Orthogonal Channel Coding for Simultaneous Co- and Cross-Polarization Measurements

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ABSTRACT

Dual-polarization weather radars typically measure the radar reflectivity at more than one polarization state for transmission and reception. Historically, dual-polarization radars have been operated at copolar and cross-polar states defined with respect to the transmit polarization states. Recently, based on the improved understanding of the propagation properties of electromagnetic waves in precipitation media, the simultaneous transmit and receive (STAR) mode has become common to simplify the hardware. In the STAR mode of operation, horizontal and vertical polarization states are transmitted simultaneously and samples of both horizontal and vertical copolar returns are obtained. A drawback of the current implementation of STAR mode is its inability to measure parameters obtained from cross-polar signals such as linear depolarization ratio (LDR). In this paper, a technique to obtain cross-polar signals with STAR mode waveform is presented. In this technique, the horizontally and vertically polarized transmit waveforms are coded with orthogonal phase sequences. The performance of the phase-coded waveform is determined by the properties of the phase codes. This orthogonal phase coding technique is implemented in the Colorado State University–University of Chicago–Illinois State Water Survey (CSU–CHILL) radar. This paper outlines the methodology and presents the performance of the cross-polar and copolar parameter estimation based on the simulation as well as data collected from the CSU–CHILL radar.

1. Introduction

A fully polarimetric radar is defined as the one that can measure the full covariance matrix of the precipitation medium (Tragl 1990; Bringi and Chandrasekar 2001). The full covariance matrix is composed of both copolar and cross-polar components. Pulse Doppler weather radars with polarization diversity can transmit and/or receive in two orthogonally polarized channels. Dual-polarization weather radars transmit and receive at more than one polarization state. Circular and linear polarization states are the most commonly used polarization states of operation. In a dual-polarized radar operating on a linear basis, the transmitted pulses are at horizontally (H) and vertically (V) polarized states. Polarization diversity refers to the ability of the system to transmit/receive orthogonally polarized waves. Such systems transmit a single polarization state and can receive co- and cross-polar components with dual-channel receivers. Thus, with a combination of both polarization agility on transmit and polarization diversity on receive, the copolar and cross-polar returns can be measured at both polarization states. Historically, weather radars have operated to make measurements at copolar and cross-polar states of the covariance matrix elements. Over two decades of measurements in this basis has led to an improved understanding of propagation characteristics in the precipitation medium (Chandrasekar et al. 1990). The cost and advancement of receiver technology over time led to implementation of different measurement schemes over a period of three decades. For example, in the early 1980s before the digital revolution, receivers were expensive, and this led to single-receiver radars with high-power transmitter switches, to enable only copolar measurements. Over the last two decades, receivers have become very advanced and primarily driven by the marketplace revolution in communication technology, making it cheaper to use multiple receivers; this has led to the introduction of the simultaneous transmit and receive (STAR) mode (Doviak et al. 2000; Bringi and Chandrasekar 2001). However, this mode prevents measure-
ments of cross-polar returns. This paper presents a phase-coding technology such that copolar and cross-polar measurements can be obtained simultaneously.

This paper is organized as follows: a brief description of the received signal and precipitation covariance matrix is given in section 2. Section 3 introduces waveform coding, the codes used to retrieve the cross-polarized signals, and gives a brief description of the properties of cross-polar received signals. Parameter estimation using advanced processing methods is described in section 4, while section 5 describes the statistics of these estimators in retrieving the cross-polar signal-based parameter. In section 6, analysis of the waveform coding with data collected from the Colorado State University–Illinois State Water Survey (CSU–CHILL) radar is presented followed by the summary and conclusions in section 7.

2. The received signal and the precipitation covariance matrix

The weather radar equation at single polarization is expressed as a well-known scalar equation whereas the corresponding equation for dual-polarization radar becomes a matrix equation expressed as (Bringi and Chandrasekar 2001)

\[
\begin{bmatrix}
    v_h \\
    u_v
\end{bmatrix}
= \frac{\lambda \sqrt{P_G}}{4\pi r^2} \left[ T \right] \left[ S_{BSA} \right] \left[ M_h \right] \left[ M_v \right]
\]

with

\[
S = \begin{bmatrix}
    S_{hh} & S_{hv} \\
    S_{vh} & S_{vv}
\end{bmatrix},
\]

where \( T \) is the transmission matrix of the uniform precipitation medium and is included for completeness to account for the propagation of electromagnetic waves through the precipitation medium but does not affect waveform coding; \( S \) is the \( 2 \times 2 \) backscattering matrix; \( M_h \) and \( M_v \) are the transmitter excitation states in horizontal and vertical ports of a dual-polarized antenna with an antenna gain \( G \), transmit power \( P_t \), and operating at a wavelength \( \lambda \). Thus it can be seen that the received signal \( (v_h, u_v) \) is a vector depending on the transmit polarization state. In a switched or alternate polarization mode the transmit states are vectors

\[
\begin{bmatrix}
    M_h \\
    M_v
\end{bmatrix} = \begin{bmatrix}
    1 \\
    0
\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}
    M_h \\
    M_v
\end{bmatrix} = \begin{bmatrix}
    0 \\
    1
\end{bmatrix},
\]

resulting in a sequence of length four vector

\[
z = \begin{bmatrix}
    v_{hh} \\
    v_{hv} \\
    v_{vh} \\
    v_{vv}
\end{bmatrix}^T
\]

corresponding to the two orthogonal transmit states. The subscripts \( \text{hh} \) (\( \text{hv} \)) refer to transmit horizontal (vertical) polarization and receive vertical (horizontal) polarization. The properties of this signal vector \( z \) can be described in terms of the covariance matrix \( \mathbf{C} \):

\[
Ezz^H = \mathbf{C} = E
\begin{bmatrix}
    |v_{hh}|^2 & \sqrt{2} S_{hh} S_{hv}^* & S_{hh} S_{vv}^* & S_{hh} S_{vw}^* \\
    \sqrt{2} S_{hh} S_{hv} & |S_{hh}|^2 & \sqrt{2} S_{hv} S_{hv}^* & S_{hv} S_{vw}^* \\
    S_{hh} S_{hv}^* & \sqrt{2} S_{hv} S_{hv}^* & |S_{hv}|^2 & S_{hv} S_{vw}^* \\
    S_{hh} S_{vw}^* & S_{hv} S_{vw}^* & S_{hv} S_{vw}^* & |S_{vw}|^2
\end{bmatrix},
\]

where \( H \) is the Hermitian operator, * indicates complex conjugate, and \( E \) represents the expectation operator. This \( 4 \times 4 \) matrix can be reduced to \( 3 \times 3 \) invoking reciprocity. This covariance matrix measurement corresponds to the covariance matrix of the scatterers in the precipitation medium defined by (Bringi and Chandrasekar 2001)

\[
\Sigma = \begin{bmatrix}
    n \left( |S_{hh}|^2 \right) & \sqrt{2} S_{hh} S_{hv}^* & S_{hh} S_{vv}^* \\
    \sqrt{2} S_{hv} S_{hv}^* & |S_{hv}|^2 & \sqrt{2} S_{hv} S_{hv}^* \\
    S_{hv} S_{hv}^* & \sqrt{2} S_{hv} S_{hv}^* & |S_{hv}|^2
\end{bmatrix}
\]

where \( S_{hh}, S_{hv}, \) and \( S_{vv} \) are the elements of the scattering matrix, \( n \) is the number of particles per unit volume, and the angle brackets denote ensemble averaging. The intrinsic backscattering properties of the hydrometeors at the two polarization states enable the measurement of characteristics such as mean size, shape, and spatial orientation of the precipitation particles in the radar resolution volume. These characteristics are described in terms of the backscattering matrix elements. The elements of the backscattering covariance matrix or some combination of them are used to compute the polarimetric variables such as differential reflectivity \( (Z_{dr}) \) and linear depolarization ratio \( (\text{LDR}) \). The differential reflectivity and linear depolarization ratio are defined as

\[
Z_{dr} = 10 \log \left( \frac{|S_{hh}|^2}{|S_{vv}|^2} \right) = 10 \log \left( \frac{|v_{hh}|^2}{|v_{vw}|^2} \right),
\]

\[
\text{LDR} = 10 \log \left( \frac{|v_{hh}|^2}{|v_{hv}|^2} \right) = 10 \log \left( \frac{|v_{hw}|^2}{|v_{vw}|^2} \right),
\]

where \( \langle |v_{hh}|^2 \rangle \) and \( \langle |v_{hv}|^2 \rangle \) are the copolar and cross-polar signal power, respectively. Polarization diversity enables us to measure differential reflectivity, differential propagation phase (or specific differential phase), and linear depolarization ratio. These multiparameter radar observations have been used to measure the bulk properties of hydrometeors in a resolution vol-
Equation (8) is obtained from Eq. (1) by choosing basis enables the retrieval of linear depolarization ratio. Waveforms in the H and V channel on a pulse-by-pulse basis introduces biases in measurements (Doviak et al. 2000; Wang and Chandrasekar 2006). The biases introduced by the cross-polar signal get larger with increases in operating frequency (Wang and Chandrasekar 2006). In a polarization diverse system operating under the STAR mode of operation, simultaneous observations of the resolution volume are made with both horizontal and vertical polarization states. STAR mode reduces hardware complexity by avoiding switching of the transmitter (switching between H and V polarization on a pulse-to-pulse basis) (Doviak et al. 2000; Bringi and Chandrasekar 2001). The radar equation for STAR mode is

\[
\begin{bmatrix}
  v_h \\
  v_v
\end{bmatrix} = \frac{\lambda P_G}{4\pi f_L} \begin{bmatrix}
  S_{hh} & S_{hv} \\
  S_{vh} & S_{vv}
\end{bmatrix} \begin{bmatrix}
  1 \\
  1
\end{bmatrix}.
\]

Equation (8) is obtained from Eq. (1) by choosing \( M_h = M_v = 1 \). It can be observed in Eq. (8) that the received signal in the h port (v port) is a linear function of \( S_{hh} \) and \( S_{hv} \) (\( S_{vh} \) and \( S_{vv} \)). Therefore, cross-polar signals \( v_{hh} \) (or \( v_{hv} \)) cannot be directly measured in STAR mode and hence measurements of LDR are not available. This paper presents a technique to separate out the copolar and cross-polar signals. The separation of copolar and cross-polar signals enables the retrieval of the linear depolarization ratio in STAR mode. In addition to the retrieval of linear depolarization ratio, the polarimetric variables are unbiased. Coding the transmit waveforms in the H and V channel on a pulse-by-pulse basis enables the retrieval of linear depolarization ratio.

3. Orthogonal channel coding for dual-polarized radars

Phase coding of transmitted signals has been suggested for various applications such as pulse compression, range ambiguity mitigation, digital communication, and cryptography. However, the implementation of phase coding requires a transmitter that is capable of controlling the phase of the transmitted pulse. Klystrons, traveling wave tube (TWT), and solid-state transmitters can control the transmit phase while magnetron-based systems cannot control the transmit phase (Skolnik 1990).

Radar signals with a single transmitter operate at two polarizations either by alternating between polarization states from pulse to pulse or transmit a slant 45° polarization and receive on two channels. Orthogonal channel coding for a single-transmitter radar operating in STAR mode would require a radio frequency (RF) phase shifter to enable phase coding. However, with a two-transmitter radar system (Klystron, TWT, or solid state) phase codes can be applied on each channel independently without an RF phase shifter. RF phase shifters traditionally have poor phase noise characteristics, and large phase errors degrade the performance of phase coding. The CSU–CHILL radar system is capable of transmitting horizontal and vertical polarization states simultaneously. Two orthogonally polarized waves are transmitted using two separate but identical Klystron transmitters with dual-channel reception of orthogonally polarized waves. Recent advances in digital technology have enabled the use of digital transmit controllers capable of generating arbitrary waveforms; the CSU–CHILL radar uses a 16-bit arbitrary waveform generator (George et al. 2006). Systems using a digital arbitrary waveform generator with two transmitters such as CSU–CHILL (Fig. 1) can generate and transmit orthogonally coded pulses in the horizontal and vertical polarization channels. The CSU–CHILL radar’s Klystron transmitters enable the use of phase coding to transmit orthogonal waveforms.

Random phase coding the transmit pulses for suppressing range overlaid echoes for weather radars was proposed in Sigia (1983). Giuli et al. (1993) suggested orthogonal signal using chirp waveforms to measure the cross-polar signal while Mudukutore et al. (1998) used independent phase coding to implement pulse compression for weather radars at both polarizations. Sachidananda and Zrnić (1999) proposed a systematic phase-coding technique to resolve range overlaid echoes and its evaluation for polarimetric radars has been de-
scribed in Bharadwaj and Chandrasekar (2007). A different coding mechanism at the high-power RF side is suggested by Stagliano et al. (2006). In this paper, the two channels (H and V) are orthogonally coded using phase sequences. The transmit polarization state for the \( k \)th transmit pulse is given by

\[
\begin{bmatrix}
M_h \\
M_v
\end{bmatrix} = \begin{bmatrix}
e^{j\phi_h(k)} \\
e^{j\phi_v(k)}
\end{bmatrix},
\tag{9}
\]

where \( \psi_h \) and \( \psi_v \) are the switching phase codes that are tagged to each transmit pulse in the horizontal and vertical polarization channel, respectively. The received signal from an orthogonally coded H-channel and V-channel transmit pulse is given by

\[
\begin{bmatrix}
s_{rh}(k) \\
s_{rv}(k)
\end{bmatrix} = \frac{\lambda \sqrt{P_i G}}{4\pi r^2} \begin{bmatrix}
T[|S|T] e^{j\phi_h(k)} \\
0
\end{bmatrix}.
\tag{10}
\]

The received signals after decoding are given by

\[
\begin{bmatrix}
s_{rh}(k) \\
s_{rv}(k)
\end{bmatrix} = \frac{\lambda \sqrt{P_i G}}{4\pi r^2} \begin{bmatrix}
T[|S|T] e^{j\phi_h(k)} \\
0
\end{bmatrix}.
\tag{11}
\]

The power levels of cross-polar signal backscattered from hydrometeors are always significantly less than the copolar signal. The lower bound for the cross-polar signal power is dictated by the polarization isolation of the antenna. The estimation of the bounds for LDR based on the antenna pattern is described by Chandrasekar and Keeler (1993). In this paper we will use the notation \( \bar{LDR} \) for estimated LDR and \( \bar{LDR} \) for mean LDR. The mean Doppler velocity of both the copolar and cross-polar signal is identical, but it has been observed that copolar spectrum is generally narrower than the cross-polar spectrum. The linear depolarization ratio is estimated from the copolar and cross-polar signal samples as

\[
\bar{LDR} = \frac{\sum_{k=1}^{N} |v_{hv}(k)|^2}{\sum_{k=1}^{N} |v_{hv}(k)|^2},
\tag{14}
\]

\[
\frac{\lambda \sqrt{P_i G}}{4\pi r^2} |\rho(n)| = \exp \left( -\frac{8\pi^2 \sigma_0^2 n^2 T^2}{\lambda^2} \right).
\tag{16}
\]

The parameter \( \alpha \) is the broadening factor of the cross-polar spectrum compared to the copolar spectrum and \( \rho_{cx} \) is the copolarization-to-cross-polarization correlation coefficient. For most types of precipitation, \( |\rho_{cx}(n)| \approx 0 \); however, \( |\rho_{cx}(n)| \) can be greater than zero for precipitation with a nonzero canting angle (Ryzhkov 2001; Moisseev et al. 2002). The standard deviation of \( \bar{LDR} \) as a function of the number of samples used in the integration is shown in Fig. 2. The standard deviations obtained from (15) are shown along with errors obtained from simulations. The procedure described by Chandrasekar et al. (1986) was extended to simulate time series data with both copolar and cross-polar signals. The standard errors in \( \bar{LDR} \) from copolar and cross-polar signals are similar to the error obtained from (15). It can be observed that the errors in \( \bar{LDR} \) are on the same order as the error in the estimated reflectivity.

a. Properties of cross-polarized signals

The received signal can now be written in terms of the copolar and cross-polar signal samples as

\[
s_h(k) = v_hh(k) + v_hv(k)e^{j\phi_h(k)},
\tag{12}
\]

\[
s_v(k) = v_vh(k) + v_vv(k)e^{j\phi_v(k)}.
\tag{13}
\]

Therefore, the cross-polar signals are phase modulated by \( \phi_h(k) = \psi_h(k) - \psi_v(k) \) and \( \phi_v(k) = \psi_h(k) - \psi_v(k) \) in the H and V channels, respectively, after recohering with the appropriate codes. The effect of decoding on the received signal depends on properties of the cross-polar signal and the codes used. In this paper we consider orthogonal systematic codes based on Hadamard matrices (Proakis 2001).

b. Walsh–Hadamard code

A Walsh–Hadamard code is obtained from a Hadamard matrix \( H_n \) \((n \times n)\) matrix (Proakis 2001) whose elements are either \(+1\) or \( -1\) with the property that any
row differs from the other rows in exactly $n$ positions. The rows of the Hadamard matrix are mutually orthogonal to each other forming Walsh codes and can be used to phase code the horizontal and vertical polarization channels. The Walsh–Hadamard code is generated as

$$H_{2n} = \begin{pmatrix} H_n & \bar{H}_n \\ \bar{H}_n & H_n \end{pmatrix},$$

(17)

where $H_n$ is the complement of $H_n$. For example, when $n = 1$, the Walsh–Hadamard code is

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$  

(18)

The binary phase codes for the H and V channels are obtained by selecting a pair of rows from the Walsh–Hadamard code matrix. The modulation property of phase sequences derived from the Walsh–Hadamard code depends on the pair of sequences selected from $H_{2n}$. In this paper the phase sequences selected are the Walsh–Hadamard codes with modulation code $\phi_h(k)$ and $\phi_v(k)$ so that modulation translates the signal by $\pi$ in the spectral domain. It is important to note that there can be more than one pair of phase codes with the modulation property of translating the signal by $\pi$ in the spectral domain. For example, a phase code for $N = 128$ pulses is obtained by selecting rows 63 and 64 from $H_{128}$. The received signals after decoding can be rewritten from (11) in terms of the copolar and cross-polar signal as given in (13). The cross-polar signal in the H and V channel is modulated by $\exp[j\phi_h(k)]$ and $\exp[j\phi_v(k)]$, respectively. For example, consider the received signal with two components such that the cross-polar signal is 20 dB below the copolar signal, a slightly wider spectral width, and centered at 10 m s$^{-1}$ Doppler velocity. Let the spectral coefficients of $v_{hh}(k)$ and $v_{hv}(k)$ be $V_{hh}(\Omega_k)$ and $V_{hv}(\Omega_k)$, respectively. In the absence of any coding, the spectral coefficients of the received signals are given by

$$S_h(\Omega_k) = V_{hh}(\Omega_k) + V_{hv}(\Omega_k),$$

(19)

$$S_c(\Omega_k) = V_{vv}(\Omega_k) + V_{vh}(\Omega_k).$$

(20)

Since $V_{hh}(\Omega_k)$ and $V_{hv}(\Omega_k)$ have identical mean Doppler velocities of 10 m s$^{-1}$ and $V_{hv}(\Omega_k)$ is significantly weaker than $V_{hh}(\Omega_k)$, cross-polar and copolar signals are indistinguishable in the spectral domain as shown in Fig. 3 (thick line). The same explanation as described above holds for the received signal in the V channel. In case of a Walsh–Hadamard coded waveform, the spectral coefficients of the recohered received signal are given by

$$S_h(\Omega_k) = V_{hh}(\Omega_k) + V_{hv}(\Omega_k + \pi),$$

(21)

$$S_c(\Omega_k) = V_{vv}(\Omega_k) + V_{vh}(\Omega_k + \pi).$$

(22)

It can be observed in Fig. 3 (thin line) that the modulated cross-polar signal $V_{hh}(\Omega_k + \pi)$ is displaced by the Nyquist velocity to $-17$ from 10 m s$^{-1}$ and is clearly separable from the stronger copolar signal in the spectral domain. A very nice feature of the code is that it provides maximum separation between the copolar and cross-polar signal in the spectral domain (i.e., $\pi$). This feature can be used for an explicit estimation of the cross-polar signal power with narrow Doppler spectrum and it has been observed that median spectrum width of widespread showers, stratiform rain, snow, and isolated severe storms is less than about 2 m s$^{-1}$ (Fang et al. 2004).

4. Copolar and cross-polar signal parameter estimation

The observed shape of the Doppler spectrum of copolar and cross-polar signal can be approximated to be Gaussian (Doviak and Zrnić 1993; Bringi and Chandrasekar 2001). Therefore, we model the observed signal as having two Gaussian spectral components, one for copolar signal and one for cross-polar signal. Let $S (v, \theta)$ denote the Gaussian spectrum model. The parameter vector $\theta = (\sigma_{co} \sigma_{cx} \sigma_{co} \sigma_{cx})^T$ describes the spectral moments of the copolar and cross-polar signals. The subscripts $co$ and $cx$ indicate copolar and cross-polar signals, respectively. The spectral model can be written as (Bringi and Chandrasekar 2001).
where $T_s$ and $\lambda$ are the pulse repetition time (PRT) and wavelength, respectively, and $\delta$ is the Kronecker delta function.

**a. Spectral processing**

The continuing advances in digital computing make the application of spectral processing in weather radars possible. For example, spectral processing is used in real time on the first-generation Collaborative Adaptive Sensing of the Atmosphere (CASA) X-band radar network (Bharadwaj et al. 2007). Advantages of spectral processing algorithms with polarimetric radars have been shown with data collected by CSU–CHILL (Seminario et al. 2001; Moisseev et al. 2006; Moisseev and Chandrasekar 2007). The modulation property of the Walsh–Hadamard code is one of the key elements in spectral processing for the retrieval of cross-polarized signal. As described in section 3b, properly selected Walsh–Hadamard codes modulates the cross-polar signal such that the cross-polar signal is translated by $p$ (or Nyquist velocity) with respect to the copolar signal in the spectral domain as given by (22).

The dwell time of the radar results in a finite length of received signal and this finite length of the signal naturally applies a rectangular window function. The application of a rectangular window leads to the

\[
S(v, \theta) = \frac{p_{co}}{\sqrt{2\pi \sigma_{co}^2}} \exp\left\{ -\frac{(v - v_{co})^2}{2\sigma_{co}^2} \right\} + \frac{p_{cx}}{\sqrt{2\pi \sigma_{cx}^2}} \exp\left\{ -\frac{(v - v_{cx})^2}{2\sigma_{cx}^2} \right\} + \frac{2T_s}{\lambda} p_n, \tag{23}
\]

where $p_n$ is the noise power. The corresponding covariance matrix is given by

\[
\gamma(k, l, \theta) = p_{co} \exp\left\{ -\frac{8\pi^2 \sigma_{co}^2 T_s^2 (k - l)^2}{\lambda^2} \right\} \exp\left\{ -\frac{j4\pi v_{co} T_s (k - l)}{\lambda} \right\} + p_{cx} \exp\left\{ -\frac{8\pi^2 \sigma_{cx}^2 T_s^2 (k - l)^2}{\lambda^2} \right\} \exp\left\{ -\frac{j4\pi v_{cx} T_s (k - l)}{\lambda} \right\} + p_n \delta(k)
\]

\[
k, l = 1, 2, \ldots, N, \tag{24}
\]

where $T_s$ and $\lambda$ are the pulse repetition time (PRT) and wavelength, respectively, and $\delta$ is the Kronecker delta function.

\[
\begin{align*}
p_{co} &= 45 \text{ dB} & p_{cx} &= 25 \text{ dB} \\
v_{co} &= 10 \text{ m/s} & v_{cx} &= 10 \text{ m/s} \\
w_{co} &= 1.5 \text{ m/s} & w_{cx} &= 1.8 \text{ m/s}
\end{align*}
\]
spreading of power in the spectral domain through the sidelobes of the window function (spectral leakage). Spectral leakage broadens the power spectrum of the signal and masks weaker signals in the spectrum. A window function is used to minimize spectral leakage due to the finite length window effect. The increase in the standard deviation of the spectral moments depends on the sidelobe suppression ability and main-lobe spectral broadening of the window function.

Since the copolar signal is always stronger than the cross-polar signal, the copolar signal is identified by a spectral peak detection. It has been observed that for Gaussian-shaped echoes this methodology provided good results. The spectral peak is used to notch filter the copolar signal with normalized width \( n_w \). A spectral notch filter is used to remove the copolar signal by removing the copolar spectral coefficients. However, the spectral peak detection and notch filter are not effective when the Doppler spectrum is bimodal, flat topped, or very wide. In cases where the spectrum is bimodal or flat topped, the location of the notch filter may be incorrectly chosen, which results in partial filtering of the copolar signal. The analysis of the impact of the bimodal and flat topped spectrum is beyond the scope of this paper. The normalized width filter is defined as the fraction of the total Nyquist band \([-\lambda/(4T_s), \lambda/(4T_s)]\). Here \( n_w = 0.5 \) is used because the Walsh–Hadamard codes separate the signal by \( \pi \) and the two narrowband signals can be considered to be in the two halves of the spectral domain relative to the copolar signal. For example, a \( \lambda = 10 \text{ cm} \) radar with \( T_s = 1 \text{ ms} \) will have a Nyquist range \((-25, 25) \text{ m s}^{-1}\) and a recohered signal with copolar signal centered at \(12.5 \text{ m s}^{-1}\) will have the cross-polar signal centered at \(-12.5 \text{ m s}^{-1}\). Therefore, the copolar signal will be in \((0, 25) \text{ m s}^{-1}\) and the cross-polar signal will be in \((-25, 0) \text{ m s}^{-1}\). The cross-polar signal power is estimated from the notch filtered signal. The inverse of the notch filter used to remove the copolar signal is used to filter out the cross-polar signal from the received signal. The copolar signal parameters are obtained from the inverse notch filtered signal. The linear depolarization ratio is then estimated from the copolar and cross-polar signal powers using (14). The impact of spectral processing with phase-coded waveform on polarimetric variables is described in section 5.

b. Maximum likelihood estimate

Maximizing the likelihood that \( \theta \) fits the estimated covariance matrix \( \hat{\Sigma} \) is a standard technique in estimation theory. It is assumed that the received signals are circularly symmetric complex Gaussian random vectors. The moments of the copolar signal and cross-polar signal are obtained by maximizing the log-likelihood function given by

\[
L(\theta) = \ln |\mathbf{R}(\theta)| + \text{Tr}\{\mathbf{R}^{-1}(\theta)\hat{\mathbf{R}}\},
\]

where \( \mathbf{R}(\theta) \) and \( \hat{\mathbf{R}} \) are the model covariance matrix and sample covariance matrix, and \( |\cdot| \) and \( \text{Tr}\{\cdot\} \) represent the determinant and trace operator, respectively. The estimated moments \( \hat{\theta} \) are given by the solution of the nonlinear optimization problem given by

\[
\hat{\theta} = \arg \min_{\theta} L(\theta).
\]

A more detailed description of the maximum likelihood method is provided by Boyer et al. (2003). The optimization problem in (26) is solved numerically to estimate the moments. The feasibility and performance of such a parametric time domain method for spectral moment estimation and clutter mitigation has been described in Nguyen et al. (2008). The likelihood function in (25) does not have a global minimum and the solution is more sensitive to the initializing seed values of the nonlinear optimizations problem in (26). It has been observed that the convergence of the minimization problem in (26) is sensitive to the mean Doppler velocities of the copolar signal. Since the range of Doppler velocities is not very large, the likelihood function in (25) is estimated at sample points uniformly spread between the Nyquist velocities. The sample points that give the minimum log-likelihood are selected as seed values for mean Doppler velocity.

5. Evaluation based on simulation study

A simulation at \( \lambda = 11 \text{ cm} \) (corresponds to the operating wavelength of the CSU–CHILL radar) was performed to evaluate the performance of the coding scheme to retrieve the linear depolarization ratio. The procedure described by Chandrasekar et al. (1986) was extended to simulate time series data with the cross-polar signal. The lag-0 copolar correlation coefficient \( \rho_{h0} = 0.99 \) in the simulations and the coto-cross polarization correlation coefficient are set to a low value between 0.2 and 0.4. A longer time series is truncated to provide the window effect. The simulated time series were phase coded with Walsh–Hadamard code, and the copolar and cross-polar signals were superimposed on a sample-by-sample basis to form the received signal. A phase error of 0.5° standard deviation is added to the transmit pulses in the two channels and true phase codes are used in decoding the received signals.

a. LDR retrieval

The spectral moments are estimated using spectral processing and the maximum likelihood method after
the received signal is decoded. The error in $\hat{LDR}$ is used to evaluate the performance of the two methods. The bias in $LDR$ with simultaneous transmission mode as a function of LDR is shown in Fig. 4. The cross-polar signal spectral width is generally larger than the copolar signal spectral width. Hence, the cross-polar signal spectral width is set to $\alpha = 1.2$ times the spectral width of the copolar signal ($\sigma_{co} = 4 \text{ m/s}^{-1}$). The bias in $LDR$ indicates that both spectral processing and the maximum likelihood method have similar performances in estimating the mean LDR. The bias in $d_{LDR}$ is within 1 dB for both the spectral and maximum likelihood method. However, the standard deviations of $d_{LDR}$ estimated from spectral processing are 0.6 dB higher than the standard deviation from the maximum likelihood method. Although the bias indicates that $LDR$ below $-34$ dB can be estimated within 1 dB, the lower bound of measurable $LDR$ is limited by the antenna integrated cross-polarization ratio (ICPR) (Chandrasekar and Keeler 1993). The CSU–CHILL system has a lower bound of $-34$ dB (Brunkow et al. 2000) based on the antenna pattern and measurements have shown values in the $-33$- to $-34$-dB range.

b. Impact on polarimetric variables

A simulation study is conducted to study the impact of phase coding on the polarimetric variables. A comparison is made between the retrievals of polarimetric variables for a phase-coded waveform and an uncoded waveform in STAR mode. It is important to note that the copolar signals from an uncoded waveform are always superimposed with the cross-polar signal. The uncoded waveform is processed with the traditional covariance algorithm to obtain the polarimetric variables. However, the phase-coded waveform is processed with spectral processing. The maximum likelihood approach for polarimetric radars is beyond the scope of this paper and is not considered in this section. The errors in the estimated polarimetric variables with phase-coded waveform and uncoded waveform in STAR mode are shown in Fig. 6 for $\sigma_{co} = 4 \text{ m/s}^{-1}$ and $\alpha = 1.2$. It can be observed in Fig. 6a that the standard deviation of $\hat{Z}_{dr}$ with a coded waveform is higher than the uncoded waveform. This increase in the standard deviation is due to the application of the window function for spectral processing. A Chebyshev window with 80-dB suppression was used in the processing. The same explanation holds for the increased standard deviation of $\hat{Z}_{df}$ and lag-0 copolar correlation coefficient ($\hat{\rho}_{hv}$) shown in Figs. 6b and 6c. There is no bias in $\hat{Z}_{dr}$ obtained from phase-coded waveform while there is a very small bias in $\hat{Z}_{df}$ obtained from uncoded waveform when $LDR$ is high. The biases $\hat{\phi}_{dp}$ and $\hat{\rho}_{hv}$ increase for an uncoded waveform when $LDR$ is high, while $\hat{\phi}_{dp}$ and $\hat{\rho}_{hv}$ remain unbiased for a phase-coded waveform. The errors in polarimetric variables are comparable to those described in Bringi and Chandrasekar (2001).

6. Waveform coding with CSU–CHILL radar

Orthogonally coded horizontal and vertical polarization channels were used to collect data from a CSU–
CHILL radar. Modulated RF drive pulses for each transmitter are synthesized by the transmitter controller. The transmitter controller uses a digital upconverter chip to synthesize a modulated intermediate frequency (IF) and the resulting IF signal is upconverted to S band. All internal data paths are 16 bits or greater and this allows the phase error to be small (determined to be 0.5°). This was verified by observing the hard target with a phase-coded waveform and decoding the received signal (see Fig. 7). Radar data on a stratiform event with a phase-coded waveform were collected from the CSU–CHILL radar on 29 May 2007. In addition, data using an alternate transmission mode were also collected. The alternate mode of operation enables the direct measurement of linear depolarization ratio. Both alternate mode and phase-coded waveform data were collected with a range–height indicator (RHI) scan at an azimuth of 80°. Figures 8a and 8c show the RHI of estimated reflectivity from the alternate mode and phase-coded simultaneous transmission mode. The increase in reflectivity from the surface up to the melting layer clearly indicates an occurrence of a bright band at an altitude of 1.8 km. The received signals near the ground are contaminated with ground clutter. The cross-polarized signals from the ground are 0–15 dB below the copolar signal from the ground and since signals close to the ground are very strong, the cross-polarized ground clutter is also significantly stronger. However, because of properties of the Walsh–Hadamard code the cross-polar clutter is not at zero velocity but is spread around $v = \pm \lambda/(4T_s)$, as shown in an example Doppler spectrum from CSU–CHILL in Fig. 9. Ground clutter is easily filtered in spectral processing because the position of the clutter spectral coefficients is known exactly. Therefore, a bandpass finite impulse response (FIR) filter with a frequency response as shown in Fig. 10 can be used to suppress both copolar and cross-polar ground clutter. The FIR filter shown in Fig. 10 is on the order of $81$ and Fig. 9 shows an illustration of the result of filtering. The spectral moments can then be estimated using the maximum likelihood method.

The LDR estimated from the alternate mode and simultaneous mode with the Walsh–Hadamard code using spectral processing is shown in Figs. 8b and 8d, respectively. The bright band can be clearly identified by the enhance depolarization ratio in Figs. 8b and 8d. The alternate mode $LDR$ is used as a reference to compare the $LDR$ estimated from the phase-coded waveform. The absolute difference in measurements between alternate mode and simultaneous mode is attributed to the time difference between data collections. There is about a 4-min difference between data collection, and the precipitation event has changed in the 4 min and therefore is not an exact match. The $LDR$ estimated from Walsh–Hadamard coded waveform is
very similar to $LDR$ estimated from the alternate mode. An infinite impulse response (IIR) filter is used to remove ground clutter in the alternate mode. The LDR measured from coded waveforms in simultaneous mode closely matches the LDR from the alternate mode and can retrieve LDR up to $-30 \text{ dB}$.

7. Conclusions

Phase coding the transmit waveform in the horizontal and vertical polarization channels on a pulse-to-pulse basis can be used to retrieve the linear depolarization ratio in simultaneous transmission mode. Walsh-Hadamard coded waveforms were considered in this paper for retrieving LDR. Two methods for estimating LDR with phase-coded waveforms were presented. A spectral method and a time domain method that used the maximum likelihood principle were presented in this paper. A simulation study was performed to evaluate the ability of the phase-coded waveform to retrieve LDR using the two methods mentioned above. Based on results from the simulation, it is found that the bias in estimated LDR is within 1 dB for both spectral processing and the maximum likelihood method. However,
it is observed that the standard deviation of estimates from the maximum likelihood method was lower than the spectral processing estimates by 0.6 dB. The impact of phase coding and associated processing on the retrieval of polarimetric variables was evaluated. Based on the results from the simulation study it can be concluded that unbiased estimates of polarimetric variables can be obtained from orthogonally phase-coded waveform. However, the standard deviations of the polarimetric variables are slightly higher because of the application of the window function.

The estimation of LDR in simultaneous transmission mode was tested with data collected by the CSU–CHILL radar. Estimates of LDR with phase-coded waveform were compared to the estimates from alternate mode. The ability to retrieve LDR is limited by the cross-polar isolation of the antenna. LDR measurements obtained by CSU–CHILL with alternate mode show values in the −32 to −34 dB interval. Data collected by the CSU–CHILL radar show that LDR estimated from phase-coded waveform is comparable to LDR estimated from the alternate mode and can retrieve LDR up to −30 dB. Based on the results from simulations and experimental data from CSU–CHILL it can be concluded that phase-coded waveforms provide a viable means to estimate both copolar and cross-polar signals in simultaneous transmission mode.

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REFERENCES


