Detecting Multiple Ground Contacts in Cloud-to-Ground Lightning Flashes

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ABSTRACT

Video recordings of cloud-to-ground (CG) lightning flashes have been analyzed in conjunction with correlated stroke reports from the U.S. National Lightning Detection Network (NLDN) to determine whether the NLDN is capable of identifying the different ground contacts in CG flashes. For 39 negative CG flashes that were recorded on video near Tucson, Arizona, the NLDN-based horizontal distances between the first stroke and the 62 subsequent strokes remaining in a preexisting channel had a mean and standard deviation of 0.9 ± 0.8 km and a median of 0.7 km. The horizontal distances between the first stroke and the 59 new ground contacts (NGCs) had a mean and standard deviation of 2.3 ± 1.7 km and a median of 2.1 km. These results are in good agreement with prior measurements of the random errors in NLDN positions in southern Arizona as well as video- and thunder-based measurements of the distances between all ground contacts in Florida. In cases where the distances between ground contacts are small and obscured by random errors in the NLDN locations, measurements of the stroke rise time, estimated peak current, and stroke order can be utilized to enhance the ability of the NLDN to identify strokes that produce new ground terminations.

1. Introduction

Most negative lightning flashes to the ground produce multiple strokes [see Schonland (1956, 1964) and Uman (1969, chapter 2) for reviews of the luminous development of cloud-to-ground (CG) lightning flashes]; typically, about half of these flashes strike the ground in more than one place. For convenience, we will refer to CG flashes that produce multiple ground terminations as multiple ground contacts (MGCs). Valine and Krider (2002) have measured the luminous characteristics of MGCs near Tucson, Arizona, and have compared their results with measurements in other geographic regions. Their dataset contained 386 video recordings of CG flashes that produced 558 different ground contacts; therefore, the average number of contacts per CG flash was 1.45, a value that is consistent with prior measurements that range from an average of 1.45 to 1.70 contacts per flash (see Valine and Krider 2002, Table 2; Saba et al. 2006a).

Because the chances of being struck by cloud-to-ground lightning and the need for lightning protection are proportional to the average area density of ground contacts in the region, there is a need to know both the number of CG flashes per unit area and the average number of ground contacts per flash over the time interval of interest. Area densities of CG flashes have been measured by the U.S. National Lightning Detection Network (NLDN) and similar systems for more than 20 yr (e.g., Orville and Huffines 2001; Schulz et al. 2005), but there is still a need to know the average number of ground contacts per flash and how this quantity varies with space and time over large regions. Until now, however, no one has used the NLDN or any other commercial lightning locating system to objectively identify the different ground terminations in each CG flash.
The primary mechanisms of producing multiple ground terminations are as follows: 1) when the leader initiating a subsequent stroke takes a new path to ground; 2) when two or more branches of the same downward-propagating leader reach the ground at about the same time and initiate two or more nearly simultaneous return strokes; and 3) “root branching,” when two or more upward discharges form during the attachment process and contact the same leader channel aloft (Schonland et al. 1935, 1938; Schonland 1956; Kitagawa et al. 1962; Berger 1967; Guo and Krider 1982; Thottappillil et al. 1992; Rakov and Uman 1994; Wang et al. 2000; Valine and Krider 2002; Saba et al. 2006a; Qie and Kong 2007; Kong et al. 2009). Other factors that can influence the formation of MGCs are (i) the order of the stroke within a flash or the number of prior strokes in the channel; (ii) the interstroke time interval; (iii) the presence of a large branch on the preceding stroke; (iv) the presence or absence of a continuing current (or luminosity) in the preceding stroke; and (v) perhaps even the peak current of the preceding stroke (Schonland 1938; Thomson et al. 1984; Rakov and Uman 1990; Thottappillil et al. 1992; Rakov et al. 1994; Valine and Krider 2002; Saba et al. 2006a,b; Ferro et al. 2009).

Prior measurements by many authors show that there are meaningful differences in the electromagnetic fields radiated by the first and subsequent strokes in negative flashes; namely, first strokes have longer rise times (RTs) and larger peaks than subsequent strokes (e.g., Nag et al. 2008; Fisher and Uman 1972; Tiller et al. 1976; Uman 1985). More recently, it has become clear that the fields produced by subsequent strokes that create new ground contacts (or that are preceded by dart-stepped leaders and are likely to have produced new ground terminations) have longer rise times and larger peaks and tend to be more similar to first strokes than to the subsequent strokes remaining in a preexisting channel (PEC; Weidman and Krider 1978; Rakov and Uman 1990; Willett and Krider 2000; Biagi et al. 2007; Fleenor et al. 2008; Ferro et al. 2009).

The horizontal distances between the different ground contacts in a flash are of fundamental importance for the physics of lightning attachment. They are also of interest when considering lightning safety and when evaluating the damage that lightning can cause to a large, spatially extended structure, such as an electric power or communications system, or buildings that have a large spatial extent. To the best of our knowledge, the measurements of Thottappillil et al. (1992) and Ishii et al. (1998) are the only data that have been published to date on the distances between different ground contacts in individual CG flashes. Thottappillil et al. (1992) measured the distances between all possible pairs of contacts in 22 single- and multiple-stroke flashes that were recorded on video cameras and had thunder ranging. The majority of the new ground terminations (i.e., those created after the first stroke) were produced by subsequent strokes that created a new ground contact (NGC); for these, Thottappillil et al. (1992) found that the average separation distance was 1.7 km, the maximum was 7.3 km, and the minimum was 0.3 km. Ishii et al. (1998) measured the distance between successive return strokes by using positions derived from the arrival times of the fields measured by a short-baseline network of fast antennas. Ishii et al. (1998) found separation distances as large as 10 km for negative flashes, and in summer the average separation was 2.1 km (22 observations) and in winter the average separation was 2.2 km (18 observations). Seventeen observations of multiple-stroke positive flashes showed an average separation of 13.4 km, and the expected location error for an individual measurement was approximately 0.5 km.

The main goal of this study will be to determine whether the NLDN and similar systems can be used to identify the subsequent strokes that create new ground terminations. In cases where the spatial separation between ground contacts is small and/or obscured by random errors in the stroke locations, we also explore whether other NLDN measurements (e.g., the stroke rise time, amplitude, order within the flash, and the interstroke interval) can be used to identify new ground contacts. We will focus primarily on the most common type of MGC flash (i.e., new ground terminations produced by subsequent strokes in negative CG flashes initiated by downward-propagating leaders). We will also require that the separations be large enough to be resolved in a video recording (i.e., tens of meters or more), and we therefore will not address root branching, wind-blown or swept attachments, or attachments created by one or more upward discharges that are not followed by a return stroke.

2. Data and methods

a. Video dataset

The lightning flashes used in this study were recorded on video near Tucson, Arizona, in 2003 and 2004 (Biagi et al. 2007). The cameras were Canon model GL1 digital camcorders that had a spatial resolution of $720 \times 480$ pixels and a time resolution of 60 video fields per second. All data were time synchronized to GPS to within 16.7 ms, the duration of one video field. First, all flashes that produced at least two separate ground contacts on different video fields were identified, and then the subsequent strokes that remained in a PEC or that created an NGC were identified. Frequently, the exact ground
terminations were obscured by obstructions near the horizon (e.g., trees and houses); in those cases, the actual contact was inferred by starting at the lowest point on the channel that was visible and extending it vertically from that point to the ground. The relative positions of the first and all subsequent ground contacts were measured manually (to the nearest millimeter) on a video monitor and then converted to a difference in azimuth angle (Δθ) by multiplying the spatial separation on the monitor, expressed as a fraction of the total field of view, by the total field of view of the camera in radians. This resulted in an azimuth resolution of about 0.5° (0.0086 rad). Next, Δθ was converted to a horizontal displacement in the image plane of the first stroke using a method that is described in the appendix. The image plane of the first stroke is a vertical plane perpendicular to the viewing direction of the camera and located at the range of the first stroke from the camera. Finally, the actual distances between the location of the first stroke and the locations of each subsequent stroke in the plane of the earth’s surface were computed using Eqs. (A2), (A3), and (A5) in the appendix. Given the 32°N latitude of our observations and the short distances involved, the simplified (flat earth) equations in the appendix will produce a distance error of less than 100 m.

b. NLDN dataset

The times of all video strokes that were coincident (to within ±17 ms) with strokes reported by the NLDN were identified by using a database that is maintained by Vaisala in Tucson, Arizona. The NLDN stroke reports included the GPS time that the stroke contacted ground (to the millisecond), the most probable stroke location (latitude–longitude), the stroke azimuth (measured in degrees clockwise from north) and range (in kilometers) from the camera location, waveform parameters (discussed below), and an estimate of the peak current of the stroke (Ip) that is based on the range-normalized peak field measured by two or more sensors. The NLDN also provides the semimajor axis (SMA) and orientation of an ellipse that describes the 50% confidence region around the most probable stroke location (Cummins et al. 1998; 2006).

Our total dataset contained 41 MGC flashes with 69 subsequent strokes that remained in a PEC and 63 subsequent strokes that produced an NGC; all were confirmed on video. We excluded strokes whose order in the flash was greater than 11 (all remained in a PEC). The NLDN locations were used to calculate the difference in azimuth angle between the first stroke and each subsequent ground contact Δθ and the horizontal distances between ground contacts both in the image plane of the first stroke and in the plane of the earth’s surface.

In the following, we compare the values of Δθ measured on video with values that were derived from the NLDN locations to see if they are consistent. Next, we compare the camera- and NLDN-based displacement d between ground contacts in the image plane of the first stroke. Finally, we compare the NLDN-based separation distances in the horizontal plane of the earth’s surface with the video- and thunder-based measurements of Thottappillil et al. (1992). In many cases, the random position errors are comparable to the separation distances between strokes; therefore, we also explore whether additional NLDN measurements, such as the threshold-to-peak rise time of the stroke waveform, the estimated peak current of the stroke, the order of the stroke within the flash, and the interstroke interval, can assist in the identification of strokes that produce NGCs.

For the rise-time analysis, histograms of stroke counts as a function of the NLDN rise time were constructed for first strokes, the subsequent strokes that produced NGCs, and subsequent strokes that remained in a PEC for all strokes in our dataset. The NLDN rise times are the values contained in the NLDN dataset that were reported by the closest time- and amplitude-consistent sensors that were at least 50 km away from the stroke location. The NLDN sensors will report a stroke impulse if the field exceeds a threshold of 8.2 Lightning Location and Protection (LLP) units (0.35 V m⁻¹) and if the peak is above 16.4 LLP units (0.7 V m⁻¹). All the peak signals in our dataset were greater than 30 LLP units, and thus there should be minimal threshold (or range) bias in the NLDN rise-time measurements. The independence of rise time on range was verified for the strokes in our dataset, and this was reasonable given the small variation in propagation distances in this dataset.

3. Results

Figure 1a shows distributions of the SMA of the 50% confidence regions that the NLDN reported at the locations of all (video confirmed) first strokes and the subsequent strokes that produced an NGC in our dataset. Note that there are two clusters of SMA distances: one less than 3 km and the other between 7.0 and 8.5 km. Because any NLDN location with a large SMA clearly has a large uncertainty in its position, we have excluded all strokes that have an SMA larger than 3 km (2 first strokes, 2 subsequent strokes, and all subsequent strokes associated with the 2 large-error first strokes) from the analysis that follows. This exclusion reduced the total number of MGC flashes in our dataset to 39, and these flashes contained 59 subsequent strokes that produced NGCs and 62 subsequent strokes that remained in a PEC. Figure 1b shows the distribution of
SMA distances for the first strokes and subsequent strokes that produced an NGC and were accepted for further analysis.

a. Comparison of video and NLDN values of $\Delta \theta$ and $d$

Figure 2 shows the separation angles, $\Delta \theta$ (in radians) that were measured on video between the first ground contact and all new ground contacts in the flash versus the corresponding values of $\Delta \theta$ that were derived from the NLDN locations using Eq. (A4). Note that the video and NLDN values of $\Delta \theta$ appear to be highly correlated with the exception of nine outliers that deviate significantly from the slope $= 1$ line (identified by open triangles in Fig. 2). All but one of these outliers was caused by flashes that struck within 9 km of the video camera, where small errors in the NLDN locations will produce large differences in $\Delta \theta$. For example, a fixed separation between two ground contacts at a range of 5 km from the camera will produce roughly a 3 times larger video $\Delta \theta$ than the same separation at a range of 15 km from the camera. At close ranges, random errors in the positions of each stroke can cause the NLDN-based values of $\Delta \theta$ to vary considerably, from very small to very large differences. One outlier was at a range of 22.6 km from the camera, but the estimated median location error (SMA) was quite large (1.6 km).

A better comparison of the video and NLDN separation distances can be obtained by converting the values of $\Delta \theta$ to an apparent horizontal displacement $d$ in a vertical plane that is orthogonal to the direction of viewing and at the range of the first stroke from the camera [Eq. (A1)]. Figure 3 shows the video- and NLDN-based displacements in the image plane together with the expected deviation (ED) defined as the rms value of the corresponding SMA distances for the first stroke SMA$_1$ and the associated subsequent strokes SMA$_s$; that is, ED = $\sqrt{(SMA_1)^2 + (SMA_s)^2}$. In this figure, error-free measurements would appear along the slope $= 1$ line. Assuming that the video measurement errors are small compared to the NLDN location errors, the SMA values accurately reflect the NLDN location errors for each stroke, and the location errors have a two-dimensional Gaussian distribution, then approximately half of the error bars in Fig. 3 should intersect the slope $= 1$ line and about 90% of the error bars should be within 2 ED lengths of this line. Figure 3a and the magnified image (Fig. 3b) are consistent with these expectations.

b. Horizontal distances between ground contacts in the plane of the earth’s surface

The consistency among the NLDN-based separation distances $D$ [Eq. (A5)] was examined by comparing the separations between the first stroke and subsequent strokes that remained in a PEC on video with the

FIG. 1. Distributions of the SMAs of the 50% confidence regions at the NLDN locations of first strokes and the subsequent strokes that produced NGCs. (a) All first strokes and NGCs. (b) Set of first and NGC strokes used in the analysis (SMA < 3 km).

FIG. 2. The $\Delta \theta$ angles measured on video vs the NLDN-based values of $\Delta \theta$ computed between the locations of 39 first strokes and 59 NGCs. The nine outliers are denoted by white triangles.
measured distances between first strokes and the strokes that produced NGCs. Ideally, the strokes in PECs would have separations of zero. Figure 4 shows normalized distributions of $D$ between the first stroke in the flash and both types of subsequent strokes in the plane of the earth’s surface. Note that the median distance between strokes that remain in a PEC in Fig. 4 (670 m) is very close to the unscaled median ($424 \sqrt{2} = 600$ m) that was found by Biagi et al. (2007) in southern Arizona. Note also that the shapes and median values of the distributions in Fig. 4 are very different; this gives us confidence that the NLDN is capable of resolving spatial separations that are larger than about 2.0 km in southern Arizona. The hypothesis of equal medians for these two distributions is rejected at the $p = 10^{-7}$ level using the (nonparametric) Wilcoxon rank-sum test.

Figure 5 shows the NLDN-based distances in Fig. 4 as a function of the rms values of the corresponding SMA distances (ED) at each NLDN location. Again, there is a good separation between the populations of strokes that remain in a PEC and the strokes that produce NGCs, particularly for ED values below 1.5 km. Figure 5 also shows that almost all of the separations between PECs are less than their associated ED values (i.e., below the dotted line indicating slope = 1.0).

To directly compare this work with the distance distribution published by Thottappillil et al. (1992), we examine the NLDN-based horizontal distances between all possible pairs of ground contacts for each of the 39 flashes. There were 98 pairs. The distribution of these distances differs somewhat from the distribution...
of distances between first strokes and strokes that produced NGCs shown in Fig. 4. These data are shown together in Fig. 6a to allow direct comparison. These populations have the same median and standard deviation values, but the means differ slightly (not significant). Figure 6b shows the normalized distance distributions (all pairs) for this study and for Thottappillil et al. (TRUBMS/1992), as a percentage of the total count in each study. Note in Fig. 6b that the mean, maximum, and minimum of the NLDN-based separations are 2.6, 7.5, and 0.1 km, respectively. These values are very similar to the mean, maximum, and minimum of 1.7, 7.3, and 0.3 km, respectively, that were reported by Thottappillil et al. The maximum values for both studies are very nearly equal; however, results from this study show a greater percentage in the range of 2–5 km and therefore exhibit a larger mean than the distribution of Thottappillil et al. Note also that the shapes of both distributions in Fig. 6b are very similar.

c. NLDN rise-time measurements

As stated earlier, the NLDN database contains the threshold-to-peak rise time of the electromagnetic impulse radiated by each stroke. The NLDN values are taken from the nearest time-consistent NLDN sensor, as long as that sensor is at least 50 km from the stroke location. Because the stroke rise time is based on the closest sensor, any threshold bias in this measurement is minimal. Figure 7 shows histograms of the rise times that were produced by first strokes, by subsequent strokes that produced an NGC, and by the subsequent strokes that remained in a PEC on video. The mean rise time and standard deviation for each of the preceding types of strokes are 5.5 ± 1.8, 5.1 ± 1.8, and 3.0 ± 0.95 µs, respectively. The mean rise times in Fig. 7 are somewhat larger than those reviewed by Uman (1985, Table 1) because the latter included 10%–90% rise times and the NLDN reports a signal threshold-to-peak value that varies with signal amplitude; it may be limited by the bandwidth of the NLDN sensors (Cummins et al. 1998) and by propagation effects (Cooray et al. 2000; Shoory et al. 2005). Because the mean NLDN rise times of first strokes and subsequent strokes that produced an NGC are quite similar (5.5 and 5.1 µs, respectively, with the same standard deviation), we have combined these into one category of first and subsequent strokes (NGC + 1st) that produce new ground terminations and replotted

![Fig. 6](image-url)  
**Fig. 6.** (a) The horizontal distances in the plane of the earth’s surface between the first stroke in 39 MGC flashes and the subsequent ground contacts in each of those flashes (59 “1st–NGC” measurements) and between all strokes with different strike locations (98 “all pairs” measurements). (b) Normalized distribution of the horizontal distances between all possible pairs of ground contacts in 39 MGC flashes and the distribution published by Thottappillil et al. (1992).

![Fig. 7](image-url)  
**Fig. 7.** Histograms of the RTs of first strokes, subsequent strokes that produced an NGC, and subsequent strokes that remained in a PEC. Note that first and NGC strokes have very similar RT statistics.
them in Fig. 8 together with the rise times of subsequent strokes that remained in a PEC.

To assess the value of stroke rise time in discriminating between first and subsequent strokes that produce new ground terminations and subsequent strokes that remain in a PEC, we have calculated the percentage of strokes in each category that would be classified correctly if their rise time was above or below a given threshold. A value of $3.7 \, \mu s$ produced a condition where the percentage of first strokes and NGCs that would be misclassified as PEC equaled the percentage of PECs that would be misclassified as NGCs (19%).

d. Estimated peak current

Because many studies have shown that there is a relationship between the amplitude of the peak field [or the estimated peak current $I_p$ that is provided by lightning detection systems (Cummins et al. 1998)] and the type of stroke in a flash (Thottappillil et al. 1992; Biagi et al. 2007; Fleenor et al. 2008), the values of $I_p$ clearly contain valuable information about whether a given stroke has produced an NGC. Figure 9 shows distributions of $|I_p|$ for all first and subsequent strokes that produced new ground terminations together with the $|I_p|$ values of the subsequent strokes that remained in a PEC. Note here that the strokes creating an NGC have a mean and standard deviation of $-21.1 \pm 11.4 \, kA$, whereas strokes that remained in a PEC have a mean of $-11.9 \pm 5.3 \, kA$. An analysis of classification thresholds similar to what was done for rise time yielded the following results: a threshold value of $-15.3 \, kA$ produced a condition where the percentage of first strokes and NGCs that would be misclassified as PEC equaled the percentage of PECs that would be misclassified as NGCs (see Fig. 9b). With this criterion, 62% of both types of strokes were classified correctly. The peak current threshold would have been $-13.9 \, kA$ using the criterion that both classes had the same number (rather than percentage) of misclassified strokes (see Fig. 9a). In this case, 29 strokes of each class were misclassified. Therefore, the magnitude of $I_p$ does contain valuable information about whether a given stroke may have produced a new ground termination, but it is not as helpful as the rise-time measurement.

e. Distribution of NGCs by stroke order

Several previous authors have noted that the second stroke in a flash is the stroke that is most likely to produce a new ground termination (e.g., Rakov and Uman 1990; Rakov et al. 1994; Valine and Krider 2002; Saba et al. 2006a), although the actual probability varies between authors because of sample size and other factors. Figure 10 shows the percentage of NGCs that were produced as a function of stroke order for all the NGC
flashes in our dataset. Note here that 59% of the time it was the second stroke that produced an NGC, and 27% of the time it was the third stroke. These percentages are generally similar to previous measurements. Because the NLDN does not report a large fraction of the lower-current subsequent strokes in a flash, it has the potential to undercount the order of a given stroke. Thus, if the NLDN indicates that the order of a stroke is greater than 5, the likelihood that the stroke is in a PEC is probably greater than 98%.

f. Interstroke intervals

We have also examined the video-based interstroke time intervals preceding strokes that produced an NGC and strokes that remained in a PEC, in order to determine whether an NLDN measurement of the interstroke interval can assist in the identification of new ground terminations. The results are shown in Fig. 11. Note that, although the numbers are limited, there does not appear to be a significant difference in the interstroke interval populations; therefore, an NLDN measurement of the interstroke interval, which is sometimes overestimated because of the nondetection of some subsequent strokes, is not likely to be useful for the identification of NGCs.

4. Summary and conclusions

We have examined the feasibility of using the NLDN to identify new ground terminations in negative cloud-to-ground lightning flashes. Our basic conclusion is that the NLDN is capable of identifying the strokes that produce new ground contacts in southern Arizona by using position information alone, as long as the NLDN-based separation distances exceed about 2 km. Because the median location error in southern Arizona is nearly
double what is found in the interior of the network in Texas–Oklahoma (Biagi et al. 2007), it is likely that NGCs can be identified with a separation criterion of about 1 km throughout most of the United States. When the separations are small or obscured by random errors in the NLDN positions, then measurements of the stroke rise time, peak amplitude, and order within the flash can assist in the identification of new ground contacts.

Because the NLDN is not able to resolve submillisecond time intervals between “simultaneous” return strokes that are produced by the same downward-propagating leader (Rakov and Uman 1994; Kong et al. 2009), this type of MGC flash cannot readily be identified by using NLDN data. Analyses of high-speed video records in Brazil by Ballarotti et al. (2005) suggest that this type of flash is rare (only 6 out of 455, or 1.3%); however, Kong et al. (2009) recently found 9 such flashes in 59 records (15%) in China. Further investigations will be required to clarify this issue.

Because each new ground termination creates a new risk to people or property, any quantitative assessment of lightning risk, such as the ones published by the National Fire Protection Association (NFPA 2004) and the International Electrotechnical Committee (IEC 2006), should certainly include the number of ground contacts per flash as a multiplying factor when estimating the probability of lightning strikes. Specifically, the frequency of strikes per unit area is the average number of flashes per unit area multiplied by the number of ground contacts per flash in the region of interest. In the future, we hope that studies such as this will enable investigators to use lightning detection systems, such as the NLDN or similar systems, to measure the number of ground terminations per flash over large areas so that statistics on the number of ground contacts per flash can be developed under varying storm conditions in different regions and seasons. Knowledge that a given lightning flash produced two or more ground terminations might also be useful in many forensic investigations.

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APPENDIX

Geometric Analyses

The purpose of this appendix is to show the geometry and equations that have been used to compute the separations between two ground contacts $P_1$ and $P_2$ in both the image plane of the first stroke $d$ and in three-dimensional space $D$. The video camera is located at the origin $C(0, 0)$.

Figure A1 shows the geometry that has been used to obtain the horizontal separations in the image plane of the first stroke $d$ and the horizontal plane of the earth’s surface $D$ from the NLDN stroke locations.

Figure A1 shows the geometry of two ground contacts, $P_1$ and $P_2$, and the horizontal distances between them in the image plane of the first stroke $d$ and in three-dimensional space $D$. The video camera is located at the origin $C(0, 0)$. Here, $P_1$ and $P_2$ are two arbitrary ground contacts that would be recorded by a video camera located at the origin $C(0, 0)$. The ground contact $P_1$ is the location of a reference stroke (typically the first stroke) in the flash, and $P_2$ is the location of any subsequent stroke in the flash. Here, $r_1$ and $r_2$ are the position vectors of $P_1$ and $P_2$, respectively, from the camera.

The angle between $P_1$ and $P_2$, $\Delta \theta = \theta_2 - \theta_1$, was measured manually on the video monitor and was also independently computed from the NLDN locations as discussed in section 3. The distance $d$ is the chord of a circle with radius $r_1$ that connects points $P_1$ and $P_2$, was measured manually on the video monitor and was also independently computed from the NLDN locations. Once $\Delta \theta$ has been measured, it can be used in conjunction with the geometric half-angle identity to obtain $d$:

$$d = 2|r_1| \sin \left( \frac{\Delta \theta}{2} \right).$$

(A1)

The computation of $\Delta \theta$ from the NLDN positions begins with the estimated latitudes and longitudes of $P_1$ and $P_2$ that were reported by the NLDN. The differences in the latitudes and longitudes relative to the camera were first converted to $X$ (east–west separation).
and \( Y \) (north–south separation) in kilometers using the approximate equations:

\[
X_i = \cos\left(\frac{\text{lat} + \text{lat}_i}{2}\right) \times 111 \text{ km} / \text{deg} \times |\text{lon}_c - \text{lon}_i| \quad \text{and} \tag{A2}
\]

\[
Y_i = 111 \text{ km} / \text{deg} \times |\text{lat}_c - \text{lat}_i|. \tag{A3}
\]

The subscripts on the latitude and longitude denote the camera \( c \) and each stroke \( i \). The average of the latitudes is used in (A2) to obtain a more accurate value for \( X \), and the factor of 111 km deg\(^{-1}\) in (A2) and (A3) converts the difference in latitude and longitude (in degrees) to \( X \) and \( Y \) in kilometers.

Once the \( X \) and \( Y \) distances to the camera have been obtained for all strokes, the values of \( \Delta \theta \) can be computed using Eq. (A4). Again, \( \Delta \theta \) is the change in azimuth between the reference and \( i \)th strokes, and \( X \) and \( Y \) are spatial separations between each stroke (or ground contact) and the camera:

\[
\Delta \theta_i = |\angle r_i - \angle r_1| = |\arctan\left(\frac{Y_i}{X_i}\right) - \arctan\left(\frac{Y_1}{X_1}\right)|. \tag{A4}
\]

This equation is simply the absolute value of the difference in the azimuthal angles of \( r_i \) and \( r_1 \), as measured from the north vertical axis. In this equation, the location of the reference stroke is the starting point for the separation distances and is depicted with a subscript 1 on the variable, and the subscript \( i \) refers to a subsequent stroke.

The horizontal distance between the NLDN-reported strokes has been calculated using the Pythagorean theorem [Eq. (A5)], where \( \Delta X \) represents the east–west displacement from the first stroke to the new ground contact and \( \Delta Y \) represents the north–south distance between the two contacts. For subsequent strokes that remained in a PEC, the calculations are the same, and again all distances are relative to the location of the reference stroke:

\[
D = \sqrt{(\Delta X^2 + \Delta Y^2)}. \tag{A5}
\]

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