Estimation of Velocity Error for Doppler Lidar Measurements

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ABSTRACT

Various methods of estimating the magnitude of the random error of Doppler lidar velocity measurements are compared for typical operating conditions using computer simulations of lidar data. Under certain conditions, the magnitude of the random estimation error can be determined from data without the need for in situ measurements for both ground-based and space-based wind measurement.

1. Introduction


Doppler lidar produces measurements of the radial velocity of the aerosols (or molecules) illuminated by the laser pulse. Coherent Doppler lidar directly measures the Doppler shift of the backscattered laser light using coherent detection, which produces a time-varying signal whose frequency is proportional to the radial velocity. Incoherent Doppler lidar determines the radial velocity from the intensity of the backscattered signal using a spectral analyzer (Fabry Perot interferometer) or edge detector. The radial velocity is produced with a single lidar pulse along a given line of sight (LOS) or with multiple pulses and fixed or scanning lidar beams.

Profiles of the horizontal velocity are produced by various scanning geometries. The velocity–azimuth display and the range–height indicator have been used for lidar measurements (Gal-Chen et al. 1992; Menzies and Hardesty 1989). More recently, fixed lidar beam geometries and step-stare beam patterns have been considered for space-based operation to improve sensitivity (Frehlich 2000, 2001, hereafter F01). The accuracy of the horizontal velocity is determined by the random error and bias of the radial velocity measurements as well as the atmospheric variability. A measurement of the bias of a Doppler lidar measurement requires an independent in situ measurement (F01). In principle, the random error can also be determined by comparisons with careful in situ measurement. Another approach is to predict the random error from the statistical properties of the estimator assuming ideal conditions such as no frequency drift of the reference laser, no contribution from random backscatter variations, and high signal levels (Drobinski et al. 2000). Accurate measurements of the random error are important to verify the quality of the measurements (Frehlich et al. 1997), to remove the contribution of the random error from measurements of wind field statistics (Frehlich et al. 1998; Drobinski et al. 2000), and to identify unknown instrumental and atmospheric sources of error. Under certain conditions, an accurate measurement of the uncorrelated random error can be produced without in situ measurements using various signal-processing techniques. The performance of the various techniques will be presented for coherent Doppler lidar. Similar analysis could be produced for incoherent Doppler lidar measurements.

2. Doppler lidar measurement errors

The definition of error for measurements of a random radial velocity requires a definition of the desired measurement $m(R, t)$ where $R$ denotes the location of the
measurement and \( t \) denotes the time. The measurement \( \hat{v}(R, t) \) is then represented as
\[
\hat{v}(R, t) = m(R, t) + e(R, t) + \text{bias}(R, t),
\]
where \( e(R, t) \) is a zero-mean random error and the bias of the measurement is defined as
\[
\text{bias}(R, t) = \langle \hat{v}(R, t) - m(R, t) \rangle,
\]
where angle brackets denote an ensemble average over many realizations of a statistically similar atmosphere. The magnitude \( \sigma_e \) of the random error is defined as
\[
\sigma_e^2 = \langle e(R, t)^2 \rangle.
\]

The velocity measurements \( \hat{v}(R, t) \) contain three sources of random error (Frehlich 1997): the “speckle” component of the lidar signal from the interference of the randomly phased backscattered fields from each individual aerosol particle, the additive detector noise, and the instantaneous random radial velocity \( v(r, t) \) as a function of the distance \( r \) along the lidar beam axis. The speckle component produces a short correlation time of the lidar signal, on the order of a few microseconds (Churnside and Yura 1983; Frehlich 1994). Because of this short correlation time, data from each lidar pulse is statistically independent and the conditional ensemble average of \( \hat{v}(R, t) \) for a given realization of the radial velocity \( v(r) \) is denoted as
\[
m_{\text{avg}}(R, t) = \langle \hat{v}[R, t \mid v(r)] \rangle
\]
and can be determined experimentally by using data from a large number of lidar pulses over a short time interval such that \( v(r, t) \) does not vary with time. This requires a high pulse repetition frequency (PRF). The conditional ensemble average is a statistically pleasing choice for the desired velocity measurement \( m(R, t) \) because the bias is zero for each realization of \( v(r, t) \) and therefore the bias defined by Eq. (2) is zero, that is, \( \langle \hat{v}(R, t) - m_{\text{avg}}(R, t) \rangle = 0 \).

The conditional ensemble average depends on the lidar parameters and the velocity estimator. It is difficult to determine the statistical properties of \( m_{\text{avg}}(R, t) \) analytically, especially when the atmospheric variability has a large contribution. However, computer simulations can produce accurate estimates of \( m_{\text{avg}}(R, t) \) by generating many estimates of \( \hat{v}(R, t) \) for each velocity realization \( v(r, t) \). It has been shown that \( m_{\text{avg}}(R, t) \) has a weak dependence on the lidar parameters (Frehlich 1997).

\[a. \text{ Doppler lidar velocity measurements with single pulses}\]

When the lidar signal energy is high, velocity measurements can be produced with each lidar pulse and the measurement time \( t \) is essentially a fixed instant in time. For typical single-shot lidar measurements of the radial velocity and for typical velocity estimators (Frehlich 1997; Banakh and Smalikh 1997), the conditional ensemble average \( m_{\text{avg}}(R, t) \) is well approximated by
\[
m_{\text{avg}}(R, t) = \int_{-\infty}^{\infty} v(r, t)W(R - r) \, dr,
\]
where \( W(r) \) is a normalized weighting function that depends on the lidar pulse and the dimensions of the measurement volume (range gate) that is centered at range \( R \). The lidar pulse spatially filters the instantaneous radial velocity \( v(r, t) \). The measurement volume is defined by the lidar pulse length \( \Delta r \) and the range-gate length \( \Delta p \) for the measurement [the distance the illuminated aerosol region travels during the measurement time (Frehlich 1997)]. The desired measurement \( m(R, t) \) is a random variable that is determined by the random velocity \( v(r, t) \) and the lidar pulse filter \( W(r) \). The random error \( e(R, t) \) is determined mainly by the random fluctuations of the backscattered laser field (“speckle field”) and the estimation algorithm.

For single-shot lidar measurements, the statistical behavior of coherent Doppler lidar measurement error has been determined using computer simulations (Frehlich and Yadlowsky 1994; Frehlich 1996, 1997, 2000, F01; Salamitou et al. 1995) for both constant radial velocity over the resolution volume and including wind turbulence where the random radial velocity \( v(r, t) \) is generated for a given spatial structure function (Frehlich 1997). If the pulse length \( \Delta r \) is much less than the range-gate dimensions \( \Delta p \), then \( m_{\text{avg}}(R, t) \) is well approximated by the linear average of the radial velocity over the range-gate length \( \Delta p \); that is,
\[
m_{\text{avg}}(R, t) = \frac{1}{\Delta p} \int_{-\Delta p/2}^{\Delta p/2} v(r, t) \, dr,
\]
which is the typical situation for space-based measurements (Baker et al. 1995; F01). For typical conditions, the Doppler lidar radial velocity measurements \( \hat{v}(R, t) \) are unbiased and can be represented as
\[
\hat{v}(R, t) = m(R, t) + e(R, t),
\]
where \( x \) can be \( \text{avg, wgt, or lin} \), and the random error \( e \) has zero mean and a standard deviation of \( \sigma_e \). The velocity measurements are unbiased for all the different desired measurements \( m(R, t) \) because of the ensemble average over a homogeneous random atmosphere. However, \( \sigma_e \) can depend on the choice of \( m(R, t) \).

\[b. \text{ Doppler lidar velocity measurements with multiple pulses}\]

Doppler lidar measurement performance is improved by fixing the laser beam and accumulating the signal from \( N \) lidar shots for each range gate (Frehlich and Yadlowsky 1994; Frehlich 1996; Frehlich et al. 1998; Rye and Hardesty 1993a,b). This also introduces an extra dimension to the spatial averaging of the velocity measurements and reduces the threshold signal energy.
required for a good measurement. In many cases, the wind variability over the measurement volume determines the performance of the velocity measurements and a full statistical description of the wind field is required to predict coherent Doppler lidar performance (Frehlich 1997, 2000; F01).

For multiple-pulse lidar measurements, an important parameter is the total measurement time or dwell time $T = N/PRF$. If the random radial velocity $v(r, t)$ does not vary over the dwell time $T$, then the single-pulse analysis of the previous section is valid. If the radial velocity changes over the dwell time, we assume that the random atmosphere is frozen and that the radial velocity as a function of time is given by the translation of a frozen two-dimensional velocity field $u(r, h)$, which is defined by the lidar beam geometry and

$$v(r, t) = u(r, Ut),$$

where $U$ is velocity transverse to the lidar beam axis and $h$ denotes the distance transverse to the lidar beam at time $t = 0$. Spatial averaging of the wind field may result from the transverse motion of the atmosphere or equivalently, the transverse motion of the lidar platform, which is the geometry for space-based or aircraft lidar measurements (Frehlich 2000; F01). The geometrical component of this spatial averaging is defined by the parameter

$$\mu = \frac{\Delta h}{\Delta p} = \frac{UT}{p},$$

where $\Delta h = UT$ is defined as the horizontal distance the atmosphere moves during the dwell time $T$ of the velocity measurement.

For a fixed lidar beam and multiple-pulse data products, the desired velocity measurements can be defined as the spatial average of the radial velocity $v(r, t)$ over the two-dimensional observation plane $u(r, Ut)$. Then

$$m_{wp}(R, H) = \int_{-\infty}^{\infty} \int_{H-\Delta h/2}^{H+\Delta h/2} u(r, h)W(R-r) \, dh \, dr,$$

and

$$m_{in}(R, H) = \int_{R-\Delta p/2}^{R+\Delta p/2} \int_{H-\Delta h/2}^{H+\Delta h/2} u(r, h) \, dh \, dr,$$

where $H$ denotes the center of the measurement region in the horizontal direction. This is also a good approximation for space-based measurements and typical velocity estimators (Frehlich 2000).

Experimental measurements of the random error statistics based on the desired velocity measurements $m_{wp}$ and $m_{in}$ are difficult because high-resolution in situ measurements of the radial velocity $v(r, t)$ are required. As noted in section 2, the measurement of the random error statistics based on the conditional ensemble average $m_{avg}$ Eq. (4) requires a high PRF lidar. For ground-based measurements of the random error, one assumes the estimation error $e(R, t)$ is uncorrelated with the random velocity field $v(r, t)$ and the aerosol backscatter. Then the magnitude of the random error $\sigma$ can be measured from the high-frequency region of the velocity spectrum (Frehlich 1996; Frehlich et al. 1994, 1997, 1998) or by interpolating the autocorrelation function to zero lag (Mayor et al. 1997; Lenschow et al. 2000). Typical velocity errors range from 0.5 to 1.0 m s$^{-1}$ for a single lidar pulse and 0.1 to 0.5 m s$^{-1}$ for measurements from multiple pulses (Frehlich et al. 1994, 1997). For multiple-pulse space-based operation, the random error can be determined from the statistics of the difference of the velocity measurements produced from the even- and odd-numbered lidar pulses (F01). All three of these methods produce an estimate of the measurement error $\sigma$, without a priori knowledge of the velocity field, that is, without in situ data. The performance of these three methods will be determined using computer simulation and compared with the results where the definition of error is defined in terms of $m_{wp}$ and $m_{avg}$. The situation for incoherent Doppler lidar is less clear because the effects of velocity and backscatter variations over the measurement cell have not been determined. However, for an ideal incoherent Doppler lidar, these three techniques should produce similar performance.

c. Statistical description of the velocity field

The performance of coherent Doppler lidar velocity measurements depends on the statistical description of the radial velocity $v(r, t)$ or equivalently, under the assumption of a frozen atmosphere, on the spatial statistics of $u(r, h)$. The spatial statistics of the radial or longitudinal velocity are described by the longitudinal spatial structure function (Monin and Yaglom 1975)

$$D_{Ls}(r) = \langle [v(r_0 + r) - v(r_0)]^2 \rangle$$

and the lateral structure function, which is given by

$$D_{Nn}(r) = D_{Ls}(r) + \frac{r}{2} \frac{d}{dr} D_{Ls}(r),$$

for incompressible isotropic turbulence.

A common model for isotropic turbulence is the von Kármán model defined by ( Hinze 1959; Lenschow and Kristensen 1988)

$$D_{Ls}(r) = 2\sigma^2 \left(1 - G \left( \frac{r}{L_o} \right) \right),$$

$$D_{Nn}(r) = 2\sigma^2 \left(1 - G \left( \frac{r}{L_o} \right) + G_0 \left( \frac{r}{L_o} \right) \right),$$

$$G(x) = \frac{22/3 x^{1/3}}{\Gamma(1/3)} K_{1/3}(x) = 0.592 548 5 x^{1/3} K_{1/3}(x),$$

$$G_0(x) = \frac{x^{4/3}}{2^{10} \Gamma(1/3)} K_{2/3}(x) = 0.296 274 26 x^{4/3} K_{2/3}(x),$$

where $L_o$ is the Obukhov length and $\sigma$ is the root-mean-square of the departure from the mean.
\( \sigma^2 \) is the variance of each velocity component, \( \Gamma(x) \) is the gamma function, and
\[
L_i = \frac{\sqrt{\pi} \Gamma(5/6)}{\Gamma(1/3)} L_{o0} = 0.746 \, 834 \, 3L_{o0}
\]
is the integral scale of the velocity, \( L_i \) is the external scale, and \( K_\nu(\nu) \) is the modified Bessel function of order \( \nu \).

For the case of infinite \( L_o \), the von Kármán model reduces to the Kolmogorov model
\[
D_{\mu}(r) = 2e^{2/3}r^{2/3}, \quad \text{and}
\]
\[
D_{N\mu}(r) = \frac{8}{3}e^{2/3}r^{2/3},
\]
where \( e \) is the energy dissipation rate. For finite \( L_o \)
\[
e = \left( \frac{2^{1/3} \pi}{\sqrt{3\Gamma(1/3)\Gamma(4/3)}} \right)^{3/2} \sigma^3 = 0.933 \, 668 \sigma^3 L_{o0}.
\]
The isotropic von Kármán model is used for all the results presented here.

d. Doppler lidar parameters

Results are presented for a 2-\( \mu \)m solid-state coherent Doppler lidar with a Gaussian temporal intensity pulse profile (Frehlich et al. 1997) with a full width at half maximum (FWHM) duration \( \tau \). The illuminated pulse volume is defined by a narrow pencil-shaped Gaussian profile in range with FWHM \( \Delta r = \tau c/2 \). Doppler lidar velocity measurements are produced from \( N \) pulses of lidar data. The range-gate location is determined by the time difference of the received signal referenced to the transmit time \( t = 0 \). The complex data from each lidar pulse is sampled at an interval of \( T_s \) and the signal statistics from \( M \) data points are accumulated for the same range-gate interval (Frehlich and Yadlowsky 1994). The range-gate length \( \Delta p = MT_s c/2 \) is defined as the distance the illuminated aerosol region moves during the time \( MT_s \). The velocity measurements span \( v_{\text{search}} = \lambda/(2T_s) \).

Another important lidar system parameter is \( \Phi_1 \), the average number of coherently detected photo-electrons per lidar shot per range gate, which is given by \( \Phi_1 = M \times \text{SNR} \), where SNR is the heterodyne signal-to-noise ratio (Frehlich and Yadlowsky 1994). The SNR is assumed constant (near-field region) over all the range gates. When \( \Phi_1 \) is small, the distribution of velocity measurements is characterized by the fraction \( b \) of uniformly distributed random outliers over the velocity search space \( v_{\text{search}} \), and a localized distribution of good estimates with standard deviation \( \sigma_e \) centered around the desired velocity. For large \( \Phi_1 \) the number of outliers approaches zero and only good estimates remain with a standard deviation \( \sigma_e \), which defines the random error of the radial velocity measurements. The fraction of outliers \( b \) and the standard deviation \( \sigma_e \) of the good estimates (rms error) can be calculated from simulated lidar data using the algorithms described in appendix E of Frehlich (1997). When there are no outliers, \( \sigma_e \) is the magnitude of the random error of the radial velocities. For the results in Figs. 1–10, high SNR was assumed, that is, there are negligible random outliers \( (b = 0) \).

The measurement of the random error of the Doppler velocity measurements is produced with \( N_{\text{rg}} \) adjacent range gates along a fixed LOS and \( L \) sequences of these LOS estimates. For a fixed horizontal velocity \( U \) the two-dimensional area sampled by the lidar is \( L_g = N_{\text{rg}} \Delta p \) along the lidar beam and \( L_u = L \Delta h \) transverse to the lidar beam. The velocity field for the lidar simulation (Frehlich 1997) is produced by an algorithm that generates accurate spatial statistics (Frehlich 2000, appendix) for an isotropic von Kármán model with \( L_o = 200 \) m and various \( e \).

e. Measurement error from the velocity spectrum

The most convenient spectral estimate of the time sequence of velocity estimates \( \hat{v}(R, kT) \), \( k = 0 \), \( L - 1 \) is the periodogram \( \hat{P}(R, nF) \) defined by
\[
\hat{P}(R, nF) = \frac{1}{L} \sum_{k=0}^{L-1} w(kT) \hat{v}(R, kT) \exp \left( -\frac{2\pi i kn}{L} \right)^2,
\]
where \( i = \sqrt{-1} \), \( |z| \) denotes the complex modulus of \( z \), \( w(t) \) is the window function, and \( F = 1/LT \) is the frequency resolution of the periodogram. (The velocity time series have been linearly detrended and the window function is unity for all the results presented here.)

An example of the average of 4000 spectra from simulations is shown in Fig. 1 for various horizontal sampling intervals \( \Delta h \) (various transverse velocities \( U \)). If the desired velocity measurements \( m(R, t) \) are uncorrelated with the random estimation error \( e(R, t) \), the spectra will consist of two distinct components: a low-frequency region from the desired atmospheric measurement \( m(R, t) \) and a constant high-frequency region from the temporally uncorrelated estimation error \( e(R, t) \) (for a given velocity field, the lidar signal is uncorrelated from shot to shot). The ensemble average of the constant high-frequency region is given by
\[
P_e = \langle \hat{P}(R, nF) \rangle = \sigma_e^2
\]
and estimates of the magnitude of the random error are produced by the average of the spectral estimates \( \hat{P}(R, nF) \) above a threshold index \( n_x \), which identifies the constant high-frequency region, that is,
\[
\hat{\sigma}_e^2 = \frac{1}{(L/2) - n_x} \sum_{s=n_x}^{L/2} \hat{P}_s(nF),
\]
where \( \hat{P}_s(nF) \) is the average of the spectral estimates from the \( N_{\text{rg}} \) range gates. The chosen threshold indices are indicated as vertical lines in Fig. 1. For this method to produce unbiased measurements, there must be a constant high-frequency region.
f. Measurement error from velocity covariance

Another measurement of the magnitude \( \sigma_e \) of the uncorrelated estimation error \( e \) is based on the unbiased covariance estimates

\[
\hat{C}(R, nT) = \frac{1}{L-n} \sum_{k=0}^{L-1-n} [\hat{v}(R, kT) - \overline{v}(R)] \times [\hat{v}(R, (k+n)T) - \overline{v}(R)],
\]

where

\[
\overline{v}(R) = \frac{1}{L} \sum_{k=0}^{L-1} \hat{v}(R, kT)
\]

is the average of the \( L \) velocity measurements \( \hat{v}(R, kT) \). Figure 2 contains the average covariance of the same simulations used in Fig. 1. The variance of the estimation error \( e(R, t) \) is contained in the discontinuity of the covariance estimates at zero lag \( (k = 0) \). A robust estimate of this variance is produced by a linear interpolation of the covariance to zero lag, that is,

\[
\hat{\sigma}^2 = \hat{C}_{0}(0) - 2\hat{C}_{0}(T) + \hat{C}_{0}(2T),
\]

where \( \hat{C}_{0}(nT) \) is the average of the covariance estimates from the \( N_{RG} \) range gates. The linear interpolation to zero lag must be accurate to produce unbiased estimates of \( \sigma^2 \).

For multiple-pulse velocity estimates, another simple algorithm is possible for estimating the random error based on consistency of the data. Consider a Doppler lidar radial velocity measurement \( \hat{v}(R, t, N) \) produced from \( N \) lidar pulses. The measurement can be represented using Eq. (1) as

\[
\hat{v}(R, t, N) = m(R, t) + e(R, t, N) + \text{bias},
\]

Produce two velocity estimates \( \hat{v}_1 \) and \( \hat{v}_2 \) from the data of even- and odd-numbered pulses, respectively. Each estimate will have data from \( N/2 \) lidar pulses. For a fixed lidar beam geometry, the two estimates will have the same desired measurement \( m(R, t) \) and the same bias for well-behaved atmospheric conditions, for example, no rare cloud events and stationary statistics for the wind field and aerosol backscatter. Then, the difference of the two estimates \( \Delta v(R, t, N/2) = \hat{v}_1(R, t, N/2) - \hat{v}_2(R, t, N/2) \) can be written as

\[
\Delta v(R, t, N/2) = e_1(R, t, N/2) - e_2(R, t, N/2),
\]

and if we assume \( e_1 \) and \( e_2 \) are statistically similar and uncorrelated, the variance of \( \Delta v \) is given by

\[
\sigma_\Delta^2 \left( R, \frac{N}{2} \right) = \left\langle \left( \Delta v \left( R, \frac{N}{2} \right) \right)^2 \right\rangle = 2\sigma_e^2 \left( R, \frac{N}{2} \right),
\]

which provides an estimate of the random error for Doppler lidar velocity measurements with \( N/2 \) pulses of data per estimate, that is,
\[ \sigma_e^2(R, N/2) = \frac{1}{2} \sum_{k=0}^{N/2-1} \Delta v \left[ R, kT, N/2 \right]^2. \]  
(31)

Consider the velocity estimate
\[ \hat{v}_{\text{avg}}(R, t, N) = \frac{1}{N} \sum_{i=0}^{N-1} \hat{v}_i(R, t), \]
(32)
which is the mean of the two estimates \( v_1 \) and \( v_2 \). If the two estimates \( v_1 \) and \( v_2 \) have the same mean value and have statistically independent random errors \( e_1 \) and \( e_2 \), the random error of \( \hat{v}_{\text{avg}}(R, t, N) \) is given by
\[ \sigma_e^2(R, N) = \frac{1}{2} \sigma_e^2(R, N/2) = \frac{\sigma_e^2}{4}. \]
(33)

We will use this result to scale the random error \( \sigma_e(R, N/2) \) to the random error \( \sigma_e(R, N) \) for velocity estimates using data from all \( N \) pulses. This algorithm is also suitable for estimating the random error contribution to other statistics such as structure functions (Frehlich 1997; Frehlich et al. 1998) and higher-order moments (Lenschow et al. 2000).

3. Results

a. High-resolution lidar

The performance of the different methods of estimating the magnitude \( \sigma_e \) of the random Doppler lidar velocity error is determined from simulations of a high-resolution coherent Doppler lidar (Henderson et al. 1993; Frehlich et al. 1994, 1997) that samples an isotropic velocity field described by a von Kármán model with an outer scale \( L_o = 200 \text{ m} \) and an \( \epsilon = 0.02 \text{ m}^2 \text{ s}^{-5} \), consistent with daytime convection. The lidar parameters are an operating wavelength \( \lambda = 2.0 \mu\text{m} \), a Gaussian transmitted pulse with a FWHM pulse duration of \( \tau = 0.16 \mu\text{s} \), which corresponds to an illuminated target region with a spatial extent of \( \Delta r = 24 \text{ m} \) (FWHM). Heterodyne signals are generated assuming a complex receiver with a sampling interval \( T_s = 0.02 \mu\text{s} \), \( M = 8 \) complex samples define the range-gate length \( \Delta p = MT_s/c/2 = 24.0 \text{ m} \), that is, \( \Delta p/\Delta r = 1 \). A total of \( N = 10 \) lidar pulses were generated for each velocity measurement. For each estimate of the random error \( \sigma_e \), \( N_{\text{rg}} = 10 \) nonoverlapping range gates were processed and a time series of \( L = 200 \) contiguous velocity measurements were generated for a total of \( N_f = N_{\text{rg}}L = 2000 \) velocity measurements.

For the results in Figs. 1–10, high SNR was assumed, that is, there are negligible random outliers \( (b = 0) \). For Figs. 1–8, a constant \( \Phi_r = 100.0 \) was assumed for all the range gates. The velocity estimates were produced with the maximum-likelihood estimator (Frehlich and Yadlowsky 1994) assuming knowledge of the SNR and 400 realizations of the full simulation \( (N = 10, N_{\text{rg}} = 10, L = 200) \) were produced to determine the mean \( \sigma_e \) and standard deviation \( \text{SD}[\sigma_e] \) of the estimates of the random velocity error.

The performance of the three methods for estimating the magnitude \( \sigma_e \) of the random velocity error is shown in Fig. 3 as a function of the parameter \( \mu = \Delta h/\Delta p \). The results in Figs. 1 and 2 correspond to the first three points of Fig. 3. The covariance-based estimate “cov” has the worst performance with a negative bias and the largest standard deviation. It is difficult to accurately interpolate the covariance to zero lag. The spectral-based estimate “spec” agrees with the velocity-difference estimate “vel diff” for \( \mu < 0.3 \) but has a larger standard deviation. As \( \mu \) becomes large, the constant spectral level at high frequency disappears as the atmospheric contribution overwhelms the contribution from the uncorrelated random error. The best performance is produced by the velocity-difference algorithm, which has a small increase in both \( \sigma_e \) and \( \text{SD}[\sigma_e] \) as \( \mu \) increases.

The three methods of estimating the random error do not require a priori knowledge of the random velocity field. The spec and vel diff methods are in excellent agreement for \( \Delta h = 0.05 \) in Fig. 3. All three methods have excellent agreement as \( \Delta h \to 0 \). The single-shot performance \( \Delta h = 0 \) of the spec method was shown to agree with the predictions of computer simulations when the conditional average \( m_{\text{avg}} \) and the pulse-weighted approximation \( m_{\text{pwt}} \) were used as the desired measurement (Frehlich 1997) for the definition of measurement error. The performance of the velocity-difference algorithm is compared with the results from computer simulation (Frehlich 1997, 2000) for these two defini-
Fig. 4. Average velocity estimation error $\sigma_v$ as a function of the parameter $\mu = \Delta h/\Delta p$ for the parameters of Fig. 1 from the velocity-difference method (open circle), referenced to the conditional mean velocity Eq. (4) (square), and referenced to the pulse-weighted velocity Eq. (10) (triangle).

The velocity-difference algorithm requires two assumptions: that the random errors $e_1$ and $e_2$ are uncorrelated and that the error scaling to the velocity estimate using $N$ pulses of data is given by Eq. (33), which is the error scaling for the estimate $\tilde{\nu}_{\text{avg}}(R, t, N)$, the average of the two velocity estimates $\nu_1$ and $\nu_2$ Eq. (32). The performance of the algorithms for estimating $\Delta \nu_{\text{avg}}(R, t, N)$ is within the error bars of the simulations. We can then conclude that the errors $e_1$ and $e_2$ are uncorrelated and the scaling of Eq. (33) has a small error that produces the disagreement in Fig. 4. When accurate knowledge of the velocity measurement error $\sigma_v$ is critical (e.g., correcting the measurement error in wind field statistics and higher-order statistics), the velocity estimate Eq. (32) is attractive for a wide variety of conditions. However, it has poorer performance in weak-signal regions because the velocity estimates from $N/2$ lidar pulses are more adversely affected by random outliers than the single velocity estimate from $N$ lidar pulses (Frehlich and Yadlowsky 1994; Frehlich 1996, 1997).

The performance of the various algorithms for $\Delta p/\Delta r = 2$ are shown in Figs. 6 and 7 with $\Phi_1 = 100.0$ and the same atmospheric conditions as Figs. 3 and 4, respectively. The velocity difference algorithm has the best performance, followed by the spectral-domain algorithm. The covariance-domain algorithm is only useful for $\Delta h < 0.05 \Delta p$. The velocity-difference algorithm

Fig. 5. Average velocity estimation error $\sigma_v$ as a function of the parameter $\mu = \Delta h/\Delta p$ for the parameters of Fig. 1 from the velocity-difference method (open circle), referenced to the conditional mean velocity Eq. (4) (square), and referenced to the pulse-weighted velocity Eq. (10) (triangle) with the velocity estimates defined as the average of the even and odd pulse estimates, Eq. (32).

Fig. 6. Average velocity estimation error $\sigma_v$ and the standard deviation of the estimates $\text{SD}(\sigma_v)$ for $\Delta p = 2 \Delta r$ and the parameters of Fig. 1 for the spectral method (square), covariance method (filled circle), and the velocity-difference method (open circle) as a function of the parameter $\mu = \Delta h/\Delta p$. 

Fig. 7. Average velocity estimation error $\sigma_v$ and the standard deviation of the estimates $\text{SD}(\sigma_v)$ for $\Delta p = 2 \Delta r$ and the parameters of Fig. 1 for the spectral method (square), covariance method (filled circle), and the velocity-difference method (open circle) as a function of the parameter $\mu = \Delta h/\Delta p$. 

Fig. 8. Average velocity estimation error $\sigma_v$ and the standard deviation of the estimates $\text{SD}(\sigma_v)$ for $\Delta p = 2 \Delta r$ and the parameters of Fig. 1 for the spectral method (square), covariance method (filled circle), and the velocity-difference method (open circle) as a function of the parameter $\mu = \Delta h/\Delta p$. 

Fig. 9. Average velocity estimation error $\sigma_v$ and the standard deviation of the estimates $\text{SD}(\sigma_v)$ for $\Delta p = 2 \Delta r$ and the parameters of Fig. 1 for the spectral method (square), covariance method (filled circle), and the velocity-difference method (open circle) as a function of the parameter $\mu = \Delta h/\Delta p$. 

Fig. 10. Average velocity estimation error $\sigma_v$ and the standard deviation of the estimates $\text{SD}(\sigma_v)$ for $\Delta p = 2 \Delta r$ and the parameters of Fig. 1 for the spectral method (square), covariance method (filled circle), and the velocity-difference method (open circle) as a function of the parameter $\mu = \Delta h/\Delta p$.
FIG. 7. Average velocity estimation error \( \sigma_v \) as a function of the parameter \( \mu = \Delta h/\Delta p \) for the parameters of Fig. 6 for the velocity-difference method (open circle), referenced to the conditional mean velocity Eq. (4) (square), and referenced to the pulse-weighted velocity Eq. (10) (triangle).

has better agreement with the robust simulation algorithms (Fig. 7) than for the case of \( \Delta p/\Delta r = 1.0 \) (Fig. 4). As the range-gate size \( \Delta p \) becomes larger than the pulse volume \( \Delta r \), Eq. (33) becomes a more accurate assumption and for typical space-based operation (Frehlich 2000) this assumption is very good.

b. Scaling of \( \text{SD}[\sigma_v] \)

The accuracy \( \text{SD}[\sigma_v] \) of the estimates for \( \sigma_v \) depends on the total number of velocity estimates \( N_T = N_{\text{RG}} L \) used for each estimate, or equivalently, the total measurement region defined by \( L_R \) and \( L_H \). If the velocity estimates are statistically independent, then

\[
\text{SD}[\sigma_v] \propto \frac{1}{\sqrt{N_T}} = \frac{1}{\sqrt{N_{\text{RG}} L}} \propto \frac{1}{\sqrt{L_R L_H}}.
\]

(34)

The accuracy \( \text{SD}[\sigma_v] \) from the simulations for the velocity-difference algorithm is compared to this error scaling in Fig. 8 for \( N_{\text{RG}} = 10 \), \( \mu = \Delta h/\Delta p = 0.1 \), and \( M = 8 \) (Figs. 1–5) and \( M = 16 \) (Figs. 6, 7). Good agreement is produced for the case of the conditional mean velocity \( m_{\text{avg}} \). The results for the pulse-weighted mean velocity \( m_{\text{wgt}} \) have a small error because of a small difference of approximately 0.01 m s\(^{-1}\) between \( m_{\text{avg}} \) and \( m_{\text{wgt}} \).

d. Performance in the weak-signal regime

The velocity estimates in the weak-signal regime consist of an extra component of random outliers that is well approximated by a uniform distribution that contains a fraction \( b \) of the total estimates. The velocity-difference algorithm can be modified to produce estimates of the fraction \( b \) and the standard deviation \( \sigma_v \) of the good estimates for velocity estimates using \( N/2 \) pulses.
es of lidar data. These results can be scaled to the case of velocity estimates using all \( N \) pulses of data using simulations or theoretical models (Dabas 1999).

The probability density function \( f(v) \) of the velocity estimates is given by

\[
f(v) = b U(v, 0, v_{\text{search}}) + (1 - b) g(v - \langle v \rangle),
\]

where \( U(v, a, b) = (b - a)^{-1} \) for \( a \leq v \leq b \) and 0 otherwise and \( \langle v \rangle \) is the mean value of the good velocity estimates. The function \( g(v) \) is the probability density function of the good velocity estimates with a mean value of zero.

The probability density function \( h(d) \) of the difference of the two velocity estimates \( d = v_1 - v_2 \) is given by

\[
h(d) = \int_{-\infty}^{\infty} f(v) f(v - d) \, dv
\]

if \( v_1 \) and \( v_2 \) are statistically independent with identical distribution functions. Substituting Eq. (35) into Eq. (36) produces

\[
h(d) = b^2 T(d) + (1 - b)^2 G(d)
+ (1 - b) \int_{-\infty}^{\infty} g(v) U(v + (v) - d, 0, v_{\text{search}}) \, dv
+ (1 - b) \int_{-\infty}^{\infty} g(v) U(v + (v) + d, 0, v_{\text{search}}) \, dv,
\]

where

\[
T(d) = \int_{-\infty}^{\infty} U(v, 0, v_{\text{search}}) U(v - d, 0, v_{\text{search}}) \, dv
\]

is the distribution function of the difference of two random outliers and

\[
G(d) = \int_{-\infty}^{\infty} g(v) g(v - d) \, dv
\]

is the distribution function of the difference of two good velocity estimates. The last two terms of Eq. (37) are the distribution functions for the difference of a random outlier and a good estimate and if the good estimates are confined to the center of the velocity search space \( \langle v \rangle = v_{\text{search}}/2 \) then these two terms can be approximated as a uniform distribution between \(-v_{\text{search}}/2\) and \(v_{\text{search}}/2\) and

\[
h(d) = b^2 T(d) + (1 - b)^2 G(d)
+ 2b(1 - b) U\left(d, -\frac{v_{\text{search}}}{2}, \frac{v_{\text{search}}}{2}\right)
\]

The distribution \( T(d) \) is a symmetric triangle function centered on \( d = 0 \) with a range from \(-v_{\text{search}}\) to \(v_{\text{search}}\).

This distribution can be converted to a uniform distribution over the interval \((-v_{\text{search}}/2, v_{\text{search}}/2)\) by the mapping

\[
z = -\frac{v_{\text{search}}}{2}, \quad d \leq -\frac{v_{\text{search}}}{2}
\]

\[
z = \frac{v_{\text{search}}}{2}, \quad -\frac{v_{\text{search}}}{2} < d < \frac{v_{\text{search}}}{2}
\]

\[
z = v_{\text{search}} - d, \quad d \geq \frac{v_{\text{search}}}{2},
\]

and the distribution of \( z \) becomes

\[
h(z) = b_{\text{diff}} U\left(z, -\frac{v_{\text{search}}}{2}, \frac{v_{\text{search}}}{2}\right)
+ (1 - b_{\text{diff}}) G(z),
\]

which has the same form as Eq. (35) with the fraction of random outliers

\[
b_{\text{diff}} = 2b - b^2,
\]

and the standard deviation \( \sigma_z \) of the good estimates given by

\[
\sigma_z^2 = 2\sigma^2.
\]

The parameters \( b \) and \( \sigma \) for velocity estimates using data from \( N/2 \) lidar pulses are then given by

\[
b = 2 - 2\sqrt{1 - b_{\text{diff}}},
\]

\[
\sigma = \frac{\sigma_z}{\sqrt{2}},
\]

where the parameters \( b_{\text{diff}} \) and \( \sigma_z \) are determined using the algorithms in appendix E of Frehlich (1997).

The results of the velocity difference algorithm is compared with previous simulation algorithms in Fig. 11. The agreement is excellent for estimates of \( b \). The estimates of \( \sigma_z \) agree for \( b < 0.5 \), the region of useful velocity estimates. Similar performance was produced for space-based operation (Frehlich 2001).

4. Summary and discussion

The measurement of the magnitude \( \sigma_z \) of the random error of Doppler lidar velocity measurements is feasible without the need for in situ measurements. The results presented in Figs. 1–10 are for high signal regimes where there are no random outliers. The low signal regime is considered in Fig. 11. The spectrum-based algorithm (spec) requires a constant spectrum at high frequencies (Fig. 1) while the covariance based algorithm (cov) requires a well-defined discontinuity of the covariance at zero lag (Fig. 2). The former condition is more robust than the latter, however, both algorithms will produce useful estimates when there is a well-defined constant spectrum at high frequencies. This requires low-turbulence conditions and a small transverse
Fig. 9. Average velocity estimation error $\sigma_e$ and the standard deviation of the estimates $SD[\sigma_e]$ for a high-PRF lidar with $N = 100$ lidar pulses per velocity estimate, $M = 32$ complex samples per range gate, $T_s = 0.02 \, \text{ms}$, $\Phi_1 = 10$, SNR = 0.3125, $\Delta r = 75.0$ m, $\Delta p = 96.0$ m, $\lambda = 2.0 \, \mu\text{m}$, $L = 20$ velocity estimates per spectra, and a von Kármán wind field with $\varepsilon = 0.002 \, \text{m}^2\text{s}^{-3}$ and $\ell_0 = 200.0$ m. The estimates of $\sigma_e$ are from the spectral method (square), covariance method (filled circle), and the velocity-difference method (open circle) as a function of the parameter $\mu = \Delta h/\Delta p$.

The third algorithm investigated is based on the difference of two velocity estimates $v_1(R, t, N/2)$ and $v_2(R, t, N/2)$, which are generated from the even- and odd-numbered lidar pulses, respectively. If the estimation error of the two estimates is uncorrelated and if the mean value of the two estimates are equal, then the variance of the difference is twice the estimation error variance $\sigma_e^2(T, t, N/2)$ for velocity estimates using $N/2$ lidar pulses. This error variance is scaled to the error variance of velocity estimates using all $N$ lidar pulses by the variance of the mean of the two velocity estimates $v_1(R, t, N/2)$ and $v_2(R, t, N/2)$ [see Eq. (32)]. The velocity-difference algorithm has the best performance (see Figs. 3, 6, 9) with the smallest estimation error $SD[\sigma_e]$.

All three algorithms produce an estimate of the magnitude of the uncorrelated random error without in situ measurements. The interpretation of the velocity error requires a choice of the desired measurement $m(R, t)$ for the definition of error. The results for the conditional average velocity $m_{\text{avg}}(R, t)$ and the pulse-weighted velocity $m_{\text{wgt}}(R, t)$ agree very well. There is a small difference between the velocity error $\sigma_e$ based on these two definitions of truth and the results from the velocity-difference algorithm for the high-resolution case of Fig. 4 (the lidar pulse volume $\Delta r$ is equal to the range-gate distance $\Delta p$). This small error is due to the error in scaling the velocity error from the two velocity esti-
mates \(v_1\) and \(v_2\) to the single velocity estimate [see Eq. (33)]. If the velocity estimate using all \(N\) lidar pulses is chosen as the average of the two velocity estimates \(v_1\) and \(v_2\), Eq. (32), then this small error is removed and excellent agreement is produced (see Fig. 5). However, this estimate has poorer performance in the weak signal regime.

For high-PRF lidar data, the velocity-difference algorithm performs best when data from a large number \(N\) of lidar pulses are accumulated for each velocity estimate and only a small number \(L\) of velocity estimates are used for the estimates of the random error. The spectral-based and covariance-based algorithms require small \(\Delta h/\Delta p\) for useful results and the covariance-based algorithm has a larger error \(SD[\sigma]\) because the discontinuity of the covariance at zero lag is very small (see Fig. 9). The velocity-difference algorithm agrees very well with the simulation algorithm based on the conditional mean velocity \(m_{av}\) (see Fig. 10). However, there is a small difference between the simulation algorithm based on the pulse-weighted mean velocity \(m_{opt}\) because of the small difference of approximately \(1\) cm \(s^{-1}\) between \(m_{av}\) and \(m_{opt}\).

The error in the estimates \(\sigma\) is defined as the standard deviation \(SD[\sigma]\). The velocity-difference algorithm satisfies the scaling \(SD[\sigma]\propto 1/\sqrt{N_{RG}}\propto 1/\sqrt{N_{RG}L}\) (see Fig. 8) where \(N_{RG}\) is the number of range gates used for each line-of-sight (LOS) and \(L\) is the number of LOSs. This scaling is also observed for the other algorithms and lidar parameters.

For weak-signal regimes (low SNR), the velocity-difference algorithm can be modified to produce useful estimates of the random error \(\sigma\) of the good estimates and the fraction \(b\) of random outliers for the velocity estimates using \(N/2\) lidar pulses (see Fig. 11). This requires a method of centering the good velocity estimates in the velocity search space. The standard method is to increase the number of lidar pulses and the range-gate length around the desired measurement to produce a good estimate for defining the center of the search space (Frehlich et al. 1997). Another option that is attractive for numerical weather prediction is using a model prediction for the center of the search space. The velocity-difference algorithm is ideally suited to space-based measurements (Frehlich 2000) because large regions of the atmosphere are sampled for each data product, which results in \(\Delta h > \Delta p\) and the spectral-based and covariance-based algorithms are not valid (F01).

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