Strategies for Circulation Evaluation of Aircraft Wake Vortices Measured by Lidar

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ABSTRACT

An assessment of different methods for circulation evaluation from lidar measurement data of aircraft wake vortices is performed. The surface integral of vorticity serves as baseline case that is compared to a method that evaluates the lidar line-of-sight velocity midway between the vortices and to another method that calculates radii averages of circulations derived from tangential velocities. Systematic deviations from nominal circulation are discussed based on analytical vortices. High-resolution numerical simulation data are applied to perform virtual lidar measurements that reproduce, explain, and quantify (i) the frequently observed initial overestimation of circulation and (ii) the scatter of circulation data caused by the genuine variability of wake vortices in the atmospheric boundary layer. The theoretically derived characteristics of the different evaluation methods are verified against lidar data recorded by several lidar teams during the Wake Vortex Forecasting and Measuring Campaign at Oberpfaffenhofen (WakeOP), performed in spring 2001 at Fairchild Dornier Airport in Oberpfaffenhofen, Germany.

1. Introduction

As an unavoidable consequence of lift, aircraft generate counterrotating pairs of trailing vortices (cf. Fig. 1), which constitute a potential hazard to following aircraft. The resulting separation distances between consecutive aircraft contribute substantially to capacity shortages of congested airports. Both the complexity of wake vortex physics and the limited experimental access impede comprehensive analyses of wake vortex physics that would allow the mitigation of wake vortex restrictions under appropriate conditions (Gerz et al. 2002). In particular, in laboratory experiments the far-field evolution of wake vortices is not accessible, and in both laboratory experiments and numerical simulations the achievable Reynolds numbers are far from reality.

The only and indispensable access to real wake vortex behavior is provided by lidar measurement techniques that trace full-scale wake vortices in the free atmosphere. Light detection and ranging (lidar) operates by transmitting a laser beam and coherently detecting the radiation backscattered by aerosols. The spectrum of Doppler shifts in the frequency of the backscattered radiation is analyzed to give the line-of-sight (LOS) velocity component of the aerosols and, hence, the air motion, along the beam. From the LOS velocities the circulation of the vortices can be deduced. Details of lidar technology can be found elsewhere (e.g., Constant et al. 1994; Harris et al. 2000).

Circulation constitutes the most important parameter for wake vortex characterization since it describes vortex strength in a form that is correlated with effects of potential wake encounters (Hinton and Tatnall 1997). However, the evaluation of circulation from lidar data involves considerable complications. First, environmental conditions that can neither be controlled nor reproduced have a strong impact on vortex evolution (Spalart 1998; Holzäpfel et al. 2003). Favorable neutral envi-
The investigations in the current manuscript are mostly restricted to kinematic effects of wake vortex characteristics on circulation evolution; aspects of lidar technology are neglected. Three different methods to derive circulation are applied to a pair of Lamb–Oseen vortices, to large eddy simulation (LES) results of wake vortices that evolve in a convectively driven atmospheric boundary layer (Holzäpfel et al. 2000), and to simulation results dealing with the roll-up of multiple vortices behind an aircraft in high-lift configuration (Stumpf 2002). The aim of the investigation is to understand the implications of the respective evaluation methods and to serve as a guideline for the proper choice of an evaluation method for circulation based on lidar data. For this purpose (i) the systematic deviation of the circulation derived with the different methods from the root circulation is discussed, (ii) the cause of the frequently observed initial overestimation of circulation is revealed, and (iii) the degree of scatter of circulation data is considered that is due to the genuine variability of wake vortices in the atmospheric boundary layer. Finally, the theoretically derived characteristics of the different evaluation methods are verified against lidar data recorded during the Wake Vortex Forecasting and Measuring Campaign at Oberpfaffenhofen (WakeOP) performed in spring 2001 at Fairchild Dornier Airport in Oberpfaffenhofen, Germany.

2. Methods to evaluate circulation

The circulation $\Gamma$ around a plane closed contour $C$ is defined as the line integral of the tangential velocity component, $v_t$:

$$\Gamma = \oint_C v_t \, ds = \int_A \omega \, dA. \quad (1)$$

Employing the Stokes theorem, circulation can equally be expressed by the integral of vorticity $\omega$ over the surface $A$. For circular integration areas, Eq. (1) can be expressed by

$$\Gamma(r) = \int_0^{2\pi} \int_0^r \omega(r, \varphi) r \, dr \, d\varphi, \quad (2)$$

where $r$ denotes the radius and $\varphi$ is the azimuthal angle in a cylindrical polar coordinate system. Equations (1) and (2) can only be applied if two-dimensional velocity data are available. For a single, axisymmetric vortex, circulation on a given radius can be simply obtained from the tangential velocity at that radius according to

$$\Gamma(r) = 2\pi v_t(r). \quad (3)$$

One appropriate method to calculate circulation of wake vortices is to average circulation over $n$ radii in a radii interval, $r_i \leq r \leq r_{i+1}$, with a lower bound $r_i$ and an upper bound $r_{i+1}$, according to

$$\Gamma_{r_i \rightarrow r_{i+1}} = \frac{1}{n} \sum_{i=1}^{n} \Gamma(r_i), \quad (4)$$

where the increment of subsequent radii typically is set to 1 m. For lidar data evaluation it is assumed that the maximum LOS velocity corresponds to the tangential velocity $v_t$ of the respective vortex, and Eq. (3) is used to determine the circulations that are averaged according to Eq. (4). It is advantageous to perform the averaging for both flanks of the vortex (cf. Fig. 2), that is, for opposite directions of the tangential velocity, because then the influence of ambient wind and self-induced descent speed are compensated automatically. This approach is termed method $v_t$.

Radii-averaged circulation according to Eq. (4) can equally be determined from two-dimensional velocity data employing Eq. (2). This is called method $\omega$. Since, however, the vorticity in Eq. (2) cannot be derived from lidar data, method $\omega$ is only used as a reference for.
circulations evaluated from virtual lidar measurements in sections 4 and 5.

For methods $v_i$ and $\omega$, the effect of the neighboring vortex should be restricted by limiting $r_i$ at most to half of the vortex spacing, $b_0/2$. This limitation also reduces disturbing effects of secondary vortices, ambient turbulence, and baroclinic vorticity. The lower limit $r_l$ should be set well above the core radius. Including radii of the order of the core radius and below would decrease the resulting circulation unnecessarily. Furthermore, velocities at small radii are difficult to measure by lidar due to the smaller volume of scatterers. The basic benefit of methods $v_i$ and $\omega$ is that the averaging of $G$ over a radii interval reduces the scatter in turbulent vortices and, thus, also enables estimations of disintegrating vortices. The radii-averaged circulation definition has a good potential as a measure for vortex strength in reduced spacing operations because it correlates well with hazardous effects induced from potential wake encounters (Hinton and Tatnall 1997).

The downdraft method $w_{d}$ proposed by Harris et al. (2000) calculates circulation from the downdraft velocity midway between the vortices, $w_m$, according to

$$\Gamma_d = \pi w_d b_0/2. \quad (5)$$

For a pair of undisturbed Lamb–Oseen vortices, $\Gamma_d$ differs negligibly from the root circulation. An advantage of this method is the high lidar signal-to-noise ratio that results from the great volume of air moving near the downdraft velocity midway between the vortices. The effects of crosswind $v$, vertical wind $w$, lidar scan angle $\theta$, and vortex tilt angle $\phi$ on the measured downdraft velocity $w_m$ can be compensated according to

$$w_d = \frac{w_m + v \cos(\theta) - w \sin(\theta)}{\sin(\theta - \phi)}. \quad (6)$$

Figure 3 illustrates that the lidar scan angle and vortex tilt angle can be combined to define an effective lidar observation angle, $\gamma = \theta - \phi$, which denotes the angle between the line that connects the vortex centers and the lidar scan direction. Figure 3 also describes the coordinate system used in this article.

3. Analytical considerations

This section aims at illustrating the effects of evaluation methods $v_i$ and $w_{d}$, averaging interval, and observation angle on the obtained results in circulation. For this purpose it is assumed that an ideal continuous-wave lidar records exact maximum LOS velocities of analytically given wake vortices. These consist of the superposition of two Lamb–Oseen vortices where the tangential velocity profile of one vortex, $v_i(r)$, is given by

$$v_i(r) = \frac{\Gamma_0}{2\pi r} \left(1 - \exp\left(-\frac{1.26r^2}{r_c^2}\right)\right). \quad (7)$$

with a core radius of $r_c = 4$ m, a vortex separation $b_0 = 23.5$ m, and a root circulation $\Gamma_0 = 283$ m$^2$ s$^{-1}$. Figure 2 sketches the respective profiles of the vertical velocity component. It is assumed that the trends observed in our study would also occur for different vortex models. The observation angle of the lidar is kept constant during measurements—an approximation that corresponds to “frozen” vortex pairs drifting through the lidar beam. We refer to Campbell et al. (1997) for a similar discussion of the effects of averaging interval and observation angle pertaining to method $v_i$.

For the following considerations the absolute value of $\Gamma_0$ is irrelevant (we employ circulations normalized by $\Gamma_0$, where the normalization is indicated by an asterisk). As a baseline case the radii-averaged circulation uses a lower bound of 5 m and an upper bound of 11 m. To demonstrate the effects of the chosen averaging

![Figure 2](image2.png)

Fig. 2. Sketch of the vertical velocity profiles of two Lamb–Oseen vortices separated by $b_0$ (fine lines) and corresponding vertical velocity envelope of the trailing vortex pair (bold line). Radii interval, $r_i \leq r \leq r_o$, used for averaging of circulation in method $v_i$ indicated for left vortex in gray.

![Figure 3](image3.png)

Fig. 3. Sketch to illustrate the definition of lidar scan angle $\theta$, vortex tilt angle $\phi$, effective observation angle $\gamma$, and the coordinate system used in the current article. The axial direction $x$ points against the flight direction.
underestimation of the circulation for a singular vortex limit of $b_0$.

As an exemplary result we find a maximum that fall below those minimum vortex spacings are not beyond the midpoint. (Therefore, the parts of the curves which can be applied without inclusion of velocities the lower limit for vortex spacing (which is twice of the averaging interval. That upper limit further dictates obtained circulation values increase with the upper limit in this case is always below $G_r$. Derestimates the circulation for between 3 and 8 m, on the other hand, nevertheless un-

interval, further results with 3–8- and 5–15-m intervals are included.

**a. Vertically viewing lidar**

We now restrict the analysis to vertical observation angles; hence, only vertical velocities are measured. Figure 4 depicts the influence of the vortex spacing on normalized circulation obtained by method $v_l$ for three different radii averages. For a singular vortex, $\Gamma^{*}_{r_{l}}$, $\Gamma^{*}_{3-11r}$, and $\Gamma^{*}_{5-15r}$ amount to 0.833, 0.967, and 0.979, respectively (horizontal lines). This underestimation of the root circulation is caused by the decreasing circulations on radii smaller than $2r_c$ [at about $2r_c$ the Lamb–Oseen vortex attains the potential flow with $\Gamma(r) = \Gamma_0$. However, when we apply the analysis to a vortex pair, we find that $\Gamma^{*}_{3-11r}$ and $\Gamma^{*}_{5-15r}$ are overestimated. Figure 2 indicates that the neighboring vortex induces superimposed downward-directed velocities in the averaging interval. In the downwash region the superimposed velocities cause an overestimation of circulation that dominates the weaker underestimation induced in the upward-directed vortex flow. An average over radii between 3 and 8 m, on the other hand, nevertheless underestimates the circulation for $b_0 > 16$ m, since $\Gamma(r)$ in this case is always below $\Gamma_0$.

It becomes evident that for a given vortex spacing the obtained circulation values increase with the upper limit of the averaging interval. That upper limit further dictates the lower limit for vortex spacing (which is twice $r_c$) for which $\Gamma^{*}_{r_{l}}$ can be applied without inclusion of velocities beyond the midpoint. (Therefore, the parts of the curves that fall below those minimum vortex spacings are not discussed.) As an exemplary result we find a maximum overestimation of $\Gamma^{*}_{3-11r} = 1.138$ at the respective lower limit of $b_0 = 22$ m. At $b_0 = 47$ m both effects, the underestimation of the circulation for a singular vortex and the overestimation caused by the neighboring vortex, compensate each other such that $\Gamma^{*}_{3-11r} = 1$.

Circulations determined by the downdraft method $w_d$ as a function of vortex spacing are shown in Fig. 5. Measuring exactly at the midpoint between both vortices (curve $b_0/2$), the normalized circulation attains a value of 1 at $b_0 = 20$ m ($b_0/r_c = 5$). Since the velocities in the downdraft region usually are not smooth, the circulation scatter may be reduced by calculating $w_d$ as an average over laterally adjacent velocities. However, this averaging also overestimates circulation (when $b_0 > 16$ m), since method $w_d$ employs the local minimum velocity and averaging and, thus, includes higher neighboring $w_d$ values (cf. Fig. 2). (When $b_0$ drops below 16 m the overestimation turns into an underestimation because for $b_0/r_c < 4$ lower velocities from the vortex core region are affecting the evaluation.) For vortex spacings of $b_0/r_c > 4$, which in general prevail, we learn that averaging causes an overestimation of circulation that increases with the averaging length and decreases with increasing $b_0/r_c$. For example, averaging along a 2-m (4 m) section causes a maximum overestimation of 0.6% (1.9%) of circulation.

**b. Oblique viewing lidar**

We now generalize our analysis and allow our ideal lidar to view at angles $\gamma$ between 90° and 45° (again with fixed beam, no scanning). Figure 6 depicts the actual positions where the continuous-wave lidar detects the maximum velocity values along its LOS in a trailing vortex pair. For a single vortex, measurements at an oblique view will not deviate from that at a vertical view as long as the vortex is axisymmetric.

For vertical observation directions ($\gamma = 90°$) the figure corroborates that the LOS velocity maxima are situated on a horizontal line. For observation angles below 90° the actual positions of the measured maximum LOS

![Figure 4](image_url)

**Fig. 4.** Here $\Gamma^{*}_{r_{l}}$, determined according to method $v_l$, for a pair of Lamb–Oseen vortices as a function of vortex spacing $b_0$, and for three different averaging intervals. The values of the corresponding single vortices are denoted by horizontal lines. Hatched lines indicate applicable lower boundaries for $b_0$.

![Figure 5](image_url)

**Fig. 5.** Circulation determined according to method $w_d$ for a pair of Lamb–Oseen vortices as a function of vortex spacing and averaging length along downdraft velocities.
velocities deviate from their nominal locations: now the lines of maximum LOS velocity position are curved. The curvature increases with increasing distance to the vortex centers and becomes strongest in the downdraft region where the influence of the neighboring vortex is most prominent.

In the regions around the vortex cores, where the rotational velocities approach zero, even the ideal lidar applied to analytical vortices finds positions with higher LOS velocities far behind or in front of the vortex centers. However, these outliers, which are found for all observation angles, are of no relevance for the evaluation of circulation, because the core region is excluded by the choice of $r_l$.

Figure 7 shows circulations determined with method $v_t$ for different observation angles and radii intervals. The overestimation of $\Gamma_{z,11}^\gamma$ at $\gamma = 90^\circ$ decreases gradually from 1.118 to a minimum of 1.029 at $\gamma = 55^\circ$, then it rises again to a maximum at $\gamma = 35^\circ$, followed by a sharp decline. The overestimation declines generally for angles that deviate from $90^\circ$ because the contribution of the velocities that are induced by the neighboring vortex decreases with decreasing observation angle: to first order, only the induced vertical velocity component contributes to the overestimation, whereas the induced horizontal velocity component has different signs on the different sides of the vortex core and therefore is compensated in the integration.

It is worthwhile to note that for geometrical reasons the LOS velocities of the adjacent vortex are included in the circulation computation when $\gamma < \gamma_t = \arcsin(2r_u/b_0)$. Then the detected maximum LOS velocity position “jumps,” for example, from the left to the right vortex (cf. Fig. 6). For the current parameters ($r_u = 11$ m, $b_0 = 23.5$ m), $\gamma_t$ is $70^\circ$. However, the overestimation of $\Gamma_{z,11}^\gamma$ grows from its minimum only again when $\gamma < 55^\circ$ (instead of $70^\circ$) because then the contribution of LOS velocities beyond the separating symmetry line $y = 0$ dominates the decreasing overestimation caused by the decreasing observation angles. For observation angles below $35^\circ$, the integration domain also includes the decreasing rotational velocities in the adjacent vortex core region and, hence, $\Gamma_{z,11}^\gamma$ decreases again. Figure 7 finally shows that the discussed characteristics of the curve for $\Gamma_{z,11}^\gamma$ are also found but shifted to higher (lower) observation angles when $r_u$ is increased (decreased).

Figure 6 indicates that the actual position of the LOS
velocity maximum that is attributed to the midpoint downdraft velocity (for method \( w_d \)) is situated considerably apart from the midpoint position for \( \gamma < 60^\circ \) (the asterisk is the first symbol that does not appear at the midpoint position). As a consequence, the corresponding velocities are adulterated. In Fig. 8 the relative deviations of the downdraft velocities \( w_d \) are plotted for the different observation angles where the LOS velocities \( w_m \) are already translated into downdraft velocities \( w_d \) according to Eq. (6). Figure 8 reveals a considerable excess of \( w_d \) at \( \gamma = 0^\circ \) and, consequently, an overestimation in circulation when \( \gamma , \gamma_{\text{min}} \). A closer inspection yields the minor relative error of 0.8% at \( \gamma_{\text{min}} = 57^\circ \). If the downdraft velocity is calculated as an average of neighboring positions in order to smooth circulation scatter, overestimations of downdraft velocities occur already for \( \gamma > \gamma_{\text{min}} \). These deviations again are caused by the offset between nominal and actual positions of the maximum LOS velocities (see Fig. 6).

4. Turbulent environment

In this section we investigate how the vortex evolution in an inhomogeneous turbulent environment affects circulation measurements by lidar. For this purpose an LES of wake vortices in an evolving convectively driven atmospheric boundary layer (Holzäpfel et al. 2000) is analyzed. The LES was performed in a domain with a uniform grid of size \( L_x = L_y = L_z = 512 \) m. The convective boundary layer simulation was driven by a constant vertical heat flux at the lower surface and three wake vortex pairs were superimposed on the turbulent flow field after the evolving convective boundary layer was well established. Figure 9 illustrates the interaction of convective cells and 5-s-old wake vortices in a perspective view of an isosurface of the upward-directed velocity \( w = 2 \) m s\(^{-1}\). The LES data are rescaled such that the dimensions of the vortices correspond to the vortices in sections 3 and 6. The rms value of the fluctuation velocities of the modeled boundary layer is \( q = 1 \) m s\(^{-1}\), which corresponds to a normalized value of \( q^* = q/w_0 = 0.54 \), where \( w_0 \) denotes the initial descent speed of the vortices.

The LES data are used to simulate observations of the upper-left vortex pair by an ideal, vertically viewing lidar. It is assumed that the maximum and minimum velocities of the LES data field in a height interval ranging from 200 to 500 m above ground correspond to the maximum and minimum velocities derived from the spectra of a continuous-wave heterodyne lidar. This implies the following assumptions and simplifications. The angle variation of the scanning lidar and the resulting trigonometrics are neglected for the sake of simplicity. The range resolution always includes the complete vortices with an equal weighting of measured velocities. The velocity maxima and minima do not include the possible biasing effect that can be caused in practice by spectral spreading due to time series windowing (Harris et al. 2000; Campbell et al. 1997). A single lidar scan detects instantaneous velocity data; that is, there is no development of the velocity field during one scan.

The resulting profiles of vertical velocities in a plane at \( x = 192 \) m (denoted by the rectangle in Fig. 9) are plotted in Fig. 10 in increments of 5 s. This section of the vortices is placed on the shoulder of an updraft and thus is exposed to strong lateral gradients of vertical wind that cause pronounced vortex tilting. The effect of the updraft causes a clearly visible variation of the velocity maxima already at \( t = 0 \) s. With ongoing time the vortex signatures are progressively eroded such that at \( t = 30 \) s the vortices can hardly be identified. Note
that at $t = 30\, s$ the vortices, nevertheless, still possess more than 50% of their initial circulation (cf. Fig. 11). This example demonstrates that the evaluation of already strongly deformed vortex signatures still may be worthwhile.

The respective circulation evolutions determined according to methods $v$, $w_d$, and $\omega$ are displayed in Fig. 11 where method $w_d$ is shown with and without the correction for vortex tilting. For method $w_d$ no lateral averaging along the downdraft velocities is performed. In the synthetic flow field the vortex positions are determined by searching the local minima of $\lambda_2$ with a resolution of 1 m. For real lidar measurements it is assumed that the triangulation method (Köpp et al. 2003, hereafter K03), which is described in more detail in section 6, would yield a similar accuracy for vortex positions and, consequently, vortex spacing and tilt angle.

The different methods yield initial deviations of almost ±20% and converge with progressing time, except method $w_d$ with tilt correction. In this situation of extreme vortex tilting [$\phi(30\, s) = 59^\circ$] the correction for vortex tilting causes an overestimation of circulation values. As it is shown in the previous section, this is due to the fact that for larger tilt angles the maximum LOS velocities prevail in front of or behind the line connecting the vortex centers, that is, closer to one of the vortices. In the current example the limiting observation angle $\gamma_{\min} = 57^\circ$ ($\phi = 33^\circ$) is reached already at $t = 11\, s$. After that time method $w_d$ with tilt correction yields too high values. In reality the variable lidar scan angle and the threshold angle for the tilting correction should always be combined to calculate the effective observation angle that can be used to rule out flawed tilting corrections.

1 The second eigenvalue $\lambda_2$ of the symmetric tensor $S^2 + \Omega^2$ is a measure for coherent vortex structures according to Jeong and Hussain (1995). Here $S$ and $\Omega$ are the symmetric and antisymmetric parts of the velocity gradient tensor $\nabla u$. 

Fig. 11. Circulation evolution for methods $v$, $w_d$, and $\omega$ at $x = 192\, m$. 

Fig. 10. Profiles of maximum and minimum vertical velocities at $x = 192\, m$ and different instants of time.
Figure 12 shows circulation evolution where the vortices are advected with an axial wind of $u = 1.6 \text{ m s}^{-1}$ along the flight direction. The initial plane corresponds to the plane evaluated in Fig. 11. The comparison of Figs. 11 and 12 indicates that axial wind may have considerable impact on estimated circulation. In particular, axial wind may cause an ostensible constancy or even increase of circulation with time when less-decayed vortex segments are advected into the measurement plane. Basically, axial wind may increase scatter provided that the spatial vortex evolution features axial gradients.

To reveal a more representative characterization of the three methods, the respective circulations are averaged over all simulation planes along the flight direction (see Fig. 13). Table 1 indicates that the mean initial circulation values correspond well to the theoretical values. In particular, method $v$ reproduces almost the theoretical value for of a single vortex; that is, method $v$ is not sensitive to the influence of the neighboring vortex.

Table 1. Theoretical initial circulation and mean initial circulation of LES determined according to the three methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\Gamma_{0,\text{theor}}$</th>
<th>$\Gamma_{0,\text{LES}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>0.967</td>
<td>0.957</td>
</tr>
<tr>
<td>$v$</td>
<td>1.113</td>
<td>1.124</td>
</tr>
<tr>
<td>$w_d$</td>
<td>1.0</td>
<td>1.018</td>
</tr>
</tbody>
</table>

Remarkably, the radii-averaging methods $\omega$ and $v$, and also method $w_d$ without tilt correction, show similar decay characteristics [$\Gamma^*(t)$ slope], where the systematic initial offset between the different methods is maintained during vortex evolution. Only the downdraft method $w_d$ yields no initial decay until $t = 5$ s. Additionally, circulation is determined by integration of vorticity on circles with a radius of $b_0/2$ as an indication of the real midpoint circulation evolution (termed method $w_d,\text{ref}$). The reference method $w_d,\text{ref}$ suggests a slower initial decay because the diffusion process that reduces circulation on smaller radii is not present at $y = b_0/2$. Until $t = 15$ s, method $w_d,\text{ref}$, which is not affected by tilting effects, follows the curve with tilt correction. Later on, when in some sections tilt angles of 30° are exceeded, method $w_d,\text{ref}$ approaches the curve without tilt correction.

Another remarkable observation is the similar magnitude of the standard deviation of the circulation, $\Gamma^*(t)$, of methods $v$ and $\omega$. Obviously the main benefit of averaging is achieved by the radii average, whereas the additional averaging of the vorticity over the integration area in method $\omega$ is of minor importance. The scatter of method $w_d,\text{ref}$ is increased compared to methods $v$ and $\omega$ because the integration over the larger area encloses more ambient turbulence. The standard deviation of method $w_d$ indicates larger scatter where the tilting correction reduces the scatter until $t = 20$ s. Later on the scatter is even increased with the correction. This again indicates that the tilt correction may improve estimated circulation for moderate tilt angles but for larger tilting may adulterate results.

5. Multiple vortex pairs

Initial overestimations of the root circulation of typically 30%–60% are frequently observed by lidar especially when method $v$ is applied (Campbell et al. 1997; Robins et al. 2001; Holzäpfel 2003). That overestimation is associated with an ostensible strong initial “decay” such that at a vortex age of roughly one timescale ($t_0 = 2\pi b_0/\Gamma_0$) normalized circulation attains a value of one. Recent analyses of numerous experimental data indicate that the completion of the roll-up for approaching aircraft equally may last about one timescale (A. Elsenaar 2001, personal communication). The correlation between circulation overestimation and roll-up process is substantiated in this section.
To reveal the mechanisms that cause the initial circulation overestimation we perform virtual lidar measurements based on high-resolution numerical simulation data that cover the generation and merger of multiple vortex pairs behind an aircraft model in high-lift landing configuration. The simulation treats the flow around the Deutsches Zentrum für Luft und Raumfahrt (DLR)–Aerodynamische Leistungsverbesserung an subsonischen Transportflugzeugen (ALVAST) model, which is similar to the Airbus A320, by solving the Euler equations up to a half-span downstream of the wing tip. The subsequent vortex evolution is modeled by a direct numerical simulation. Details of the near-field simulation approach are described in Stumpf (2002). Figure 1 illustrates the resulting topology of multiple vortex pairs. The contours of axial vorticity indicate the vortex sheet that forms immediately downstream of the trailing edge of the wing and then becomes organized in two corotating vortices from outer flap edge and wing tip. The counterrotating vortices on smaller lateral positions stem from the inner edge of the outer and inner flaps, respectively, whereas their centric corotating counterpart detaches from the outer edge of the inner flap.

Figure 14 shows the lateral profiles of maximum and minimum LOS velocities that are measured by the ideal vertically viewing lidar at different downstream positions. With increasing downstream distance the separation of the primary vortices decreases driven by the mutual velocity induction of the multiple vortex system. At a downstream position of one span ($x/B = 1$) the two closely spaced velocity minima at $y/B = 0.8$ indicate the merger process of the corotating outer flap edge vortex and the wing tip vortex. The velocity extrema on smaller lateral positions represent the vortices stemming from the inner edge of the outer flap and from the edges of the inner flap. The observed multiple vortex topology clearly illustrates that the assumption of axisymmetric primary vortices that is applied in method $v_t$ [Eq. (3)] is heavily violated as long as the roll-up process to a single vortex pair is not completed. If method $v_t$ is applied nevertheless, the velocity minima corresponding to the secondary vortices are interpreted as high tangential velocities on large radii appendant to the primary vortex, hence, high circulation. Figure 15 shows that the resulting overestimation, which varies between 67% and 29%, is of the order of overestimations found in field measurements. The reduction of the overestimation with increasing distance $x/B$ is caused less by the gradually proceeding roll-up but rather can be explained as follows. Since the triplet of secondary vortices is rotated below the primary vortex outward, the vertical projection of the distance to the primary vortex decreases (see Fig. 14), which, in turn, reduces circulation. This example demonstrates that the manifold of vortex topologies that evolve during roll-up may cause a considerable variability of the initial overestimation of circulation. In contrast, the ambiguity of the actual vortex center location of the merging wing tip and outer flap edge vortex at $x/B = 1$ (see Fig. 14) leads to a comparatively small circulation variation (see Fig. 15).

Method $w_d$ is less sensitive to remaining secondary vortices as long as these are sufficiently separated from the midpoint position. At $x/B = 1$ we state an overestimation of up to 28% and an increased sensitivity to the determination of vortex core position. From $x/B = 3$ onward the small decay rate is caused by the slowly approaching primary vortices. Method $\omega$ obtains an almost constant circulation of 0.9 because 10% of the circulation was dissipated in the preceding simulation of the flow around the aircraft.
6. Field measurements

The Wake Vortex Forecasting and Measuring Campaign at Oberpfaffenhofen (WakeOP) was performed at the airfield of Fairchild Dornier in Oberpfaffenhofen, Germany, from 29 March to 4 May 2001. An outline of the campaign is given in Gerz (2001). Besides a large variety of meteorological measurement systems, three 10-\(\mu\)m continuous-wave lidars were operated to trace the wake vortices generated by DLR’s VFW 614 aircraft ATTAS. The circulation of the ATTAS varied between 113 and 168 m\(^2\) s\(^{-1}\) with a mean value of \(\Gamma_0 = 142\) m\(^2\) s\(^{-1}\), and the reference timescale was \(t_0 = 12.6\) s on average.

It was successfully demonstrated that vortex spacing and trajectories can be obtained with significantly higher accuracy when triangulating the vortex core intersections of two or three simultaneously measuring lidars (K03). For this purpose the lidars were placed at a lateral offset of approximately 80 m and were scanned in a vertical plane, aligned perpendicular to the aircraft’s flight direction. Vortex core intersections of a lidar give accurate angular information but poor range information. The triangulation method combines two series of accurate angular measurements with an extended Kalman filter to produce the horizontal and vertical positions of the vortices with high accuracy. Here we employ WakeOP lidar data to verify our theoretical considerations concerning the evaluation strategies of circulation. For this purpose vortex positions determined with the triangulation method are applied.

Figure 16 illustrates several interesting features of the lidar spectra that are already translated into LOS velocities. The aircraft passed almost directly above the lidar at an altitude of 146 m. The lidar scan range was from 60° to 95° with a scan rate of 8° s\(^{-1}\). The mean crosswind was approximately 2.5 m s\(^{-1}\), measured by a separate wind-profiling lidar, and this was consistent with the observed horizontal vortex drift velocity. For full details of the measurement geometry see K03.

The background wind field with its fluctuations is evidenced in Fig. 16 by the width and the noisy appearance of the oscillating trace. The tiny solid bar at \(t = 0\) s that lasts for a small fraction of a second indicates that the lidar beam strikes the aircraft. The flow observed immediately afterward shows high velocities and complex structure, which represents the multiple vortices that form during roll-up. Clearly, the evaluation of circulation of such structures is not feasible. The subsequent vortex pairs display the classic vortex pattern. Eventually the vortices drift out of the scanning region. The drift leads to an apparent stretching of the profiles when scanning with the wind, and a contraction when scanning against the wind.

Figure 17 shows lidar spectra of five intersections with the decaying vortex pair. The lidar settings were analogical to that in Fig. 16. The comparison of simulation and measurement (Figs. 10, 17) clearly points out similar characteristics of the LOS velocity evolution during vortex decay. At earlier times the vortex signatures are distinct albeit not perfectly symmetric, whereas at later times the vortex signatures are pro-
progressively eroded such that, for example, the two downdraft peaks vanish in favor of an unstructured downwash region.

Figure 18 depicts the evolution of mean normalized circulation determined from data of the three lidar teams with methods $v_t$ and $w_d$ together with the respective standard deviations. The circulations are normalized by theoretical circulations that are calculated individually from aircraft weight and flight speed of every overflight (OF). The different curves denote averages of the calculations of all evaluated OFs that are combined regardless of the prevailing weather conditions and adjusted aircraft configurations. In spite of the unspecific combination of the respective samples, the statistics corroborate the previously described characteristics of the different evaluation methods.

1) Initial overestimation (cf. section 5). Method $v_t$ applied to QinetiQ and the Office National d’Etudes et de Recherches Aéronautiques (ONERA) data clearly exhibits an initial overestimation of nominal circulation by roughly 40%. At about one timescale (12 s) the curves reach a circulation of one. For individual measurements maximum overestimations of 90% are found.

2) Scatter. The standard deviations of the lidar field measurements (Fig. 18) are of the same order of magnitude as found in the LES (Fig. 13). Since in the LES the roll-up is not considered, standard deviations start from a lower level and increase with time driven by atmospheric turbulence. The higher initial standard deviations and, in particular, the maximum of ONERA’s standard deviation at 3 s reflect the enhanced variability of the initial overestimation that can be caused by the complex topology of multiple vortex pairs during roll-up. Method $w_d$ applied to QinetiQ data yields, indeed, increased standard deviations compared to method $v_t$ for data of the same lidar team.

Since in the field data the average environmental turbulence intensity is even higher than in the sim-
Circulations evaluated by DLR show less systematic overestimation and reduced scatter. However, such circulation data are only achieved in an interactive and iterative evaluation procedure where an experienced scientist is in the loop and introduces adequate threshold levels for the signal intensity attributed to the LOS velocity and rejects spurious data. Nevertheless, these data follow the expected trends discussed below.

3) The effect of neighbor vortex and internal diffusion. Method \( v_1 \) initially yields slightly higher circulation values compared to method \( w_d \) caused by the superimposed velocities of the neighbor vortex. These are followed by a slightly steeper circulation decline, which originates from diffusion processes in the vortex. The final decay phase is not observed because in most cases the vortices were already advected out of the observation area covered simultaneously by both lidar systems before the final decay.

7. Summary

The evolution and decay of circulation of aircraft trailing vortices constitutes an essential issue of wake vortex research. For full-scale vortices this key parameter can, to date, only be derived from lidar measurements. The current manuscript investigates the impact of different effects that inherently modify circulation values during the different phases of vortex evolution in the atmospheric boundary layer. To understand the nature of the circulation, assessment is a vital interest for all those who aim at safely reducing aircraft separations.

For this purpose, different circulation evaluation methods are applied to (i) analytically given Lamb–Oseen vortices and to virtual lidar velocity data established from (ii) a numerical simulation of the flow around an aircraft in high-lift configuration, which provides the peculiarities of a near-field multiple vortex topology; and to (iii) LES of wake vortices in a convectively driven atmospheric boundary layer, which represents vortex evolution in an inhomogeneous turbulent environment. The investigations reveal specific characteristics of the different evaluation methods and explain and quantify systematic deviations from root circulation.

Averaging over a radii interval of a single vortex leads to a small underestimation of the root circulation due to reduced circulation values at small radii. This underestimation is reproduced well also for vortex pairs when circulation is determined by the integration of vorticity. The degree of the underestimation depends on the averaging interval, the core radius, and the vortex spacing. On the other hand, the evaluation of radii-averaged circulation from vertical maximum LOS velocities according to method \( v_t \) leads to an overestimation of the root circulation due to the impact of the neighboring vortex. The degree of the overestimation again is a function of the last named parameters and the observation angle \( \gamma \). The overestimation decreases slightly when the angle of observation deviates from 90°. In the quite large angle domain \( 90° \geq \gamma \geq 40° \) it varies between 2.9% and 11.8%.

The downdraft method \( w_d \) evaluates circulation from the tangential velocity midway between the vortices. This position corresponds to the radius where the determined circulation comes most close to root circulation. For sufficiently tight vortex cores, method \( w_d \) causes a slight overestimation of the root circulation only when the downdraft velocities are smoothed by averaging. However, the correction for vortex tilting may significantly adulterate circulation values when the (observation) angle between the line connecting the vortex centers and the scan direction falls short below 57°. In a real turbulent vortex already smaller tilt angles may cause ambiguous tilt corrections.

During the roll-up of the complex near-field wake topology generated by a high-lift wing the assumption that the sensed maximum line-of-sight velocities can be attributed to the tangential velocities of a pair of axisymmetric vortices does not hold. As a result the circulation evaluation from lidar data can be flawed. Fortunately, the roll-up phase typically is accomplished after one timescale and therefore is irrelevant for separations to following aircraft. As long as the roll-up to a single vortex pair is not completed, the radii-averaging method \( v_t \) interprets vorticity stemming, for example, from the edges of a flap as high tangential velocities on large radii, hence, high circulation. Initial overestimations of the root circulation of 30%–70% are found in measurements and simulation. Method \( w_d \) is less sensitive to remaining secondary vortices as long as these are sufficiently separated from the midpoint position.

LES data indicate that considerably eroded vortices, which already have lost the classic signature and therefore can scarcely be identified as wake vortices, nevertheless may still possess more than 50% of their initial circulation. This result emphasizes the difficulties associated with the investigation of final vortex decay. Since, on the other hand, vortex decay is of primary interest for wake vortex separations, we recommend to push the evaluation of already strongly deformed vortex signatures as far as possible.

A comparison of measurements accomplished during the WakeOP campaign and the LES data corroborates our theoretical findings. In particular, we show that the degree of scatter of circulation data observed after the completion of roll-up appears to be mostly due to the genuine variability of wake vortices in the atmospheric...
boundary layer. This scatter, consequently, neither can be avoided nor should be artificially smoothed. The only way to further reduce this intrinsic scatter is to average over several nominally identical measurements with respect to aircraft configuration and atmospheric conditions where the measurements can be based on multiple overflights and/or multiple lidars.

We have shown that the two investigated circulation evaluation methods are of complementary character and therefore should both be applied if possible. It was further shown that the circulation of the primary vortices may be best approximated when an experienced scientist evaluates data in an interactive and iterative evaluation procedure. However, for circulation evaluation in an operational reduced spacing system, time-consuming interactive procedures are inapplicable. Here the radii-averaged approach seems most appropriate. Sure enough, the method suffers from a systematic overestimation of the root circulation but there is no overestimation in terms of encounter metrics for follower aircraft. The velocities that apparently increase circulation would also be sensed by a following aircraft in an encounter. Further advantages are that no ambient wind data are needed to evaluate circulation, the radii averaging reduces scatter, and the observation angle domain is less restricted.

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REFERENCES


