Computationally Efficient Methods of Collocating Satellite, Aircraft, and Ground Observations

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(Manuscript received 17 June 2008, in final form 20 October 2008)

ABSTRACT

The usefulness of measurements from satellite-borne instruments is enhanced if these measurements can be compared to measurements from other instruments mounted aboard the same or different satellite, with measurements from aircraft, or with ground measurements. The process of associating measurements from disparate instruments and platforms is referred to as collocation. In a few cases, two instruments mounted aboard the same spacecraft have been engineered to function in tandem, but commonly this is not the case. The collocation process may then become an awkward geometric problem of finding which of many observations within one dataset corresponds to an observation in another set, possibly from another platform. This paper presents methods that can be applied to a wide range of satellite, aircraft, and surface measurements that allow for efficient collocation with measurements having varying spatial and temporal sampling. Examples of applying the methods are presented that highlight the benefits of efficient collocation. This includes identifying the occurrence of simultaneous nadir observations (SNOs); collocation of sounder, imager, and active remotely sensed measurements on the NASA Earth Observation System (EOS); and collocation of the polar orbiting imager, sounder, and microwave measurements with geostationary observations. It is possible, using an inexpensive laptop computer, to collocate Moderate Resolution Imaging Spectroradiometer (MODIS) imager observations from the \textit{Aqua} satellite with geostationary observations rapidly enough to deal with these measurements in real time, making either dataset, enhanced by the other, a potentially operational product. A “tool kit” is suggested consisting of computer procedures useful in collocation.

1. Introduction

For more than 40 yr, satellites have monitored the earth’s weather and climate with significant advancements in the quality and scope of the observations. However, no single measurement provides the necessary information to characterize all relevant atmospheric properties. For this reason it is desirable to combine multiple measurements often on different satellites with diverse viewing geometries and sampling characteristics. This collocation process can be time consuming if the two instruments have not been engineered to function in tandem, as is often the case. This paper presents methods that allow a user to quickly find relevant observations within one set of data with matching or collocated data within another set from another instrument. The algorithms presented provide the capability to collocate two or more measurements, mounted aboard different spacecraft with different viewing geometries and temporal sampling.

The collocation methods can be applied to a wide variety of satellite and aircraft measurements including imager, infrared sounder, microwave, and active sensors. Recent applications include combined sounder and imager retrievals using collocated Atmospheric Infrared Sounder (AIRS) (Aumann et al. 2003) and Moderate Resolution Imaging Spectroradiometer (MODIS) (Justice et al. 1998) observations (Li et al. 2005), and global comparisons of cloud detection and height between the Cloud-Aerosol Lidar with Orthogonal Polarization (CALIOP) and MODIS (Holz et al. 2008). For these applications the multiple spacecraft and different spatial and temporal sampling required an efficient and accurate collocation provided by the collocation methods presented in this paper. Applications to aircraft measurements have facilitated comparisons between hyperspectral sounder, lidar, and imager retrievals (Holz et al. 2006). The methods can also be applied to collocate geostationary
and polar orbiting measurements and to determine simultaneous nadir observations (SNOs), and can be extended to the case in which one of the two or more instruments to be utilized is mounted aboard an aircraft or ground station. This is accomplished by creating a fixed master field of view (FOV) with a radius corresponding to the sampling of the ground observation.

2. Navigation

a. Master and slave

We shall use the term master to denote the instrument onto whose footprint the observations of a second instrument, the slave, are to be projected. A typical example might be the collocation of AIRS and MODIS instruments, both carried aboard the Aqua spacecraft. The subsatellite diameter of the AIRS FOV is roughly 13 km, whereas the MODIS has a much higher spatial resolution, with more than 150 MODIS observations within a single AIRS quasi-oval. In this case, we consider AIRS to be the master, and MODIS the slave. As it happens, AIRS and MODIS are comounted aboard the same spacecraft, but in the following discussion no use is made of this fact. One might even envision a case of self-location in which the same instrument is both master and slave. For instance, one could seek a set of AIRS observations today and another set as nearly as possible over the very same set of geographical points a week later for the express purpose of observing the weekly variation.

b. Coordinate systems

Vector notation is used in much of what follows because of its facility in representing geometric relationships. The manner of implementing such relationships in a computer program is discussed presently. Vectors are indicated by bold roman type.

It is almost impossible to deal with satellite navigation without becoming involved in three different coordinate systems, as presented in Fig. 1. By the celestial frame of reference (CFR), which some may call an inertial system, we mean an orthonormal coordinate set, or basis, defined by three unit vectors, \( \mathbf{i} \), \( \mathbf{j} \), and \( \mathbf{k} \) (which, respectively, lie along the \( x \), \( y \), and \( z \) axes in the figure), in which the \( \mathbf{i} \) vector is directed from the center of the earth as origin toward the vernal equinox, an imaginary point in the sky defined as one of two intersections of the plane of the ecliptic with the plane of the equator. [The plane of the ecliptic would be seen to be the apparent annual path of the sun about the earth if the earth’s daily rotation did not mask the sun’s annual motion (see Smart 1977).] The \( \mathbf{j} \) vector is 90° to the east of the \( \mathbf{i} \) vector, in the plane of the equator. The \( \mathbf{k} \) vector is directed from the center of the earth toward the North Pole. The terrestrial frame of reference (TFR) is similar in structure but with the \( \mathbf{i} \) and \( \mathbf{j} \) vectors fixed to the rotating earth, with \( \mathbf{i} \) pointing from the earth's center toward the Greenwich meridian, \( \mathbf{j} \) pointing to a longitude 90° east of Greenwich, and the \( \mathbf{k} \) axis again pointing through the North Pole. Both these systems are dextral (right hand) orthonormal bases. The third system is the familiar latitude/longitude and central distance (LLD), that is, the distance from the earth’s center to an object, with the caveat that we must be careful to distinguish between geodetic and geocentric latitude (see below). There is a fixed association between LLD and TFR, since they are both fixed to the earth. Time is not involved in this relationship.

However, the relation between the celestial and terrestrial bases is a function of time because of the apparent rotation of the vernal equinox about the earth once per sidereal day, and in fact this rotation defines the sidereal day. Montenbruck and Pfleger (2000) have provided an algorithm for computing the longitude of the vernal equinox as a function of time, which in turn allows simple interconversion between celestial and
terrestrial coordinates. With this longitude known, the reader can think of the transformation of a vector from the TFR to the CFR as

\[ V_C = \begin{bmatrix} \cos(L) & \sin(L) & 0 \\ -\sin(L) & \cos(L) & 0 \\ 0 & 0 & 1 \end{bmatrix} V_T \quad (1a) \]

and the inverse as

\[ V_T = \begin{bmatrix} \cos(L) & -\sin(L) & 0 \\ \sin(L) & \cos(L) & 0 \\ 0 & 0 & 1 \end{bmatrix} V_C. \quad (1b) \]

where \( L \) is the instantaneous longitude of the vernal equinox and hence a function of time. These matrices are orthogonal, implying that their inverses are also their transposes. Aitken (1956) discusses orthogonal matrices.

Many polar orbiting weather satellites are sun synchronous, and hence they pass over a given region at roughly the same local time each day. Given the apparent annual revolution of the sun about the earth, the right ascension of the ascending node of such satellites (the celestial point where they cross the equator flying north), measured in degrees, must increase about 360/365.25, or roughly 1°, per day. Since such satellites typically make about 14 orbits per day, the instantaneous plane of the satellite’s motion precesses about \( \frac{1}{14} \) of a degree per orbit in celestial space. This fact permits a simplification in estimating satellite positions in the absence of a formal prediction model. With negligible (and easily corrected) error we can speak of the orbital plane of a sun-synchronous orbit over a limited time (e.g., 25 min) in the CFR. The rotation of the earth beneath the satellite invalidates a similar assumption in the TFR, and the subsatellite track traced onto the rotating earth departs markedly from a great circle.

Satellite navigation uncertainties arise from failures to discriminate carefully between geodetic and geocentric latitude. The earth-located coordinates of data-sets are usually rendered in geodetic latitude, while satellite navigation software typically returns its results in geocentric. For a discussion of geodetic and geocentric latitude, see Bowditch (1977). For the earth model used by the authors, the equation

\[ b^2 \tan(d) = \tan(c) \]

can be used to interconvert between geodetic and geocentric latitude, where \( d \) is geodetic, \( c \) is geocentric, and \( b = 6 \times 356 \times 911.946/6 \times 378 \times 388 \), the ratio of polar to equatorial radius.

c. Forward and inverse navigation

It is no surprise that the collocation of satellite observations requires the ability to specify the location of a satellite as a function of time. We refer to this computation as forward navigation. It is equally important to employ some technique of inverse navigation, by which we mean determining the time when a satellite is over or abeam of a given point on the surface. Inverse navigation may not always be needed (e.g., in the case when master and slave are mounted aboard the same spacecraft, so that the times of the observations are the same or nearly so). However, if the instruments are mounted aboard different spacecraft, then we must know the time when the slave satellite is in a position to observe the master field of view. This problem is straightforward if the slave instrument is a cross-scanner, for example, MODIS, AIRS, etc., but can become a bit awkward otherwise, as with a conical scanner.

Satellite forward navigation software is available from sundry sources, but the authors developed the inverse navigation software. For reasons of policy or convenience, one may decide to use only the satellite positioning data contained within the data files delivered to the investigator from the source agency [i.e., National Aeronautics and Space Administration (NASA), National Oceanic and Atmospheric Administration (NOAA), etc.,] without recourse to formal navigation routines that would also demand the availability of concurrent orbital parameters. (However, if orbital parameters are wanted, the user can visit the Web site celestrak.com.) The satellite orbital location at the time of an observation is not always provided in the data files. For these cases, the satellite location must then be approximately inferred from the distribution of the geolocated FOVs. Knowledge of the master satellite’s location at an observation time need not always be known to extreme accuracy. The reason for wanting the satellite’s approximate position is to provide an estimate of the size, orientation, and elongation, or eccentricity, of a quasi-ellipsoidal master FOV as it is projected onto the surface at a high scan angle. Often we assume that a satellite’s spatial attitude is nominal, so that it views its own nadir when the instrument scan angle is zero, and for this case the position of the satellite is taken to be the zenith of the center point of a scan at a known or presumed altitude. If the satellite exhibits a known pitch, roll, or yaw, the needed correction is not difficult. For this case the most difficult component to estimate is the satellite’s altitude if it is not provided with the observations. It has been found, at least in the case of a cross-scanning instrument like MODIS or AIRS, that the altitude can be computed to
an accuracy of about 0.05 km by examining the locations of the FOVs along a scan line, and driving the projection geometry backward to infer the satellite’s altitude (i.e., triangulating on the satellite from various earth locations). This scheme presupposes that the satellite’s attitude is nominal or nearly so.

**INVERSE NAVIGATION**

Inverse navigation is a major portion of collocation. If we can estimate closely the time at which a slave satellite passes abeam of a master FOV on the surface, we have greatly restricted the amount of slave data that must be searched to find collocations.

Let a satellite’s position at a given moment be expressed by the vector \( S(t) \), and its velocity by \( V(t) \) as presented in Fig. 2. The satellite’s angular momentum vector \( A \), which is normal to the quasi plane of the orbit in celestial space, is given by \( A = S \times V \); \( A \) can also be thought of as the vector orbital plane, since knowledge of \( A \) defines the orientation of the orbital plane in the CFR. As noted, the direction of \( A \), which is all that interests us, is almost invariant over a brief period (i.e., 25 min or a quarter-orbit). Let \( G(t) \) be the position vector of an arbitrary point on the ground in the CFR. It is a function of time in celestial space because of the earth’s rotation.

Let \( R(t) = |G(t) - S(t)| \) at time \( t \) denote the slant vector from the satellite to this ground point. We seek to know when the satellite is over or abeam of this point on the ground in the CFR. It is a function of time in celestial space because of the earth’s rotation.

Let \( A(t) = S(t) \times R(t) \cdot A = 0 \).

Equation (2) is the governing equation for inverse navigation. We can solve the inverse navigation problem if we have any scheme for finding the value of \( t \) that makes Eq. (2) a true statement.

If approximating polynomials are used to estimate the satellite’s position and velocity (see below), and because the rate of the earth’s rotation is a known constant, then (2) can be formally differentiated to find \( dP/dt \), and a Newton–Raphson iteration used to seek the value of time \( t \) that satisfies (2):

\[
0 = \frac{dP}{dt} \bigg|_{t(t_0)} = (dP/dt)(t_0).
\]

If we have no approximating polynomials, but possess only a full-scale forward navigation model for obtaining \( S \) and \( V \), then formal differentiation of (2) is not feasible and we then begin with some initial guess for \( t(0) \), and use finite differences in consecutive iterations of the value of \( P \) to approximate the rate of change of \( P \) with respect to \( t \). Note that a Newton–Raphson approximation as expressed by (3) displays geometric convergence, meaning that with each iteration we roughly double the number of correct digits, so that convergence is rapid.

To create a “quick-and-dirty” navigation model that lends itself either to forward or inverse navigation, we can proceed as follows: We compute four satellite positions in the celestial system (the earliest, the latest, and two intermediate points) based on the scan line centers from our dataset, encompassing not more than 25 min, by converting the four earth locations to celestial coordinates and projecting them upward to a computed or nominal altitude. We can make this conversion using the transform in Eq. (1a), assuming the times of these four earth locations are provided with the data. Let these four celestial positions be \( V_i \), where \( i = 1, 4 \). We can then create two cubic polynomials, shown in nested form:

\[
D(t) = a_0 + t[a_1 + t(a_2 + a_3t)],
\]

\[
C(t) = c_0 + t[c_1 + t(c_2 + c_3t)].
\]
Here $D(t)$ is the angular displacement in the orbital plane from the initial position $S_0$, and $C(t)$ is the central distance of the satellite as presented in Fig. 3. Let $R(S, A, x)$ be an operator that rotates vector $S$ clockwise about $A$ in the amount of $x$ degrees. In Fig. 3 the satellite's vector orbital plane, or angular momentum vector, is given by the cross (vector) product $A = S \times V$, and points out of the paper toward the reader. Further, let $U$ be a unitizing or normalizing operator that reduces its vector argument to a unit vector. Since $D(t)$ and $C(t)$ are differentiable functions of time, we can use their derivatives to find the velocity $V$ at a given moment. The desired satellite position at an arbitrary time is given by

$$S(t) = C(t)U\{R[S_0, A, -D(t)]\}. \quad (4)$$

This approximation of the satellite's position involves little more than evaluating the two polynomials, and since these polynomials are easily differentiated, we effectively have an inverse navigation model using a Newton–Raphson inverse solution for $t$, given $S(t)$.

An alternate scheme, instead of using a subset of points chosen from the dataset, is to use all satellite locations derived from the subsatellite data track and then to compute a least squares cubic or quadratic fit as a function of time for both the angular displacement along the track and the radius vector of the satellite.

3. Simultaneous nadir observations

The term simultaneous nadir observation (SNO) is something of a misnomer, because it rarely happens that two satellites will pass over any point on the ground at precisely the same moment. In using this term, we mean only ascertaining within some acceptable time window for example, 15 min, when and where two satellites will pass over a common point on the earth’s surface. The SNOs thus identified may be scattered in a seemingly random manner over the earth, or if the orbits have nearly identical periods, the SNO points may be confined to restricted latitude bands in both the Northern and Southern Hemispheres. Satellites with slightly differing orbital periods will experience numerous SNOs for a period of time, and then, like two clocks ticking in the same room at slightly different rates, they will pass through a prolonged period of many weeks during which there are no SNOs within the stated time window. The choice of time window is a scientific judgment dictated by the perishability or timeliness of data that are not coincident in time with other data.

Ascertaining the occurrence of SNOs for a given pair of satellites is actually rather rapid. For this purpose we need a fairly general orbital prediction model (OPM), as well as a general inverse navigation algorithm, and orbital parameters for a time period covering the period of interest, say several months to a year. Again, we use the convenient fact that the orbital plane of a synchronous satellite has an almost constant orientation in the CFR over a time span of a single orbit, that is, its motion is essentially planar over a limited time span.

Let us choose two satellites, satellite 1 and satellite 2, and select an arbitrary initial time $t_0$. At this time we obtain the position $S$ and velocity $V$ of both satellites, and from these we obtain their two vector orbital planes (or angular momentum vectors).

$$A_i = S_i \times V_i = 1, 2.$$

Refer to Fig. 4. In celestial coordinates, two possible SNO points will lie along the intersection of the two orbital planes, that is, they will lie on the vector cross product $P = A_1 \times A_2$ and on diametrically opposite sides of the earth in the CFR. This expression for $P$ presupposes that the two orbital planes are not nearly coincident as in the cases, say, of Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observation (CALIPSO) and Aqua, for in such cases the angle between the two planes is so small that the cross product $A_1 \times A_2$ is ill defined.

Let us select one of the two possible SNO points, say the one most closely following time $t_0$. Let us call this point $P$ in Fig. 5. The angular speed of a satellite is only slightly variable, about $3.66 \text{ min}^{-1}$ for most of the
satellites of interest, so that we can compute the time $t_1$ of the arrival of satellite 1 at P by some scheme of successive approximation. In like manner we find the time $t_2$ of arrival of satellite 2 at this point. From our orbital model we also know the velocity of each satellite at point P. Keep in mind that P is defined in celestial coordinates, not terrestrial. If the difference $|t_1 - t_2|$ is greater than the acceptable time window, we discard this case and advance the time $t_0$ by about 50 min, or half an orbit, and repeat the process.

But assume the satellites both reach point P within a suitable time interval of each other. We seek to learn the unique point on the earth’s surface over which both satellites will pass, though in general not at precisely the same moment. For each satellite we convert its position and velocity at its arrival time $t_i$ to the TFR, using the relation (1b). Since the satellites in general will not arrive at the celestial point P at the same moment, the subsatellite points on the earth beneath P at $t_1$ and $t_2$ will be slightly different, owing to the rotation of the earth during the interval between $t_1$ and $t_2$. Transforming a satellite’s position to terrestrial coordinates is straightforward, but in converting velocity we face the complication that the satellite’s apparent velocity over the earth is altered by the eastward rotation of the earth, so that as seen by an earth-bound observer the subsatellite track has a westward component equal to the eastward speed of a point of the earth at its given latitude $H$. In other words, the satellite’s apparent velocity vector is

$$V_A = V_i + E \cos(H) W,$$

where $V_A$ is the apparent velocity over the earth, $V_i$ is the celestial velocity converted to the terrestrial frame [based simply on (1b) and unadjusted for the speed of the rotating earth], W is a unit vector pointing toward local west at the given ground point, and $E$ is the eastward speed of rotation of the earth at the equator. At time $t_i$, we can regard the velocity over the ground as described by $V_A$ as lying in the osculating plane of the satellite’s motion as seen by an earth-bound observer. The orientation of this osculating plane is defined by expressing it as the cross product

$$O_i = S(t_i) \times V_A,$$

for each satellite. The intersection of the two osculating planes, one for each satellite, represents an approximate
point in the TFR on the earth’s surface over which the satellites will fly at times \( t_1 \) and \( t_2 \), respectively. This intersection \( \mathbf{I} \) of the two osculating planes is found from

\[
\mathbf{I} = (\mathbf{S}_1 \times \mathbf{V}_{A1}) \times (\mathbf{S}_2 \times \mathbf{V}_{A2}),
\]

where \( \mathbf{V}_{A1} \) and \( \mathbf{V}_{A2} \) are found from (5) for each satellite, and the vector \( \mathbf{I} \) is an approximate point on the earth overflown by both satellites. Using the inverse navigation scheme mentioned above, that is, solving (2) for \( t \), we compute for each satellite new values of \( t_1 \) and \( t_2 \), the times when they are over or abeam of \( \mathbf{I} \), and repeat the process described above, this time with more accurate values of \( t_1 \) and \( t_2 \). That is, at the new times \( t_1 \) and \( t_2 \) we compute new satellite positions, convert these to a terrestrial frame, recompute osculating planes, obtain an improved value of \( \mathbf{I} \), etc., until the changes in \( \mathbf{I} \) from one iteration to another fall below some threshold. Three iterations of this procedure are quite adequate.

### 4. Intersecting data swaths

To hasten the collocation process, it is desirable to know if two data swaths intersect at all. For instance, the nadir-viewing lidar instrument aboard the CALIPSO traces a data swath along the subsatellite track. It may or may not intersect a data swath created by the cross-scanning MODIS instrument carried aboard Aqua. Plotting the swaths on a map would show instantly that they do or do not intersect, but what is a convenient way of determining this analytically?

Imagine two couples playing bridge, seated around a table, with east facing west and north facing south. Let their vector positions be \( \mathbf{E}, \mathbf{W}, \mathbf{N}, \) and \( \mathbf{S} \) as presented in Fig. 6a. The position of the origin is arbitrary. East sees north on his right and south on his left. East turns his head to the right to look at north, then leftward to look at west, and still more to the left to look at south. In vector terms the scalar product

\[
(\mathbf{N} - \mathbf{E}) \times (\mathbf{W} - \mathbf{E}) \cdot (\mathbf{W} - \mathbf{E}) \times (\mathbf{S} - \mathbf{E}) > 0. 
\]

Both cross products in (6a), by the customary right-hand rule of vector algebra, are directed upward toward the ceiling, and hence their scalar product is positive. By the same token,

\[
(\mathbf{E} - \mathbf{S}) \times (\mathbf{N} - \mathbf{S}) \cdot (\mathbf{N} - \mathbf{S}) \times (\mathbf{W} - \mathbf{S}) > 0,
\]

and because both (6a) and (6b) are positive, the segment connecting east and west intersects the segment connecting north and south at the center of the table.

After the game the couples go to lunch, and sit at different tables (Fig. 6b). In such a case, there will be no intersection because at least one player will see both his opponents on his left or both on his right.

In the case of data swaths, let the latitude/longitude of the end points of swath A and swath B be converted to terrestrial vectors (see the tool kit below). Then the beginning and end points of swath B can be viewed analogously with east and west, and the end points of swath A with north and south. In analogy with the bridge partners, the swaths intersect if the two scalar products are both positive:

\[
(\mathbf{B}_1 - \mathbf{A}_1) \times (\mathbf{A}_2 - \mathbf{A}_1) \cdot (\mathbf{A}_2 - \mathbf{A}_1) \times (\mathbf{B}_2 - \mathbf{A}_1) > 0
\]

\[
(\mathbf{A}_2 - \mathbf{B}_1) \times (\mathbf{B}_2 - \mathbf{B}_1) \cdot (\mathbf{B}_2 - \mathbf{B}_1) \times (\mathbf{A}_1 - \mathbf{B}_1) > 0.
\]

### 5. Detecting overlap

In some approaches to collocation, an investigator is satisfied to know merely if a slave observation does or does not overlie a master oval. For example, the template approach suggested by Aoki (1980) produces a yes/no response to this question. However, in the case of most sounders, the instrument spatial response function is strongly dependent on the distance from the center of the FOV. Often the half-power distance from the center is used arbitrarily to define the radius at nadir. In such a case, an investigator may prefer to assign a lesser weight to an overlying slave observation that falls near the circumference of a master oval and greater weight near
the center. The techniques described in this article determine not only if a slave observation falls within a selected master oval, but also assign a linear weight to each slave observation, varying from one at the center to zero at the edge. A user can choose to use only those slave observations whose weights exceed some threshold or apply the weighting to a predetermined spatial response function.

**a. The quasi-elliptical approach**

If a satellite-borne earth-observing scanning instrument has a circular field of view when aimed at a point directly beneath the satellite, the fields of view become egg-shaped, or oviform, as the scan moves away from nadir. The FOVs are not exactly elliptical, owing to the fact that a perceptible curvature of the earth exists even within a small FOV. However, the approximation to an ellipse is often so close that the FOV may be regarded as quasi-elliptical for purposes of collocation, provided the scan is not close to the limb of the earth, where the FOV ceases to be even quasi-elliptical. The quasi-elliptical assumption is more valid the smaller the FOV. It is not difficult to compute the size, orientation, and eccentricity of a quasi-elliptical FOV. See Fig. 7a.

In the following, we assume that a master FOV is not at the master satellite’s nadir. If we let \( \mathbf{F} \) be the vector position of the nominal center of the master FOV, computable from its latitude, longitude, and earth radius, and \( \mathbf{S} \) the vector position of the satellite, then the slant range vector, earth to satellite, is the vector difference \( \mathbf{R} = \mathbf{S} - \mathbf{F} \). The vector or cross product \( \mathbf{F} \times \mathbf{R} \) is oriented along the minor axis, and its length is the diameter of the FOV at nadir multiplied by the ratio of slant range to satellite altitude. We use the vertical bars \( |\cdot| \) to denote the scalar magnitude of a vector, and \( \mathbf{U} \) to be a unitizing or normalizing operator that produces a unit vector in the direction of its argument:

\[
\mathbf{U}(\mathbf{F} \times \mathbf{R}) / z,
\]

where \( z \) is the altitude of the satellite, the scalar \( D \) is the diameter of the FOV at nadir, and \( \mathbf{B} \) is the vector minor axis. Figure 7b enlarges the quasi-elliptical field of view. The major axis \( \mathbf{A} \) is of course normal to both \( \mathbf{B} \) and \( \mathbf{F} \), and hence its direction is given by their cross-product \( \mathbf{B} \times \mathbf{F} \). If \( \mathbf{A} \) is thus computed, it will point in the general direction of the subsatellite point. The length of the major axis equals the length of the minor axis, multiplied by the secant of the satellite zenith angle as seen at the FOV. Let us introduce the vertical caret \( ^\wedge \) to denote the unsigned angle in degrees between two vectors. Then

\[
\mathbf{A} = \mathbf{U}(\mathbf{F} \times \mathbf{B})|\mathbf{B}| \sec(F^\wedge \mathbf{R}).
\]

The angle \( F^\wedge \mathbf{R} \), indicated as \( z \) in Fig. 7b, is the zenith angle of the satellite seen from the FOV. The eccentricity of the quasi-elliptical oval, if it is wanted, is implied by the values of the major and minor axes \( |\mathbf{A}| \) and \( |\mathbf{B}| \), respectively.

We have now defined the quasi ellipse on the earth’s surface of the master observation. Again let the vector \( \mathbf{F} \) represent the terrestrial position vector of the center of the master quasi ellipse. Assume we are given the coordinates of a slave observation from which we are to decide if the slave does or does not fall within the master, and if so with what weight. First, we convert the coordinates of the slave observation to a terrestrial vector \( \mathbf{P} \). See Fig. 7b. We define the \( x \) coordinate of the slave observation as the component of its displacement along the major axis and the \( y \) component along the minor axis:

\[
x = \mathbf{U}(\mathbf{A}) \cdot (\mathbf{P} - \mathbf{F})
\]

\[
y = \mathbf{U}(\mathbf{B}) \cdot (\mathbf{P} - \mathbf{F}),
\]

where the dot \( \cdot \) denotes the dot or scalar product of two vectors. If the distance \( D \) given by \( |\mathbf{P} - \mathbf{F}| \) is less than the semiminor axis, the slave spot falls within the master FOV. If it is greater than the semimajor axis, it falls outside the master FOV. In the intermediate ambiguous case, the slave spots falls outside if \( y^2 > b^2(1 - x^2/a^2) \). Letting \( a = |\mathbf{A}|/2 \) and \( b = |\mathbf{B}|/2 \), the semimajor and semiminor axes, the weight assigned to an overlapping slave observation is

\[
w = 1 - (x^2 + y^2)/[x^2 + b^2(1 - x^2/a^2)].
\]
This represents a linear weighting from zero at the edge of the FOV and one at the center.

Figure 7 is somewhat misleading in that it suggests that the slave instrument, whose FOV is the point $P$, is mounted on the same satellite as the master. This condition, however, is not a requirement.

As a means of verifying the accuracy of collocated slave points, it is a simple matter to compute the angular distance between the centroid of the slave points within the master FOV, and the center of the master FOV. This centroidal distance should typically equal a small fraction of a kilometer. The centroid is found from the sum of the overlapping vector slave positions. This method is sufficiently rapid to allow for collocation in real time of geostationary and polar orbiting imager measurements.

The quasi-elliptical approach applied to MODIS 1-km-resolution imager measurements with AIRS is presented in Fig. 8 for a nadir, 34°, and 49° AIRS scan angles. The figure presents the MODIS 11-μm brightness temperature (BT) measurements found to be collocated within each AIRS FOV. Notice that as the AIRS scan angle increases, the projected AIRS FOV becomes increasingly more quasi-elliptical at large scan angles. Using this methodology, AIRS retrievals of temperature and water vapor within partially cloudy FOV have been improved by integrating the MODIS cloud mask using the collocation (Li et al. 2005).

b. Quasi-conical approach

Another approach may be easier than the quasi-elliptical method, especially if both master and slave instruments are mounted aboard the same spacecraft. It avoids any exercise in analytic geometry involved in determining whether a slave observation falls within a master oval on the surface (see Fig. 9).

Let us make the assumption, not always strictly correct, that the master instrument views the underlying earth as if through a cone whose angular opening is determined by the size of the master FOV when seen at nadir, with the apex of the cone at the satellite. Then any slave observation viewed within the solid angle of the cone overlaps the master FOV on the ground. We need not concern ourselves with the size, shape, or orientation of the master FOV on the surface. Let $A$ be the angular half-width of the master FOV, $S$ the vector position of the satellite, and $F$ and $E$ be the surface vector positions of a master and slave FOV, respectively. Then if

$$(F - S) \cdot (E - S) < A,$$

the master FOV is overlapped by the slave observation, that is, if the angular difference between the satellite-to-ground slant ranges is less than the half-aperture of the master FOV. If desired, a weight can be assigned to an overlapping slave observation by the amount of the angular difference. If the difference is zero, then we assign a weight of one. If the angular difference of the master and slave slant ranges is at the angular limit $A$, then we would assign a weight of zero.

The quasi-conical approach suffers from the drawback that computing a small angle between two nearly parallel vectors demands an accurate arccosine routine,
given that the cosine function has a zero derivative when its argument is zero, so that the inverse function is ill defined. This problem is amplified for the case when one satellite is geostationary, with a very high altitude, and in such a case the quasi-elliptical approach is not recommended.

6. General procedure

Let us sketch a sequence of events in collocating a set of MODIS slave points (imager) over an AIRS (sounder) FOV. In this example the slave instrument is a cross-scanning instrument.

1) We are given an arbitrary master FOV. The diameter or radius of this FOV at nadir is known, for example, from the half-power point of the signal. Using an inverse navigation scheme such as described above, we estimate the time at which the satellite carrying the slave instrument is over or abeam of the master FOV. The slave-carrying satellite may or may not be the same as the master-carrying satellite.

2) Knowing the approximate time of transit (i.e., time over or abeam), we now know the approximate row or rows of the slave observations that are likely to overlie the master FOV, since each slave observation has an associated time, and shall confine our search to these rows. It may be adequate initially to inspect all the slave observations in the row for possible overlap, but on subsequent passes we shall already know approximately which points along the slave scan line will overlap.

3) Using one of the overlap-detecting schemes outlined above, we identify those slave observations, and assign weights to them, that overlap the given master FOV. The indices of these slave observations (row and spot indices) are then recorded.

4) Another master FOV is selected, and if it is contiguous to the previously chosen master FOV, we already know the general location of the slave observations that are likely to overlap it, thus hastening our search.

The time of an observation may be provided to an investigator in a variety of ways. The authors have often been given time as the number of seconds elapsed since some epoch, usually 0000 UTC 1 January 1993, including intercalated leap seconds. Conversion to the civil calendar and coordinated universal time is straightforward, with each day consisting of 86 400 s, but with adjustment for the intervening leap seconds.

The authors use a flat time field (i.e., free of years, months, days, hours, leap years) based on the Julian day number (JDN), which is itself defined as the number of mean solar days elapsed since 1 January 4713 BCE. This is a large number in the modern era, being 2454101 on 1 January 2007, and for this reason the authors use a modified Julian day number (MJDN) in which the JDN of 1200 UTC 1 January 1970 is subtracted. (The reason for using 1200 UTC 1 January 1970 is that a Julian day begins at noon UTC, not midnight.)

7. Collocation software

In preparing a set of programs to deal with collocation, the authors recommend that whatever the computer language being used, routines be available to compute the following:

1) The sum, difference, dot product, cross product, and angle between two 3D Cartesian vectors, so that
vectors can be used as entities, and not dealt with by their three separate components.

2) The position in either the CFR or the TFR of a meteorological satellite at a given time, given the orbital parameters.

3) From a position in either the CFR or TFR the corresponding position in the other frame of reference, with latitude/longitude as a variant in the TFR.

4) The position on the surface of the earth directly beneath the known position of a satellite, considering the ellipsoidal shape of the earth, and the possibility that a satellite may experience an attitudinal perturbation;

5) Either geodetic or geocentric latitude from the other;

6) The time at which a satellite is directly over or abeam of a given point on the earth’s surface, that is, a general inverse navigation algorithm, or a solution of Eq. (2).

7) Time expressed in one system (civil calendar, coordinated universal time, elapsed seconds from an epoch, Julian day number or a variant of it, etc.) from its value in another system.

The computer programming needed to accomplish collocation can be done in any language with which the programmer feels comfortable. The language used by the authors was Meteorological FORTRAN (Meteo-For), an enhancement of FORTRAN-77 developed at the Space Science and Engineering Center, Madison, Wisconsin, that incorporates vector and matrix algebra within its syntax. This allows, for instance, lengthy vector expressions to be contained within a single line of code.

**Acknowledgments.** We would like to acknowledge the NASA Atmospheric Product Evaluation and Test Element (PEATE) Grant number NNX07AR95G for supporting this research. This research was funded under NASA grants.

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