The Retrieval of Stratus Cloud Droplet Effective Radius with Cloud Radars

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ABSTRACT

In situ samples of cloud droplets by aircraft in Oklahoma in 1997, the Surface Heat Budget of the Arctic Ocean (SHEBA)/First ISCCP Regional Experiment (FIRE)-Arctic Cloud Experiment (ACE) in 1998, and various other locations around the world were used to evaluate a ground-based remote sensing technique for retrieving profiles of cloud droplet effective radius. The technique is based on vertically pointing measurements from high-sensitivity millimeter-wavelength radar and produces height-resolved estimates of cloud particle effective radius. Although most meteorological radars lack the sensitivity to detect small cloud droplets, millimeter-wavelength cloud radars provide opportunities for remotely monitoring the properties of nonprecipitating clouds. These high-sensitivity radars reveal detailed reflectivity structure of most clouds that are within several kilometers range. In order to turn reflectivity into usable microphysical quantities, relationships between the measured quantities and the desired quantities must be developed. This can be done through theoretical analysis, modeling, or empirical measurements. Then the uncertainty of each procedure must be determined in order to know which ones to use. In this study, two related techniques are examined for the retrieval of the effective radius. One method uses both radar reflectivity and integrated liquid water through the clouds obtained from a microwave radiometer; the second uses the radar reflectivity and an assumption that continental stratus clouds have a concentration of 200 drops per cubic centimeter and marine stratus 100 cm⁻³. Using in situ measurements of marine and continental stratus, the error analysis herein shows that the error in these techniques would be about 15%. In comparing the techniques with in situ aircraft measurements of effective radius, it is found that the radar radiometer retrieval was not quite as good as the technique using radar reflectivity alone. The radar reflectivity alone gave a 13% standard deviation with the in situ comparison, while the radar–radiometer retrieval gave a 19% standard deviation.

1. Introduction

A number of procedures have been developed recently to estimate the microphysical properties of clouds from millimeter-wave radar observations. In this article we restrict our attention to liquid water clouds; retrievals for ice clouds are described in other studies (e.g., Matrosov 1997). Approaches for retrieving ice and liquid water content were suggested by Liao and Sassen (1994), whose work was expanded on and validated by Sassen et al. (1999). Another retrieval for stratocumulus cloud properties using solar radiation, microwave radiometer, and millimeter-wave cloud radar was developed by Mace and Sassen (2000). Retrieval methods for marine boundary layer cloud microphysics were also developed by Dong et al. (1997) and Frisch et al. (1995). Gossard et al. (1997) approached the problem by using radar measurements of the full spectrum of measured Doppler vertical velocities with deconvolution adjustments to address the effects of atmospheric turbulence. Further work using spectra has been done by Babb et al. (1999). In this study, we assess the quality of two effective radius retrievals; method 1 uses radar reflectivity only, and method 2 is the one described in Frisch et al. (1995).
2. Methods

A method for determining the modal radius from cloud radar and microwave radiometer measurements was developed in Frisch et al. (1995). This method used a lognormal model of the droplet distribution to relate the modal radius to radar reflectivity and liquid water path. They assumed a value for the droplet spread and that the droplet concentration was constant with height. The validity of these assumptions was noted by Davidson et al. (1984), who observed that in stratus clouds the number of droplets per unit volume is almost constant with height. He based this observation on the works of Slingo et al. (1982a,b), Nicholls and Leighton (1986) and Nicholls (1987) also support this assumption. We used the lognormal distribution to represent the droplet distribution rather than the gamma distribution since it is computationally more convenient and was a good approximation for droplet distributions (Borovikov 1961; Levin 1961; Atlas et al. 1989; White et al. 1991). The lognormal cloud droplet distribution is written as

\[ n = \frac{N}{\sqrt{2\pi r_0^2}} \exp\left[\frac{(x - x_0)^2}{2r_0^2}\right], \]

where \( N \) is the cloud droplet concentration, \( x = \ln(r) \), \( x_0 = \ln(r_0) \), \( r \) is the droplet radius, \( r_0 \) is the median radius, and \( \sigma_x \) is the logarithmic spread of the distribution. The moment of the distribution is

\[ \langle r^4 \rangle = r_0^6 \exp\left[\frac{k^2}{2\sigma_x^2}\right]. \]

One important variable in cloud radiation transfer is the effective radius. It is defined as the third moment over the second moment, and for a lognormal drop size distribution it is related to the median radius by

\[ r_e = r_0 \exp\left[\frac{5}{2}\sigma_x^2\right]. \]

The radar reflectivity factor for a lognormal drop size distribution is

\[ Z = 2^6N\langle r^4 \rangle = 2^6Nr_0^6 \exp 18\sigma_x^2. \]

Solving for \( r_e \) in Eq. (4) gives retrieval method 1:

\[ r_e = \frac{1}{2}\left(\frac{Z}{N}\right)^{1/6} \exp(-0.5\sigma_x^2). \]

Using aircraft measurements of cloud stratocumulus, Fox and Illingworth (1997) noted an almost one-to-one relationship between \( r_e \) and the reflectivity factor. From Eq. (5), we can see that relatively large changes in \( N \) or \( \sigma_x \) will produce small changes in the effective radius; therefore, if we have an estimate of the droplet concentration and the droplet spread, \( r_e \) can be retrieved from \( Z \). If microwave radiometer measurements are available for estimating the integrated liquid water, and \( N \) and \( \sigma_x \) are constrained to be constant with height, we can use our Eq. (3), Eqs. (3) and (4) of Frisch et al. (1995), and Eq. (9) of Frisch et al. (1998) to solve for the effective radius as a function of height. This gives us retrieval method 2:

\[ r_e(h) = \frac{Z_{1/6}^{1/6}(h)}{2Q^{1/3}} \left\{ \frac{\pi \rho}{6} \left[ \sum_{i=1}^{n} Z_{1/2}(h_i)\Delta h \right] ^{1/3} \right\} ^{1/3} \exp(-2\sigma_x^2), \]

where \( h_i \) is height in the cloud, \( i = 1 \) and \( i = m \) represent the radar range gate at cloud base and cloud top, respectively. \( \Delta h \) is the radar range gate thickness, \( \rho \) is the water density, and \( Q \) is the microwave radiometer-derived integrated liquid water through the depth of the cloud. Note that the additional measurement eliminates the need to know the value of \( N \); however, we still need an estimate of \( \sigma_x \).

To evaluate the retrieval errors introduced by our assumptions and measurement errors, we derive the errors in Eqs. (5) and (6). The relative error in retrieval method 1 [Eq. (5)] is

\[ \varepsilon = \frac{\Delta r_e}{r_e} = \pm \left[ \frac{(\Delta N)^2}{6N} + (\sigma_x \Delta \sigma_x)^2 + \left(\frac{\Delta Z}{6Z}\right)^{3/2} \right]. \]

Evaluating the error in retrieval method 2 [Eq. (6)] is a little more complicated. The error can be written as

\[ \Delta r_e(h) = \left[ \left(\frac{\Delta Z}{6Z(h)}\right)^2 + \left(\frac{\Delta}{\int_{h_1}^{h_2} Z(h)^{1/2} dh}\right)^{1/2} \right]^{1/2} \]

\[ + (4\sigma_x \Delta \sigma_x)^2 + \left(\frac{\Delta Q}{3Q}\right)^2 \right]^{1/2}. \]

Equation (8) shows that even though there is no longer an error due to \( N \), we have increased the contribution of the error in \( \sigma_x \) by a factor of 4 and have added some error due to the measurement of the integrated liquid water. There is an added complication resulting from the second term under the radical involving the height integral of the reflectivity factor. In order to evaluate the error in Eq. (8), we can approximate Eq. (8) by assuming that the second term is negligible. This is a reasonable approximation if the error in \( Z \) is random and the cloud thickness is several range gates thick. However, there could be a bias in the reflectivity, but if this bias is small compared to the actual reflectivity, then its contribution to the integral can be neglected also. This should be the case with most calibrated cloud...
radars with a good cloud signal. With this term eliminated, Eq. (8) becomes

\[
\frac{\Delta r_e(h)}{r_e(h)} = \pm \left[ \frac{\Delta Z}{6Z(h)} \right]^2 + \left( 4\sigma_e \Delta r_e \right)^2 \left( \frac{\Delta Q_e}{3Q_e} \right)^{1/2} \raiseshaded(9)
\]

3. Measurements

In order to determine the values in the parameters needed for the two retrievals, we used measurements from both continental and marine stratocumulus clouds. The first set of measurements was obtained during an April 1997 Intensive Observation Period (IOP) at the Atmospheric Radiation Measurement (ARM) Program’s Southern Great Plains (SGP) site near Ponca City, Oklahoma. The second set was obtained from instrumented aircraft that flew from April to July 1998 during the First International Satellite Cloud Climatology Project (ISCCP) Regional Experiment (FIRE)-Arctic Cloud Experiment (ACE) program in the Arctic. In both experiments the droplet size distributions were measured with a forward scattering spectrometer probe (FSSP). For the SGP IOP, an FSSP was installed on the University of North Dakota’s Citation aircraft. For the FIRE-ACE campaign, FSSP measurements were made by the University of Washington’s Convair-580 and the National Center for Atmospheric Research’s C-130. In order to compare reliable statistics, the in situ data were only considered when the liquid cloud particle concentrations were greater than 10 cm\(^{-3}\) and the ice particle concentrations measured by other probes were sufficiently low. The FSSP droplet spectra were used to calculate the particle effective radius, the radar reflectivity, the droplet concentration, and the logarithmic spread of the radii distribution.

For the first retrieval, we used Eq. (5) and the assumption of constant \(N\) and \(\sigma_e\) for marine and continental stratus clouds. We also used Matrosov’s (2000) suggestion of using one value of \(N\) for continental stratus and another value for marine stratus. Measurements of droplet concentration in continental stratus at the ARM SGP site during IOP varied from a low of 25 cm\(^{-3}\) to a maximum of about 400 cm\(^{-3}\), with a few measurements having higher concentrations. The average was 212 cm\(^{-3}\) with a standard deviation of 107 cm\(^{-3}\). From these data we calculated \(\sigma_e = 0.32 \pm 0.09\). We used about 11 000 1-s spectra in these calculations. For the FIRE-ACE marine stratus data, the average \(\sigma_e = 0.34 \pm 0.09\), and the range for \(N\) was from 10 cm\(^{-3}\) to 400 cm\(^{-3}\), with a mean \(N\) of 98 and standard deviation of 74 cm\(^{-3}\). Here we had about 48 000 1-s droplet spectra available. If we assume a reflectivity factor of \(-30\) dBZ, and that we can measure dBZ to \(\pm 1\) dBZ, then its contribution to the error in \(r_e\) will be about 0.08 [this can be derived from the relationship between \(Z\) and dBZ shown by Eq. (8) in Frisch et al. (1995)]. From Eq. (7), by using a value of \(N = 200\) cm\(^{-3}\) for the continental stratus and a standard deviation of 100 cm\(^{-3}\), the error would be about 10% for this effective radius retrieval. Of course, this assumes that our approximation of a lognormal droplet distribution is a reasonable approximation. The marine retrieval error is larger, about 14%, due to the smaller droplet concentration. For the second retrieval, using the same values for \(Z\), \(\sigma_e\), and their standard deviations, plus the assumption that we can measure the liquid water to 20%, the calculated error is about 16%. This liquid water error is an estimate for clouds where the radiometer has a good signal. This error in the liquid water measurement will be a function of the liquid water amount in the cloud plus other variables. Westwater et al. (2001) give a discussion of various radiometer errors.

4. Retrieval comparisons with in situ measurements

We used the measured cloud droplet size distributions to compute both radar reflectivity and particle effective radius size. We plotted the effective radius versus the calculated radar reflectivity for the FIRE-ACE measurements (Fig. 1a), and for the SGP IOP measurements (Fig. 1b). The curves represent Eq. (5) for different values of \(N\) with a value of \(\sigma_e = 0.32\). The measurements from FIRE-ACE show that most of the droplets fall between 10 and 200 cm\(^{-3}\), while results from the ARM SGP IOP show higher droplet concentrations, with most values between 100 and 400 cm\(^{-3}\).

Figure 2 shows the measured values of \(\sigma_e\) for both continental and marine stratus clouds versus calculated values of radar reflectivity. There is a spread in \(\sigma_e\) from about 0.1 to 0.7, similar to the results of Miles et al. (2000); however, most of the values lie in a much smaller band. In order to test retrieval method 1, we used the marine and continental aircraft FSSP data to compute reflectivities for use in Eq. (5). The plot of retrieved cloud droplet effective radius versus FSSP-measured effective radius is shown in Fig. 3. We used a droplet concentration of \(N = 200\) for the continental retrievals and \(N = 100\) for the marine droplet retrievals. Values of \(\sigma_e = 0.32\) and \(\sigma_e = 0.34\) were used for the continental and marine stratus retrievals, respectively. The standard deviation between retrieved cloud droplet effective radius and measured effective radius is about 12% for continental stratus, while the marine standard deviation is about 16%.

During the April 1997 IOP at the SGP site, we had an aircraft instrumented with an FSSP and a two-dimensional precipitation probe (2DP). We compared radar- and radar–radiometer-retrieved cloud droplet effective radius with the in situ FSSP measurements of effective radius. The radar was the National Oceanic and Atmospheric Administration (NOAA) Ka-band cloud radar and the radiometer was the SGP microwave radiometer. We also used the 2DP to determine the number of events that had particles large enough to cause
significant errors in our radar reflectivity estimates of cloud particles. Because of the height error in the aircraft altitudes and the sharp vertical gradients in the radar reflectivity measurements of clouds, the aircraft heights were adjusted by as much as 200 m (see Frisch et al. 2000). We set an arbitrary horizontal circle of 1.5 km around the radar for our comparisons. If the aircraft was within this circle, then we would compare the FSSP and radar-based retrievals. These comparisons were made on 9 April 1997, from 1533 to 1731 UTC.

Figure 4 shows a time series plot of the aircraft FSSP-calculated reflectivity factor along with the radar-measured reflectivity factor for measurements within a 1.5-km horizontal distance from the radar. The measurements track very well until about 1648 UTC, when the radar reflectivity becomes much lower relative to the FSSP-calculated reflectivity. At this time the cloud was rapidly dissipating and probably becoming less horizontally homogenous and not suitable for a comparison. Retrieval method 1 [the radar-only effective radius re-
Fig. 2. Logarithmic standard deviation ($\sigma_x$) of stratus cloud droplets vs calculated radar reflectivity from aircraft measurements with an FSSP. (a) Continental cloud droplets; (b) marine stratus cloud droplets. The blank horizontal stripes in (a) are due to roundoff in the calculations of $\sigma_x$, which is not present in (b).

Fig. 3. Retrieved cloud droplet effective radius values vs calculated radar reflectivity from Eq. (5) using aircraft FSSP measurements of (a) continental stratus and (b) marine stratus. For the continental retrieval, $\sigma_x = 0.32$ and $N = 200$ cm$^{-3}$. For the marine stratus cloud retrieval, $\sigma_x = 0.34$ and $N = 100$ cm$^{-3}$.

As another verification of retrieval method 1, we used data from Pinnick et al. (1985), which were obtained from some other parts of the world. A plot of radar
reflectivity versus particle effective radius calculated from these droplet spectra is shown in Fig. 7. We can see that there is good correspondence between the two variables. We used a droplet concentration of 200 drops per cubic centimeter and $\sigma_x = 0.35$ in these retrieval calculations.

A potential problem with both of these retrievals is that large droplets can occur in the cloud, even for clouds with low reflectivities. Using the ARM SGP IOP datasets from 9 April, we examined 2DP measurements for large particles and found about 20 events during
which large particles were present. The total aircraft flight time in cloud for this analysis was about 90 min, and the sampling rate was 1 sample per second. During this time, there were over 5000 samples, so the 20 or so events appear to be negligible for the continental stratus case.

5. Conclusions

We have compared the results of two methods for determining cloud particle effective radius with in situ aircraft measurements. The first method uses only radar reflectivity factor. The second method is based on Frisch et al. (1995) and uses radar reflectivity factor and a measurement of the integrated cloud liquid water. In both methods, an estimate of the logarithmic spread of the cloud droplet distribution is required. However, large uncertainties in this spread contribute small errors to retrieved effective radius in the technique using radar alone (method 1). In this retrieval, an estimate of the droplet concentration is also required, although large uncertainties in the concentration give small errors in the effective radius. Our examination of the Pinnick et al. (1983) data from other locations also shows the strong correlation between radar reflectivity and effective radius. In retrieval method 2, even though the droplet concentration no longer contributes, we have considerably increased the error in retrieved effective radius due to the cloud droplet logarithmic spread. In addition, we have introduced an error due to the integrated liquid water measurements.

An error analysis based on in situ measurements of both marine and continental stratus clouds shows that the effective radius retrieval accuracy is on the order of 15%. From our limited comparisons between radar retrievals of cloud effective radius and in situ aircraft, the reflectivity-only technique appears to be somewhat superior. Method 1 gave a 13% standard deviation between radar-retrieved effective radius and FSSP measurements, while method 2 gave a 19% standard deviation. The difficulties of the radar–radiometer technique are due to a combination of larger error contributions from the logarithmic spread, errors in the microwave radiometer–derived integrated liquid water, and differences in the beamwidths of the radar and the microwave radiometer.

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