Impact of the Incremental Analysis Updates on a Real-Time System of the North Atlantic Ocean

M. BENKIRAN AND E. GREINER

CLS/DOS/OO, Ramonville Saint Agne, France

(Manuscript received 25 October 2006, in final form 28 March 2008)

ABSTRACT

Incremental analysis updates (IAUs) are a procedure by which analysis increments can be incorporated into a model hindcast and forecast in a smooth manner. It is similar to nudging but has a better response, particularly in regions of missing data. The IAU procedure was popular in the late 1990s in weather forecasting centers, because it acts as a low-pass filter. The impact of the IAU is examined in the context of a real-time, eddy-permitting ocean forecasting system in the North Atlantic from Mercator Océan. Forecast scores and ocean physics are compared for the following three companion runs: a forced mode, a sequential analysis, and IAU. These comparisons confirm that the IAU is beneficial because it removes spinup effects such as spurious waves and tropical convective cells. Forecast scores are also slightly improved. In addition, contrary to the weather forecasting case where the model and data are fairly unbiased, the IAU has the advantage of correcting the systematic biases in the ocean data assimilation system.

1. Introduction

Data assimilation in meteorology has generally been performed in an intermittent manner over the last decades. Data are gathered over a given period of time (say 6 h), and some optimal interpolation or 3D variational analysis is performed to produce the best analyzed fields. The model is then restarted from these fields, and run in a forecast mode, over a few days. However, the imbalance between the analysis increment and the model physics generally produces model shock. This creates spurious oscillations and, ultimately, leads to data rejection. For the atmosphere, the main nonphysical adjustment process is excessive rainfall during the first model hours. The effect of a valid but localized data point is also often radiated away in the form of gravity waves. This reduces the benefit of the data, because part of the analysis increment is rejected by the model.

This severe problem has been solved by operational weather centers in different ways. Early solutions relied on the damping properties of the Euler-backward time scheme (Matsuno 1966) or the filtered leapfrog scheme (Asselin 1972) to control the high-frequency shocks generated by the insertion of observations during model integration. However, this was not efficient enough to filter out all of the spinup effects in the model forecast. Proper initialization procedures were then added somewhere between the analysis and the forecast. The normal-mode initialization (Machenhauer 1977) was widely used in the 1980s and early 1990s, but still with unwanted side effects (nondiagonal tropical winds, remote corrections in data-free areas). The latest improvement came with the use of a digital filter with a diabatic or adiabatic initialization (Lynch and Huang 1992), and many weather centers use some variant of this initialization procedure. It is even included in the minimization of the four-dimensional variational data assimilation (4DVAR) analysis at the European Centre for Medium-Range Weather Forecasts (ECMWF) and Météo-France, and this solves the theoretical problem of finding the analyzed state on a slow manifold (close to the atmospheric attractor).

However, intermittent data assimilation schemes present discontinuities at the analysis time that do not facilitate the analysis of time series. They also tend to degrade the coupling with specific models (either regional or biogeochemical). Continuous data assimila-
tion would reduce this problem. A repeated insertion method was indeed in use at the Met Office in the 1990s (Lorenc and MacPherson 1991). However, instantaneous data were quite sparse, and the continuous data assimilation forced the model with corrections that were local in space and time, without correcting the places where data were recently used or absent. In those regions, spinup effects develop and lead to a large model bias. The result is a decrease of the impact of valid observations, and, ultimately, data rejection.

Bloom et al. (1996) proposed an alternative solution with the incremental analysis updates (IAUs). This is a procedure that incorporates analysis increments into the model hindcast in a smooth manner. It bears some resemblance with the repeated insertion or nudging, but with important differences leading to very different results. The IAU does not allow corrections where there are no data, and it does not dissipate the model’s mesoscale signal. The IAU was popular in the late 1990s in weather centers [e.g., with the three-dimensional variational data assimilation (3DVAR) at the Met Office], because it acts as a low-pass filter. In particular, spinup effects such as excessive rainfall during the first hours of forecast were filtered out, and the forecast skill was improved. The IAU also has been used in the atmosphere for the purpose of either reanalysis [National Aeronautics and Space Administration (NASA) Goddard Space Flight Center (GSFC)] or regional weather forecasting [Lee et al. (2006); either the Catalan Meteorological Service or MeteoGalicia, Spain].

In meteorology, the lack of humidity observations led to a significant inaccuracy of the analysis. Spinup effects have a strong impact because of the complex physics and the change of state from vapor to liquid water (or snow). In addition, both the public and clients perceive errors in rainfall prediction as more serious than similar errors in temperature. In short, humidity is the prognostic variable of the atmospheric models that has the most complex physics, the least data, and the strongest impact on customers. The corresponding variable in the ocean is salinity. However, the physics of salinity is much simpler (no change of state like solid snow, liquid snow, etc.) than its atmospheric counterpart, humidity. The scarceness of observations of salinity is comparable, but oceanographers (navy, fisheries, scientists, ships, exploitation of the deep ocean, etc.) are not highly sensitive to errors in salinity. Spinup effects on salinity are thus less critical than in meteorology.

Recently, the IAU procedure has been employed in the context of ocean forecasting [e.g., Ocean Data Acquisition System (ODAS) El Niño monitoring from the Japan Meteorological Agency, and Japanese Coastal Ocean Predictability Experiment System from Japan’s Marine Science Technology Center], as well as ocean reanalysis (Huang et al. 2002; Carton et al. 2005). However, only Huang et al. (2002) have examined in detail the effects of IAU on a tropical model (resolution about $1.5^\circ \times 1^\circ$ with 20 levels) against Pacific TAO data. Their model has a relatively coarse resolution and short cycling (1 day), and only in situ temperature data are analyzed above 400 m and inserted into the model. The analysis is done independently for each model level, and there are no corrections for salinity or currents. Hence, the interest of the IAU is more to extend the temperature correction below 400 m, and to propagate (extrapolate) it to the other variables. Still, their results are similar to Bloom et al. (1996). Their resulting time series are smoother with the IAU, and present a frequency spectrum that is similar to the in situ observations. The analyzed fields are less accurate because the residuals are larger. However, a more appropriate balance is achieved, and the equatorial subsurface currents are slightly more realistic. Recently, Ourmières et al. (2006) have tested the IAU with an eddy-permitting model of the North Atlantic. Only satellite altimetry and sea surface temperature are assimilated in this study. They use a reduced-order Kalman filter with a 3-day cycle, and a flat-time weighting of the increments of temperature and salinity. Again, they report a filtering of the spurious oscillations, at the expense of their forecast scores that are slightly degraded and the strong gradients that are smoothed.

In the present paper, the impact of the IAU is examined in the context of the real-time, eddy-permitting ($1/3^\circ$) ocean forecasting system of the tropical and North Atlantic from Mercator Océan. This system assimilates both in situ (temperature and salinity) and satellite data (sea level and sea surface temperature) in a multivariate way. Hence, the extrapolation over undetermined variables is less a problem here, and the paper focuses on the control of the spinup effects and the systematic biases. To this purpose, a new variant of the IAU with two sharp time weighting functions is introduced. This system is described in section 2. The assimilation cycling and the experimental set up are presented in section 3. The results are presented in section 4, and a discussion follows in section 5. Conclusions are summarized in the last section.

2. Description of the assimilation system

Mercator Océan has operated a multivariate multiple data assimilation system in real time since January 2004 (http://www.mercator-ocean.fr). This system, called
PSY1v2, provides an oceanic large-scale analysis and a 2-week forecast of the North and tropical Atlantic (70°N–20°S) on a weekly basis. The model is eddy permitting (1/3°, 27 km) for basin-scale studies, biogeochemical coupling, and embedding of area-limited models. Satellite altimetry and sea surface temperature (SST) are assimilated, as well as in situ data. The assimilation is based on a reduced-order optimal interpolation (ROOI). The real-time system uses a simple sequential 1-week cycle (one analysis each Wednesday). These components, as well as the IAU procedure, are detailed in the following sections.

a. Model

The PSY1v2 system uses the ocean model Océan Parallélisé (OPA) 8.0 (http://www.lodyc.jussieu.fr/opa) developed at Laboratoire d’Océanographie et du Climat: Expériences et Approches Numériques (LOCEAN; Madec and Imbard 1996). The model uses primitive equations with the Boussinesq approximation. Details about the model formalism and discretization are given in Blanke and Delecluse (1993). The model configuration was developed by the CLIPPER project (Tréguier et al. 2001). It includes a rigid lid and vertical mixing with a 1.5 turbulence closure scheme. Free slip is used as lateral friction whereas bottom friction is quadratic. Velocity and tracer diffusion are bi-harmonic. The horizontal grid is a 1/3° Mercator grid, with an average resolution of 27 km. This is not fine enough to resolve the eddies, but eddies introduced by the assimilation have a reasonable lifetime, although their trajectories are not always correct. The grid is stretched near Gibraltar for a better resolution of the Mediterranean outflow. The vertical grid is a “Z”-type grid. The 43 vertical levels are distributed so that 20 levels are in the first 1000 m of the ocean. The levels vary in depth from 12 m at the surface to 200 m below 1500-m depth. The depth of the bottom level is 5600 m. Note that the first model temperature node is at 6-m depth. The temperature and salinity near the artificial boundaries are restored to the seasonal climatology (Reynaud et al. 1998). A monthly climatology of attenuation depth for the solar-penetrating flux is derived from Sea-viewing Wide Field-of-view Sensor (SeaWiFS; 1997–2003), and used as a proxy for the water type (Murtugudde et al. 2002). There is no proper ice model, but a diagnostic ice concentration is deduced from the SST data. All forcings are cut proportionally to the ice concentration as with a real ice model (wind and thermohaline fluxes do not affect the water under the ice). Salt rejection during ice formation and freshwater production during melting are parameterized (Greiner et al. 2006). The daily surface forcing comes from the ECMWF operational outputs. The 12–24-h forecasts from the ECMWF 0-h run are cumulated with the 12–24-h forecasts from the 12-h run in order to have the best balanced daily wind stress, heat flux, precipitation, and evaporation. Hence, there is no diurnal signal in this configuration. Thirty-two climatological runoffs (including the Baltic outflow) are input at the surface. There is no restoring term to the climatology for the model sea surface salinity. A 40 W m$^{-2}$ °C$^{-1}$ restoring term is used to constrain the model SST to the observed SST.

b. Data

The Ocean Topography Experiment (TOPEX)-Poseidon (T/P), Jason-1, European Remote Sensing Satellite (ERS), Environmental Satellite (Envisat), and Geosat Follow-On (GFO) altimeter data are assimilated along tracks. Data processing is performed by Segment Sol Multimissions d’Altimétrie, d’Orbitographie et de Localisation Précise (SSALTO; http://www.aviso.cnes.fr/Developing Use of Altimetry for Climate Studies (DUACS; see also http://www.aviso.oceanobs.com/en/data/product-information/duacs/index.html) and is distributed through the Archiving, Validation, and Interpretation of Satellite Oceanographic data (AVISO) user service. Processing includes a correction for the long wavelength biases, except on the shelf (depth less than 200 m), where the altimetry contains high-frequency noise, mostly resulting from the aliased wind and atmospheric pressure effects. The along-track resolution is typically 20 km. The altimeter errors are set at 2 cm for T/P and Jason-1, and 3.5 cm for ERS, GFO, and Envisat. Note however, that only the observed sea level anomalies (SLAs) are kept as final data for the assimilation [relation (1) below]. In other words, the model (superscript $m$) computes an absolute sea surface height (SSH), but it can only be compared to the SLA data (superscript $d$) after an estimate of the mean SSH (MSSH) has been subtracted:

$$\text{SLA}^d = \text{tracks anomalies},$$  \hspace{1cm} (1)

$$\text{SLA}^m = \text{SSH}^m - \text{MSSH}^d.$$  \hspace{1cm} (2)

More precisely, in (1), the altimetry data are track anomalies relative to the 1993–99 period. The unknown difference, namely, the MSSH, is the height corresponding to the mean oceanic circulation, and to the geoid’s variations (Fig. 1). It is estimated with an objective analysis mapping including altimetry, hydrology, drifters, and gravimetry from Gravity Recovery and Climate Experiment (GRACE; Rio and Hernandez 2004). The MSSH is the most critical parameter of a
multidata system, but today it is still not accurately known (the latest version, version 5.1 from June 2005, is used in this system). The MSSH used determines the mean circulation of the model, as well as the thermocline depth and the mean Gulf Stream path. Sensitivity tests to various MSSH products, as well as data withholding (in situ and/or altimetry) suggest that, at low latitudes, an MSSH inaccuracy of 2 cm can typically lead to a temperature bias of 1°C at 100 m. The MSSH error typically ranges from 3 to 6 cm (Fig. 1), which is slightly higher than the SLA error, and sometimes even larger than the sea level variability. This moderate signal-to-noise ratio of the sea level data is probably the biggest difference with data assimilation in the atmosphere. Note finally that the SLA is not used under the model ice.

Another important type of satellite data is SST. The daily, high-resolution, real-time, global, sea surface temperature (RTG_SST) 0.5° × 0.5° analysis (Thiébaux et al. 2003) is produced daily by the National Oceanic and Atmospheric Administration (NOAA)/National Centers for Environmental Prediction (NCEP; information online at http://polar.ncep.noaa.gov/sst/). Details of the procedure can be found in the above reference, but the main points are listed here. It is a refined version of the 1° × 1° weekly Reynolds analysis (Reynolds and Smith 1994). This analysis uses in situ SST and NOAA-16/National Environmental Satellite, Data, and Information Service (NESDIS) satellite SST, plus SST simulated by Special Sensor Microwave Imager (SSM/I) sea ice cover. Daytime and nighttime data are treated separately, and known biases are corrected, but cloud cover and aerosols are still a source of uncertainty. Ship and buoy data are used to remove the remaining biases. Data are combined through a 3DVAR system to give a unique “bulk” temperature on a 0.5° grid (roughly the temperature at 50-cm depth, and not the “skin” temperature corresponding to the first millimeters). The daily analysis is used once in the ocean analysis at the end of the cycle (Wednesday), at a degraded resolution of 1° × 1°. SST error is set to 0.7°C.

In situ observations include low- and high-resolution profiles of temperature and salinity. Depths vary from a few tens of meters to 2000 m for the Argo profiles. Data acquisition and quality control (Carval et al. 2002) are performed by the Coriolis Data Centre (http://www.coriolis.eu.org). The data are gathered for Mercator Océan by the Analyse de Routine Multivariée des Observations MERCATOR (ARMOR) processing system (Guinehut et al. 2004) where other checks and data thinning are performed.

As for many systems in oceanic data assimilation, the analysis includes a horizontal interpolation but no vertical interpolation. Consequently, we project the in situ profiles onto the model levels. The nearest data are associated with those levels. To prevent large errors near the pycnocline, we project the data here only if the data depth is close to the model depth (not farther than
half the model-level thickness). If the nearest interpolation is not possible, a linear interpolation point is performed when the distance between the two data depths is less than the model-level thickness. No extrapolation is performed at the top or at the bottom of the profile. The resulting error is negligible on average, and it is similar to the error in the depth of the observations (e.g., error in the pressure captor or XBT fall rate). We do take into account that the model representativity varies with depth for the temperature. This representativity error is dominated by the inaccuracy of the thermocline position given by the model and the data (e.g., data profiler depth uncertainty can reach 5 m, or 2% of the depth). The following error profile is used for temperature ($z$ is the depth in meters):

$$\text{error}_T(z) = 0.3 - 0.2(z - 200) / \sqrt{[(z - 200)^2 + 400]}.$$

It ranges from 0.5°C at the surface to 0.1°C below 500 m. The higher surface values correspond to mixed layer processes or thermocline motions that are not resolved. The salinity error is defined as constant on the vertical (0.1 psu). A depth-varying error was tested but actually leads to a small degradation. Unlike the SST, which is affected by large-scale heat flux uncertainty, the model error in salinity more locally is due to runoff and precipitation. The halocline gradient is generally weaker than the thermocline gradient, which leads to a more homogeneous error in the vertical (the uncertainty of the pressure sensor has a smaller impact in salinity for the same reason). Finally, only innovations less than 6°C and 1.5 psu are kept.

c. Assimilation

PSY1v2 is the first Mercator Océan system that performs multivariate assimilation of altimetry data and in situ data. This system uses the SOFA3.0 analysis tool developed by De Mey and Benkiran (2002) at the Laboratoire d’Etudes en Géophysique et Océanographie Spatiales (LEGOS), and Project d’Assimilation par Logiciel Multiméthode (PALM) from the Centre Européen de Recherche et de Formation Avancée en Calcul Scientifique (CERFACS; Lagarde et al. 2001; see also http://www.cerfacs.fr/globe/PALM_WEB), which is the generalized coupler used to resolve large-scale assimilation problems on distributed memory computers. The standard optimal linear estimation theory is used (De Mey and Benkiran 2002). The following equations are presented only to introduce the quantities that will come into play in the results. If the true state is $x^f$, $H$ is the observation operator, and $\varepsilon$ are the errors, then the observation $y^o$ (measurement) at this time can be written as

$$y^o = H(x^f) + \varepsilon.$$

We can write the misfit (innovation) as a function $x^f$ the forecast model and the observation,

$$d = y^o - H(x^f).$$

Assuming that the forecast and observation errors are unbiased and have a normal distribution and known covariances ($B^f$ and $R$, respectively), the best estimate of $x$ is the analysis $x^a$, which uses the Kalman gain $K$,

$$x^a = x^f + K[y^o - H(x^f)].$$

As in optimal interpolation methods used in meteorology, we assume that the horizontal covariances can be separated from the vertical:

$$B^f = (D^f)^{1/2}C(D^f)^{1/2},$$

$$D^f: \text{guess variance error},$$

$$C: \text{correlation matrix, constant in time.}$$

The relevance of this parameterization for operational oceanography is discussed in Etienne and Benkiran (2007). We assume that the correlations go to zero over a set distance, according to a bell-shaped function without negative lobes [other correlation structures were tested in Etienne and Benkiran (2007)]. The differences between the data and the model, which are represented by the innovation vector, are therefore only taken into account within an area of influence around each analysis point. The size of this influence bubble is taken as twice the spatial decorrelation scales (see below). This makes the method less than optimal, but the calculations are easier and can be done in parallel.

The state reduction tries to satisfy both the need for robustness and for a reduced computing cost. The problem is truncated by using only the dominant modes for the analysis, which avoids projecting the innovation onto a broad and unknown set of modes. The relationship between the gain in the full space $K^\text{ROOI}$ and the gain in the reduced space $K_r$ can be written using $S$, the matrix consisting of the empirical orthogonal functions (EOFs) of the error covariance:

$$K^\text{ROOI} = S^T K_r,$$

$$K_r = B_r H_r (H_r B_r H_r^T + R_r)^{-1}.$$

The observations are related to the reduced space by the observation operator $H_r = HS^T$. The forecast error covariance matrix in the reduced space can be written in the same way:

$$B^f = S^T B_r / S.$$
The increment in the reduced space is projected onto the full space with the EOFs ($S^T$). In addition to the assumption of the horizontal–vertical separation of the forecast covariance, the EOF basis is the most important parameter of the ROOI. The basis contains all of the complex information of the vertical crossed covariances. It is computed from 1-day model averages of streamfunction ($\Psi$), temperature ($T$), and salinity ($S$) at each model grid point, using all profiles within a radius of 150 km, with a variance maximizing decomposition. Note that it is essential to normalize the variables and express all of them in terms of sea level. This is achieved by using the coefficients of the thermal ($\alpha$) and haline ($\beta$) expansion of the density, the model-level thickness ($\delta z$), in order to get the thermosteric and halosteric heights, which are respectively $\alpha$, $\delta z$, $T$, and $\beta$, $\delta z$, and $S$. The barotropic height $f \Psi gH$ is obtained using the gravity ($g$), the Coriolis force ($f$), and the bottom depth ($H$). The sum of the barotropic height and the thermosteric and halosteric heights over all model levels is the best 1D local model equivalent of the sea level. It is this property that ensures the basis efficiency and its compatibility with the model equations. All of the other normalizations that we tried led to a degradation of the system.

Another important point is that the basis is computed for each season. The idea is that the model is good enough to represent the seasonal signal, but misses the intraseasonal fluctuations. This was first used successfully in studies of the Mediterranean (De Mey and Benkiran 2002). In practice, the structure of the first EOF corresponds principally to the steric effect, whereas the second EOF can be associated with the vertical structure of the pycnocline. Higher EOFs are generally very complex and difficult to interpret. The EOF basis has been built iteratively over the Argo years 2002–04. Each refinement of the EOF basis improved the results, and in particular the pycnocline gradient in the tropics. A maximum number of 20 EOFs is generally used. This number is indeed dynamically determined in order to explain 99% of the signal. The forecast error is set to 10% of the EOF standard deviation [see Etienne and Benkiran (2007), about this choice].

The imposed correlation scales (longitude, latitude, time) are the last important parameters of the ROOI. In the present PSY1v2, we have chosen to use a unique set of parameters for all EOFs. The zonal and meridional scales are deduced a posteriori from iterative PSY1v2 experiments (as for the EOFs). The zonal scale is typically 150 km, except at the equator where it is larger. Meridional scales are somewhat smaller. The time scale is also deduced a posteriori from time series lag correlations (Fig. 2). ITCZ unpredictability and fast Kelvin waves lead to a 10-day time scale near the equator, whereas the Azores region goes up to 30 days.

d. Initialization

Optimal interpolation is entirely a statistical method, as opposed to a 4DVAR method. However, the order reduction favors the major physical directions as expressed by the multivariable EOF basis. Contrary to a classical optimal interpolation, working level per level, or using some analytical vertical correlation structure, the ROOI has implicit vertical covariances corresponding to the ocean physics. Hence, the ROOI produces increments that have quite meaningful physics. It is not as good as it would be if the correlations were built on a full 4D dataset, but the iterative construction of the EOFs is certainly beneficial for this point. As a consequence, initialization, which is the procedure between the analysis and the model restart, is not as critical as with a classical optimal interpolation. It is actually possible to restart the model from the analysis without initialization, and forecast statistics will be minimally affected.

However, in order to reduce the data storage and the run-time memory allocation for very large configurations (such as the $1/4^\circ$ resolution Atlantic–Mediterranean system), the baroclinic velocity is not part of the state vector. Hence, it is necessary to initialize the velocity given the barotropic and temperature–salinity initial conditions. Finally, being attentive to some technical details can already reduce the spinup effects, in particular around the islands, and near the seamounts. The initialization procedure used in this system is detailed in the appendix. It mostly relies on geostrophy and a shallow-water approximation near the equator. Other procedures have been tested, but this one is slightly better in terms of statistics and physical balance.

3. Cycling and experimental setup

In this section, we describe the cycling of the operational suite, and the cycling of the IAU experiment. We then introduce the three different runs that will be discussed in the study.

a. Cycling

The cycling of the operational suite is sequential (SEQ). Basically, this is a simple forward sequential scheme (C1 in Fig. 3). It produces an analysis at time $t_1$ (today) starting from an analysis at $t_0$ (1 week ago) and is followed by a model forecast (Fig. 3). This forward cycling is a simple and cheap mode of production that is convenient when analysis and forecast have to be car-
ried out within a limited production time. However, it has the flaw of giving a discontinuous time series, with spinup effects after each analysis. Time consistency is not taken into consideration, but the analysis of the time series is not an issue, contrary to the 7-day forecast scores. The SEQ is instead optimized to produce the best forecast, not the best analysis.

A smoother cycling is obtained with the classic technique of incremental analysis updates (Bloom et al. 1996). The IAU is a low-pass filter, which gives smooth model integration, without the jumps at the analysis time for the SEQ. The IAU also reduces spinup effects after the analysis time. It is fairly similar to nudging but it does not exhibit its weaknesses (frequency aliasing and signal damping). Following an analysis at $t_1$, a classical forward scheme would continue straight from this analysis, integrating until $t_2$. Instead, the IAU scheme rewinds the model; it starts again from $t_0$ and integrates the model until $t_2$ with a tendency correction centered around $t_1$ (Fig. 3). Note in particular that the big jump

![Fig. 2. Decorrelation time scale (days) corresponding to a 0.4 lag correlation.](image)

![Fig. 3. The sequential cycle (C1) is made of a forecast from $t_0$, a model integration, and an analysis at $t_1$. In real time, this could be continued by a pure model forecast. The IAU cycle (C1) complements the SEQ cycle by rewinding the model at $t_0$, and integrating with a tendency correction (green arrows) until $t_1$. The model forecast over the interval $t_1 - t_2$ serves as the forecast for the SEQ cycle C2.](image)
at the analysis time in the forward scheme is replaced by many small corrections all along the model integration process. In practice, the IAU scheme is more costly than the SEQ mode because of the additional model time integration over the C1 time window.

If \( x(t) \) is the model state and \( F \) is the tendency, we can write this as

\[
\frac{dx(t)}{dt} = F(x, t) + \gamma(t)(x^a - x^s).
\]

The tendency correction term on the right-hand side is the analysis increment (analysis minus guess) modulated in time by the IAU time weighting of the increment \( \gamma(t) \). It looks like a nudging term, but it is very different in detail because the model state is not part of it. Hence, it is not dissipative. It is more like a state-independent error/forcing correction, and the high frequencies generated by the model itself are not damped. The forcing intensity \( \gamma(t) \) either may be a constant over the whole window or over half of it, or have a bell shape. The constant flat function is generally employed for the sake of simplicity, but it gives a severe attenuation for periods less than the window size; it also introduces a time discontinuity in the forcing at each analysis date. A very sharp bell function (Dirac) gives results that are very similar to the SEQ. Finally, the IAU forcing is significant only in regions where the observations induce an analysis increment, whereas nudging impacts each model point with dissipation.

Another very important point is which part of the increment is used to correct the model tendencies. In this primitive-equation model, the barotropic current (2D field), the baroclinic current (3D), temperature, salinity, and turbulent kinetic energy are prognostic variables. There is no analysis in turbulent kinetic energy, so this variable is not corrected by the IAU. In this paper, we will present the results when only the analyzed values (barotropic velocity, temperature, and salinity) are used to correct the model tendency. We have also tested correcting only the temperature and salinity, but this leads to a great loss in the mid- and high latitudes, where the barotropic signal is significant (e.g., more than 20% of the sea level signal). Another test was to include the initialized baroclinic velocity in the IAU. However, the improvement was small, and only in the tropics and where nonlinearities and ageostrophy matters (i.e., Gulf Stream).

In this study, we will use a new variant of the IAU with two complementary tendency corrections (Fig. 4), one for each end of the model integration interval:

\[
\frac{dx(t)}{dt} \leftarrow \gamma_0(t)(x^a_0 - x^s_0) + \gamma_1(t)(x^a_1 - x^s_1),
\]

\[
\gamma_0(t) = 2/(t_1 - t_0) \cos^2[\pi/2(t - t_0)/(t_1 - t_0)],
\]

\[
\gamma_1(t) = 2/(t_1 - t_0) \sin^2[\pi/2(t - t_0)/(t_1 - t_0)].
\]

The two sharp weightings \( \gamma_0(t) \) and \( \gamma_1(t) \) avoid the excessive damping obtained with a flat function. The choice of the function and the link with a digital filter have been analyzed by Polavarapu et al. (2004). The present function is not as sharp as it could be, but it is much better than a flat function. Moreover, the tendency correction term is continuous in time, contrary to the flat function, which introduces discontinuities (similar to wind bursts). It also has a valuable property concerning the system bias that will be examined below.

The operating mode is the following (Fig. 4). Given an analysis at \( t_1 \) the model is restarted 1 week earlier, and integrated in a hindcast mode (H1). The first tendency corrections term is obtained from the previous analysis. Its impact decays with time, and is nil at \( t_1 \). The second tendency correction reaches a maximum at \( t_2 \). Then, the model is integrated in a forecast mode (F1), with only one tendency correction. Eventually, a longer forecast would be performed without any correction (F2), but this is not discussed in this study. Only scores relative to the date \( t_2 = t_1 + 7 \) (today plus 7 days) will be considered in the following analysis.

b. Experimental setup

In this study, we will consider three different runs. A free run (FREE) is integrated without data assimila-
tion. It starts in January 2003 from the climatology data. The SEQ and IAU runs start in January 2004 from the FREE run. Most of the results will be presented for the year 2004 only for the sake of clarity, and in order to cover the assimilation spinup. Results over 2005 are similar to those of 2004.

4. Results

In this section, we address the following two questions: are the scores improved or degraded by the IAU, and does the IAU have some positive impact on the spinup effects? To answer the first question, we will look at the forecast scores. We will answer the second question by looking at physical quantities that are not directly observed.

a. Forecast scores in sea level

A disturbing feature of the IAU is that the model state at the analysis time after the extra model integration differs from the IAU analysis (Fig. 3). Because of the following property

$$\int_{t_0}^{t_1} \gamma(t) \, dt = 1,$$

the model has integrated the entire analysis increment, but the increment noise has been filtered. The final state may thus slightly differ from the original analysis. Previous authors (Bloom et al. 1996; Huang et al. 2002) have already noted that the SEQ state is closer to the data. In other words, the residuals (observation minus analysis) can be smaller with the SEQ.

Instead of looking at the residuals, which is not very intuitive to start, we consider the 7-day forecast of SLA (end of F1, at $t_f$). The root-mean-square (rms) misfit (Fig. 5) over the whole domain (shelf included) is typically 7.5 cm for the SEQ state, whereas it is 7 cm for the IAU state. This means that the IAU forecast is “better” in term of SLA rms. The low-pass filtering of the IAU does not have a negative impact of the forecast, as often mentioned (Huang et al. 2002; Ourmières et al. 2006) when a flat weighting function is used. We have checked (not shown) that the IAU with a flat function actually leads to a degrading of the SLA forecast rms (8–8.5 cm). This result, in combination with the work of Polavarapu et al. (2004), confirms the interest of the sharp weighting function used here. The flat function is efficient in a system where some extrapolation of the increment is necessary (temperature onto salinity, 0–700 m onto the whole profile). It is detrimental to the timing of the corrections in this multivariate eddy-permitting system.

Another clear feature of the behavior of the IAU is given by the tendency correlations. The repetitiveness of satellite altimetry gives us the possibility to measure the ability of the system to forecast the sea level change from one cycle ($n$) to another ($n + 1$): if the satellite has observed a 5-cm rise since the last satellite track, what was the change in the forecast? If one gathers all of the points where a sea level tendency was observed,
then the tendency correlation can be computed as follows:

\[ \text{tendency correlation} = \text{cor}(\text{SLA}_n, \text{SLA}_{n+1}) \]

The striking point about this score (Fig. 6) is that the IAU ability to forecast a sea level change is near 0.6 in correlation, and only 0.3–0.4 with the SEQ run. At first sight, this jump in performance seems much better than the improvement in rms. Going back to the definitions of section 2b, one finds

\[ \text{rms(misfit)} = \sqrt{\sum (\text{SLA}^d - \text{SLA}^m)^2} \]
\[ = \sqrt{\sum (\text{SLA}^d + \text{MSSH}^d - \text{SSH}^m)^2} \]

The big difference is that the MSSH is not part of the tendency correlation score (sea level change). An inaccurate MSSH could then contaminate the SLA rms statistics of the IAU run in Fig. 5. This point is illustrated in Fig. 7 by the comparison of the model SSH for the SEQ run and the IAU run with the sum of the SLA and the MSSH. The SSH of the SEQ run (blue) is often 1 cm below the data (SLA + MSSH), with a lot of oscillations. The IAU run (green) is not biased and has very few oscillations. It has a better skill in forecasting the sea level fluctuations, and this is why the tendency correlations are improved a lot with the IAU. The fact that the SEQ run has a minimal bias, and the rms misfit is less improved than the tendency correlations, suggests that there might be a model bias in the mean SSH, or an MSSH bias. In both cases, the IAU makes the system less sensitive to this compatibility problem.
b. Forecast scores against in situ data

The forecast scores are slightly improved in terms of in situ temperature. This is summarized by the yearly rms of the innovation over the basin (Fig. 8). This improvement is moderate (about 0.05°C), but can be felt down to 700 m. It concerns all regions, and seemingly has more impact in regions of high error growth [the Gulf Stream, subpolar gyre, Loop Current, and Brazil retroflection, as shown in Etienne and Benkiran (2007)]. This improvement in rms is partly explained by a bias reduction. A larger change can be seen on the score in forecast salinity (Fig. 9). The bias has been reduced in the top meters, and the improvement in rms can be felt down to 500 m (not shown, 0.3 psu at 0 m, less than 0.13 psu below 200 m). The Gulf Stream area is the region where the improvement is most clear, but this moderate improvement is general.

c. SST bias

This section is dedicated to an unexpected feature of the IAU. Even if a restoring term is used to constrain the model SST to the observed SST, the FREE run (not shown) still exhibits a cold bias in the western tropics, a Gulf Stream bias dipole resulting from a steady meander north of Cape Hatteras (model overshoot), and a warm bias along the Labrador Current. The SEQ run succeeds in reducing this bias (Fig. 10), which rarely exceeds the prescribed error (0.7°C), but the SEQ assimilation fails to correct this bias entirely. A justification is that the initial conditions are less responsible for this bias than the forcing error and the mixed layer physics (Roquet et al. 2003). The IAU more effectively reduces both cold and warm biases (Fig. 10). The Gulf Stream dipole is greatly reduced. This improved result was surprising, because the IAU was introduced to low pass the high frequencies induced during the spinup, and not to solve a very low-frequency problem.

d. Currents

The IAU avoids the use of a velocity initialization (see the appendix). For instance, near the equator, the IAU increments include waves and meanders of the Equatorial Undercurrent that are not in the SEQ initialization. However, subsurface moorings or drifters are too rare for us to make a significant validation.

The surface eddy kinetic energy (EKE) is often used as a basic criterion to evaluate the performances of a system (Ourmieres et al. 2006). However, both the SEQ and the IAU runs present an EKE map that is very close to the DUACS estimates (not shown). This means that the variability of the surface current is only slightly affected by the IAU.

The comparison over the Atlantic domain with the Coriolis surface drifters reveals only a small improvement. Rms differences of zonal or meridional velocities are 17 cm s^{-1} for the SEQ, and 16 cm s^{-1} with the IAU. Outside of the tropics, geostrophy strongly dominates, and increments are very similar. However, the eddies and meanders are more clearly marked with the IAU run. In particular, the thermal front of the Gulf Stream is too broad in the SEQ run, but it is tighter with the IAU (SST from Fig. 10). The temperature increment of the SEQ tries to sharpen the front. However, the model bias is strong, and the benefit of the improved initial conditions is already fading at the end of the 7-day forecast. In the IAU run, the increment is active all along the model integration. It acts like a flux correction at the surface, but it is more than this, because a similar sharpening of the thermal front is imposed at 100 m, 300 m, and deeper. Note that there is the following chain reaction: as the pressure gradient is stronger, the Gulf Stream jet is more energetic and strength-
ens the front farther east. Carsten et al. (2004) obtained similar results (their Fig. 5) by modifying the pressure gradient with climatology. Here, the IAU brings a correction that is analyzed (data errors are taken in account), time dependent, and has a more general form than a density gradient. In summary, the IAU can be successful in correcting the heat flux errors, and unknown model deficiencies (bathymetry, density gradient, etc.). Note, however, that this does not necessarily improve the skill in turbulent regions. Our model is only eddy permitting, and this method does not help to resolve the eddy dynamics or forecast the exact position of the eddies.

In the tropics, the wind stress has a dominant impact on the model surface currents. On average, over the year, the SEQ run presents a South Equatorial Current (SEC) with two well-marked branches in the west (35°–20°W). The IAU slightly weakens the SEC branches.

**Fig. 9.** Basin average of the innovations in salinity for the (top) SEQ run and (bottom) IAU run.
and reinforces both the North Equatorial Countercurrent (6°N) and the Equatorial Undercurrent. The drifter positions do not match the regions of clear changes in the Gulf of Guinea. Positions are more interesting in the west (14°S–14°N, 60°–15°W, as in Fig. 11). The zonal current is only slightly improved (correlation of 0.79 for the SEQ and 0.81 for the IAU), but it is stronger for the meridional velocity (19 cm s\(^{-1}\) rms and 0.46 in correlation for the SEQ, 17 cm s\(^{-1}\) rms and 0.57 in correlation for the IAU run). Figure 11 offers a typical illustration of the impact of the IAU on surface currents, which will also help for the next section. The SEQ run presents two strong branches of the SEC near 25°W on 20 April 2004 (top left). The westward flow at the equator is not supported by the two drifters between 25° and 20°W. In the IAU run (bottom left), the SEC branches are less strong, and there is an eastward flow at the equator coming from the North Brazil Current, according to the drifters. Ten days later, the flow crosses the equator southward, both in the SEQ (top right) and in the IAU runs (bottom), barely in agreement with the drifters. Note, however, that the cross-equatorial flow is stronger in the SEQ run.

e. Low-pass filter

The main expected result of the IAU is to remove the spinup effects. In Fig. 12, we examine a rather extreme example of the bad consequences that sequential data assimilation can have on the model physics. The FREE run shows a surface westward (blue) current, namely, the South Equatorial Current, pushed by the easterlies. Near 50 m, the eastward (yellow) Equatorial Undercurrent undulates around the equator, and strengthens in September. There are some high frequency changes induced in July when the equatorial wind reinforces and creates the equatorial upwelling. This time series from Sao Tome is just at the end of the equatorial waveguide (0°, 6°E), where inertial gravity and Kelvin waves are naturally present. However, the velocity of the SEQ run is too noisy by far. This is the result of the local corrections introduced by the analysis increment, but also the remote contribution of the waves generated farther west along the equator. The IAU run has a frequency spectrum that is very similar to the FREE run. Artificial high frequencies are filtered out, whereas the waves that are naturally present in the model in July are not dissipated. The corrections resulting from the assimilation are clearer (eastward core near 400 m in April, or near 100 m in September). There is more kinetic energy (shear) in the IAU run than in the SEQ run, and the noise level is much lower.

f. Vertical velocity

High-frequency gravity waves are actually not a major problem. They do not interact much with the rest of the model physics. The vertical diffusion is reduced in the SEQ run, because the pycnocline is actually sharper than in the FREE run. Problems are more apparent when the model outputs are coupled with a bio-
geochemical model. The primary production was much too high near the equator with the SEQ run, whereas it was normal with the FREE run. A careful examination revealed that this problem was due to an anomalous mean vertical velocity (not the high frequencies). Near 100 m, the vertical velocities of the FREE run are generally moderate (Fig. 13). The main feature is the equatorial divergence (upwelling). On both sides of the equator, the countercurrents create two convergence zones (downwelling). Note that Sao Tome (0°N, 60°E) is an area of upwelling. The SEQ run does quite well north of 20°N: mesoscale information is added without disrupting the overall structure of the vertical motion. In the tropics, however, patches of strong vertical velocities appear, and it is difficult to link these structures with any equatorial current. The analysis of the 3D fields reveals that meridional cells are created within the SEQ run (wavelet meridional structure near 25°W). There, a strong equatorial upwelling is redistributed north and south of the equator by two strong downwellings, plus some other wave effects farther north and south. In addition, the SEQ vertical velocities at Sao Tome show downwelling. The IAU has little impact on the mid- and high latitudes vertical velocities, but the impact in the tropics is spectacular. The overall structure of the FREE run is recovered, with some local details well depicted like near the Ivory Coast. Sao Tome is once again a place of upwelling in the IAU run.

Convective cells have been found in the Met Office (UKMO) system in the equatorial regions (Bell et al. 2004). It was shown that the systematic errors were due to the incompatibility of erroneous wind forcing and accurate ocean observations. It could be mitigated by correcting the pressure gradient with the analysis mean increment (Huddleston et al. 2004). The meridional convective cells that we are facing here are essentially of the same kind, but with a different origin. For the UKMO, zonal wind was responsible for the zonal convective cells. The same problem has been recently presented by the Japan Meteorology Agency system (Ishizaki 2006). Mercator Océan is fortunate that the ECMWF wind stress is well balanced with this model configuration at the equator. We have checked this important point by withholding the altimetry data. The vertical velocity of a SEQ run with only SST and in situ data is very similar to the FREE run. This proves that what is responsible for the imbalance is not the wind.
stress, but the altimetry and MSSH inputs; we will come back to that point. A clear result is that the IAU is successful in solving this imbalance (no more convective cells).

We can make a parallel between the pressure gradient correction from Bell et al. (2004) and the IAU. The former method assumes that the system error can be controlled by only the pressure gradient term. The IAU is more general, because a gradient form is not assumed. The IAU can also correct for heat and freshwater fluxes errors, as well as model physics (horizontal advection, vertical diffusion, etc.). Controlling the systematic bias of the system is only possible because the EOF basis includes a good part of the bias. This is a lucky catch because nothing guarantees that using intraseasonal model anomalies will produce this property. Technically, the details that make it work, is that

![Fig. 12. Profile of zonal velocity plotted as a function of depth and time near Sao Tome (0°, 6°E) for (top) the FREE run, (middle) the SEQ run, and (bottom) the IAU run.](image1)

![Fig. 13. Map of vertical velocity near 100 m for the (top) FREE run, (middle) SEQ run, and (bottom) IAU run. Positive values indicate a downwelling (convergence).](image2)
Fig. 14. Equatorial temperature for the (top) SEQ and (bottom) IAU runs.
we use two complementary tendency corrections, and one always has the following property:

$$\gamma_0(t) + \gamma_1(t) = 2(t_1 - t_0) = \text{constant.}$$

Hence, the part of the bias included in the increments will be corrected continuously. A steeper time weighting $\gamma(t)$ would provide a sharper frequency response of the IAU, but it would be detrimental to the control of the bias.

5. Discussion

The exact origin of the incompatibility between the altimetry input and the system is still an open problem, because we are close to the error level. The fact that the IAU run does a good job at forecasting the sea level fluctuations demonstrates that the model has “some” skill. However, the SEQ has small ($< 2$ cm) regional biases in SLA in the tropics. Part of the problem may be due to the MSSH, which is known to have errors of this order, in particular in the tropics. However, this is not to say that these data are the origin of the system bias. This medium-resolution model ($\frac{1}{6^\circ}$) is satisfactory in reproducing equatorial waves and the overall thermocline structure. However, comparisons with drifters show that the eastward North Equatorial Countercurrent ($3^\circ$–$7^\circ$N) and the westward South Equatorial Current ($4^\circ$N–$7^\circ$S) are both underestimated, and this correlates well with the SST bias in the Gulf of Guinea (Fig. 10). This indicated that the model fails to reproduce the top stratification and the shear in these two major currents. The model dynamics near the equator (roughly $5^\circ$N–$5^\circ$S) produce thermocline slopes that are not compatible with the steeper meridional structures of the MSSH. Consider the mean equatorial temperature of the SEQ run (Fig. 14); there are four spikes within the $20^\circ$C isotherm. It corresponds exactly to the Pirate Research Moored Array in the Tropical Atlantic (PIRATA) moorings ($35^\circ$W, $23^\circ$W, $10^\circ$W, and $0^\circ$E). The model thermocline is a little too deep (warm) and not steep enough, and this bias is corrected (cooled) by the SEQ run. The ripples in vertical velocity that are symmetric off the equator (Fig. 13) show that the corrections of the SEQ introduce some equatorial waves that propagate along the equatorial waveguide (Fig. 12), confirmed by Hovmoeller diagrams (not shown). Moreover, the cooling introduced by the SEQ is not compatible with the sea level data (Fig. 7). An increase is expected at the equator to satisfy the sea level data. It is indeed imposed by the SEQ, except at the PIRATA moorings, where the two kinds of data are not compatible with this model. Comparisons with the few salinity observations suggest that a larger correction of the halocline structure (a freshening of the 150–400-m slab) helps in raising the equatorial sea level, and it is partly achieved by the IAU. With the IAU, the spikes disappear as part of the PIRATA mooring information is discarded (the thermocline bias is larger with the IAU than with the SEQ). The IAU run also has an SLA that is more coherent with the data. Hence, it seems that the vertical gradient of the thermocline and the halocline, as well as the horizontal currents’ shear, are too weak in this model configuration. Error covariances are not accurate enough to correct for these biases, and the systematic biases are strong enough so that the corrections are quickly forgotten, unless the IAU is used to correct for the model weaknesses.

It is not still possible to determine to what extent the bias comes from the model or from the MSSH data. The increase of real-time profilers and drifters near the equator in the Atlantic, and the results of the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) mission will help us to address this question in a near future (Vossepoel 2007). In the meantime, the IAU seems to provide a good solution in the presence of these biases. If the MSSH has an error, and the model SSH is not biased, both the IAU and the SEQ will be wrong and introduce a bias in the least observed variables (typically the salinity at depth, or the barotropic motion). When the MSSH is correct and the model thermocline (and thus SSH) is biased, as seems to be the case near the equator, the IAU gives much better performances, adding a corrective term in all the model equations. In the Gulf Stream, the model predictability is weak, and benefits concern only the average quantities.

6. Conclusions

The IAU procedure performs well in the context of the large-scale oceanography with an eddy-permitting model if the time weighting functions and the control variables are carefully chosen. High-frequency gravity waves are well filtered with the IAU, and the forecast scores are slightly improved. More encouragingly, spurious convective cells are removed from the run, and the SST bias is also significantly reduced. These last two properties are linked with the systematic biases of the system that are probably more important than in an atmospheric context. In summary, the IAU is even more useful in oceanography as long as the sources of systematic bias remain (model physics, forcing, MSSH).

Acknowledgments. The authors thank Marianna Childress and Rosemary Morrow for their help.
APPENDIX

Initialization Procedure

The first point concerns the initialization of the barotropic current for this rigid-lid model. To perform this initialization is necessary because of two points that are not well handled during the analysis. First, data from behind islands are used in the influence bubble as if no land were present (taking the coastline into account is too costly). The other problem is that the observability of the barotropic motion is very poor on the shelf with altimetry, and the uncertainty of the analysis there is quite large. Local bathymetric details can lead to an imbalance within the analyzed state. In meteorology, an orographic initialization is required. In oceanography, transports on the shelf or around islands are the issue.

Given some streamfunction analysis increment \( \psi^a \), the problem is to find a close initialized increment \( \psi^i \) that gives a moderate increment in barotropic velocity \( \hat{u} \) for a given water column height \( H \). This can be written as

\[
\psi^i = \psi^a, \\
\pi = (1/H) \nabla \psi^i
\]

is not large. The balanced increment is obtained by solving iteratively the following elliptic problem:

\[
-(\kappa/2)(1/H) \nabla \cdot [(1/H) \nabla \psi^i] + \psi^i = \psi^a.
\]

If one multiplies both sides by \( \psi^i \), after integration of the left-hand side, and uses the Cauchy-Schwarz and Young inequalities for the right side, one obtains the classical energy equation of the elliptic problem,

\[
\kappa \| (1/H) \nabla \psi \|^2 + \| \psi^i \|^2 \leq \| \psi^a \|^2.
\]

If \( \kappa \) is small, one obtains the first wanted property from the elliptic problem. If \( \kappa \) is larger, the second property is obtained from the above inequality. In practice, \( \kappa \) is chosen so that a little smoothing of the barotropic increment can give a moderate barotropic velocity. Moreover, this is exactly coherent with the model discretization.

It is also worthwhile to initialize the baroclinic velocity. However, because the baroclinic velocity is not part of the state vector, the baroclinic increment has to be determined from the temperature and salinity increments only. Geostrophic equilibrium is a big help, except it collapses near the equator. Here, we tested several initializations in the real-time systems. The first one used a first-order geostrophy, which was gently cut off near the equator (nil velocity increments). However, the attenuation of the increments was too artifi-

cial. We then used a second-order geostrophic term, which had the advantage of avoiding an artificial cutoff. However, the tilting of the thermocline was not balanced near the equator. We then searched for an algorithm that could balance the thermocline motions by convergence or divergence. Our most recent and efficient initialization scheme is nonstationary. It is based on the time integration of the shallow-water equations embedded in the model dynamics. More precisely, we integrate the linear equation of baroclinic motion \( \hat{u} \) over 1 day:

\[
\frac{\partial \hat{u}}{\partial t} - \nu \nabla \cdot (\nabla \hat{u}) + \frac{f k}{\theta} \times \hat{u} + g/p \nabla p = 0.
\]

The initial condition is zero. The only forcing term is \( p \), the increment of baroclinic pressure at a given level, scaled by a linear function increasing from zero to one at the analysis time. In short, we assume that the model forecast of baroclinic motion is good up to 1 day before the analysis and then it has to be corrected according to the pressure increment and the model dynamics (and discretization). This technique ensures that off the equator the response is geostrophic. Near the equator, a warming is equilibrated by a convergence, and vice versa. Note, however, that it does not include the tropical waves. The positive part is that the transition between the equator and the tropics is invisible, and it satisfies a good part of the model equations. This method shares some points with the pressure correction (Bell et al. 2004). Among the three methods we tested, the latter gives the best statistics near, or away from, the equator. Finally, we note that the baroclinic increments are not used with the IAU, and only the barotropic part is used.

The last ingredient of the initialization is the choice of the time scheme for the model restart. As the model time scheme has two stages (a leapfrog scheme with an Asselin filter), there are two solutions: either one restarts from the analysis with an Eulerian forward scheme, or we carry the two stages of the leapfrog scheme all along the analysis, and we add the analysis to both stages. The first solution saves inputs/outputs and memory allocation, which is valuable for very large configurations. In the present configuration, the second solution is used because it preserves the model tendency at the analysis time, and avoids the instabilities linked with the Eulerian scheme of the first model time step.

REFERENCES

Bell, M., M. Martin, and N. Nichols, 2004: Assimilation of data


