( S \) is about twice as large in the 500-m resolution simulation as in the 33-m simulation. Thus \( S \) can be used to diagnose to what extent the inverse cloud model is representative of the data.

The shape parameter of the cloud droplet distribution selected for the inverse model is not a very critical factor in the algorithm retrieval of parameter values. The impact of entrainment is more problematic. Although it
exerts a moderate influence on the cloud-base retrieval, it exerts a strong influence on $N_T$ as soon as $\alpha > 1$ $\text{km}^{-1}$. As noted before, in stratocumulus, $\alpha$ ranges from 0 to 2 $\text{km}^{-1}$, which means that the results of the algorithm should be taken with caution. Concerning the subsequent errors in the LWC and $r_e$ retrievals, the same considerations as in the previous section apply; that is, the LWC estimate, which is closely connected to $h_b$, will be only weakly affected by the presence of entrainment, whereas the $r_e$ estimate, which is also dependent on $N_T$, will be severely affected.

4. Testing the algorithm with ground-based 94-GHz radar data

Although the along-path attenuation does not act in the same way for nadir-pointing airborne radar and ground-based zenith-pointing radar measurements, the same algorithm may be applied in the latter configuration. This section reports the results of an analysis of stratocumulus data obtained with the 94-GHz radar from
The Pennsylvania State University (Clothiaux et al. 1995) during the Continental Stratus Experiment. Attenuation is negligible at cloud base when looking to zenith; nevertheless, in essence, a radar cannot detect cloud base because the particle sizes there are the smallest, which implies that the reflectivity gets necessarily below the radar detection threshold. Our algorithm provides a “natural” extrapolation to retrieve the cloud base.

A stratiform case that occurred over central Pennsylvania at Rock Springs on 24 October 1996 was chosen. Radiosoundings were available that allowed us to determine the wet-bulb temperature profile needed to set the relationship between $T_b$ and $h_b$. Ceilometer measurements were used to determine cloud-base altitude ($h_{ceilo}$) and to provide verification data to the $h_b$ estimate retrieved by the algorithm.

The specifications of the radar (which include detection threshold at 0-dB SNR and at 3-km range), the number of independent pulses averaged, and range resolution are given in Table 5. The processing of about 200 profiles of radar reflectivity was performed using the improved version of the algorithm (which includes the correction for attenuation). The value of the cost function $S$ was used to diagnose the “success” or “failure” of the algorithm (the threshold considered here is 10 (dBZ)$^2$ since fluctuations of the signal due to noise are much larger here than in the simulations). For 43% of the profiles, the algorithm was “successful,” and the PDF and $r_e$ at cloud top were subsequently derived.

Figure 6 illustrates three cases of success of the algorithm for profiles observed at 1120:00, 1806:22, and 1809:28 UTC. Note that the boundary layer echoes (probably insects) and the measurement points corresponding to entrainment at cloud top are not considered in the adjustment. At 1120:00 UTC, the retrieved cloud base is at 1095 m, while the ceilometer measures it at 1052 m; at 1806:22 UTC the difference is 22 m, and at 1809:28 UTC, only 3 m. For these three situations the reflectivity profile is close to that computed for an adiabatic profile, and apparently no drizzle occurs. The total droplet concentration values determined by the method are 804, 377, and 343 cm$^{-3}$. Figure 7 displays the corresponding retrieved profiles of LWC and $r_e$.

An example of algorithm failure is illustrated in Fig. 8 (corresponding to 1258:00 UTC), showing large deviations between the measurement points and the adjusted profile. For the same case, the cloud-base estimate is in large discrepancy with that obtained from the ceilometer: $h_b$ derived from the algorithm is 500 m, while the ceilometer estimate is 899 m (i.e., at the peak of the reflectivity profile). This is typical of the presence of drizzle. Drizzle drops are expected to initiate near cloud top, grow by collection of cloud droplets during their fall, reach their maximum size at cloud base, and evaporate below cloud base. Drizzle contributes negligibly to the liquid water content, but it dominates in the radar reflectivity because of the associated particle size. The presence of drizzle invalidates the assumption of a $K$–$Z$ relationship on which the algorithm is based.

An attempt of validation of the algorithm is presented in Fig. 9 which compares a scatterplot of the cloud base retrieved by the algorithm $h_b$ with the one measured by the ceilometer $h_{ceilo}$ for the 86 successful cases. The agreement is reasonably good and the standard deviation between the two estimates is 85.4 m.

Figure 10 displays histograms of the retrieved param-
Fig. 6. The 24 Oct 1996 results of the algorithm for successful cases: reflectivity profiles (a) at 1120:00, (b) 1806:22, and (c) 1809:28 UTC.

Parameters for the successful cases: (a) total number of particles \( N_T \), (b) the LWC at cloud top (LWCmax), and (c) the \( r_e \) at cloud top (\( r_{e,max} \)). In interpreting the \( N_T \) histogram, it should be remembered that the effect of entrainment is to bias positively the estimate, so the tail of the histogram (with \( N_T > 600 \) cm\(^{-3}\)) is probably not meaningful. The most frequent values occur around 300–350 cm\(^{-3}\), which is consistent with observations and other retrievals made in continental stratiform clouds (Dong et al. 1997). The LWCmax histogram (Fig. 10b) has two peak values near 0.65 and 0.85 g m\(^{-3}\). The smallest values (0.2 g m\(^{-3}\)) correspond to very thin clouds, and LWC reaches more than 1.2 g m\(^{-3}\) when the cloud layer is about 600 m deep. The values of maximum effective radius (Fig. 10c) lie between 5 and 13 \( \mu \)m, with most of the values falling between 7 and
10.5 μm. No microphysical data were available to validate these results.

5. Conclusions

In the problem of determining the cloud radiative properties from a nadir-looking W-band radar, the method described here met with some success, at the expense of restricting it to particular cloud types, and by using a priori knowledge of the vertical cloud structure.

The cloud model introduced in the algorithm retrieval may represent warm stratiform clouds only. It assumes that the lateral entrainment is negligible and that all condensation nuclei are activated at cloud base. The
The basic parameters of the cloud model that are recovered by the algorithm are the altitude of cloud base and the total concentration of cloud particles. The algorithm includes a stable scheme for correcting the observed radar reflectivity for attenuation.

The algorithm was tested using a numerical simulation that helped to investigate the sensitivity of the retrieval to measurement noise, degradation of the range resolution, shape of the cloud droplet size distribution, and presence of entrainment. For realistic simulations of an airborne experiment (33-m resolution, integration over 2000 pulses), the retrieval of cloud base, total num-
ber concentration of particles, and profiles of the liquid water content and effective radius can be performed with good accuracy. With the sampling characteristics of a spaceborne radar, retrievals of the cloud base and liquid water content remain reasonably accurate, but the estimates of \( N_T \) and \( r_e \) are degraded to a level where they become meaningless. The shape parameter of the cloud droplet size distribution is not critical for the retrieval, but the presence of entrainment rapidly induces severe bias in the \( N_T \) (and subsequently \( r_e \)) estimate. Nevertheless, as long as the entrainment coefficient does not exceed 1 km\(^{-1}\) (which is common in stratocumulus), the retrieval is acceptable.

A test of the algorithm with real data has been performed using a dataset from the zenith-pointing ground-based 94-GHz radar of The Pennsylvania State University, obtained during the Continental Stratus Experiment. A selection of 200 radar reflectivity profiles in a stratocumulus layer was analyzed. Using the cost function to diagnose the failure or success of the algorithm, we found that the algorithm performed successfully in 43% of the cases. An evaluation of the cloud-base retrieval was made by comparison with ceilometer data. The retrieved values of the liquid water content, total number concentration, and effective radius of particles seemed reasonable. Most failures are probably due to the presence of drizzle. Despite the above-mentioned limitations, it should be pointed out as an advantage of our algorithm that it determines the LWC and \( r_e \) profiles through LWC–Z and \( r_e \)–Z relationships adapted to each situation through the adjustment of the \( N_T \) parameter.

Spaceborne data will generate more problems, especially due to the large range resolution, but nevertheless it may constitute an approach for estimation of the cloud base and liquid water content.

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**REFERENCES**


