Optimization of Dynamic Retrievals from a Multiple-Doppler Radar Network

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ABSTRACT

Recently, Protat and Zawadzki described an analysis method to retrieve the three wind components and their temporal derivatives from measurements collected by a bistatic multiple-Doppler radar network deployed around Montreal for nowcasting and research purposes. In the present paper, an extension of this method to retrieve the corresponding pressure and potential temperature perturbations is presented. The method consists of adding the three projections of the momentum equations as weak constraints to the minimization procedure, as is classically done. An evaluation of the performance of this basic constraining model indicates that, after minimization, the residuals of the horizontal momentum equations are of the same order of magnitude as the dominant terms of these equations. It is then shown that including the vorticity equation as an additional constraint substantially improves the perturbation pressure and temperature solutions, leading to negligible residuals of the horizontal momentum equations. This is due to the fact that the vorticity equation is equivalent to the condition that pressure derives from a potential (the second-order horizontal cross-derivatives of pressure are equal), which ensures that the problem has a mathematical solution.

1. Introduction

Acquiring a complete knowledge of the three-dimensional (3D) kinematic and thermodynamic fields at different scales of motion is the first step of a wide range of meteorological applications, including diagnostic research studies, hazard warning, nowcasting, and data assimilation. The Doppler radar (ground-based, shipborne, airborne, on wheels, etc.) is currently the only instrument capable of sampling the 3D structure of the convective-scale and mesoscale flows in time, although it measures only one component of the wind out of three. As a result, a number of research activities have been lately oriented toward the retrieval of the kinematic and thermodynamic properties of the flow using a multiple-Doppler radar network. This research area has been extensively explored for the cases of a traditional ground-based multiple-Doppler radar network, using the basic methods of Ray et al. (1980) for the retrieval of the 3D wind field, and of Gal-Chen (1978) and Hane et al. (1981) for the indirect determination of the pressure and potential temperature perturbations from a reference state (referred to as the “dynamic perturbations” herein). However, the operational deployment of multiple radar networks is not currently practical, as recently pointed out by Wurman et al. (1993), because of the expense associated with the deployment of two or more transmitting radars in a given area. These problems are now addressed through development of the bistatic multiple-Doppler radar networks, an old idea made possible by recent technological developments, as described by Wurman et al. (1993). A bistatic multiple-Doppler radar network (see Fig. 1) consists of one traditional transmitting Doppler radar and one or more passive, low-cost (less than 2% of a typical weather radar, according to Wurman et al. 1993), nontransmitting receivers with a broadbeam antenna at remote sites (the so-called bistatic receivers). Each bistatic receiver measures the Doppler shift of the radiation emitted by the transmitting radar and scattered obliquely by precipitation toward the bistatic receiver. Therefore, it provides a new look angle at a weather volume, just as a Doppler radar would do. In addition, observations of a given volume are collected quasi-simultaneously by all receivers, since there is only one source of illumination, which solves the well-known problem of measurement nonsimultaneity encountered in the case of traditional Doppler radar networks.

Such a bistatic network has been recently implemented at the J. S. Marshall Radar Observatory of Mc
Gill University in collaboration with the University of Oklahoma and the National Center for Atmospheric Research staff. This network consists of the McGill S-band Doppler radar and two passive remote receivers (see Fig. 1). The broad scientific objectives linked to the deployment of this bistatic network around Montreal are to provide operational nowcasting products for the Dorval International Airport, and to address the crucial scientific issues of initialization and data assimilation in a mesoscale model. The first natural step of this work is to access the most accurate real-time estimate of the 3D wind field and associated dynamic perturbations. A method particularly appropriate to this real-time processing of the bistatic network data has been developed by Protat and Zawadzki (1999, hereafter referred to as PZ99). This method was itself an extension of the single-Doppler retrieval method described by Laroche and Zawadzki (1994). Nevertheless, in the method of PZ99 only the three wind components and their temporal derivatives were retrieved. The extension of this method for retrieval of the dynamic perturbations as well is presented in this paper. In section 2, the basic constraining model used to recover the pressure and temperature perturbations is presented, and its performance evaluated. In section 3, this basic constraining model is improved by introducing the vorticity equation as an additional constraint in the minimization procedure. Conclusions are given in section 4.

2. The basic constraining model

a. Principle of the retrieval method

To allow for measurement error, the method described in PZ99 uses the Doppler velocities of two or three receivers of the bistatic network as weak constraints and the continuity equation as a strong constraint in a cost function in which the two horizontal wind components are the variables to be retrieved (the control variables). To account for errors due to the time required to sample a complete weather volume with a Doppler radar (typically 5 min), linear time interpolation of the measurements to a single reference time is used. Thus, the local time derivatives of the three wind components are retrieved at the same time as the wind components themselves. For this linear interpolation, two successive volumetric radar scans are used. The local time derivative of the vertical component of the 3D wind is not constrained much by the measurements. Hence, the local time derivative of the continuity equation, which relates the local time derivatives of the three wind components, is used as a strong constraint, in the same way as the continuity equation itself.

If the bistatic network consists of $P$ bistatic receivers (where $P > 1$, and $P = 2$ in the case of the McGill bistatic network, see Fig. 1), the problem is over determined using the $(P + 1)$ velocities as weak constraints, leading to the following cost function to be minimized:

$$
\mathbf{J}_1 = \sum_{n=1}^{N} \left( (\mathbf{V}_n - \mathbf{\tilde{V}}_n)^T \mathbf{W}_1 (\mathbf{V}_n - \mathbf{\tilde{V}}_n) \right) + \sum_{p=1}^{P} (\mathbf{V}_{n\beta_p} - \mathbf{\tilde{V}}_{n\beta_p})^T \mathbf{W}_2(p) (\mathbf{V}_{n\beta_p} - \mathbf{\tilde{V}}_{n\beta_p}) \right) + \sum_{n=1}^{N} \left[ \frac{\partial \mathbf{V}_n}{\partial t} - \frac{\partial \mathbf{\tilde{V}}_n}{\partial t} \right] ^T \mathbf{W}_3 \left( \frac{\partial \mathbf{V}_n}{\partial t} - \frac{\partial \mathbf{\tilde{V}}_n}{\partial t} \right) \right] 
\times \mathbf{W}_4(p) \left[ \frac{\partial \mathbf{V}_{n\beta_p}}{\partial t} - \frac{\partial \mathbf{\tilde{V}}_{n\beta_p}}{\partial t} \right] ,
$$

where $N$ is the number of time levels used to recover the 3D wind field, vectors $\mathbf{V}_n$ and $\mathbf{\tilde{V}}_{n\beta_p}$ represent the values of the radar radial velocities and of the velocities measured by the $P$ bistatic receivers in space, respectively, $T$ stands for the transpose, and the tilde denotes the observations. The local time derivatives of the vectors $\mathbf{V}_n$ and $\mathbf{\tilde{V}}_{n\beta_p}$ are obtained using the linear time interpolation discussed previously. Here, $\mathbf{W}_1$, $\mathbf{W}_2(P)$, $\mathbf{W}_3$, and $\mathbf{W}_4(P)$ are weighting matrices that give the strength of the constraint. Presently, the weighting matrices $\mathbf{W}_1$ and $\mathbf{W}_2(P)$, as well as $\mathbf{W}_3$ and $\mathbf{W}_4(P)$, are taken as equal so that all the receivers of the network are equally important in the constraining model. Work is in progress.
to determine optimally these matrices (De Elia and Zawadzki 1999).

The adaptation of the original method for retrieval of the dynamic perturbations [that is, the nondimensional pressure $\pi^*$ and “virtual cloud” potential temperature perturbations $\theta^* = \theta^* - q_\circ (\theta^*_w)_\circ$, where $\theta^*_w$ is the virtual potential temperature and $q_\circ$ the cloud water mixing ratio] is now presented. This extension consists in adding to the constraining model (1) the three projections of the momentum equation under the anelastic approximation, according to the works of Gal-Chen (1978), Hane et al. (1981), and Hane and Ray (1985, hereafter referred to as HR85). The retrieved quantities are the deviations of pressure and “virtual cloud” potential temperature from their horizontal limited-area average at each retrieval level (as explained in HR85). The momentum equations used are the same as those presented in HR85, including the same parameterizations of the turbulent force $\mathbf{F} = (K_w, K_v, K_u)$ (the parameterization of $K_w$, $K_v$, and $K_u$ is as in Smagorinsky 1963 and Klemp and Wilhelmson 1978) and of the rainwater mixing ratio $q_w$. These equations may be written as

$$\frac{\partial \pi^*}{\partial x} = \frac{1}{c_p \theta^*_w} \left[ \frac{D\mathbf{u}}{Dt} - f \mathbf{u} - K_u \right] = A \quad (2)$$

$$\frac{\partial \pi^*}{\partial y} = \frac{1}{c_p \theta^*_w} \left[ \frac{D\mathbf{v}}{Dt} + f \mathbf{u} - K_v \right] = B \quad (3)$$

$$\frac{D\mathbf{w}}{Dt} = -c_p \theta^*_w \frac{\partial \pi^*}{\partial z} + g \left( \frac{\theta^*_w}{\theta^*_w} - q_\circ \right) + K_w \quad (4)$$

where $(u, v, w)$ are the three wind components, $D/Dt$ the Lagrangian derivative (following an air parcel), $c_p$ the specific heat at constant pressure, $\theta^*_w$ the potential temperature of the reference state, and $f$ the Coriolis parameter.

It must be noted that, contrary to HR85, the local time derivatives of the wind components are taken here in the moving frame of reference of the storm. We use the assumption that the wind components are varying linearly between two consecutive radar scans (5 minutes in the case of the McGill bistatic network) in this moving frame, as discussed previously (PZ99). The new cost function to be minimized including the momentum equations may be written as

$$J = J_1 + \| \mathbf{e}(2) \|^2 \mathbf{W}_d [\mathbf{e}(2)] + \| \mathbf{e}(3) \|^2 \mathbf{W}_c [\mathbf{e}(3)]$$

$$+ \| \mathbf{e}(4) \|^2 \mathbf{W}_w [\mathbf{e}(4)], \quad (5)$$

where vectors $\mathbf{e}(2)$, $\mathbf{e}(3)$ and $\mathbf{e}(4)$ are the residuals of (2), (3), and (4), respectively, and $\mathbf{W}_d$, $\mathbf{W}_c$, $\mathbf{W}_w$ are weighting matrices that give the strength of the constraint. In practice, if the cost function (5) is minimized so as to obtain the 3D wind, the temporal derivatives of the wind components, and the dynamic perturbations simultaneously, the convergence is relatively slow, which is likely due to the important number of control variables to retrieve and of constraints to satisfy. The convergence speed has therefore been increased as follows: the 3D wind field and temporal derivatives of the wind components are retrieved first using (1), and dynamic perturbations are subsequently retrieved by minimizing (5) without $J_1$ (only the momentum equations as weak constraints). The resulting set of control variables is then used as an initial guess of (5). During this last step, the 3D fields of wind, temporal derivatives of the wind and of pressure and temperature perturbations are retrieved simultaneously. This implies that the three wind components and their temporal derivatives are constrained by the three momentum equations as well. This is not the case in a sequential retrieval (e.g., Hane et al. 1981; Brandes 1984; HR85; Roux et al. 1993; Protat et al. 1998) in which the three wind components and their temporal derivatives are retrieved first, and the thermodynamic variables are retrieved in a second step while the three wind components are not allowed to change. Using this three-step procedure results in a much faster convergence, compatible with operational purposes (roughly two minutes for a domain of $30 \times 30 \times 10$ grid points on a low end 1998 workstation).
The next subsection evaluates the performance of this basic constraining model (5).

b. Performance of the basic constraining model

The basic retrieval procedure (described in section 2a) and computer code were verified using a simulated data set described in Protat et al. (1998). A relative error of less than 1% was found for the dynamic perturbations. These errors are attributed to the discretization of the momentum and continuity equations involved in the retrieval procedure.

In a second step, internal consistency checks were carried out in order to evaluate the quality of the retrieved pressure and temperature perturbations. A commonly used indicator of this quality is the so-called momentum checking introduced initially by Gal-Chen and Hane (1981). This quantity may be written for \( \pi^* \) [for \( E_p \), simply replace \( \pi^* \) by \( \theta^* \) in (6)] as

\[
E_p = \frac{\iint \left( \left( \frac{\partial \pi^*}{\partial x} - A \right)^2 + \left( \frac{\partial \pi^*}{\partial y} - B \right)^2 \right) dx \, dy}{\iint (A^2 + B^2) \, dx \, dy}. \tag{6}
\]

According to the numerous authors who used this parameter to validate their pressure and temperature solutions (e.g., Gal-Chen and Hane 1981; HR85; Roux et al. 1993), values up to 0.25±0.5 (depending on the authors) are indicative of a good fit. We applied the retrieval procedure described in section 2a to process data collected within a shallow supercell hailstorm that passed over the McGill bistatic radar network on 26 May 1997 (documented in Protat and Zawadzki 2000, manuscript submitted to J. Atmos. Sci.), and computed the momentum checking parameter. Vertical profiles of momentum checking are shown in Fig. 2. Both momentum checking parameters \( E_p \) and \( E_u \) are found to meet the “good fit” criterion, with values less than 0.2 at all heights. However, it must be pointed out that the corresponding residuals of the two horizontal momentum equations (2) and (3) are at all heights of the same order of magnitude as the horizontal derivatives of pressure perturbations and the right-hand sides of (2) and (3), despite a small momentum checking. This result is illustrated in Fig. 3, showing these different terms at the 2-km altitude. In contrast, the vertical momentum equation (4) is almost perfectly satisfied (not shown) by the pressure and temperature solutions. This implies that the momentum checking parameter, which is a horizontally integrated
quantity, may not be considered as an indicator of the goodness-of-the-fit in all cases. It is therefore necessary to add additional constraints so as to reduce the residuals of the horizontal momentum equations and improve the pressure and temperature solutions. A procedure is proposed in section 3.

3. Improvement of the constraining model:

The vorticity equation

As discussed in earlier studies (e.g., Hane et al. 1981; Gal-Chen and Kropfli 1984; HR85; Protat et al. 1998), the system of Eqs. (2)–(3) will have a solution if, and only if

$$\frac{\partial^2 \pi^*}{\partial x \partial y} = \frac{\partial^2 \pi^*}{\partial y \partial x}. \quad (7)$$

Mathematically, the equality of the second-order horizontal cross-derivatives of pressure implies that the pressure derives from a potential. If the measurements were error-free, the modeling assumptions were exact (turbulent friction parameterization, anelastic formulation of the continuity equation), and the grid spacing infinitely small, then (7) would be satisfied. In practice, (7) is not satisfied, and hence the system of Eqs. (2)–(3) does not have solutions in a strict sense. Interestingly, it may be shown that this condition is implicitly equivalent to the condition that the vorticity equation be satisfied by the wind solution. In fact, by taking the curl of the two horizontal momentum equations (2) and (3) and using condition (7), this vorticity equation may be written as follows:

$$\frac{D\zeta}{Dt} = \omega_H \cdot \nabla_H w - \zeta \nabla \cdot v_H + f \frac{\partial w}{\partial x} + F_v, \quad (8)$$

where $\omega_H$ is the horizontal vorticity vector, $\zeta$ its vertical component, $v_H = (u, v)$ the horizontal wind vector, $\nabla_H$ the horizontal gradient operator, and $F_v = k \cdot (\nabla \times F)$ the turbulence term. This formulation contains implicitly the hypothesis that (7) is satisfied, which removes the pressure dependence in (8). Hence, if the wind satisfies the vorticity equation, condition (7) should be satisfied. On the other hand, if the 3D wind retrieved using the basic constraining model (5) is used to evaluate the left-hand and right-hand sides of (8) it appears clearly that the vorticity equation is not satisfied (as seen in Figs. 4a,b, respectively), despite of the fact that the 3D wind is constrained by the momentum equations, from which the vorticity equation is derived. Condition (8) is therefore introduced in the minimization procedure (5), which leads to the following new cost function to be minimized:

$$J_3 = J_2 + [e(8)]^T W_s [e(8)], \quad (9)$$

where vector $e(8)$ is the residual of (8), and $W_s$ the weighting matrix that gives the strength of the constraint. First, it must be noted that the 3D wind solution is not fundamentally changed. However, the first- and second-order derivatives of the three wind components have slightly changed in such a way that the vorticity equation (8) is now well-satisfied by the wind solution, as shown by the computation of the left-hand and right-hand sides of (8) displayed in Figs. 5a,b, respectively.

Computation of the momentum checking parameters (Fig. 6) shows a substantial improvement of the fit ($E_p$ values ranging from $3 \times 10^{-3}$ to $1.5 \times 10^{-2}$ in Fig. 6a,
as opposed to the values in Fig. 2). To our knowledge, such a small value has not been obtained previously. A slight improvement is also seen for the $E_\theta$ parameter (compare Fig. 6b and Fig. 2), which is likely due to the better estimate of the pressure perturbation term used in the vertical momentum equation (4). Let us recall, however, that the momentum checking parameter may be misleading in some cases, as discussed in section 2, and that only an inspection of the residuals of the horizontal momentum equations at all heights would clearly evidence the quality of the retrieval. These residuals are given in Figs. 7a,b at the 2-km altitude, and may be compared with the residuals obtained using the basic constraining model (5) [displayed in Figs. 3e,f, respectively for Eqs. (2) and (3)]. It is clearly seen from this comparison that these residuals have been significantly reduced. More importantly, the residuals become much smaller than the horizontal pressure gradients (Figs. 3a,b) and forcing terms A (Fig. 3c) and B (Fig. 3d) of (2) and (3), which evidences the substantial improvement brought to the dynamic retrieval. It must be noted as well that these results systematically apply at all heights (not shown) and for the 14 successive dynamic retrievals performed using the measurements collected within the shallow supercell hailstorm sampled by the bistatic network on 26 May 1997.

4. Conclusions

The purpose of this work is to extend the real-time retrieval method of the 3D wind described by Protat and Zawadzki (1999) to include the retrieval of the dynamic perturbations from the measurements collected by a bistatic Doppler radar network. The original method uses the Doppler velocities of two or three receivers of the bistatic network as weak constraints, and the air-mass continuity equation as a strong constraint (using its adjoint equation). Our retrieval of the dynamic perturbations basically follows the ideas already described in literature (e.g., Gal-Chen 1978; Hane et al. 1981; Roux 1985; HR85; Roux et al. 1993; Protat et al. 1998). The momentum equations are introduced as weak constraints in the previous minimization procedure. It is then shown that a significant improvement is brought to the perturbation pressure and temperature solutions by adding the vorticity equation as a weak constraint in the minimization procedure. This condition, which is equivalent to the equality of the second-order horizontal cross-derivatives of pressure, is mandatory to ensure the existence of a mathematical solution for the pressure perturbation using the horizontal momentum equations, as also discussed in earlier studies (e.g., Gal-Chen and Kropfli 1984). The 3D fields of wind and dynamic perturbations retrieved using this additional constraint are found to satisfy both the momentum and vorticity equations. Although the vorticity equation is implicit in the momentum equations, the explicit addition of the vorticity equation as a constraint leads to a faster convergence of the algorithm and a better fit of the model. The use of the vorticity equation, together with the simultaneous retrieval of the 3D fields of wind, temporal derivatives of the wind, and dynamic perturbations, are the main differences between the proposed method and previous retrieval works. The advantage of our procedure is that the three wind components and their temporal derivatives are also constrained by the three momentum equations. The real-time application of the algorithm is assured by a fast convergence: for a domain of $30 \times 30 \times 10$ grid points the solution for the 3D wind and dy-

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**Fig. 6.** Same as Fig. 2, but using the constraining model (9).

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analyzed so as to study the internal dynamics and life cycle of a shallow supercell hailstorm that passed over Montreal on 26 May 1997 (Protat and Zawadzki 2000, manuscript submitted to *J. Atmos. Sci.*).

**REFERENCES**


