Low-Frequency Resonant Scattering of Bubble Clouds*

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ABSTRACT

The acoustic properties of water can be drastically modified by a small amount of air content in the fluid. In the ocean, bubble clouds are long lasting, and most of the time can be considered as passive scatterers. Solutions of the key parameters such as resonance frequencies, sound speed, and target strength can be derived from treating the bubble cloud as a homogeneous medium with proper effective bulk properties. In this paper, the resonance frequency and target strength are derived from computations based on a classical solution for acoustic scattering of elastic spheres. The range of void fractions covers four orders of magnitude and includes single air bubbles as its asymptotic condition of void fraction equals unity.

Based on these computations, it is found that the isothermal condition is approached only at very low void fraction levels ($<10^{-4}$). At high void fraction ($>3 \times 10^{-1}$), the cloud oscillation gradually approaches adiabatic condition. Within the broad range of void fraction from $2 \times 10^{-4}$ to $3 \times 10^{-1}$, the effective polytropic coefficient of the bubble cloud is approximately 1.2, which is halfway between adiabatic and isothermal conditions. Also, two simple scaling laws for the resonance characteristics of a spherical bubble cloud are revealed: (i) the dimensionless resonance wavenumber is uniquely determined by the void fraction, and (ii) the backscattering cross section is uniquely determined by the resonance frequency.

1. Introduction

The study of resonant scattering of bubble clouds is stimulated by research in sea surface sound generation. The publications on this topic are quite extensive (e.g., the proceedings of the series of Sea Surface Sound Symposia: Kerman 1988, 1993; Buckingham and Potter 1995 and the references therein). In the oceanographic environment, acoustic properties of bubbly water involve both situations of active sound generation and passive acoustic scattering. Near the sea surface in the active breaking region, void fractions (fraction of air in water) of 0.01 and higher are common (e.g., Lamarre and Melville 1994a, b). Their measurements suggest that the duration of a high void fraction event is usually on the order of minutes in field conditions (e.g., Figs. 10 and 11 in Lamarre and Melville 1994a), and on the order of wave period in laboratory simulations (e.g., Figs. 4 and 6 in Lamarre and Melville 1994b).

In contrast to the long life time of bubble cloud events, the duration of active breaking in a random wind-generated wave field is typically a small fraction of a wave period (e.g., Longuet-Higgins and Smith 1983; Xu et al. 1986; Hwang et al. 1989). Considering that the duration of active breaking is such that certain key wave properties exceed some threshold conditions (e.g., the water particle speed exceeds the wave phase speed thus the water particle overtakes and detaches from the wavefront; the particle acceleration at the wave crest exceeds the gravitational acceleration thus the water particle at the surface penetrates into the water column; or the local slope of the wave profile exceeds a stability limit thus causing instability of the waveform), the active duration of breaking wind-generated waves in the laboratory is found to be on the order of 30 ms (Hwang et al. 1989). There is a weak dependence of breaking duration on the wind speed, and the magnitude of breaking duration appears to scale with the breaking wave period (approximately 5% of the wave period, see Fig. 7 in Hwang et al. 1989). The authors are not aware of similar measurements reported from oceanic conditions, however, it is generally accepted that the active breaking duration is a small fraction of the wave period. If the duration of active sound generation from entrained bubbles is of the same order as the active breaking duration, which is many orders of magnitude shorter than those of the bubbly events, it can be deduced that during...
most of the time the bubble clouds in the ocean are passive scatterers of ambient sound.

The population of large bubbles in the surface region is also much greater. Bubble size distribution measured in the laboratory has shown a systematic change of size spectrum from $a^{-4}$ in the deep region (relative to the wave height) to $a^{-2}$ near the air–water interface, where $a$ is the bubble radius (Baldy and Bourguet 1987; Baldy 1988; Hwang et al. 1990). The combination of larger bubble size and higher void fraction near the air–water interface represents a favorable condition for the generation and scattering of low-frequency sound from the surface layer of the ocean, as has been discussed in great details over the past 10 years.

In this study, two of the most interesting properties of bubble cloud scattering, the lowest mode resonance frequency and the scattering coefficient, are investigated using the effective medium approach. Also discussed are the similarities of a bubble cloud and a single bubble in terms of their resonance behavior, the proper value of the polytropic coefficient of a bubble cloud as a function of void fraction when using the effective medium approach, and two convenient scaling laws governing the resonance properties of a spherical bubble cloud.

As a matter of terminology, Prosperetti et al. (1993) adopt the term “cloud” for the “fresh” bubbles that produce active generation of sound and “plumes” for “old” bubbles that passively scatter sound. In the literature on ocean waves, plume has been used in the active sense as in Longuet-Higgins and Turners’ article (1974). We adopt the ocean wave convention and denote “bubble clouds” as passive groups of air bubbles in the water column. We note also that Morse and Ingard (1968) use the term “cloud of scatterers” to describe air bubbles in water or fog droplets in air that scatter sound waves in the passive sense.

2. Resonance frequency of a bubble cloud

The acoustic resonance frequency of a single bubble in water neglecting the effects of surface tension and viscous attenuation given by Minnaert (1933) is

$$f_b = \frac{1}{2\pi a} \left( \frac{3\gamma P_a}{\rho} \right)^{1/2}, \quad (1)$$

where $a$ is the bubble radius, $\gamma$ the polytropic coefficient, $P_a$ the ambient pressure, and $\rho$ the density of water. There is a great degree of similarity between a spherical bubble cloud and a single air bubble. Because the compressibility of air and water differs by a factor of more than four orders of magnitude, the compressibility of the air–water mixture is significantly modified by even a very small amount of air. For example, the acoustic velocity decreases from approximately 1500 m s$^{-1}$ in “pure” water to approximately 100 m s$^{-1}$ in water with 1% air (Wood 1930). Because of the large mismatch of compressibility between a bubble mixture and its surrounding water, the acoustic scattering of a spherical bubble cloud is essentially identical to that of a single air bubble. Treating the bubble cloud as an effective medium, the resonance frequency of a spherical bubble cloud given by Carey and Fitzgerald (1993) is

$$f_k = \frac{1}{2\pi R_b} \left( \frac{3\gamma P_a}{\rho \chi} \right)^{1/2}, \quad (2)$$

where $\chi$ is the air fraction in the fluid and $R_b$ is the bubble cloud radius. Equations (1) and (2) are very similar. In particular, (1) can be considered as a special case of (2) for $\chi = 1$, that is, an air bubble with radius $R_b$.

Because the size of a bubble cloud is much larger than the size of constituent bubbles, the resonance frequency of a bubble cloud is much lower than the resonance frequency of the constituent bubbles. Comparing (1) and (2), this ratio is $a/(R_b^{1/2})$. For example, bubble clouds of 0.1-m radius and containing 100-μm-radius bubbles have a resonance frequency 10 times lower than that of the constituent bubbles for $\chi = 10^{-4}$, and 100 times lower for $\chi = 10^{-2}$. This characteristic has stimulated the recent development of an artificial bubble technology to produce bubble-based acoustic targets for low-frequency underwater acoustic research (Hwang et al. 1995a,b).

Figure 1 shows the dependence of resonance frequency on the void fraction for three near-surface bubble clouds with radii of 0.07, 0.2, and 0.4 m. The range of void fractions used in this computation is from 0.001 to 0.2. For each cloud size, two curves are given, show-
ing the upper and lower bounds of resonance frequency corresponding to $\gamma = 1.4$ and 1.0, respectively. The acoustic scattering in the low hundred hertz range can be produced by small clouds (less than 10-cm radius) of high void fraction (on the order of 0.01), or medium-size clouds (on the order of 20-cm radius) and less void fraction (on the order of 0.001).

The numerical value of $\gamma$ has been discussed at great length for a single bubble or a bubble cloud in the literature. In the case of an air bubble, the bubble oscillation is close to adiabatic ($\gamma = 1.4$) except for very small bubbles, where the isothermal condition ($\gamma = 1.0$) is more appropriate (Clay and Medwin 1977). For bubble clouds the oscillation is frequently considered isothermal (e.g., Carey and Browning 1988) due to the low resonance frequency. Different choices of the numerical values of $\gamma$ may introduce up to 20% differences in the computed resonance frequencies. The quantitative dependence of $\gamma$ on void fraction and cloud size will be investigated in section 3 based on numerical computations of acoustic scattering of elastic spheres.

3. Bubble cloud resonant scattering

a. The Anderson (1950) solution on the sound scattering from a fluid sphere

The general solution for the scattering of sound from a homogeneous spherical obstacle is given in Anderson (1950). The procedure is to solve the three-dimensional wave equation (formulated in terms of the acoustic pressure) inside and outside of the sphere. At the boundary of the sphere, both the pressure and the normal component of the particle velocity are matched. The resulting axial symmetrical solution is given in terms of the series summations of the products of the Legendre function, the spherical Bessel function, and the spherical Neumann function. In the limiting case of scattering from small particles, the solution is shown to be identical to the Rayleigh scattering solution (Rayleigh 1945).

For the limiting case of rigid spheres, the solution is shown to be identical to the earlier solution of Stenzel (1938). For the general case, there are three parameters: the acoustic radius (dimensionless) of the spheres ($ka$, where $k$ is the acoustic wavenumber and $a$ is the radius of sphere), the relative density ($g = \rho' / \rho$, where $\rho'$ is the density of the internal mixture and $\rho$ is the density of the surrounding fluid), and the relative sound speed ($h = c'/c$, where $c'$ is the sound speed in the internal mixture and $c$ is the sound speed in the surrounding fluid).

The general solution given by Anderson (1950) is sufficiently complicated and requires numerical evaluation. While such tasks have become routine today, it was a rather major achievement in the late forties, as evident in the following quotation from Anderson (1950, p. 429):

The numerical evaluation of the preceding equation giving the scattering from a fluid sphere whose dimensions are comparable to a wavelength would not have been feasible except for the recent publication of spherical Bessel functions. In spite of the aid of these tables, considerable computation was still required. Exclusive of checking, the numerical examination presented in the accompanied graphs, consisting of more than 600 points, required the full time services of two computers for a period of about two months. The possibility of using one of the large automatic computing machines was investigated but it was soon discovered that the problem did not lend itself feasible to automatic computation.

Computer technology has progressed tremendously since then. The same computations for each case (a datum point) can now be accomplished on a desktop PC within less than a minute even without significant code optimization. Despite the primitive computation power of his time, Anderson (1950) presents a marvelous and enlightening discussion on the acoustic scattering of fluid spheres, and covers a broad range of acoustic parameters (the relative density, the relative sound speed, and the acoustic radius). The primary result relevant to this paper is that the reflectivity ($R$) becomes large without limit as the relative sound speed ($h$) and the relative density ($g$) both become small; of the two factors, the dependence on the relative sound velocity $h$ is more pronounced. For very small values of $g$ and $h$, $R$ increases as $1/gh^2$. This quantity, $1/gh^2$ is simply the ratio of compressibility inside the sphere to that of the surrounding medium. On the other hand, for $g$ and $h$ large (an incompressible fixed particle), the reflectivity approaches the finite limit, $R \rightarrow 5(ka)^2/6$.

Bubble clouds in the ocean fall into the category of $g \rightarrow 1$ and $h < 1$, thus very large reflectivity and scattering cross sections are expected. The computed target strength of bubble clouds will be presented in section 3c.

b. The magnitude of polytropic coefficient

In presenting their results, Carey and his colleagues (Carey and Browning 1988; Carey and Fitzgerald 1993; Carey and Roy 1993) emphasize that their solutions of resonance frequency and backscattering cross section are identical to those of Anderson (1950), Rschevkin (1963), and Morse and Ingard (1968) when the Wood (1930) approximation for the mixture compressibility is used. The argument for the isothermal assumption, however, is more or less based on an intuitive ground (Carey and Browning 1988). In the following, the Anderson (1950) solution is used to calculate the resonance frequencies of bubble clouds with the Wood (1930) approximation for the density and compressibility of the bubble mixtures. An example of the computed acoustic scattering of a bubble cloud in a few selected elevation...
angles (at 45° intervals) is shown in Fig. 2. The resonance characteristics of the bubble cloud scattering is quite obvious as revealed in the feature of resonance spikes. The squared reflectivity coefficient is in excess of 100 for the lowest resonance mode.

The frequencies of the lowest mode resonance of three bubble clouds (0.07-, 0.2-, and 0.4-m radii) over a wide range of void fractions (0.001–0.2) are given in Fig. 3. For reference, the two curves corresponding to the theoretical upper and lower bounds (Fig. 1) for each bubble cloud size are also superimposed. The numerically calculated resonance frequencies stay well above the lower bound defined by the isothermal condition over the range of void fractions shown.

To further investigate the transition of the polytropic coefficient $\gamma$ from the adiabatic to the isothermal condition, the numerical calculations were carried out over an even wider range of $\chi$, from $5 \times 10^{-3}$ to 1. The latter condition, $\chi = 1$, corresponds to a spherical air cavity, that is, a single air bubble. As discussed earlier, the adiabatic condition ($\gamma$ equals 1.4) is appropriate for the resonant oscillation of large bubbles. This is correctly predicted by the numerical calculation, showing that $\gamma \to 1.4$ as $\chi \to 1$ (Fig. 4). It is found that the oscillation of bubble clouds approaches isothermal only at very low void fractions, say $\chi < 10^{-4}$. Note that the computed value of $\gamma$ continues to decrease for $\chi < 10^{-4}$. Since as $\chi \to 0$, the properties of the mixture approach those of the surrounding fluid. To the limit of $\chi = 0$, the bubble cloud loses its identity, resonant scattering becomes meaningless and the concept of effective medium should not be applicable.

c. Scaling laws

In presenting the resonance frequency of a bubble cloud using (2), there are two obvious dependent variables, the void fraction and the bubble cloud radius. The equation can be rendered dimensionless for more convenient applications. With a simple rearrangement, the dimensionless expression for the resonance frequency becomes

$$\frac{2\pi f_{\chi} R_b}{c} = k_{\gamma} R_b = \frac{1}{c} \left( \frac{3 \gamma P_a}{\chi \rho} \right)^{1/2},$$

(3)
where $k$ is the wavenumber and $c$ is the speed of sound in water. In this form, the system of curves shown in Fig. 3 are consolidated into a single curve (Fig. 5a). For comparison, Fig. 3 is reproduced in Fig. 5b. Superimposed on these curves are the lowest mode resonance frequency and dimensionless wavenumber obtained from numerical calculations using the Anderson (1950) solution. As discussed earlier, these data points fall between the upper and lower bounds defined by the adiabatic and isothermal conditions. For the single bubble resonance, the dimensionless wavenumber is $k_{c,1} = 0.0135$, and corresponds to the intercept of the upper bound curve with $x = 1$ in Fig. 5a.

The backscattering intensity of a spherical bubble cloud can also be defined uniquely by a simple parameter. For an acoustically compact spherical bubble cloud, the relative backscattering intensity, $I_s/I_i$, given by Carey and Roy (1993) is

$$I_s/I_i = \frac{K^6}{9(kr)^2 \left[ 1 - (x/x_m)^2 \right]^2 + \left[ (x/x_m)^2 \right]} \left[ 1 - (x/x_m)^2 \right]^2,$$

(4)

where $r$ is the distance between the acoustic receiver and the bubble cloud, $K = kr_{b,1}$, $x = c^2$, and subscript $m$ denotes the effective properties of the air–water mixture. At resonance, the first bracketed term in the denominator of (4) vanishes. The relative intensity can be expressed as (Hwang et al. 1995b)

$$\left( \frac{I_s}{I_i} \right)_R = \left( \frac{x_m - 1}{x} \right)^2 \approx \left( \frac{c_m^2 - 1}{c^2} \right)^2.$$

(5)

The scattering intensity at resonance is critically affected by the resonant acoustic wavelength (proportional to the inverse of wavenumber) and the ratio $c_m/c$ (5). The dependence on the cloud size is implicit through the resonance frequency relationship (2). Because the sound speed in a liquid with air bubbles is much lower (approximately 100 m s$^{-1}$ with a 0.01 void fraction) than that in the bubble-free fluid (on the order of 1500 m s$^{-1}$ in water), the target strength (in decibels, defined as $10 \log (I_s/I_i)$ at $r = 1$ m) and the backscattering cross section of a bubble cloud at resonance reduces simply to $k_{cr}^2$, or equivalently, $c^2(2\pi f_R)^{-2}$,

$$\left( \frac{I_s}{I_i} \right)_R = \frac{1}{k_{cr}^2} = \frac{c_m}{c} \left( \frac{c}{2\pi f_R} \right)^2.$$

(6)

Fig. 5c presents the numerical results of the lowest-mode resonance backscattering cross sections using the Anderson (1950) solution for the three bubble clouds ($R_b = 0.07, 0.2, \text{and } 0.4$ m) with void fractions ranging from $10^{-2}$ to 0.2. All the computational results fall on a single curve corresponding to (6).

In presenting the scattering properties of an elastic sphere with a similar density and acoustic velocity as the surrounding fluid, Anderson (1950) shows that two parameters (the density ratio and the velocity ratio of the sphere and the surrounding fluid) govern the acoustic scattering properties of the elastic body. These two parameters can be combined to represent the ratio of the compressibility inside and outside of the sphere. His scaling is presented in Fig. 5d, showing that in addition to the compressibility, the size of the sphere is an additional parameter, resulting in a system of curves. With the present analysis given above (6), these dependent...
variables are consolidated into a single parameter, the resonance frequency or resonance wavenumber, as shown in Fig. 5c.

4. Summary

A small fraction of air bubbles in water can modify significantly the acoustic properties of the air–water mixture. In the ocean, bubbles entrained by breaking waves remain in the water column for an extended period of time (order of hours) after a stormy event. The clouds of bubbles are excellent scatterers of ambient sound, especially in the low-frequency range. The resonance of a bubble cloud is very similar to the resonance of a single bubble in the liquid. Using the concept of effective medium, a simple dimensionless formula (3) can be used to calculate the resonance frequency of a bubble cloud. The range of applicability of the formula is greater than four orders of magnitude in void fraction (10^{-2} \leq \chi \leq 1). Similarly, the backscattering cross section (target strength) can be derived from a single quantity (the bubble cloud resonance frequency) from (6).

These two simple scaling laws for the resonance characteristics of a spherical bubble cloud indicate that (i) the dimensionless resonance wavenumber is uniquely determined by the void fraction (Fig. 5a), and (ii) the backscattering cross section at resonance is uniquely determined by the resonance frequency (Fig. 5c).

Also presented in the discussion is the numerical value of $\gamma$, which can produce a 20% difference in the numerical value of the resonance frequency depending on whether an adiabatic or an isothermal condition prevails. Within the framework of employing the Anderson (1950) solution on the scattering of elastic spheres and adopting the Wood (1930) approximation for the mixture density and compressibility, the numerical calculations show that the lowest mode bubble cloud resonance approaches isothermal only at very low void fractions, say $\chi \approx 10^{-4}$. Within the range of great interest in oceanographic applications, say $2 \times 10^{-4} < \chi < 3 \times 10^{-1}$, the value of $\gamma$ is approximately 1.2. For $\chi > 3 \times 10^{-1}$, $\gamma$ increases monotonically to 1.4, which corresponds to an adiabatic condition (Fig. 4).
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